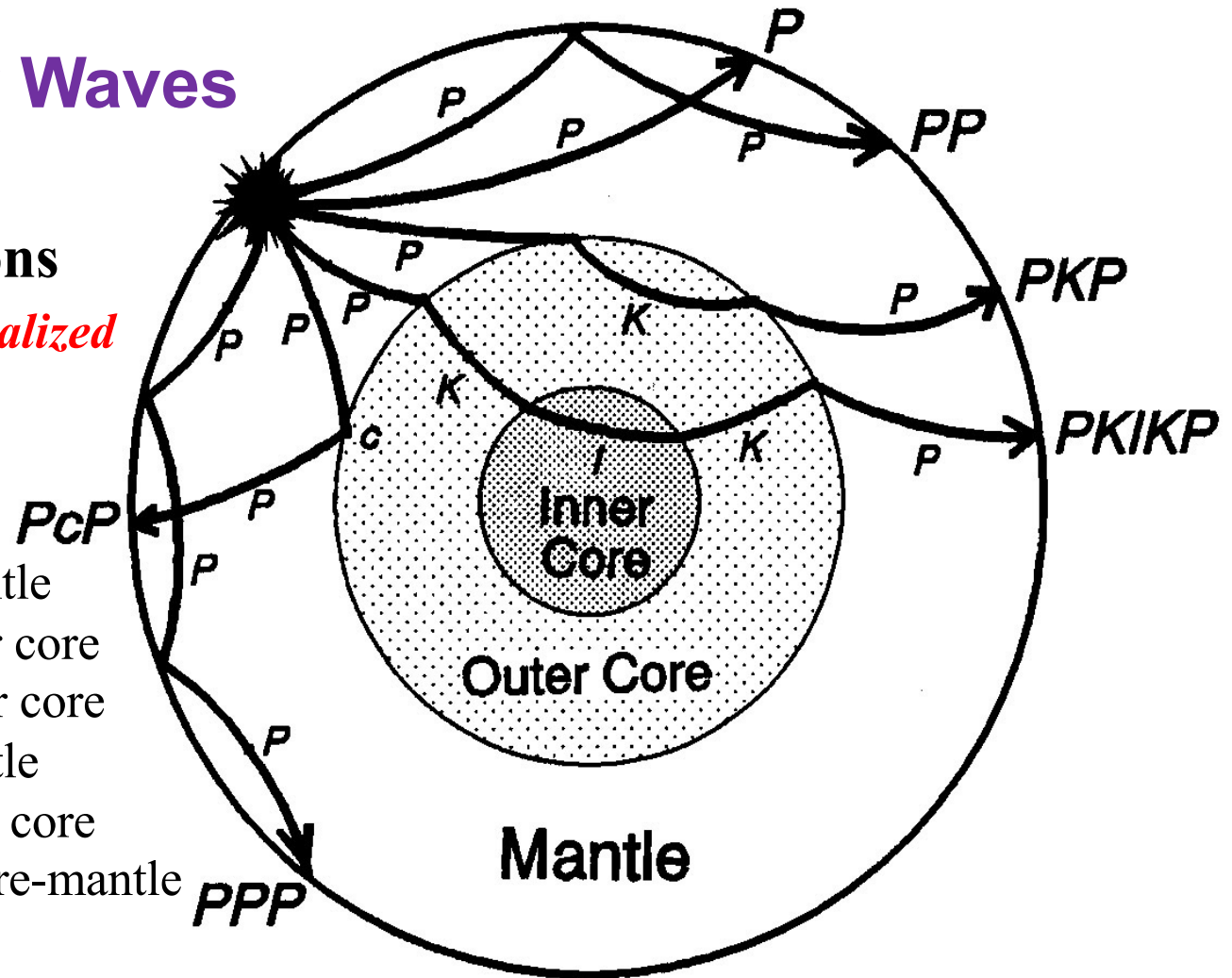


Seismic Body Waves

Naming conventions

(Note: small and capitalized letters do matter!)

- P: P- wave in the mantle
- K: P-wave in the outer core
- I: P-wave in the inner core
- S: S-wave in the mantle
- J: S-wave in the inner core
- c: reflection off the core-mantle boundary (CMB)
- i: reflection off the inner-core boundary (ICB)
- PmP: Reflection off of Moho
- Pn: refracted wave on Moho
- Pg: direct wave in the crust.



**Different phases have different paths,
hence are sensitive to different parts of the Earth**

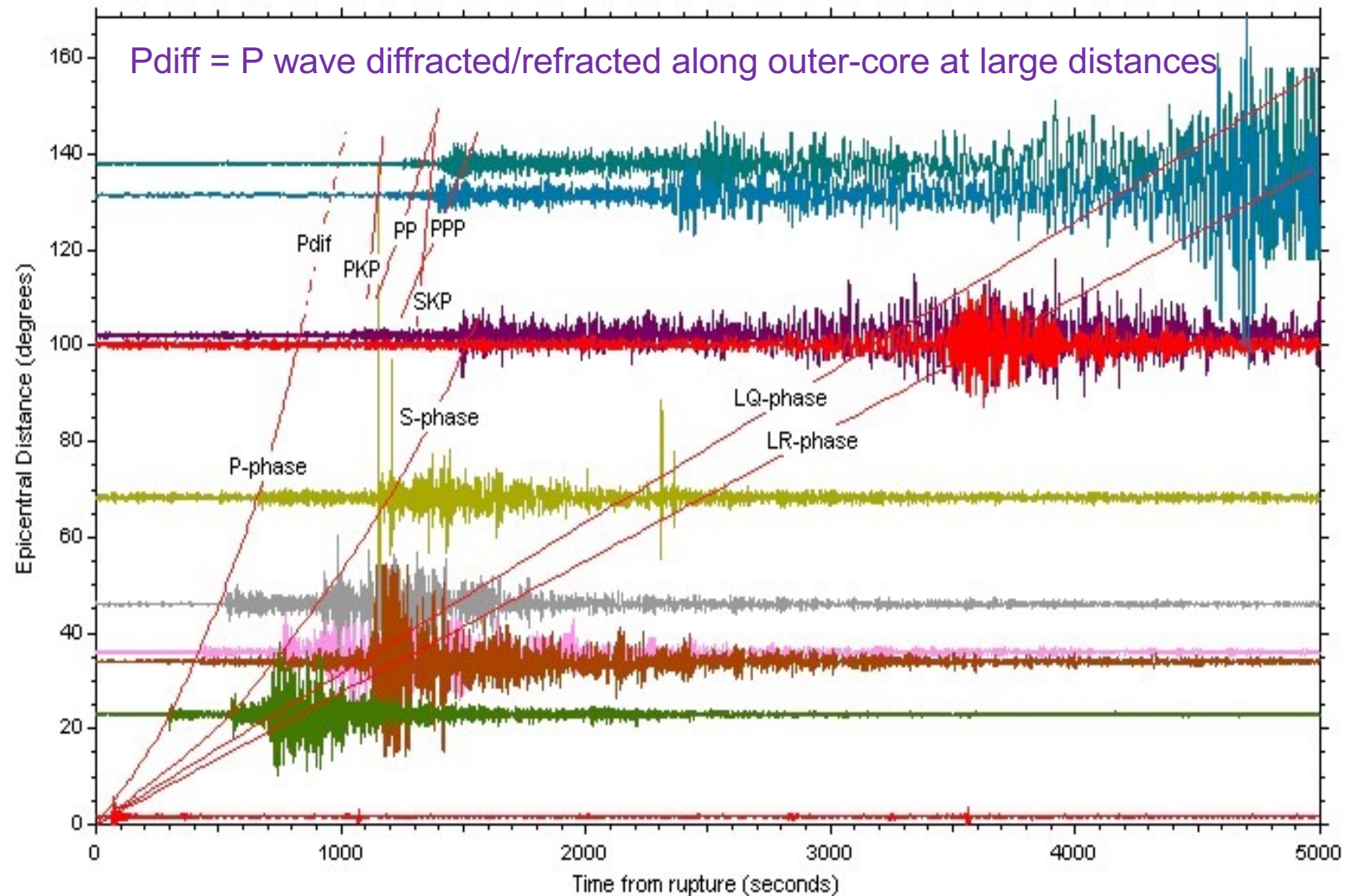
Phase vs. Wave

PcP and PP are different 'phases', but they are both P waves (particle motions)

Some clarifications:

1. 'Phases' are not 'waves'. P and PP are different 'seismic phases', but they are both P waves. Same goes for ScS and Ps, which are both phases, but the former is an S wave and the latter is a P-to-S converted wave.
2. Just because a vertical line intersects a specific seismic phase does not mean it is observable. It could be too small to be observed. The general rule of seismic phases is 'survival of the fittest'.
3. At the source, all types of waves are generated, though a source such as a weight drop from airplanes tends to have more S energy than a pure explosive source.
4. Despite the garden variety of seismic waves going out at all 'takeoff angles', only certain ones are received at certain stations due to their fitness. All S waves are land animals that can't swim.
5. Rays can bundle, e.g., surface wave at the source or at the antipode. Bundling = constructive interference = greater amplitude. This is why the antipode has a great amplitude despite being far from the source (hence greater amplitude loss).

Arrival Time of Seismic Phases



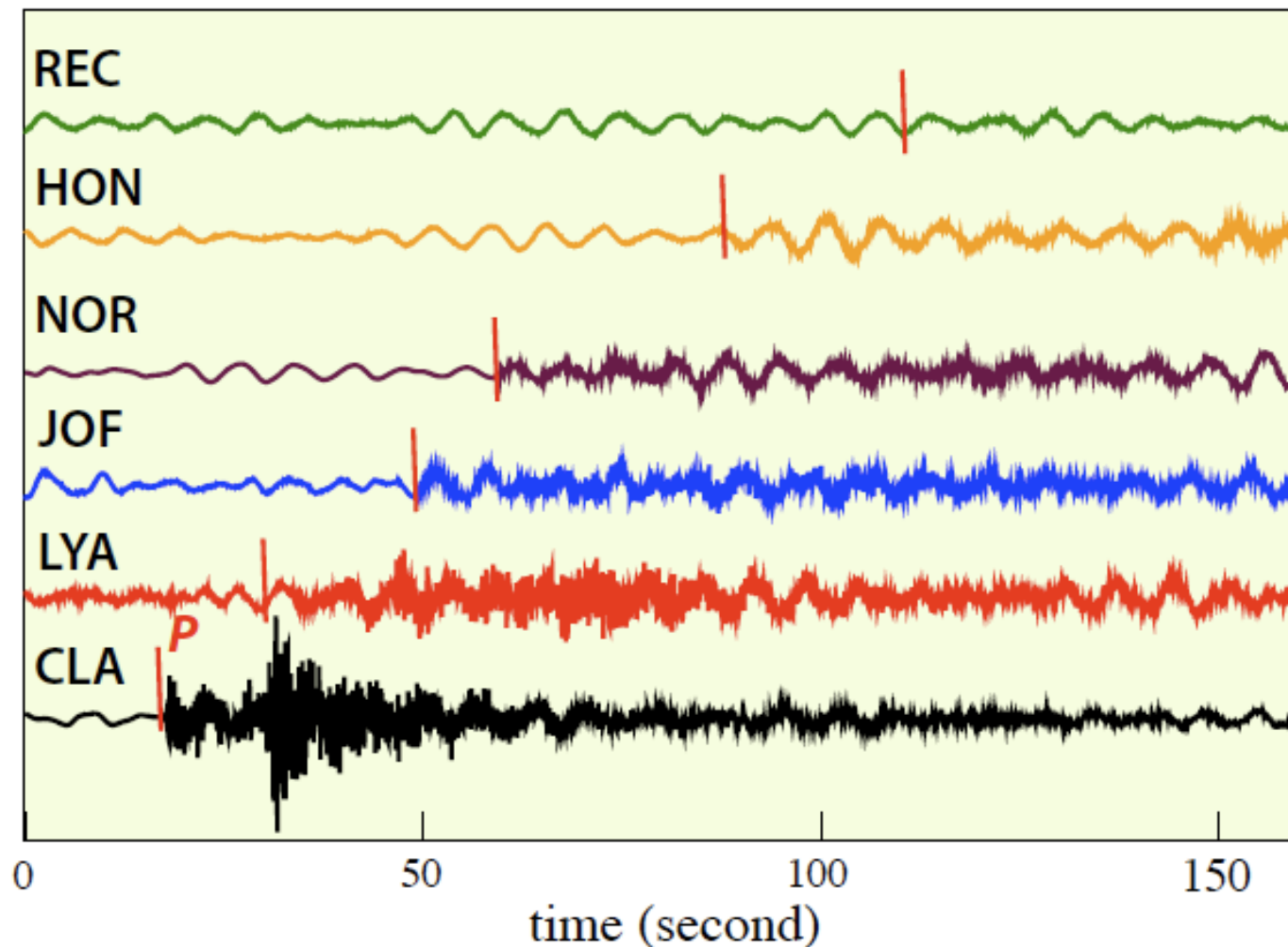
Terminology: *Onset time* (time for the beginning, not max, of a seismic phase. It is where waves begins to take off.)

