Condition based maintenance optimization for multi-component systems using proportional hazards model

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Abstract

The objective of condition based maintenance (CBM) is typically to determine an optimal maintenance policy to minimize the overall maintenance cost based on condition monitoring information. The existing work reported in the literature only focuses on determining the optimal CBM policy for a single unit. In this paper, we investigate CBM of multi-component systems, where economic dependency exists among different components subject to condition monitoring. The fixed preventive replacement cost, such as sending a maintenance team to the site, is incurred once a preventive replacement is performed on one component. As a result, it would be more economical to preventively replace multiple components at the same time. In this work, we propose a multi-component system CBM policy based on proportional hazards model (PHM). The cost evaluation of such a CBM policy becomes much more complex when we extend the PHM based CBM policy from a single unit to a multi-component system. A numerical algorithm is developed in this paper for the exact cost evaluation of the PHM based multi-component CBM policy. Examples using real-world condition monitoring data are provided to demonstrate the proposed methods.

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1. Introduction

Within the condition based maintenance (CBM) framework, the health condition of a piece of equipment is monitored via collecting and analyzing the inspection data, such as vibration data, acoustic emission data, oil analysis data and temperature data. Future health condition can be further predicted, and optimal maintenance actions can be scheduled for preventing equipment breakdown and reducing total operation costs [9, 13]. Various CBM policies and optimization methods have been proposed [9, 11, 21]. Proportional hazards model (PHM) has been widely used in biomedicine field for lifetime data analysis since it was introduced. Since 1990s, PHM has been used in reliability analysis by using covariates to describe different operating conditions [5-7, 12], and in maintenance optimization by combining the age data and the condition monitoring data so as to more accurately represent the equipment health condition and failure probability [9, 14-15, 20]. A CBM optimization approach based on PHM has been developed, aiming at determining an optimal replacement policy, that is, an optimal risk threshold control limit in this approach, for minimizing the expected replacement cost [2, 8, 16]. This approach was developed into the CBM optimization software EXAKT [2], and it has been applied in many industries, such as mining industry, food processing industry, and utility industry [18].

However, the reported CBM methods mainly focus on a single unit or component. That is, replacement and other maintenance decisions on components are made individually, based on the age of each component, condition monitoring data and the CBM policy. However, for a system consisting of multiple components, there are typically economic dependencies among the components. Consider for example the replacement of bearings on a set of pumps at a remote location. The fixed maintenance cost, such as sending a maintenance team to the site, is incurred whenever a preventive replacement is performed. Thus, it would be more cost-effective if we replace multiple components at the same time to reduce the unit fixed replacement cost. A system consisting of multiple components is referred to as a multi-component system in this paper if economic dependency exists among the components.

Studies on time-based maintenance of multi-component systems have been reported in the literature [4]. Economic dependency and stochastic dependency among components were considered. The most basic multi-component time-based replacement policy is the group replacement policy, also known as the standard block-replacement policy, where the components are preventively replaced in group at pre-specified intervals [10]. The modified block replacement policy for multi-component systems was developed by Archibald and Dekker [1], where preventive replacement is performed at constant intervals and only on the components that are older than a pre-specified age limit. Schouten and Vanneste [17] proposed two simple control policies for multi-component systems by considering several intermediate states. Some other reported studies on time-based multi-component system maintenance are summarized in Cho et al. [4] and Wang [19]. However, the aforementioned maintenance models were time-based and did not consider the condition monitoring information. Castanier et al. [3] presented a condition based maintenance policy with non-periodic inspections for a two-unit series system, and developed a stochastic model based on the semi-regenerative properties of the maintained system state for cost evaluation. The control limits appeared to be on the observed condition monitoring measurements, and no real-world data were used to demonstrate the approach.

In this paper, we propose a PHM based CBM policy for multi-component systems, where economic dependency exists among different components. The cost evaluation of the PHM based CBM policy becomes much more complex when we extend the CBM policy from a single unit to a multi-component system. A numerical algorithm for exact cost evaluation is developed in this paper. Examples based on real-world data are used to demonstrate the proposed PHM CBM policy and the cost evaluation method.

Acronyms:

CBM: condition based maintenance;
PHM: proportional hazards model.

Notations:

h	Hazard value;
$z_i(t)$	Covariate value for covariate *i* at time *t*;
β, η, γ	Parameters of the proportional hazards model;
m	The number of covariates considered;
$Q(d)$	Probability that failure replacement will occur;
$W(d)$	The expected time until replacement;
d_1	Level 1 risk threshold;
d_2	Level 2 risk threshold;
C_f	The cost of performing a failure replacement;
C_p	The variable cost of performing a preventive replacement;
K	Constant parameter used in the PHM policy decision rule, representing the difference between the preventive replacement cost and the failure replacement cost, and the unit is $;
C_{p0}	The fixed cost of performing a preventive replacement;
Pr_S	Total probability of preventive replacement;
Pr_F	Total probability of failure replacement;
T_S	Expected component age at preventive replacement;
T_F	Expected component age at failure replacement;
C_{ES}	Total expected replacement cost for preventive replacements;
C_{EF}	Total expected replacement cost for failure replacements;
C	Total expected replacement cost;
N	The number of components in the multi-component system;
$\mathbf{PrS}$	System probability matrix indicating the probabilities of the system in different possible states;
$\mathbf{PrS}^k$	Matrix $\mathbf{PrS}$ at inspection point *k*;
$\mathbf{PrS}^{kt}$	A temporary matrix for calculating matrix $\mathbf{PrS}^k$;

$\Pr_{AB}$	Probability of the system in the absorbing state;
i	Component i;
$\boldsymbol{k}$	A vector indicating the component age combination, $\boldsymbol{k} = (k_1, k_2, \dots, k_N)$;
$\boldsymbol{j}$	A vector indicating the component state combination, $\boldsymbol{j} = (j_1, j_2, \dots, j_N)$;
K_a	The largest possible component age index;
J	The highest possible component state;
M	The $J + 1$ by $J + 1$ transition probability matrix;
L	The inspection interval;
ff	A N-element vector indicating whether a component is failed.
λ	The ratio between the fixed preventive replacement cost and the total fixed and variable preventive replacement cost.

2. PHM based CBM policy for a single unit

A widely used PHM model combines a Weibull baseline hazard function with a component considering the covariates which affect the time to failure, which is shown as follows [9]:

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left\{\sum_{r=1}^{m} \gamma_r z_r(t)\right\} \quad (1)$$

where $h(t, Z(t))$ is the hazard value, or failure rate value, at time t, given the covariate values $z_1(t)$, $z_2(t)$, …,$z_m(t)$. The first part of this model is the baseline hazard function $\beta/\eta(t/\eta)^{\beta-1}$, which takes into account the age of the equipment at time of inspection, given the values of parameters β and η. The second part $\exp\{\gamma_1 z_1(t) + \gamma_2 z_2(t) + ... + \gamma_m z_m(t)\}$ takes into account the covariates which can be considered to be the key condition monitoring measurements reflecting the health condition of the equipment. The covariate state transition matrix, denoted by M, is used to describe the transitions of the covariate states. Transition probability matrix indicates the probabilities of a covariate in different ranges at the next inspection time given its current range at the current inspection point. Transition probability matrices are estimated using the historical inspection data.