LQ, LR are surface wave types (*no need to memorize*)

Not all phases are easily distinguishable: Below is the result from a 3.5 event in Lethbridge. Only P and surface waves are seen near Claresholm region. Strong decay due to sedimentary cover and attenuation.

Regional Earthquake

(Dec 30, 2006, Lethbridge, Mag=3.5)



Differential time between two phases (arrivals) are particularly useful for many reasons, some listed here:

- (1) Their time difference is essentially INSENSITIVE to source effect (since they came from the same source, regardless of the complexity of source. This means less error!
- (2) Their difference usually can be used to approximate the depth of major jumps (discontinuities) inside the earth.

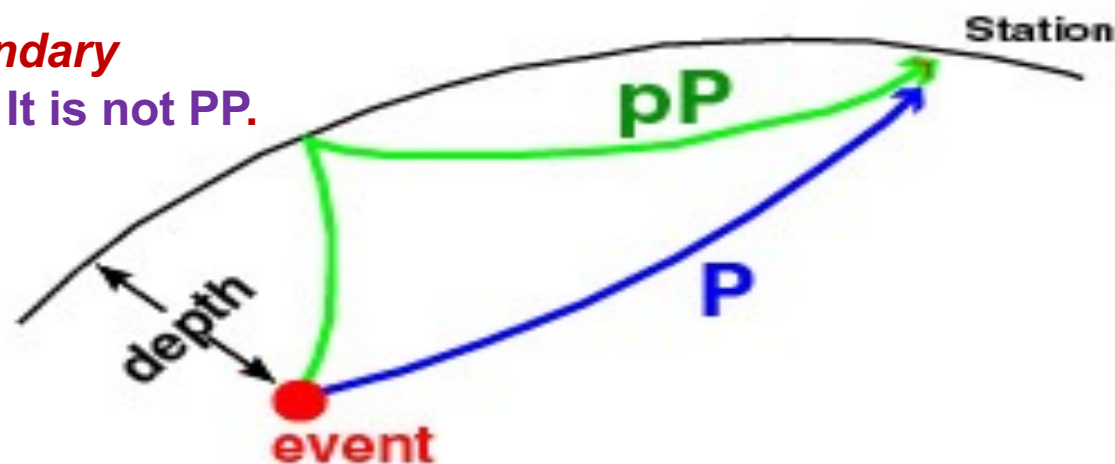
Example 1

pP-P time indicates depth

pP = 'depth phase', a 'secondary phase' with low amplitude . It is not PP.

Hypocenter: True location of earthquake

Epicenter: Projection of earthquake on the Earth's surface



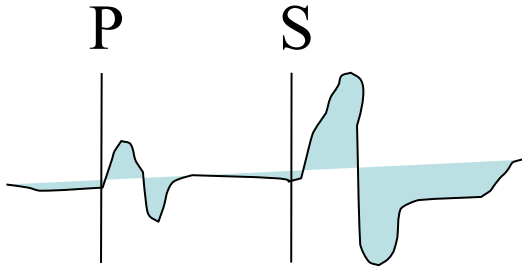
Highly exaggerated image!!

Example 2

$$\Delta t = T_s - T_p$$

magic 8: distance vs. S – P time difference

Similar to finding the timing difference between thunder and lightening!



Timing difference:

$$T_s - T_p$$

Now, assume typical
crustal speeds:

$$V_p = 5.5 \text{ km/s}, V_s = 3.2 \text{ km/s}$$

Need to find distance X.

$$T_s - T_p = X/3.2 - X/5.5$$

$$= X(1/3.2 - 1/5.5)$$

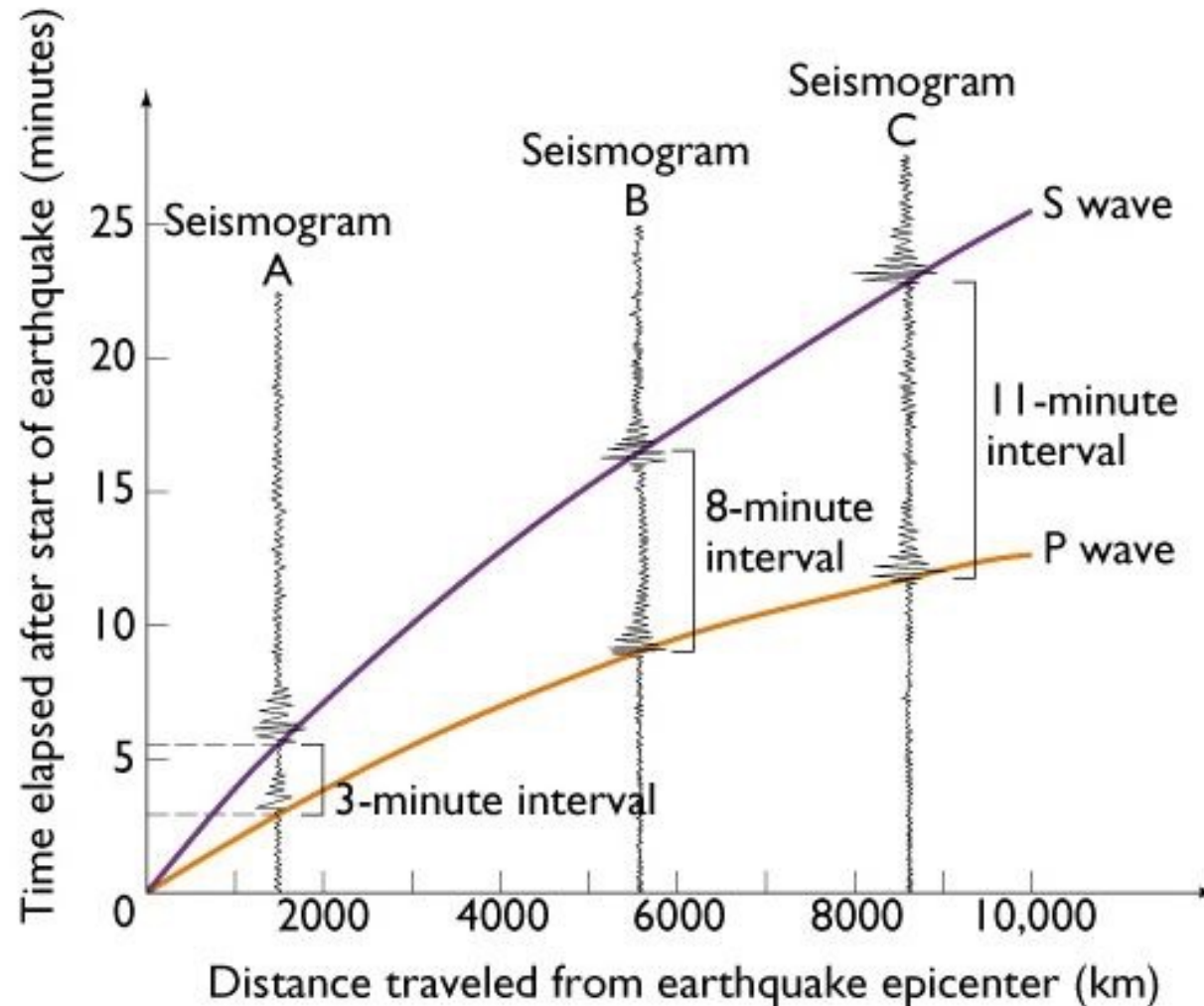
$$\sim X(0.3 - 0.18) \sim \mathbf{X/8}$$



$$\text{Dist} \sim \Delta t_{s-p} * 8$$

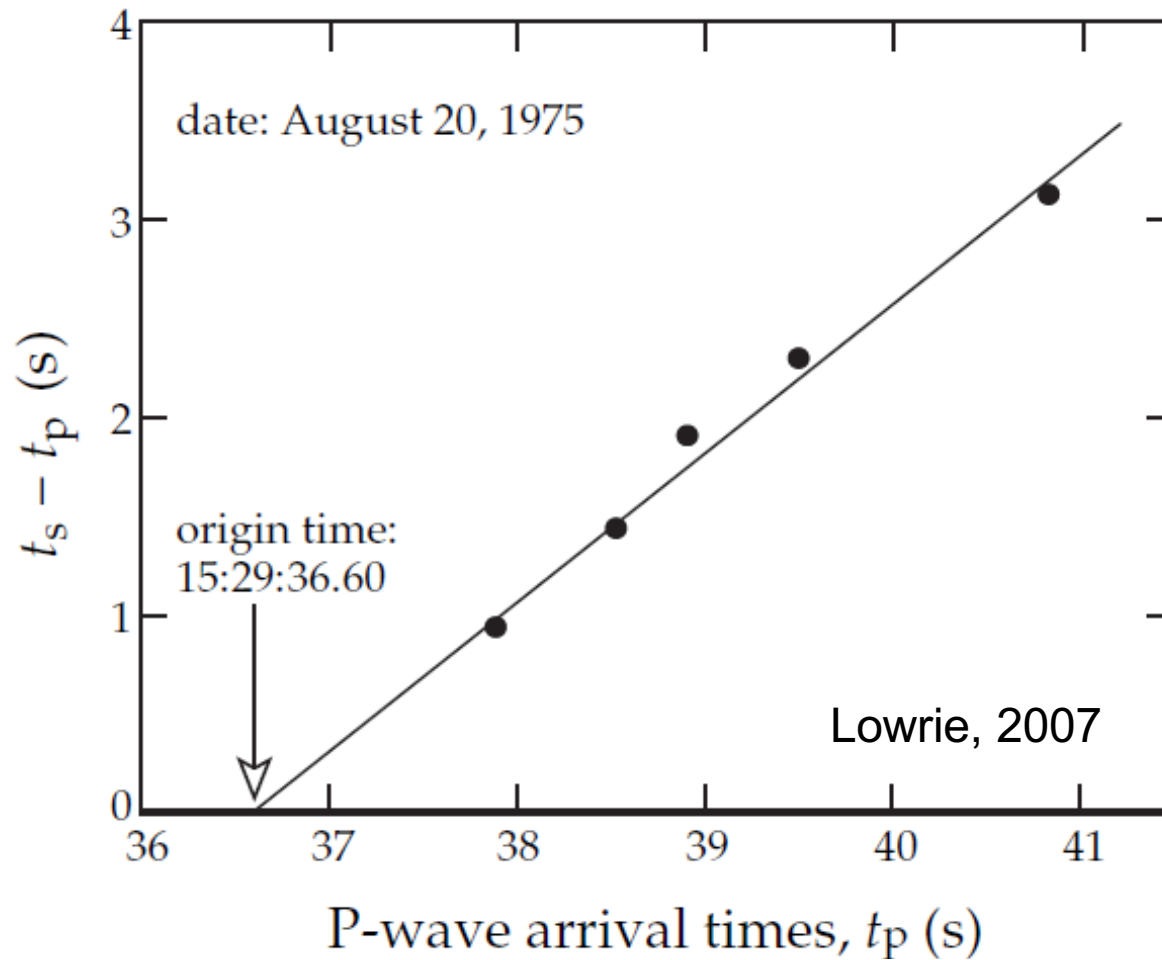
This works for short distance events (< 1000 km). ⁶Why?