The CBM optimization approach based on proportional hazards model, and the method for calculating the cost and reliability objective function values, were developed in Ref. [2, 16]. A summary of method is given in this section. In the PHM based CBM policy, if the observed hazard rate $h(t, Z(t))$ multiplied by K at the given inspection point is greater than a certain risk threshold d, preventive replacement action should be taken. If a failure occurs, a failure replacement will be performed. Thus, the risk threshold d determines the PHM based CBM policy. The objective of CBM optimization is to find the optimal risk threshold to optimize the cost and reliability objectives.

The cost objective C, i.e., the total expected cost per unit of time, can be calculated based on the following formula [2]:

$$C = \frac{C_p(1-Q(d)) + (C_p + K)Q(d)}{W(d)} = \frac{C_p(1-Q(d)) + C_f Q(d)}{W(d)}, \tag{2}$$

where C is the average cost per unit of time. C_p is the preventive replacement cost, and $C_p + K$ is the failure replacement cost, denoted by C_f. K is a constant cost value. It is assumed that the cost due to unavailability, that is, loss of productivity, is a part of the failure replacement cost, and it has been considered when estimating the failure replacement cost. $Q(d)$ is the probability that failure replacement will occur, that is, $Q(d) = P(T_d \geq T)$. $T_d = \inf\{t \geq 0: K \cdot h(t, z(t)) \geq d\}$ is the preventive replacement time at the risk level d. $W(d)$ is the expected time until replacement, regardless of whether it is a preventive action or failure, that is, $W(d) = E(\min\{T_d, T\})$, where T is the failure time. Once the optimal risk level, d*, is determined, the item is replaced at the first moment t when the condition $\beta/\eta(t/\eta)^{\beta-1}\exp(\gamma Z(t)) \geq d^*/K$ is met. A numerical algorithm was developed by Banjevic et al. [2] for the exact cost evaluation of a CBM policy with respect to a certain risk threshold value d.

3. PHM based CBM policy for multi-component systems

3.1. The policy

The following assumptions are made in this paper regarding the multi-component systems under discussion:

- The components in the system are identical, and are independent in their degradation and failure process.
- The components are economically dependent. Specifically, a fixed preventive replacement cost, denoted by C_{p0}, is incurred if a preventive replacement is performed on any component. If preventive replacement is performed on multiple components simultaneously, the fixed preventive replacement cost is incurred only once.
- Similar to the PHM based CBM policy for a single unit, it is assumed that inspection points are discrete and equally spaced.
- The inspection methods may include vibration monitoring, oil analysis, etc., depending on the specific problems. The covariates are obtained via analyzing the inspection data.
- The time required for performing replacement actions is assumed to be relatively small and it can be ignored in maintenance optimization.
- We focus on the replacement optimization in this study, and the inspection interval is not a design variable in the optimization problem. The inspections are assumed to be performed at zero costs.

In this work, we extend the PHM based CBM policy from a single unit to multi-component systems. The objective is to take into consideration the economic dependency among the components, and reduce the overall long-run replacement cost by performing preventive replacements for multiple components simultaneously if certain conditions are met. In the proposed CBM policy, we introduce an additional level of risk threshold so as to determine which components should be preventively replaced given that a preventive replacement is performed at a certain inspection point. The PHM based CBM policy for multi-component systems are proposed as follows:

(1) Perform failure replacement if a failure occurs;

(2) For component i, preventively replace the component if $K \cdot h_i > d_1$, where K is a constant, h_i is the hazard value of component i, and d_1 is the level-1 risk threshold;

(3) If a replacement (preventive or failure) is performed on any component in the system, perform preventive replacement on component l if $K \cdot h_l > d_2$, where d_2 is the level-2 risk threshold, and $d_2 \leq d_1$.

As can be seen from the proposed policy, two levels of risk threshold are used in order to deal with the economic dependency among different components in the multi-component systems. The level-1 risk threshold, d_1, is used to determine if preventive replacements should be performed on the system because some of its components should be preventively replaced. If a preventive replacement is to be performed on a component, the level-2 risk threshold, d_2, is used to determine if other components should be preventively replaced so as to take advantage of the economic dependency among different components.

The PHM based CBM policy is determined once the two risk threshold levels, d_1 and d_2, are determined, and the cost measure can be evaluated for the CBM policy using an algorithm to be presented in the next section. Thus, the objective of the CBM optimization is to find the optimal risk threshold values to minimize the long-run expected replacement cost per unit of time. The optimization model can be formulated as follows:

$$\begin{aligned} &\min \ C(d_1, d_2) \\ &\text{s.t.} \\ &\quad C \leq C_0 \\ &\quad d_1 \geq d_2 \geq 0 \end{aligned} \tag{3}$$

where C_0 is the cost constraint value. $C(d_1, d_2)$ is the total expected replacement cost, and the algorithm for evaluating it will be presented in Section 3.2.

3.2. Cost evaluation for the PHM based CBM policy

The cost model for the PHM based CBM policy is summarized as follows based on the discussion in the previous section. At a certain inspection point, if a failure replacement is performed on a component, the failure replacement cost C_f is incurred. If a preventive replacement is performed on a component, both fixed preventive replacement cost C_{p0} and variable preventive replacement cost C_p are incurred. However, if preventive replacements are performed on multiple components, the fixed preventive replacement cost is incurred only once.

A numerical algorithm is developed in this section for the exact cost evaluation for the PHM based CBM policy for multi-component systems. The system probability matrix $\mathbf{PrS}$ is introduced, which indicates the probability distribution of the multi-component system at a certain inspection time point. The sum of all the matrix element values is equal to 1. $\mathbf{PrS}(\boldsymbol{k},\boldsymbol{j})$ denotes the probability of the system in state $(\boldsymbol{k},\boldsymbol{j})$, where $\boldsymbol{k}$ is the component age combination vector, $\boldsymbol{k} = (k_1, k_2, \dots, k_N)$, $0 \le k_i \le K_a$ $(1 \le i \le N)$. K_a is the largest possible component age index, and N is the number of components in the system. The age of a component represents the duration between the current time and the point when it was put into use. $\boldsymbol{j}$ is the component state combination vector, $\boldsymbol{j} = (j_1, j_2, \dots, j_N)$, $0 \le j_i \le J$ $(1 \le i \le N)$. We assume there is only one covariate for now, and will present the method to extend to consider multiple covariates later in this paper. A covariate can be divided into a number of ranges, and these ranges are referred to as the component states, including state 0, 1, …, J, where J is the highest possible component state. The initial value of $\mathbf{PrS}$ at inspection time 0 is: $\mathbf{PrS}(\mathbf{0},\mathbf{0}) = 1$, where $\mathbf{0} = (0, 0, \dots, 0)$, and $\mathbf{PrS}(\boldsymbol{k},\boldsymbol{j}) = 0$ for all the other elements. This indicates that all the components are in the best states at inspection time 0.

The basic idea of the proposed numerical algorithm is that the system probability matrix $\mathbf{PrS}$ is updated at each inspection point based on the component state transitions and the CBM policy. The probability of the system in different states is re-distributed among different elements of the matrix, and the expected replacement costs are updated based on the replacement costs incurred during this process. At the end of the computation at the largest inspection point, the probability of the system in the absorbing state, in which all the system components are replaced, is very close to 1. The performance measures such as total expected replacement cost per unit of time can be determined. The flow chart of the proposed algorithm is shown in Figure 1, and detailed explanations are given in the remainder of this section.

Figure 1. Flow chart for the proposed cost evaluation algorithm

In the initialization process, the initial value of $\mathbf{PrS}$ at inspection 0 is specified as mentioned above. The initial values of the total expected failure replacement cost C_{EF}, the total expected preventive replacement cost C_{ES}, the expected component age for failure replacements, T_F, and the expected component age for preventive replacements, T_S, are all set to be 0. The initial value of the probability of the system in the absorbing state, $\mathrm{Pr}_{\mathrm{AB}}$, is 0 too.

Once the initialization process is completed, we will go from inspection point 0 to inspection point K_a, the highest possible component age index. At each inspection point starting from inspection point 1, the following operations are performed.

(1). State transitions. Based on the probability matrix **PrS** at inspection point *k*-1, matrix **PrS** is updated at inspection point *k* by only considering the state transitions. From inspection point *k*-1 to inspection point *k*, each component might transit from the previous state to a new state or remain in the same state. We need to go through all the possible transitions to compute the matrix **PrS** at inspection point *k*, denoted by $\mathbf{PrS}^k$.

Consider the transition from component state combination $\boldsymbol{j1} = (j_{11}, j_{12}, \ldots, j_{1N})$ to $\boldsymbol{j2} = (j_{21}, j_{22}, \ldots, j_{2N})$. The transition probability is $\mathrm{Pr}_{\boldsymbol{j1j2}} = \prod_{i=1}^{N} M(j_{1i}, j_{2i})$, where matrix M is the covariate state transition matrix. Let $\mathbf{PrS}^{kt}$ be a temporary matrix to be used to calculate $\mathbf{PrS}^k$. First, the probabilities associated with component state combination $\boldsymbol{j1}$ at inspection point *k*-1**,** multiplied by the transition probability $\mathrm{Pr}_{\boldsymbol{j1j2}}$, are added to the corresponding elements of **PrS** associated with component state combination $\boldsymbol{j2}$ at inspection point *k***.** That is,

$$\Delta\mathbf{PrS}^{kt}(:, \boldsymbol{j2}) = \mathrm{Pr}_{\boldsymbol{j1j2}} \cdot \mathbf{PrS}^{k-\mathbf{1}}(:, \boldsymbol{j1}). \tag{4}$$

The initial value of $\mathbf{PrS}^{kt}(:, \boldsymbol{j2})$ is $\mathbf{0}$, and its value can be obtained by going through all possible $\boldsymbol{j1}$ vectors and applying the above-mentioned equation. Thus, $\mathbf{PrS}^{kt}(:, \boldsymbol{j2})$ can be determined using the following formula:

$$\mathbf{PrS}^{kt}(:, \boldsymbol{j2}) = \sum_{\boldsymbol{j1}} \mathrm{Pr}_{\boldsymbol{j1j2}} \cdot \mathbf{PrS}^{k-\mathbf{1}}(:, \boldsymbol{j1}). \tag{5}$$

The component age indexes are increased by 1 when we move from inspection point *k*-1 to inspection point *k*. Thus, for any component age vector $\boldsymbol{k}$, we have

$$\mathbf{PrS}^{k}(\boldsymbol{k} + 1, \boldsymbol{j2}) = \mathbf{PrS}^{kt}(\boldsymbol{k}, \boldsymbol{j2}). \tag{6}$$

(2). Failure replacement and preventive replacement operations. After Step (1), we obtain the probability matrix $\mathbf{PrS} = \mathbf{PrS}^k$ at inspection point *k*. Now we need to go through each element of matrix $\mathbf{PrS}$, and re-distribute the probabilities values based on whether failure replacements and/or preventive replacements should be performed according to the CBM policy. For a certain element corresponding to vector $(\boldsymbol{k},\boldsymbol{j})$, where $\boldsymbol{k}=(k_1,k_2,\ldots,k_N)$ and $\boldsymbol{j}=(j_1,j_2,\ldots,j_N)$, the associated probability is given by $\mathbf{PrS}(\boldsymbol{k},\boldsymbol{j})$, where the ages of the components are indicated by vector $\boldsymbol{k}$, and the states of the components are given by vector $\boldsymbol{j}$.

Let L be the inspection interval, say 20 days. The hazard value for component i can be calculated using Equation (1):

$$h_i(\boldsymbol{k},\boldsymbol{j})=\frac{\beta}{\eta}\left(\frac{L\cdot k_i}{\eta}\right)^{\beta-1}\exp\left\{\gamma\cdot z(j_i)\right\}, \tag{7}$$

where $z(j_i)$ represents the covariate value corresponding to state j_i. Again here it is assumed that there is only one covariate. Vector $\boldsymbol{h}(\boldsymbol{k},\boldsymbol{j})$ is used to represent the hazard value vector corresponding to vector $(\boldsymbol{k},\boldsymbol{j})$:

$$\boldsymbol{h}(\boldsymbol{k},\boldsymbol{j})=[h_1(\boldsymbol{k},\boldsymbol{j}),\ h_2(\boldsymbol{k},\boldsymbol{j}),\ldots,h_N(\boldsymbol{k},\boldsymbol{j})]. \tag{8}$$

The failure probability for component i during the interval between inspection points k-1 and k can be calculated as follows:

$$F_i(\boldsymbol{k},\boldsymbol{j})=1-\exp\left\{-h_i(\boldsymbol{k},\boldsymbol{j})\cdot L\right\}. \tag{9}$$

State $(\boldsymbol{k},\boldsymbol{j})$ is further divided into 2^N cases, based on whether a component is working or failed. Each case is represented by a vector $\boldsymbol{ff}=[ff_1,\ ff_2,\ldots,ff_N]$, where $ff_i=0$ if component i is failed, and $ff_i=1$ otherwise. The probability corresponding to vector $\boldsymbol{ff}$ can be calculated as follows:

$$\mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})}=\mathbf{PrS}(\boldsymbol{k},\boldsymbol{j})\cdot\prod_{i=1}^{N}\Big(ff_i\cdot\big(1-F_i(\boldsymbol{k},\boldsymbol{j})\big)+(1-ff_i)\cdot F_i(\boldsymbol{k},\boldsymbol{j})\Big). \tag{10}$$

For case $\boldsymbol{ff}$ for state $(\boldsymbol{k},\boldsymbol{j})$, based on the PHM based CBM policy described in Section 3.1, we can determine if failure replacement or preventive replacement should be performed on a certain component i: (1) failure replacement will be performed if $ff_i=0$; (2) preventive replacement will be performed if $ff_i=1$ and $K\cdot h_i(\boldsymbol{k},\boldsymbol{j})\geq d_1$; (3) preventive replacement will be performed if $ff_i=1$, $K\cdot h_i(\boldsymbol{k},\boldsymbol{j})\geq d_2$, and replacement is performed on at least one component m other than the current component.