Finding distance from P-S travel time difference



It turns out that the travel time differences between P and S phases are related to the **epicentral distance** (distance between source and receiver), **the larger the time difference the greater the distance!**

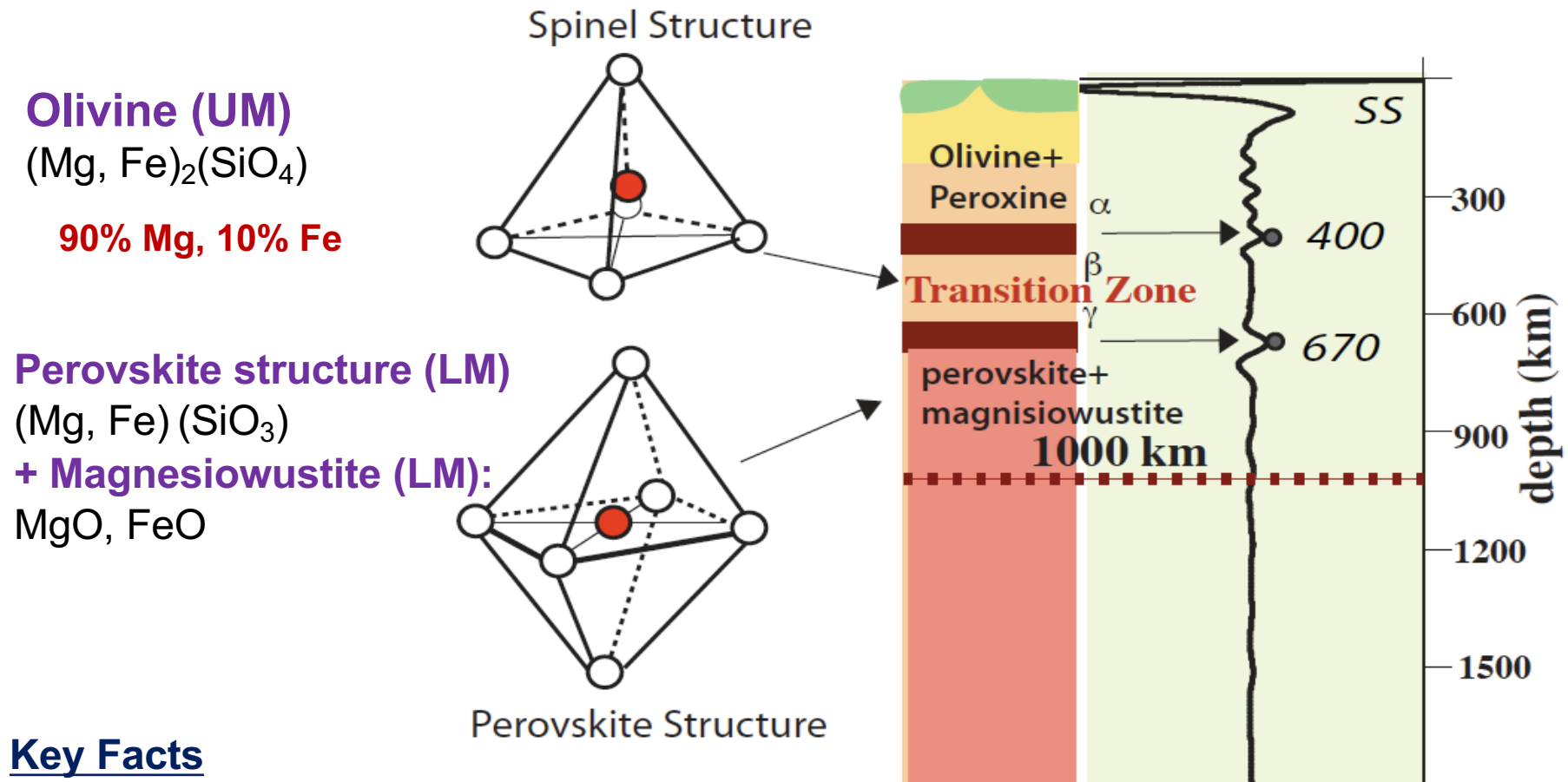
Distance Determination



$$t_s - t_p = D \left(\frac{1}{\beta} - \frac{1}{\alpha} \right) = t_p \left(\frac{\alpha}{\beta} - 1 \right)$$

This says that the P-S times (y) are linear functions of independent variable t_p (P wave travel times)

Seismology & Mineralogy of the “Transition Zone”



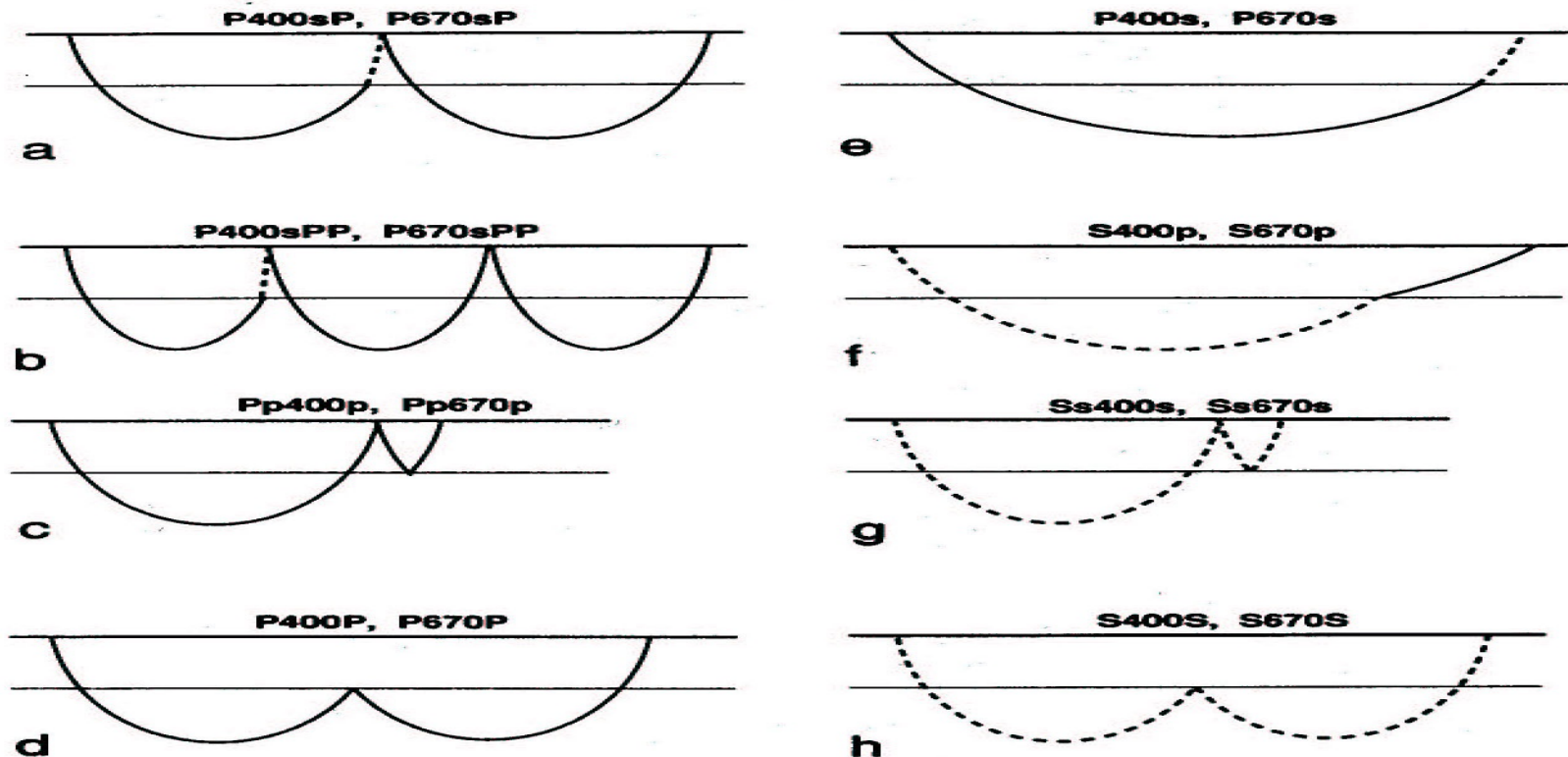
Key Facts

- * Coarse grained mantle rocks rich in olivine & pyroxene (silicate minerals) are called ‘**peridotite**’
- * Perovskite structure = any mineral with ABX_3 formulation (e.g., MgSiO_3)
- * **Breakdown** of **olivine** (upper mantle) to **perovskite** (lower mantle) is considered a ‘**phase transition**’, where *nothing is added to make this happen*.
- * Perovskite structure (**lower mantle**) is more compact, more dense, more stable, faster seismically and more viscous than the Olivine (**upper mantle**)

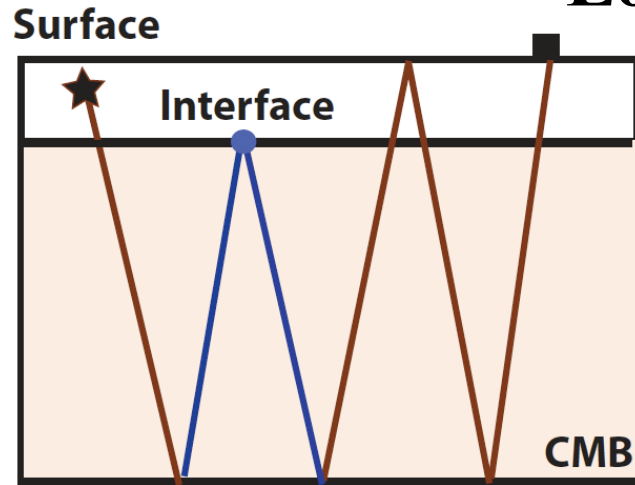
Seismic Imaging: Mapping thermal or compositional anomalies or stratifications using *major seismic arrivals* (P, S, SS, PkP, etc.) and *Secondary arrivals*.

Secondary arrivals: small phases that are useful to constrain internal boundaries. In other words, they do not result from Earth's surface or any large boundaries such as Core or free surface.

Example 1 : Reflections from upper mantle interfaces



“Low Res” approaches



ScS reverberations

Period: 15 sec +

Distance: varies

Plus: versatility, path sensitivity to mantle

Minus: Low resolution

SS/PP precursors (e.g., S670S)

Period: 12 sec +

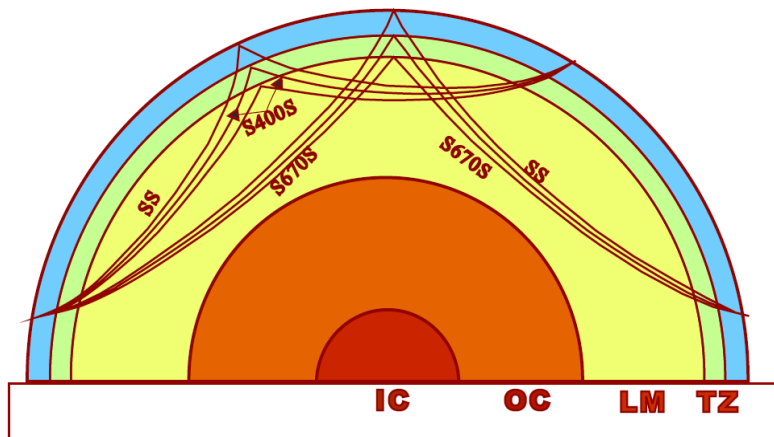
Distance: 90-170 deg

Plus: True global coverage

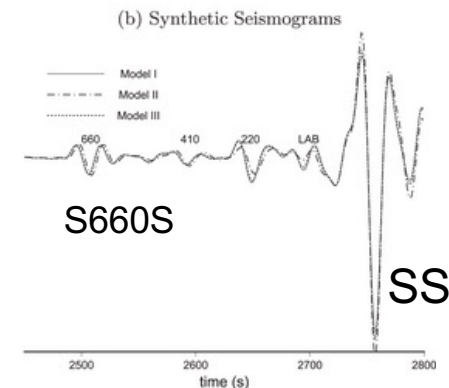
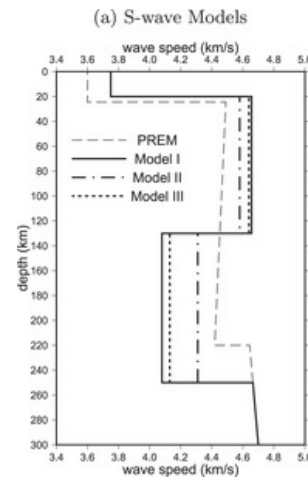
Minus: Low resolution