We go through each component, and update the total expected replacement cost values and the expected component age values based on whether failure replacements and preventive components will be performed. For a certain component *i*, if failure replacement is to be performed, the total expected failure replacement cost, C_{EF}, and the expected component age for failure replacements, T_F, will be updated:

$$\Delta C_{EF} = \mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})} \cdot C_f$$

$$\Delta T_F = \mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})} \cdot k_i \cdot L \qquad (11)$$

where C_f is the cost of performing a failure replacement, and k_i is the *i*th element of the component age combination vector $\boldsymbol{k}$. If preventive replacement is to be performed on a certain component *i*, the total expected preventive replacement cost, C_{ES}, and the expected component age for preventive replacements, T_S, will be updated:

$$\Delta C_{ES} = \mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})} \cdot C_p$$

$$\Delta T_S = \mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})} \cdot k_i \cdot L \qquad (12)$$

where C_p is the variable cost of performing a preventive replacement. If preventive replacement is performed on at least one component, the fixed preventive replacement cost will be added in the end:

$$\Delta C_{ES} = \mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})} \cdot C_{p0} \qquad (13)$$

where C_{p0} is the fixed cost of performing preventive replacements, which is added only once if preventive replacements are performed when we go through the components.

Probability $\mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})}$, corresponding to case $\boldsymbol{ff}$ for state $(\boldsymbol{k}, \boldsymbol{j})$, will be re-distributed. If a failure replacement or preventive replacement is performed on any component, the probability will be deducted from the current state $(\boldsymbol{k}, \boldsymbol{j})$, that is:

$$\Delta \mathbf{PrS}(\boldsymbol{k}, \boldsymbol{j}) = -\mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})} \qquad (14)$$

If at least one replacement is performed, and at least one component is not replaced, probability $\mathrm{Pr}_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})}$ will be assigned to state $(\boldsymbol{k1}, \boldsymbol{j1})$, where if a replacement is performed on component *i*,

$$k1_i = 0,\ j1_i = 0, \qquad (15)$$

and if no replacement is performed on component *i*,

$$k1_i = k_i,\ j1_i = j_i. \tag{16}$$

And $\mathbf{PrS}(\boldsymbol{k1}, \boldsymbol{j1})$ will be updated as follows:

$$\Delta\mathbf{PrS}(\boldsymbol{k1}, \boldsymbol{j1}) = \Pr_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})}. \tag{17}$$

If all the components are replaced, probability $\Pr_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})}$ will be assigned to the absorbing state, and $\Pr_{AB}$ will be updated:

$$\Delta\Pr_{AB} = \Pr_{\boldsymbol{ff}}^{(\boldsymbol{k},\boldsymbol{j})}. \tag{18}$$

At a certain inspection point k, we need to go through every possible state $(\boldsymbol{k}, \boldsymbol{j})$, and continuously update probability matrix **PrS**, the total expected replacement cost values and the expected component age values.

(3). Total expected replacement cost calculation. Once we reach inspection point K_a, the highest possible component age, the total expected cost can be calculated using the following formula:

$$C(d_1, d_2) = \frac{C_{EF} + C_{ES}}{T_F + T_S} \tag{19}$$

Value K_a can be carefully chosen so that after the computation at inspection point K_a, the probability of the system in the absorbing state, $\Pr_{AB}$, is close to 1. However, if $\Pr_{AB}$ is much smaller than 1, the resulting expected cost value will not be accurate enough. Thus, we check the value of $\Pr_{AB}$ after the computation at inspection point K_a. If $\Pr_{AB}$ is found to be less than 0.95, we will raise the value of K_a and re-evaluate the cost of the CBM policy using the proposed algorithm. The total expected cost value obtained in Equation (19) will be used in the CBM optimization model presented in Equation (3) for the calculation of $C(d_1, d_2)$. As can be seen, C_{EF}, C_{ES} T_F, and T_S in Equation (19) need to be computed using the algorithm in developed in this section. Due to the complexity of the computation process, it is impossible to present an explicit expression of $C(d_1, d_2)$ as a function of d_1 and d_2, and use it in the CBM optimization model described in Equation (3).

As mentioned before, we assume that there is only one covariate in the discussions above. However, the method presented above can be easily extended to deal with the case where there are multiple covariates. Specifically, we need to expand vector $\boldsymbol{j}$ to incorporate multiple covariates as follows:

$$\boldsymbol{j} = [\boldsymbol{j}^{\mathbf{1}}, \boldsymbol{j}^{\mathbf{2}}, \dots, \boldsymbol{j}^{\boldsymbol{m}}] \tag{20}$$

where *m* denotes the number of covariates, and

$$\boldsymbol{j}^{\boldsymbol{r}} = [j_1^r, j_2^r, \dots, j_N^r],\ r = 1, 2, \dots, m.$$

The hazard value for component *i* can be calculated using Equation (1):

$$h_i(\boldsymbol{k}, \boldsymbol{j}) = \frac{\beta}{\eta}\left(\frac{L \cdot k_i}{\eta}\right)^{\beta-1} \exp\left\{\sum_{r=1}^{m} \gamma_r z_r(j_i^r)\right\} \tag{21}$$

where $z_r(j_i^r)$ denotes the value of covariate *r* corresponding to state j_i^r. The rest of the method for cost evaluation is the same as that for the single covariate case presented above.

4. Examples

Two examples are used in this section to illustrate the proposed PHM based CBM policy for multi-component systems, and the method for the cost evaluation of the CBM policy. Comparative studies are conducted between the CBM policy for multi-component systems and that for a single unit to demonstrate the advantage of the proposed CBM policy for achieving lower total expected replacement cost.

4.1. Example 1

4.1.1. Example introduction

In this example, we consider a multi-component system consisting of two bearings under vibration monitoring. We use two bearings in order to simplify our discussion. The bearing vibration monitoring data reported by Banjevic et al. [2], which were collected from shear pumps in a food processing plant, are used in this example. It is assumed that a certain fixed preventive replacement cost will be incurred if preventive replacement is performed on one or both of the bearings in the system. The objective is to find an optimal condition based replacement policy, i.e., the optimal risk threshold values, to minimize total expected long-run replacement cost, given the vibration monitoring data, the replacement histories, and the cost data.

21 vibration measurements were collected from the bearings using accelerometers, including vibration data in axial, horizontal and vertical directions for the overall velocity, velocities in 5 bands and acceleration. There are 25 histories in the recorded data, including 13 failure histories replacements, where bearings ended with failure replacements, and 12 suspension histories, where bearings ended with preventive replacements. Significance analysis was performed using the software EXAKT developed by OMDEC Inc. [2] to identify the significant covariates. To simplify the problem, we only keep the most significant covariate, which was identified to be VEL#1A, representing the band 1 velocity in the axial direction. The PHM parameters can thus be estimated using software EXAKT, and the resulting hazard function is given as follows:

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \cdot \exp\{\gamma_1 z_{1A}(t)\} = \frac{3.046}{667.6}\left(\frac{t}{667.6}\right)^{3.046-1} \cdot \exp\{5.14 z_{1A}(t)\} \qquad (22)$$

The transition probability matrix is required for calculating the cost evaluation of the PHM based CBM policy. The transition probability matrix gives the probabilities of a covariate in different ranges at the next inspection time given its current range. Assume the inspection interval is 20 days. The transition probability matrix for covariate VEL#1A can be estimated using software EXAKT, and the estimated matrix is shown in the Figure 2. As can be seen, the covariate is divided into 5 ranges, and the transition probability values are shown in the figure. These 5 ranges are also referred to as 5 states: state 0, 1, 2, 3, and 4. Thus, the highest possible component state is $J = 4$, and a component is in state 0 if the covariate falls into range [0, 0.035266), and so on.

The failure replacement cost is estimated to be \$16,300. In this example, suppose that the fixed preventive replacement cost is \$3,000, and the variable preventive replacement cost is \$1,800. Thus, constant K can be calculated as follows:

$$K = C_f - C_{p0} - C_p = \$16{,}300 - \$3{,}000 - \$1{,}800 = \$11{,}500, \qquad (23)$$

and K is used in the condition for performing preventive replacements in the CBM policy [2]. Based on the length of the available failure histories and suspension histories, K_a , which is the largest component age index, is chosen to be 48, i.e., the highest possible age is 960 days, to make sure that the probability that the component age exceeds the highest possible age is near zero. The computation results show that this choice of K_a value is reasonable because the probability of the system in the absorbing state, $\Pr_{AB}$, is 1 or very close to 1 in all of the CBM policy cost evaluations.

Figure 2. The transition probability matrix for covariate VEL#1A

4.1.2. An example of the PHM based CBM policy

Given the data presented in Section 4.1, we consider an example of the PHM based CBM policy for the system consisting of two bearings. Suppose the risk threshold values are given as follows: $d_1 = 20\$/\text{day}$, $d_2 = 10\$/\text{day}$. According to the PHM based CBM policy for multi-component systems, at a certain inspection point *k*, the following actions will be performed:

(1) If a bearing failed, perform failure replacement for the bearing;

(2) For bearing i ($i = 1, 2$), preventive replacement will be performed if the bearing has not failed, and the following condition is met:

$$K \cdot h_i = \$11{,}500 \cdot h_i > d_1 = 20\$/\text{day},$$

where h_i is the hazard value of bearing *i* at inspection point *k* calculated using Equation (22).

(3) If one bearing meets the conditions in (2) and will be preventively replaced, the other bearing will be preventively replaced too if it has not failed and the following condition is met:

$$K \cdot h_i = \$11{,}500 \cdot h_i > d_2 = 10\$/\text{day}.$$

If preventive replacement is performed only on one bearing, the total preventive replacement cost would be $\$3{,}000 + \$1{,}800 = \$4{,}800$. If preventive replacement is performed on both of the bearings, the total preventive replacement cost would be $\$3{,}000 + 2 \times \$1{,}800 = \$6{,}600$.

4.1.3. CBM optimization and the comparative study

Because $K_a = 48$ and a component has 5 possible states based on the covariate value, the probability matrix **PrS** is a $48 \times 48 \times 5 \times 5$ matrix. Matrix **PrS** is updated at every inspection point. Given the data presented earlier in this section, using the algorithm described in Section 3.2, the cost of a certain CBM policy can be evaluated given certain risk threshold values d_1 and d_2. The cost as a function of d_1 and d_2 is plotted in Figure 3, where the risk threshold values are given in the logarithm scale. It can be observed from the cost values and the figure that the optimal risk threshold values exist, where the lowest cost is achieved. Optimization is implemented using optimization functions in Matlab. The optimal risk threshold values are found as follows:

$$d_1^* = 10.0\$/\text{day}, d_2^* = 0.5\$/\text{day},$$

and the corresponding lowest cost is $C^* = 35.69\$/\text{day}$. At the optimal risk threshold values, we also show in Figure 4 the progression of the expected cost and age values, C_{ES}, C_{EF}, T_S and T_F, with respect to the inspection time. It can be seen that all of these four values reach steady states in late stage of the cost evaluation algorithm. We also fix d_1 at the optimal value 10.0$/day, and investigate the change of the cost value with respect to risk threshold value d_2. The results are shown in Figure 5(a). It can be seen that the cost value is sensitive to risk threshold value d_2.

Figure 3. Cost versus risk threshold values in the logarithm scale

Figure 4. Progression of C_{ES}, C_{EF}, T_S and T_F

Figure 5. Cost versus risk threshold value d_2 while d_1 is fixed at the optimal value

We perform a comparative study between the proposed CBM policy for multi-component systems and the CBM policy for single unit. In the CBM policy for single unit, which is described in Section 2, replacement decisions are made on components individually, and there is only one risk threshold value d. The single-unit CBM policy, using one covariate, is applied to the current bearing replacement problem. The failure replacement cost is the same at $16,300. The preventive replacement cost is set to be $4,800, since preventive replacements are performed on components separately, and thus both the fixed cost and variable cost will be incurred when a preventive replacement is performed. Other data are the same as those in the multi-component CBM policy. In the single-unit PHM based CBM policy, the cost, which is a function of risk threshold d, is calculated using the algorithm developed by Banjevic et al. [2]. The cost as a function of the risk threshold d in the logarithm scale is shown in Figure 6. The cost curve is not so smooth due to the fact that the covariate is discretized into 5 ranges and only one covariate is considered in this problem. The optimal risk threshold is found to be $d^* = 11.8\$/\text{day}$, and the corresponding lowest cost is $C^* = 39.92\$/\text{day}$. Comparing to the lowest cost obtained using the multi-component CBM policy, which is $35.69\$/\text{day}$, the optimal cost corresponding to the single-unit CBM policy is about 10.6% higher. This comparative study demonstrates that by taking advantage of the economic dependency in the multi-component systems, the proposed multi-component CBM policy can lead to lower total expected replacement cost.

Figure 6. Cost versus risk threshold value for the single-unit CBM policy

Suppose the total fixed and variable preventive replacement cost is fixed at $4,800, we can investigate how the fixed replacement cost value affects the optimal CBM policy and the optimal total expected replacement cost. When the fixed preventive replacement cost is 0, the multi-component CBM policy is reduced to the single-unit CBM policy. We use λ to denote the ratio between the fixed replacement cost and the total fixed and variable replacement cost. For example, if $\lambda = 2/3$, the fixed preventive replacement cost will be $2,400, and the variable preventive replacement cost will be $1,200. With different λ values, optimization is performed and the optimal CBM policies are obtained. With respect to different λ values, the optimal cost values and the optimal CBM policies are listed in Table 1. We can see that when ratio λ is 0, the optimal cost is the same as that for the single-unit CBM policy, since the multi-component CBM policy is reduced to the single-unit CBM policy when the fixed preventive replacement cost is 0. When ratio λ is larger than 0, the optimal cost for the multi-component CBM policy is lower than that for the single-unit CBM policy. And with the increase of ratio λ, the optimal cost becomes lower, and the resulting cost savings by utilizing the multi-component CBM policy rather than the single-unit policy become higher, rising to as much as 25.68% when ratio λ equals 1. In another word, when the economic dependency among components becomes stronger, the benefit of utilizing the multi-component CBM policy becomes higher. When the fixed preventive replacement cost ratio λ is equal to 0.3, we also fix d_1 at the optimal value 11.5$/day, and investigate the change of the cost value with respect to risk threshold value d_2. The results are shown in Figure 5(b). We can observe again that the cost value is sensitive to risk threshold value d_2.