Complex Fresnel zone

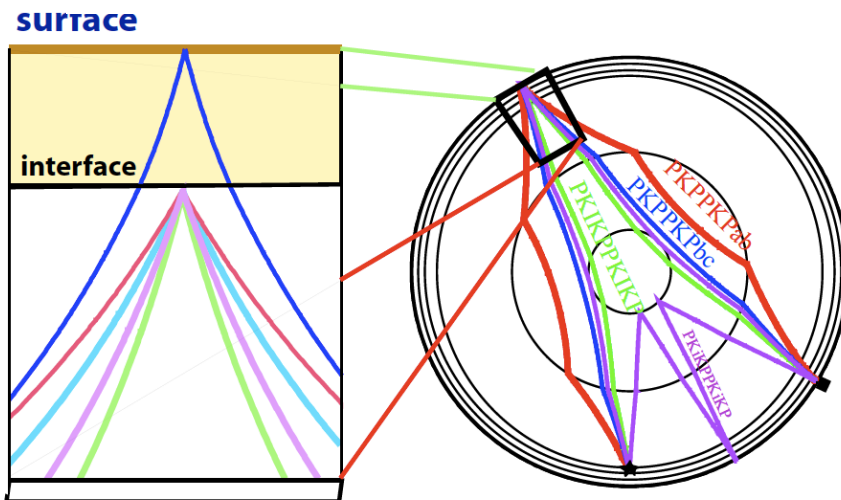
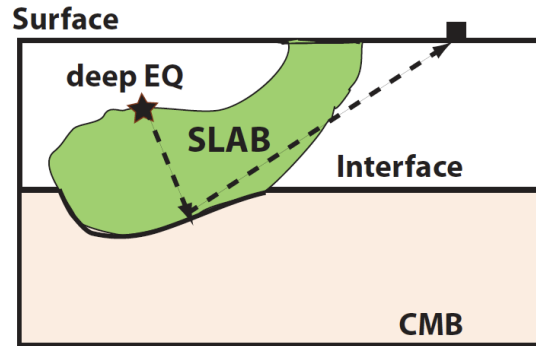
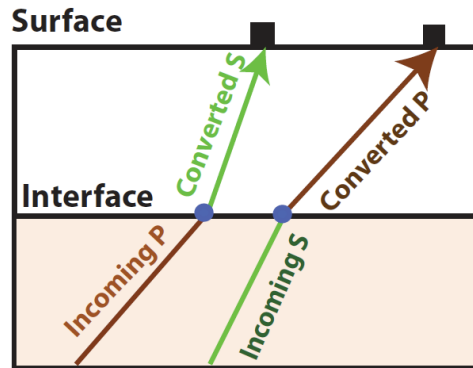
Sun&Zhou, 2023



The right hand image is produced by earthquake synthetic record (noise free). The main phase is SS, and the precursory arrivals are reflections from 410, 660, and lithosphere depths.



Other Secondary Waves



“High Res” approaches

P-to-S or S-to-P conversions

Period: 1-5 sec (high-freq study)

10+ sec (global survey)

Distance: 30-90

Plus: high-resolution

Minus: poor coverage in oceans

Scattered Phases

Period: 1 sec +

Distance: varies

Plus: high resolution

Minus: limited spatial/depth coverage
hard to identify

P' P' (aka PKIKPPKIKP) Precursors

Period: 1 sec

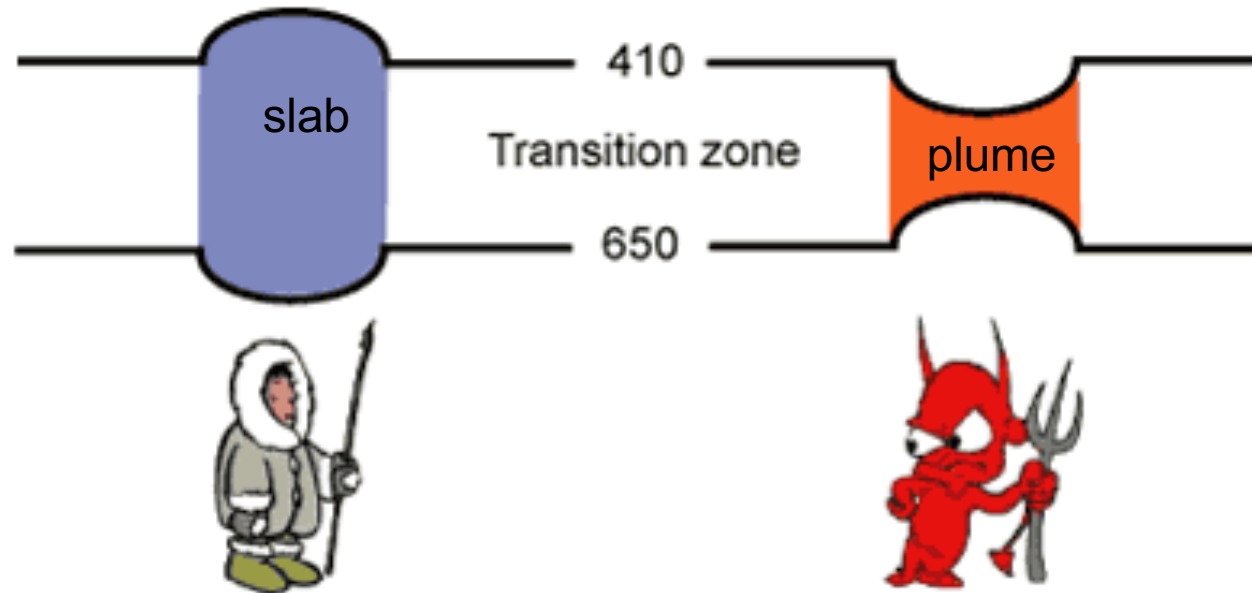
Distance: 60-75 deg

Plus: high resolution

Minus: limited spatial/depth coverage
hard to identify identification

Imaging the mantle transition zone (MTZ) using secondary reflection/conversion

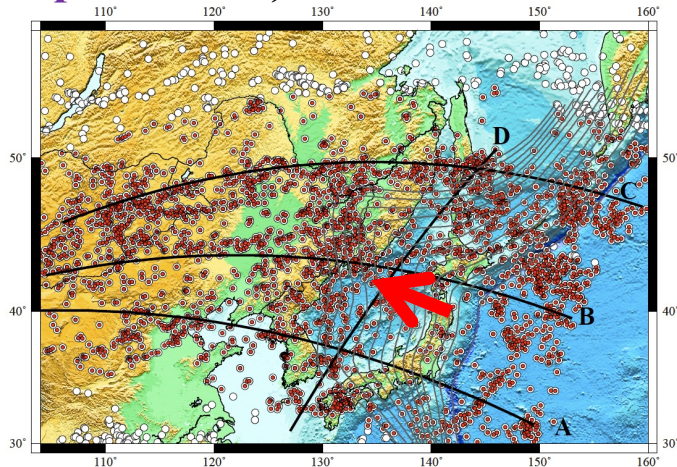
Key: temperature dependence



<https://www.mantleplumes.org/TransitionZone.html>

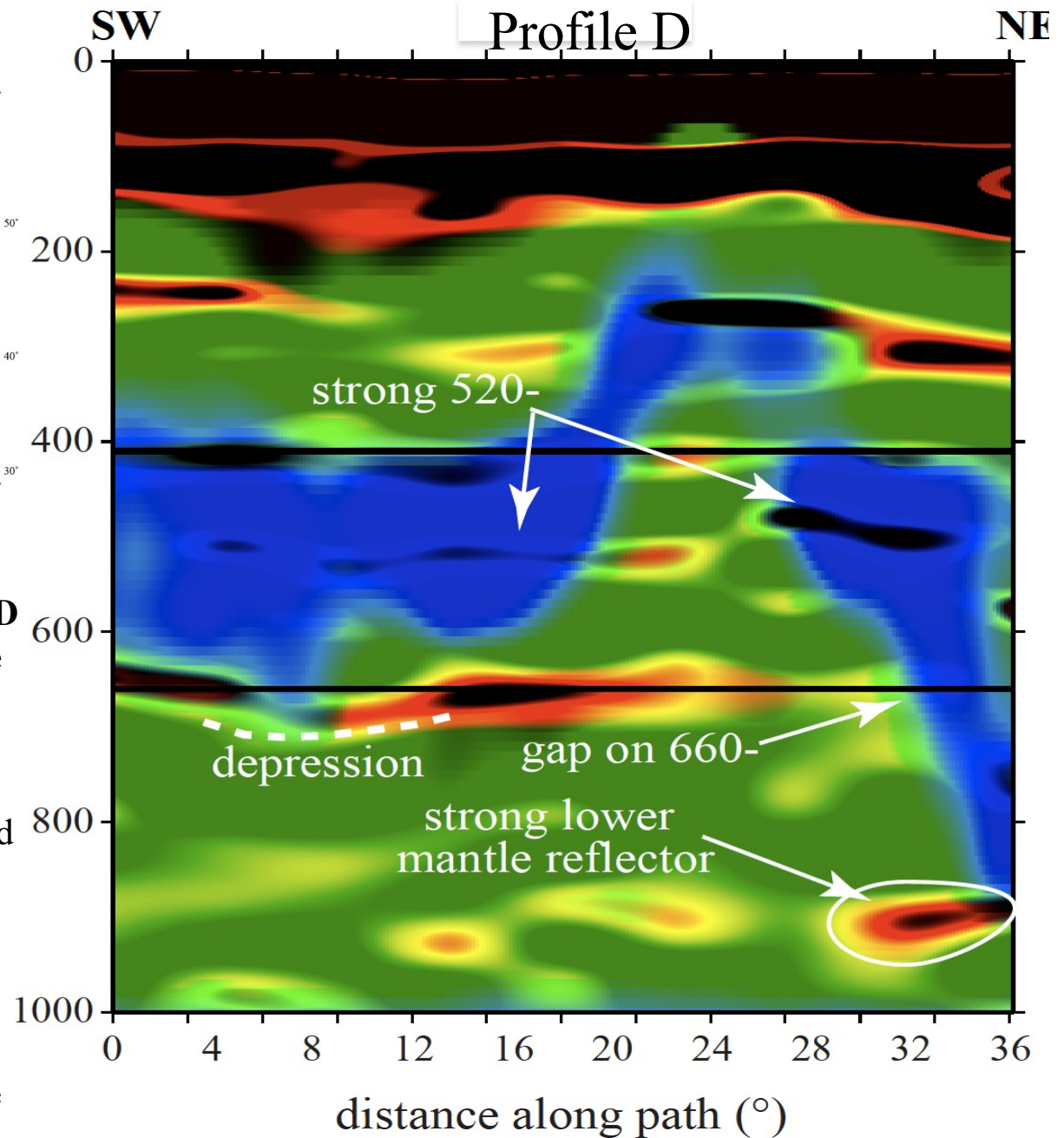
Mapping the boundaries of 410 and 660 km phase boundaries works because of their dependence on mantle temperature. In hot regions, the 410 becomes deeper and 660 becomes shallower, leaving a thin transition zone. In cold regions, such as a slab region, the opposite occurs, which produce a thick transition zone. By mapping the depths of the 660 and 410, we can effectively map the velocities, which are indicators of temperature, in the mantle.

Data & Cross-section (SS precursors)



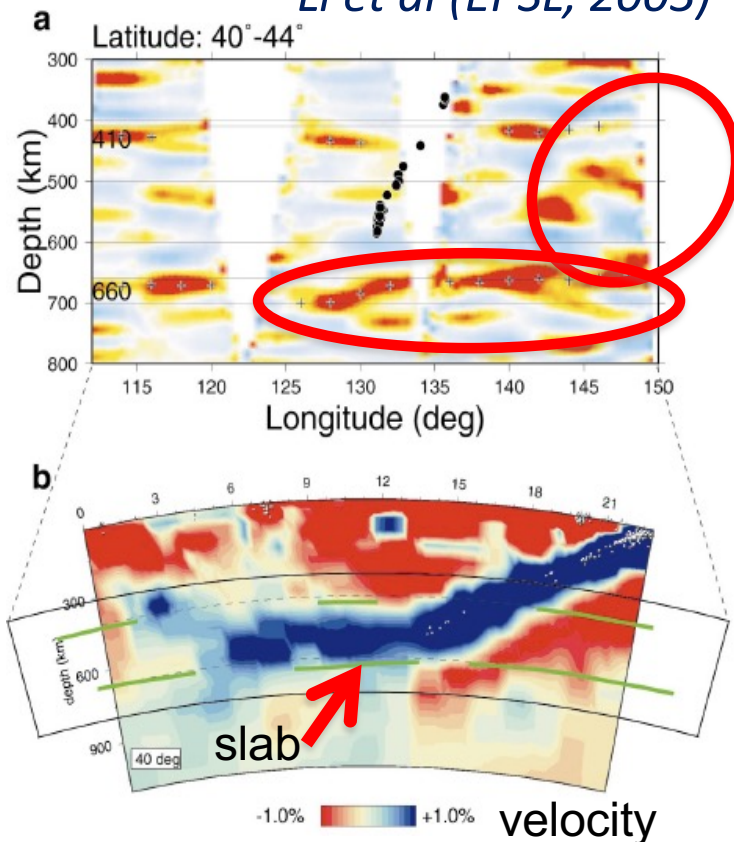
The above plot shows the reflection points of S670S waves. The lines indicate cross-sections (**note profile D where the reflection amplitudes are shown on the right**)

There is a remarkable connection between seismic velocity (background colors on the right) and the major reflectors (red or yellow anomalies). While slab ‘ponding’ is suggested in central Honshu (south), the Kuril subduction zone (north) shows clear signs of a slab that penetrates into the lower mantle.



P-to-S converted Wave

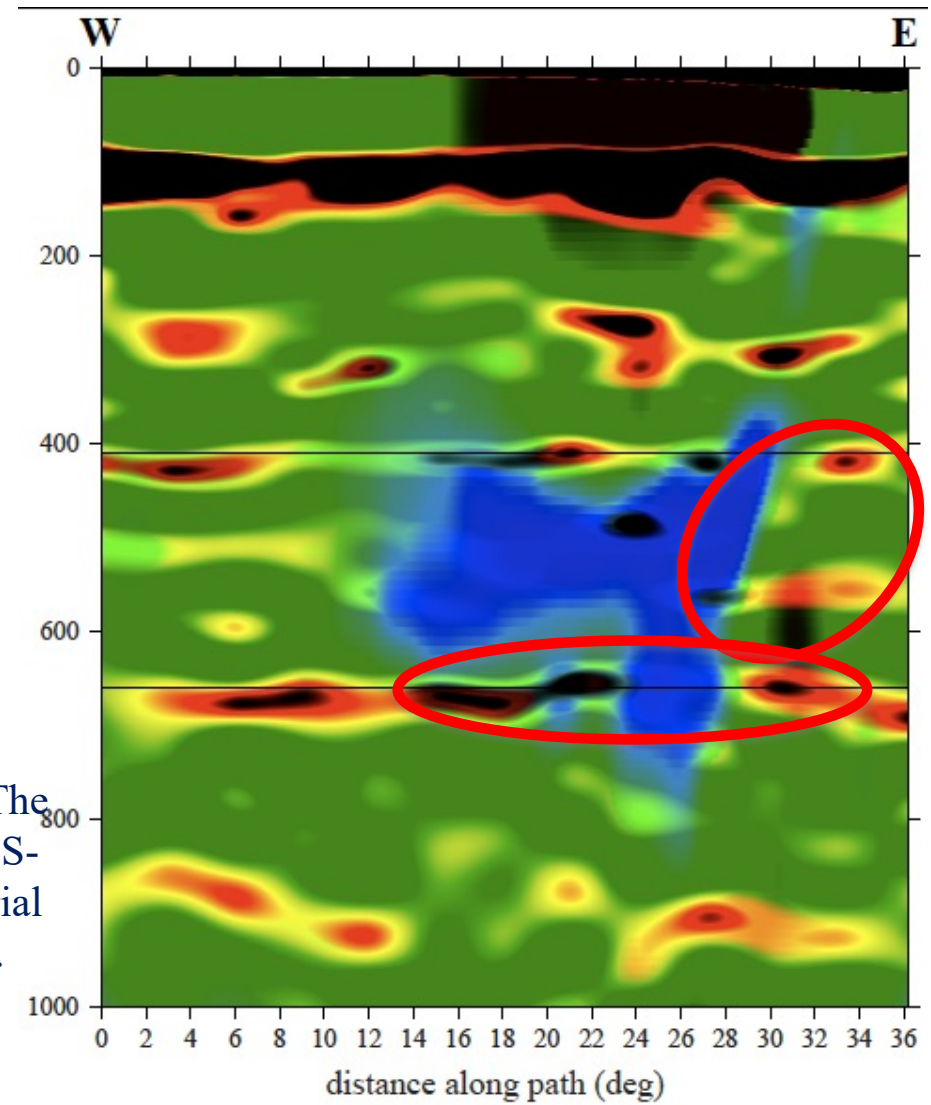
Li et al (EPSL, 2003)



Consistency between the above methods really provides solid evidence for mantle geodynamics. The top figure shows mantle reflection structures using S-to-P converted waves. The high velocity slab material is shown by the shaded colors in blue in both maps. **Different approaches, but the circled features in both images are consistent.**

SS Precursor Image

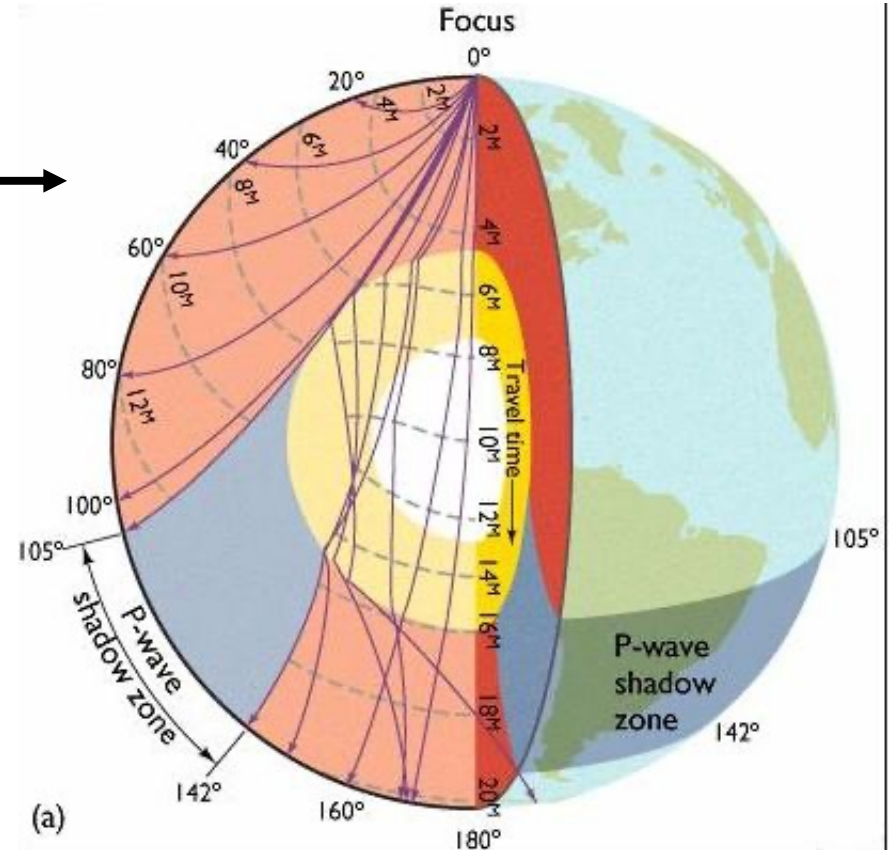
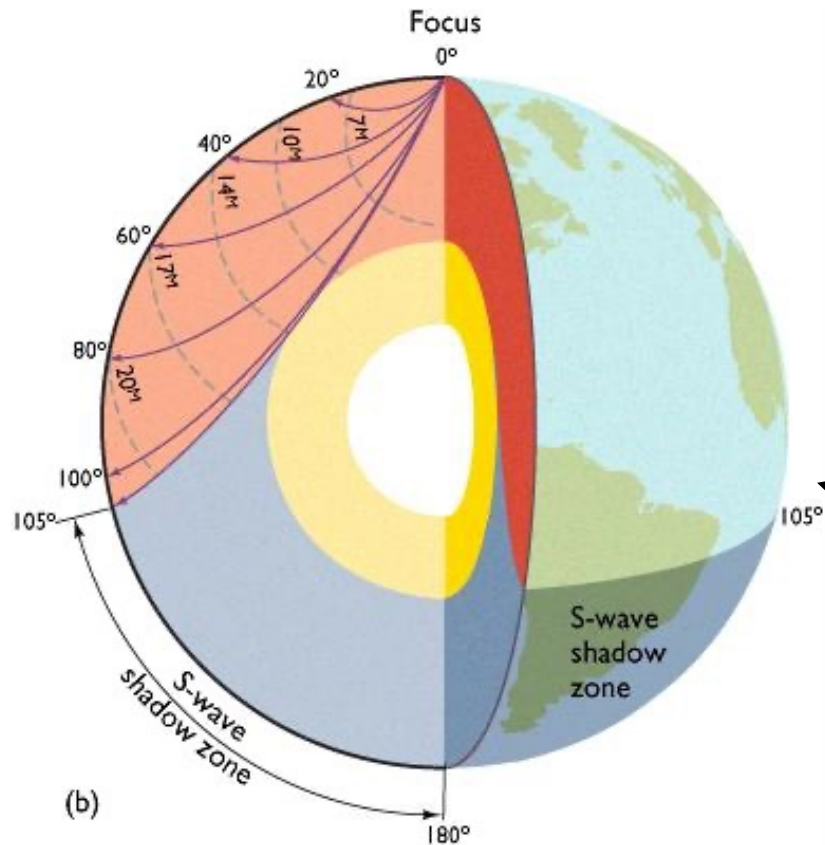
Blue/red background, velocity
Front, reflectors, *Gu et al. (2011)*