Table 1: Cost versus fixed preventive replacement cost ratio λ for Example 1

It is also interesting to compare the proposed CBM policy with the corrective maintenance policy, under which replacements are performed only when failures occur, i.e., only failure replacements are performed. Again, the failure replacement cost is assumed to be $16,300. Based on the data in this case, it is found that the average failure replacement time is 363.32 days. Thus, for the system with two components, the average replacement cost per unit of time is 89.73 $/day. Thus, comparing to the corrective maintenance policy, approximately 60% cost saving can be achieved using the proposed CBM policy.

4.2. Example 2

In this example, we consider another multi-component system consisting of three bearings, and use the vibration monitoring data collected from bearings on a group of Gould pumps at a Canadian kraft pulp mill company [22]. Totally 36 failure and suspension histories are available. Again, EXAKT was used to identify the most significant covariate among all the collected vibration measurements, and the vibration magnitude in the horizontal direction in the 5th vibration frequency band was found to be the most significant. The PHM parameters can be estimated using software EXAKT, and the resulting hazard function is given as follows:

$$h(t,Z(t))=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\cdot\exp\{\gamma_1 z_{1A}(t)\}=\frac{2.244}{2,362}\left(\frac{t}{2,362}\right)^{2.244-1}\cdot\exp\{42.23 z_{1A}(t)\} \quad (24)$$

Assume that the inspection interval is 100 days in this example. The transition probability matrix for the covariate can be estimated using software EXAKT. The failure replacement cost is estimated to be $12,000, and the total preventive replacement cost is $4,000. First, the single-unit

PHM based CBM policy was applied to the system, and the cost was evaluated using the algorithm developed by Banjevic et al. [2]. The optimal risk threshold is found to be $d^* = 6.31\$/\text{day}$, and the corresponding lowest cost is $C^* = 9.85\$/\text{day}$.

Now we investigate cases where economic dependency exists, and apply the proposed PHM based CBM policy for multi-component systems. Again, λ is used to denote the ratio between the fixed replacement cost and the total fixed and variable replacement cost, which is $4,000. For example, if $\lambda = 0.2$, the fixed preventive replacement cost will be $800, and the variable preventive replacement cost will be $3,200. With respect to different λ values, optimization is performed, and the resulting optimal cost values and the optimal CBM policies are listed in Table 2. When $\lambda = 0$, the multi-component PHM based CBM policy is reduced to the single-unit policy. When $\lambda > 0$, i.e., economic dependency exists, the optimal cost for the multi-component CBM policy is lower than that for the single-unit CBM policy, and the cost saving increases with the increase of the λ value. This example demonstrates again that for multi-component systems where economic dependency exists, the proposed multi-component PHM based CBM policy is more effective and can lead to lower overall replacement cost.

Table 2: Cost versus fixed preventive replacement cost ratio λ for Example 2

5. Conclusions

The existing work reported in the literature only focuses on determining the optimal CBM policy for a single unit. However, for systems consisting of multiple components, economic dependency exists among different components subject to condition monitoring, and thus it might be cheaper to preventively replace multiple components at the same time. In this work, we propose a multi-component system CBM policy based on proportional hazards model. The cost evaluation for the PHM based CBM policy becomes much more complex when we extend the policy from a single unit to a multi-component system. A numerical algorithm is developed for the exact evaluation of the PHM based CBM policies for multi-component systems. The optimal

CBM policy can be obtained through optimization. Examples are used to demonstrate the proposed methods.

The algorithm developed in this paper for the cost evaluation of the multi-component CBM policy can provide accurate expected replacement cost, which is important for finding the trend in the cost as a function of the risk threshold values and thus finding the optimal CBM policy corresponding to the lowest cost. However, the cost evaluation algorithm is computationally intensive, particularly when the number of components and the number of covariates become large. More efficient cost evaluation algorithms are desired in future research. Another future research topic is to develop CBM methods for multi-component systems with different types of components instead of identical components, and we need to build PHM models for different types of components and likely need to introduce more decision variables to deal with different component types.

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List of Figure Captions

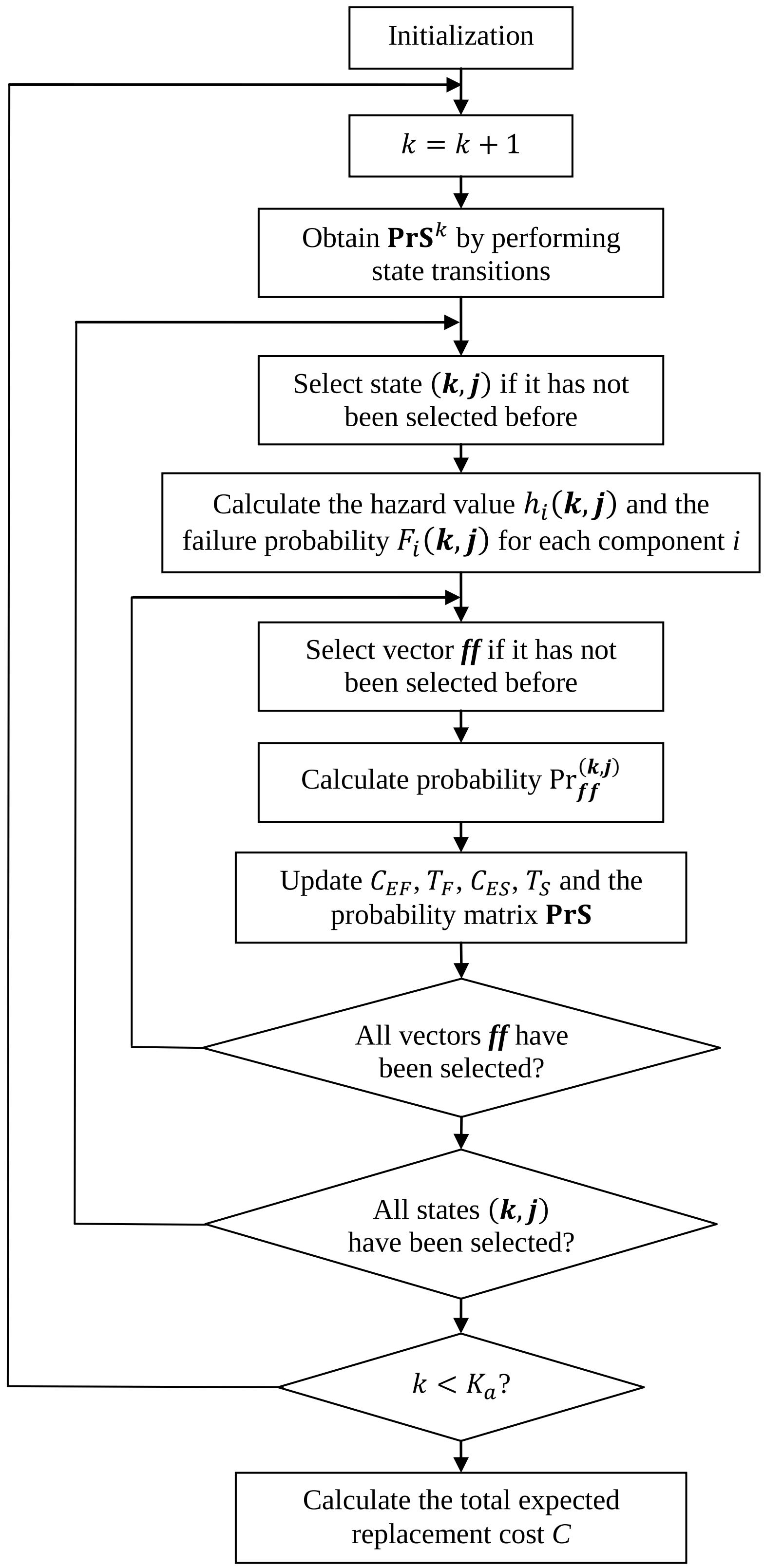


Figure 1. Flow chart for the proposed cost evaluation algorithm

VEL_1A	0 to 0.035266	0.035266 to 0.2519	0.2519 to 1.08821	1.08821 to 2.51648	Above 2.51648
0 to 0.035266	0.765522	0.214501	0.0187137	0.00123314	3.01141e-005
0.035266 to 0.2519	0.0419512	0.809202	0.134907	0.0134952	0.000445182
0.2519 to 1.08821	0.00436408	0.160862	0.683157	0.144277	0.00734044
1.08821 to 2.51648	0.000138356	0.00774194	0.0694142	0.838071	0.0846349
Above 2.51648	0	0	0	0	1

Figure 2. The transition probability matrix for covariate VEL#1A

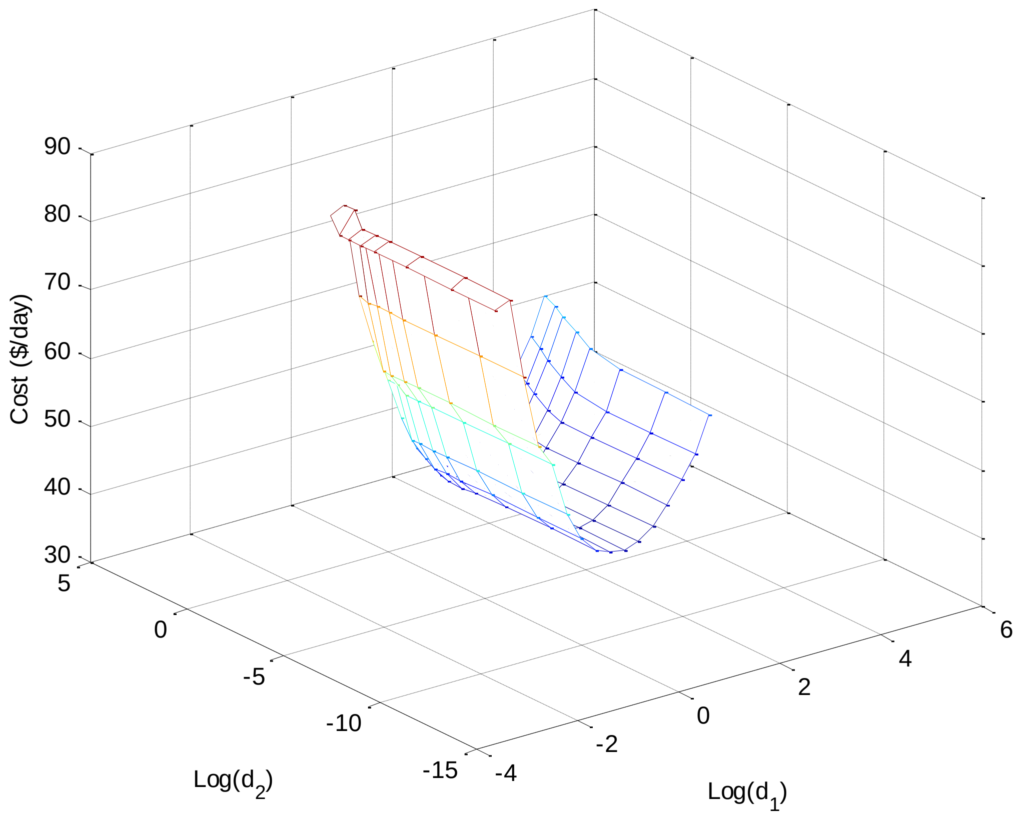
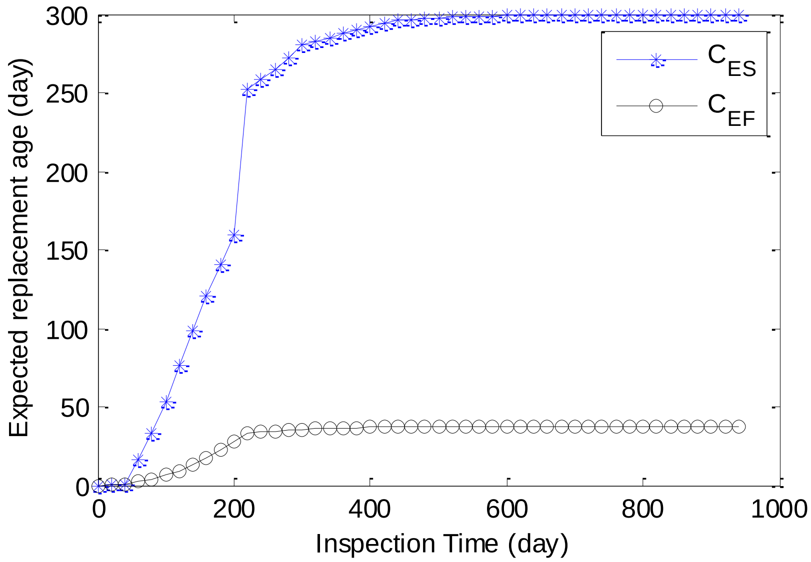
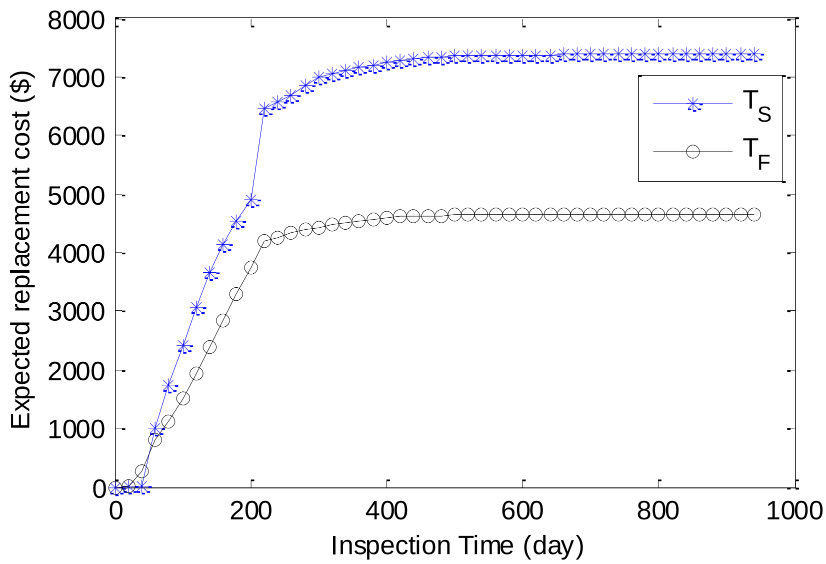