Complexities: 1. Major Low Velocity Zones

First-order Low velocity Zones

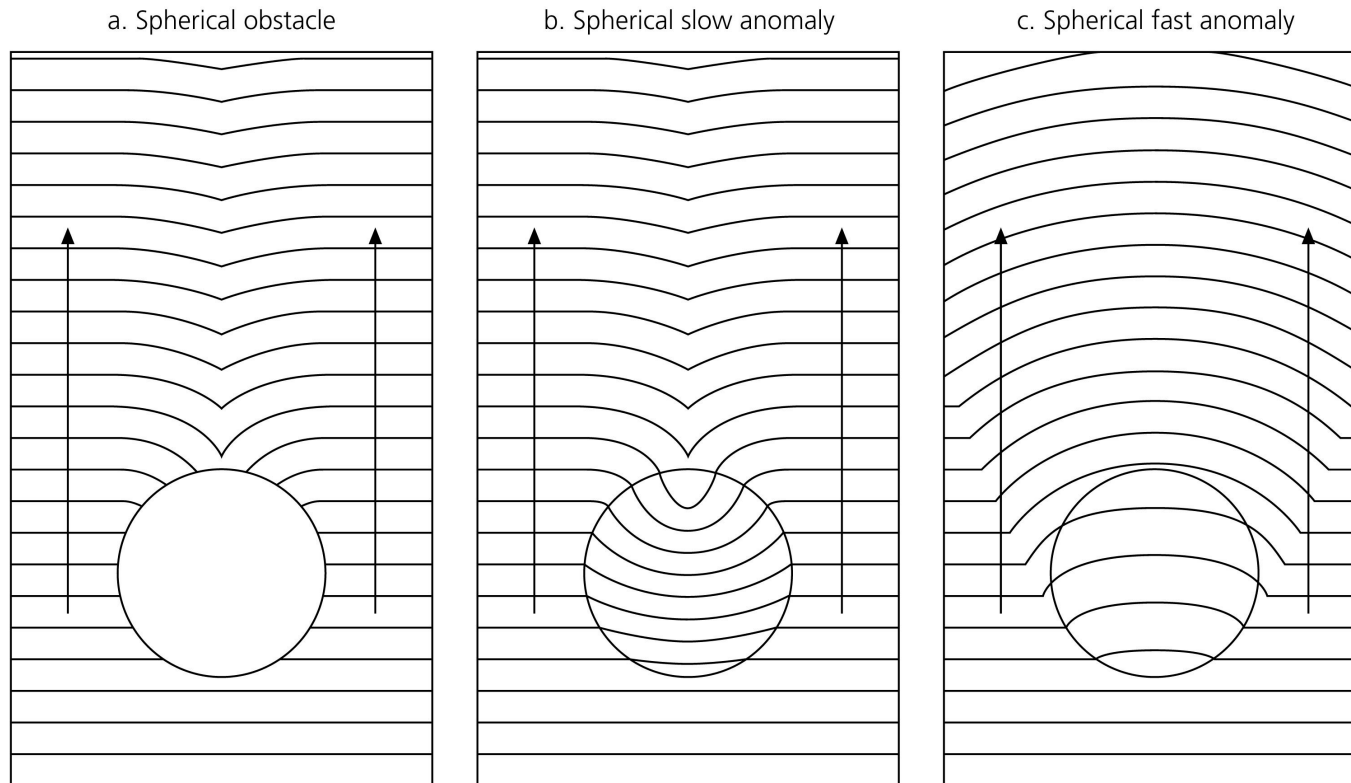
P wave shadow zone is caused by a decrease of P speed in the outer core.



S wave shadow zone is caused by the fact no shear wave propagates through the outer core, which is liquid.

Complexity 2: Wavefront healing

Figure 2.5-19: Waves interacting with a spherical anomaly.



Case 1. Huygen's will allow waves to diffract around a solid obstacle.

Case 2. Small effect of wave distortion and slow-down inside gets "healed" once ray passes the slow anomaly.

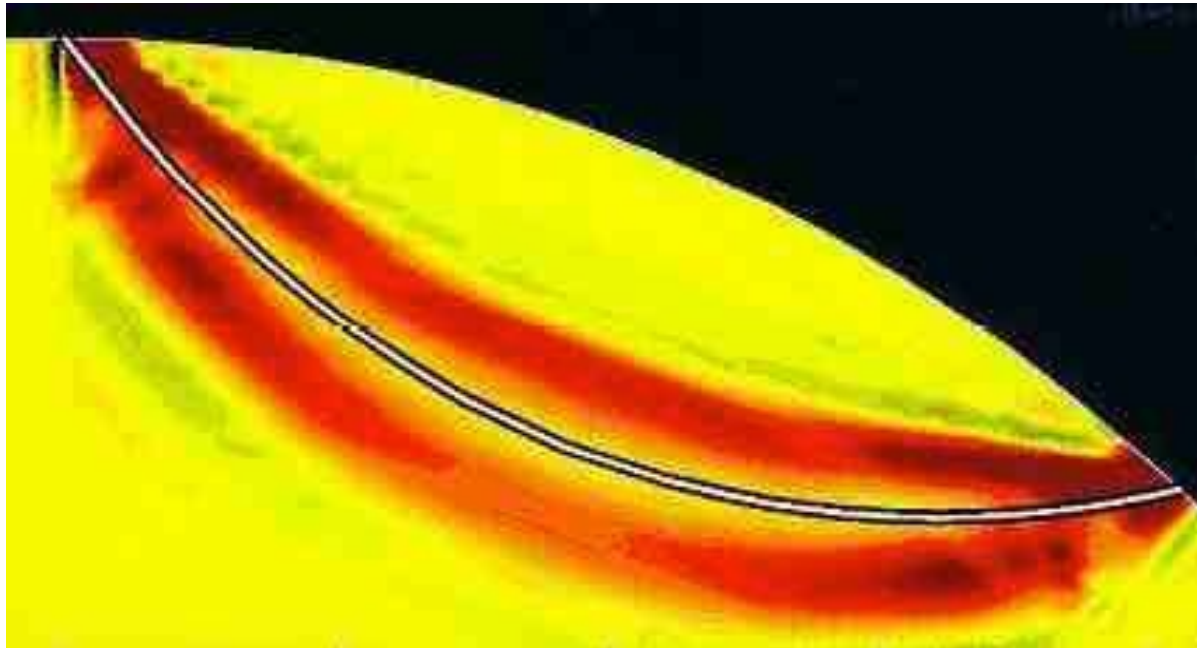
Case 3. Fast anomalies are not easily "healable".

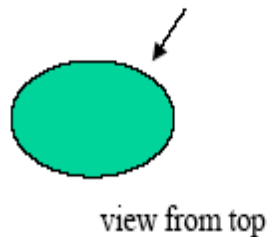
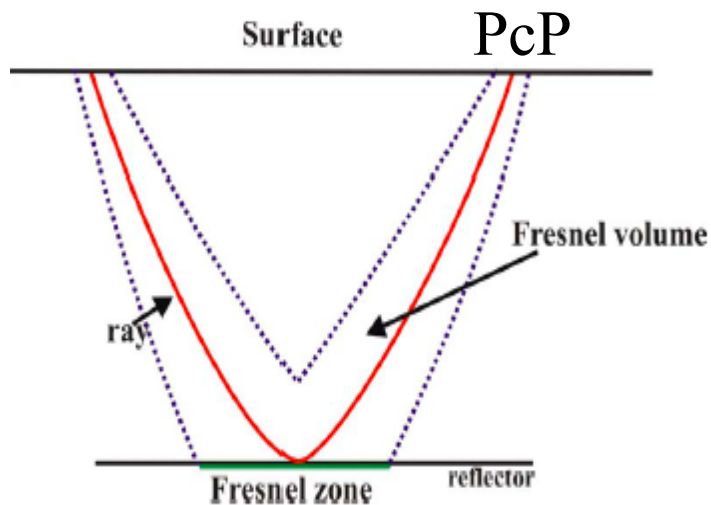
Moral: Subduction zone signatures are easier to observe than plumes!

Complexity 3: Banana and Donut Wave Volume

Drawback of geometrical ray theory (i.e. Snell's law):

- (1) no amplitude information
- (2) only good to elastic limits, geometrical ray.
- (3) Doesn't work well in 3D aspherical phenomenon.
- (4) Rays are wide and banana looking

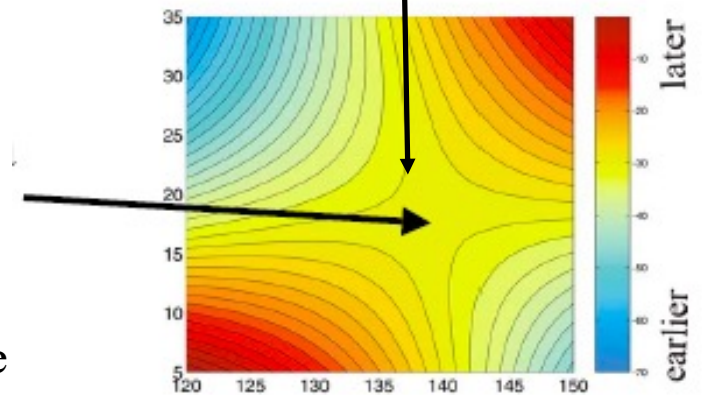
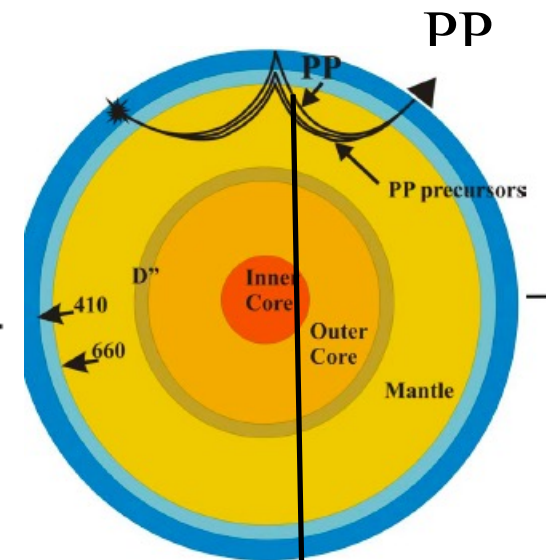




Volume around ray contributes to signal:
Fresnel volume.
 At reflector this volume (in 1D) is the **Fresnel zone.**

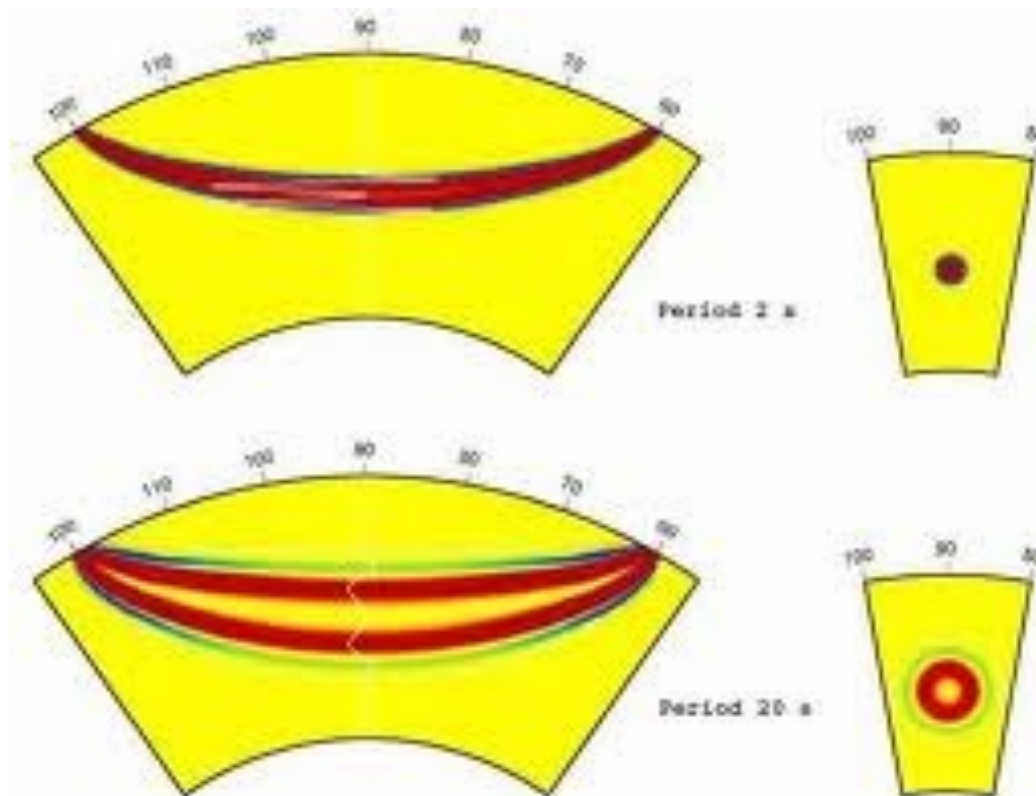
Size of Fresnel zone depends on:
 frequency, depth of reflector, and
 seismic velocity ($\Rightarrow$ wavelength).