Figure 3. Cost versus risk threshold values in the logarithm scale

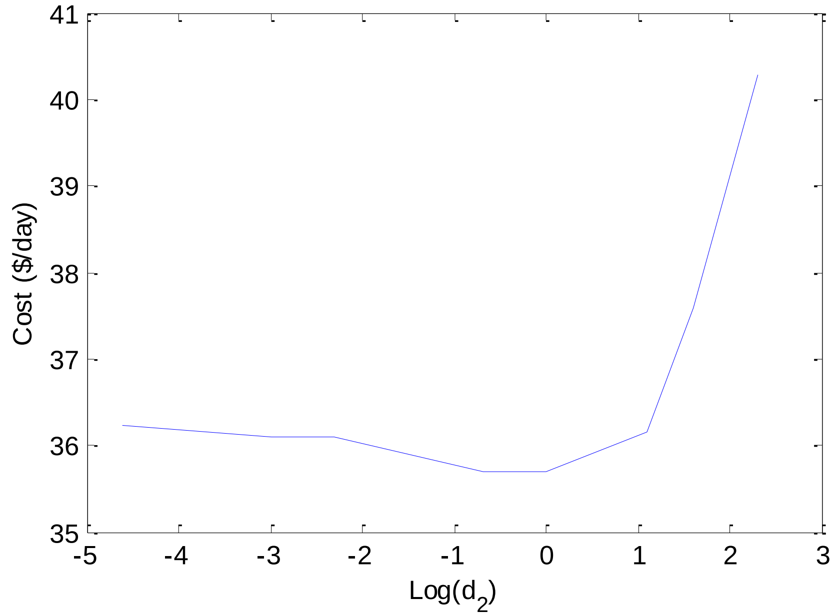


(a)

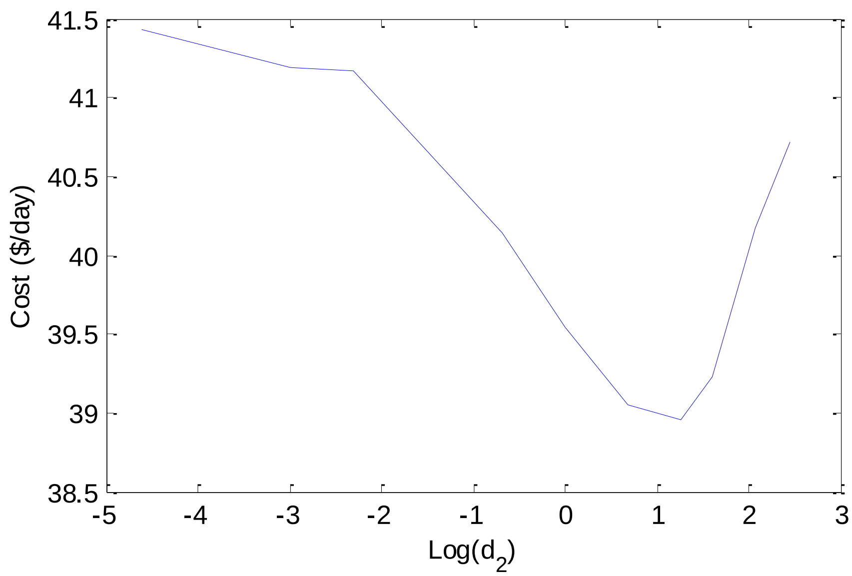


(b)

Figure 4. Progression of C_{ES}, C_{EF}, T_S and T_F



(a) $\lambda = 0.625$



(b) $\lambda = 0.3$

Figure 5. Cost versus risk threshold value d_2 while d_1 is fixed at the optimal value

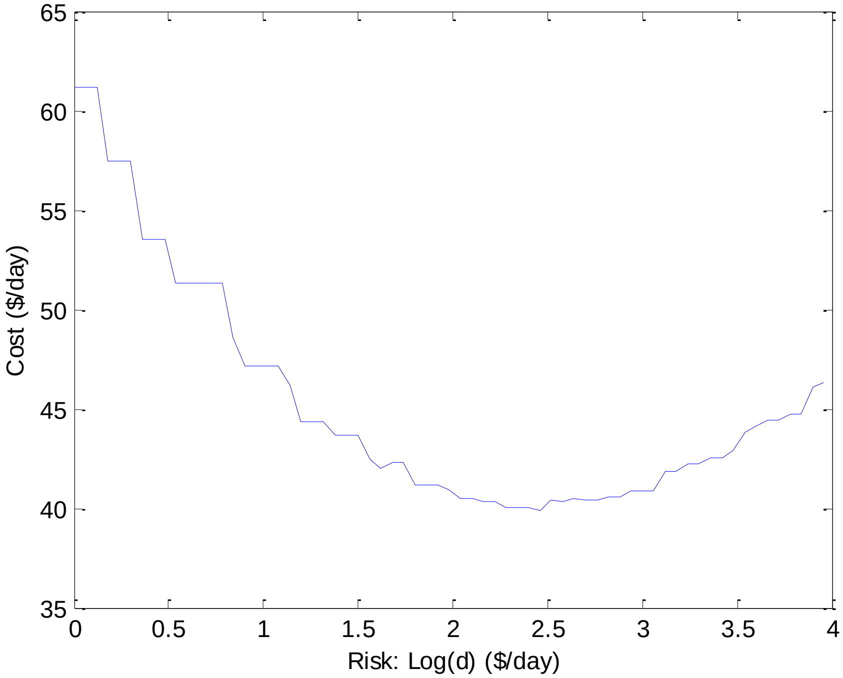


Figure 6. Cost versus risk threshold value for the single-unit CBM policy

Table 1: Cost versus fixed preventive replacement cost ratio λ for Example 1

ratio λ	Optimal CBM policy: $[d_1, d_2]$	Cost ($/day)	Cost savings in percentage
0	[11.8, 11.8]	39.92	0%
0.1	[11.5, 8.1]	39.83	0.23%
0.2	[11.5, 5.8]	39.37	1.38%
0.3	[11.5, 3.5]	38.74	2.96%
0.4	[11.5, 1.2]	38.04	4.71%
0.5	[10.0, 1.0]	37.07	7.14%
0.6	[10.0, 0.5]	36.00	9.82%
0.7	[10.0, 0.5]	34.67	13.15%
0.8	[10.0, 0.5]	33.30	16.58%
0.9	[5.0, 0.05]	31.60	20.84%
1.0	[5.0, 0]	29.67	25.68%

Table 2: Cost versus fixed preventive replacement cost ratio λ for Example 2

ratio λ	Optimal CBM policy: $[d_1, d_2]$	Cost ($/day)	Cost savings in percentage
0	[6.31, 6.31]	9.85	0%
0.2	[5.37, 1.61]	9.56	2.94%
0.4	[4.47, 0.67]	8.95	9.14%
0.6	[3.63, 0.44]	8.22	16.55%
0.8	[3.60, 0.18]	7.31	25.79%