Fresnel zones can be viewed at different points of the ray, for example, the Fresnel zone of PcP viewed from the top has an oval shape, which is very different from an “underside” reflection, such as PP wave Fresnel zone viewed near the Earth’s surface. The pattern depends on where we make a cut. The whole envelope surrounding the ray at any point is part of the Fresnel volume.



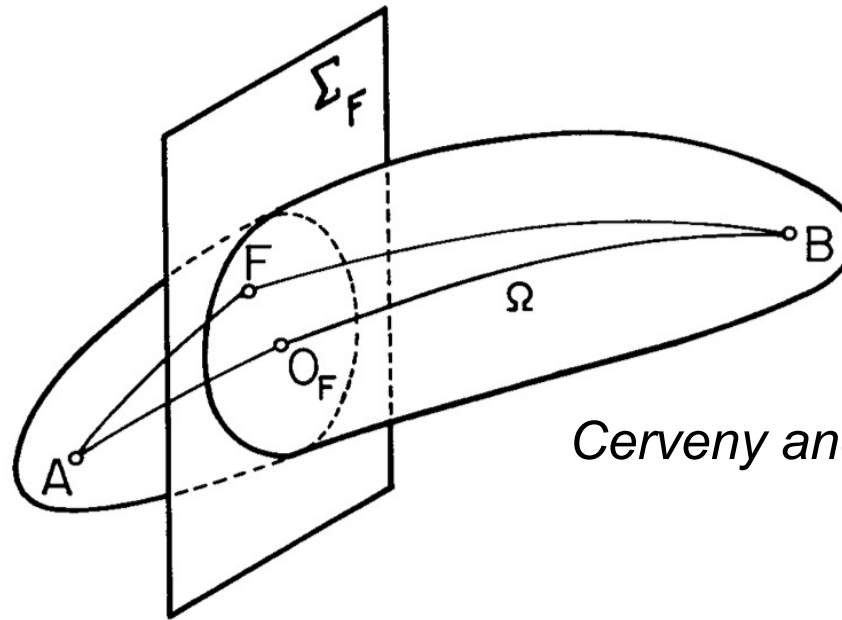
In the Earth, seismic waves travels more like *banana* (along path) and *donut* (cut surface)

These are called Fresnel Zones, or 'footprints' of rays



Princeton seismology group

‘footprints’ of rays (based on timing)



Cerveny and Soares, 1992

FIG. 1. Schematic presentation of a Fresnel volume for a point source at A and receiver at B . The point F belongs to the Fresnel volume if and only if it satisfies equation (1). The cross-section of the Fresnel volume by a plane Σ_F perpendicular to the ray at a point O_F represents the first Fresnel zone at O_F . Assume that the point F is situated in the plane Σ_F . Then it also belongs to the first Fresnel zone at O_F if it satisfies equation (1).

$$|\tau(F, A) + \tau(F, B) - \tau(B, A)| \leq \frac{1}{2} T.$$

T = period (sec), τ are travel times of ray segments.
 $\tau(B,A)$ is the geometrical ray.

Frequency/Period dependence

Top to Bottom frequencies (Hz):

0.75 Hz

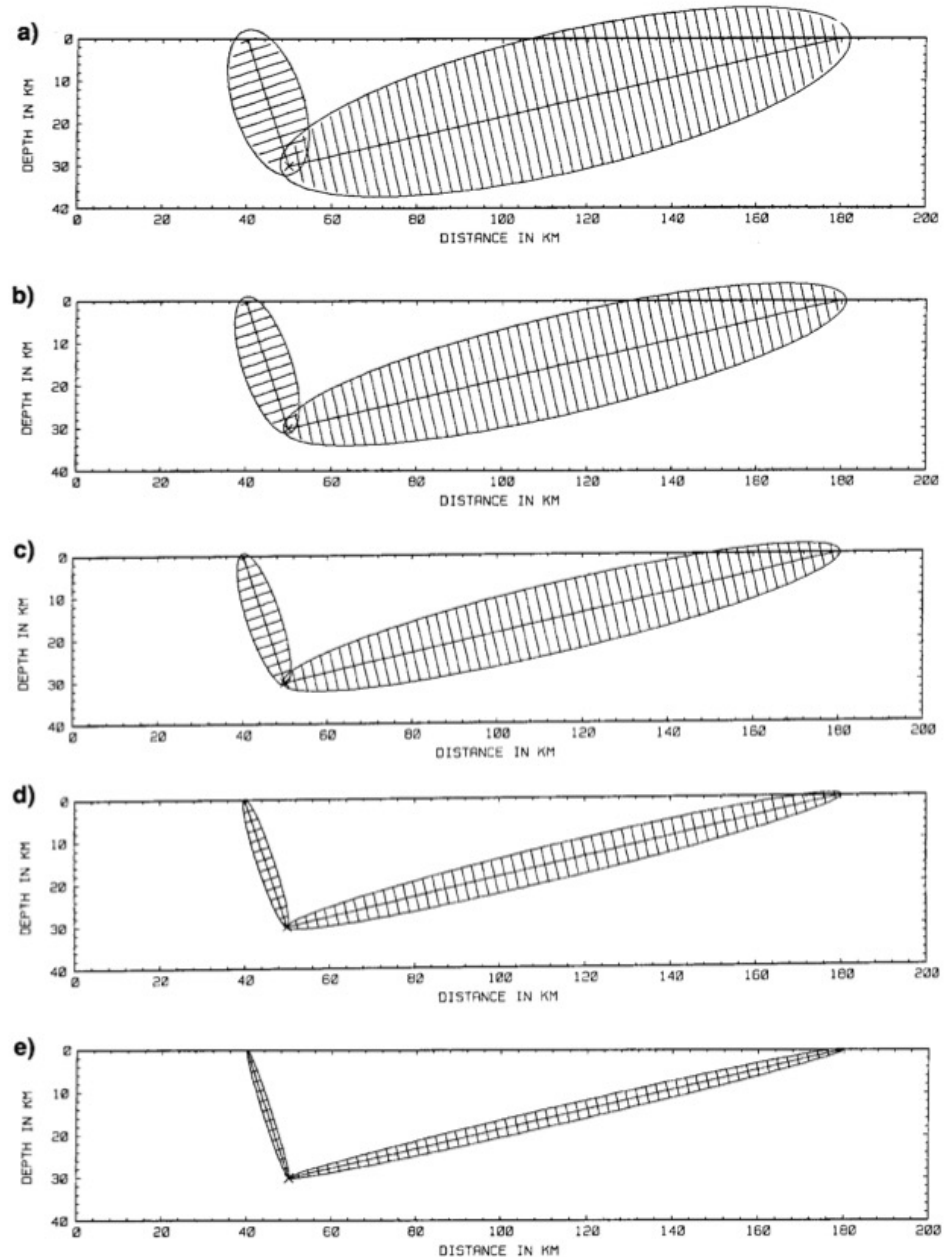
1.5 Hz

3.0 Hz

12 Hz

48 Hz

Last one looks like ray,
hence a *ray theory* is
called
“*infinite frequency
approximation*”



Cerveny and Soares, 1992

Surface Wave & Earth's Normal Modes

Dispersive Properties of Surface waves (Love & Rayleigh):

The following traces show a seismogram (containing body waves too) in both time domain (filtered at different frequencies) and in frequency domain (velocity differs at different frequencies).

Figure 2.8-4: Example of Love wave group velocity dispersion through bandpass filtering.

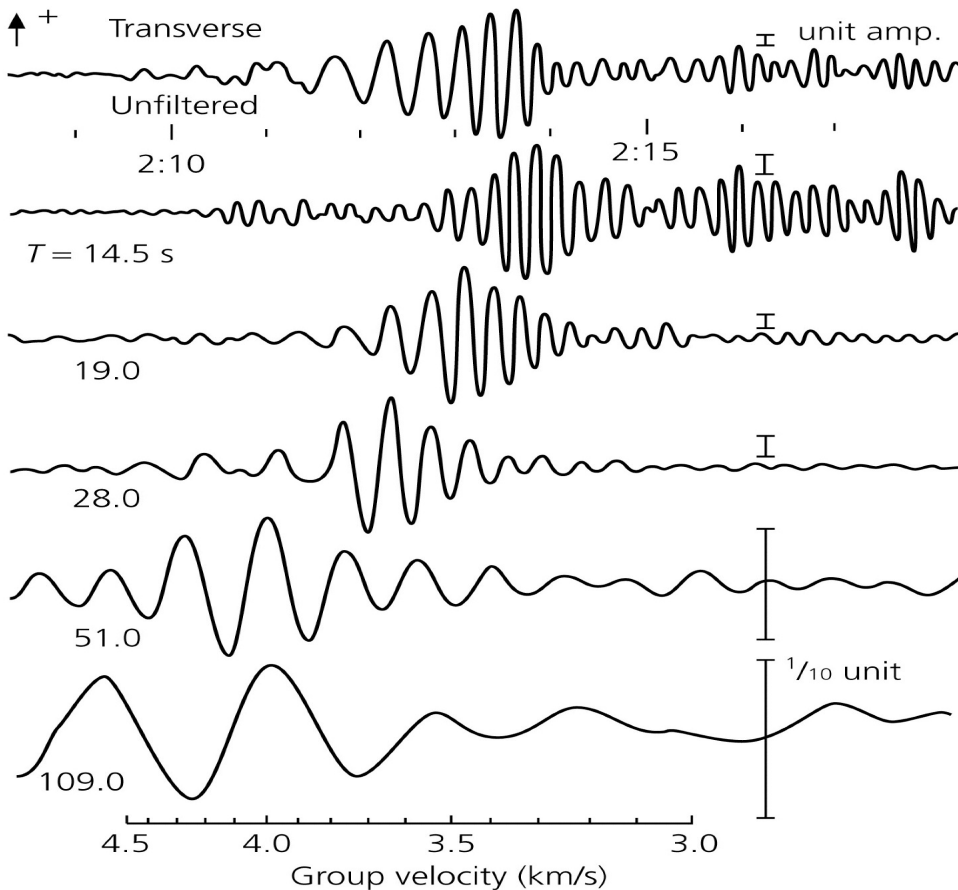


Figure 2.8-3: Example of calculating Love wave group velocity dispersion.

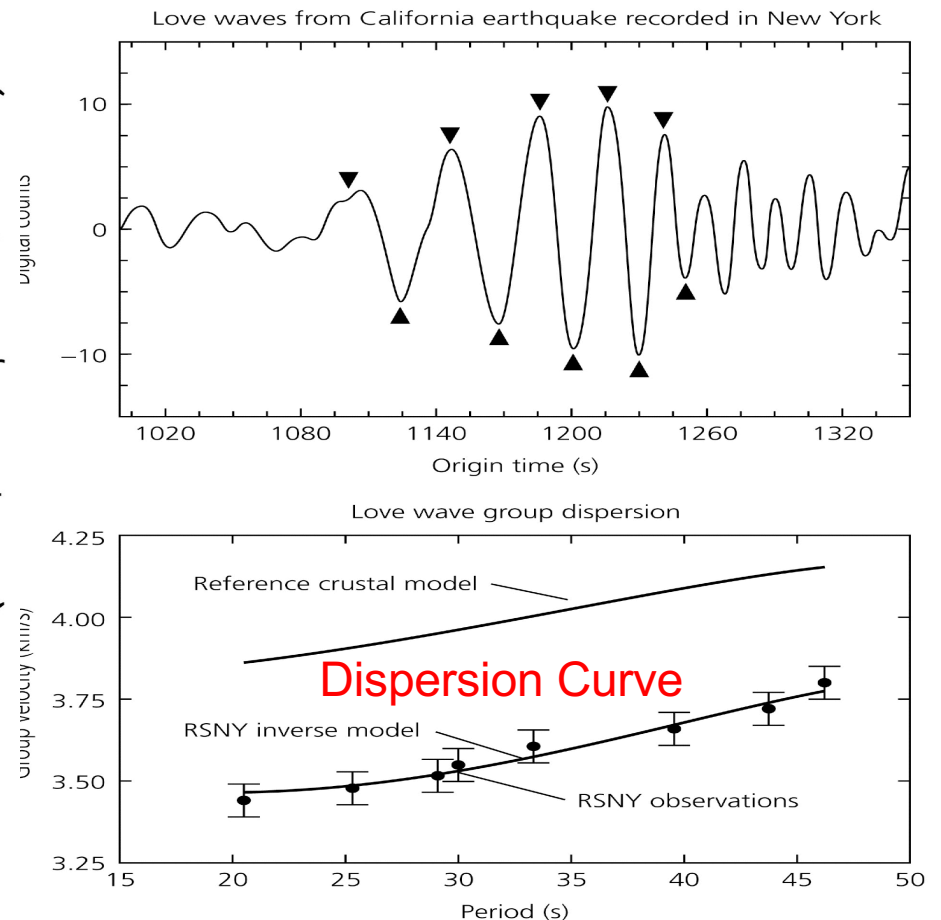
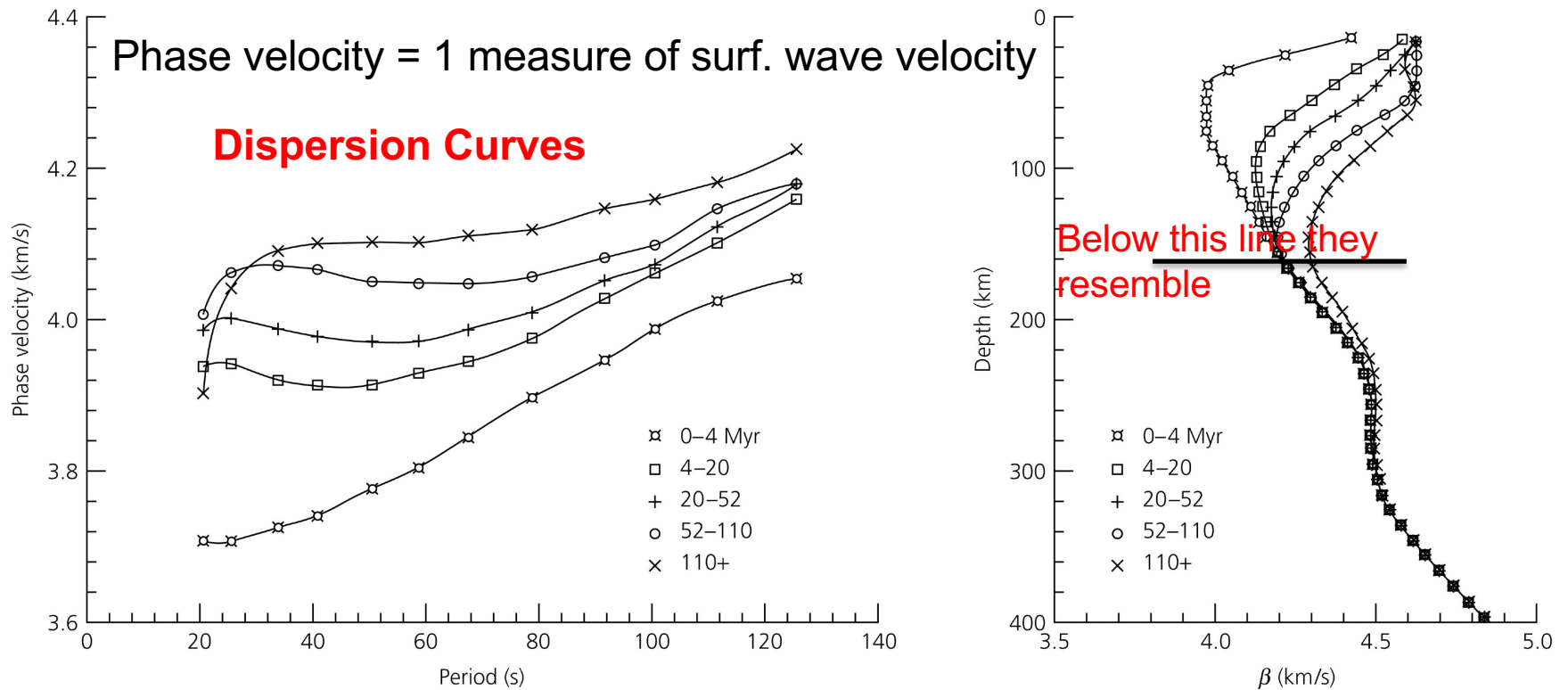


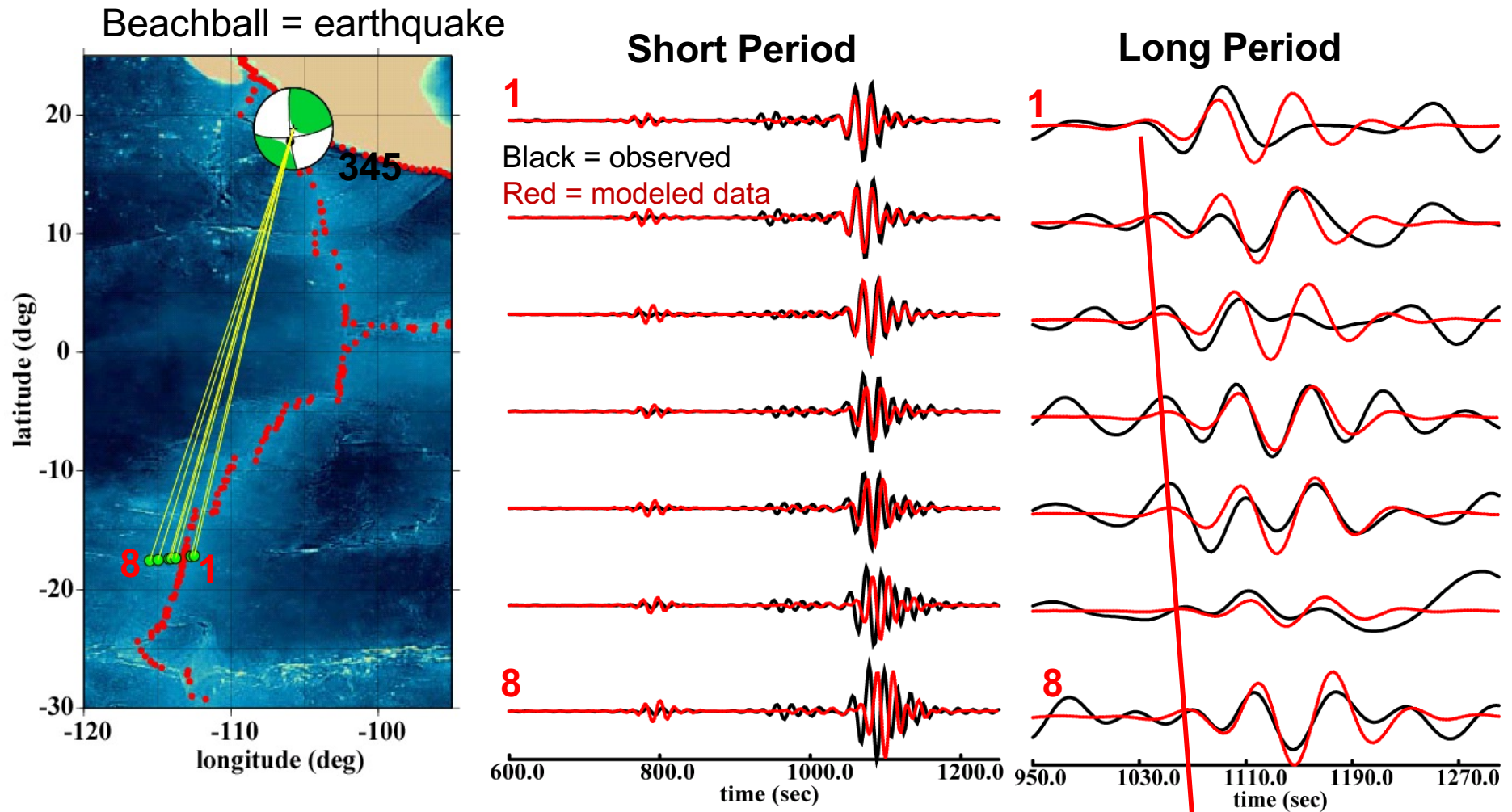
Figure 2.8-7: Rayleigh wave phase velocity dispersion as a function of oceanic plate age.



Obtained by modeling shear speeds using the surface waves that travel near the East Pacific Rise, the fastest spreading ridge on Earth (*Nishimura and Forsyth, 1989*).

Moral:

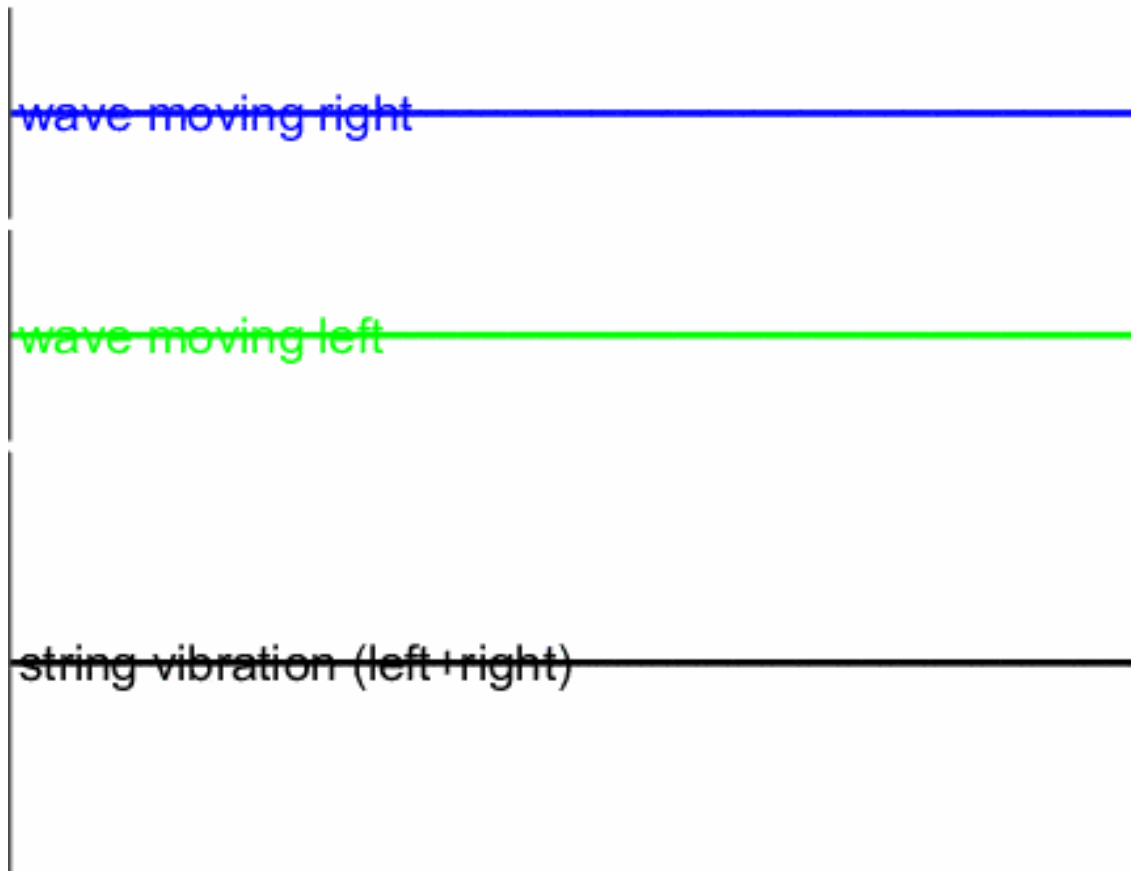
1. the closer to the ridge, the younger and slower!
2. Magma chamber near 50 km depth



Effect of ridge (time-wise) is strong in the shallow mantle, but is greatly reduced below ~80 km (Gu et al., 2005).

In this example, ocean bottom seismic instrument ‘sees’ slower speeds near ridge (< 40 km) in short-period data (left panel), but it fails to detect any difference in long-period data *that are sensitive at ~80 km depth* (right panel).

Standing waves are caused by interference of waves of similar frequencies.



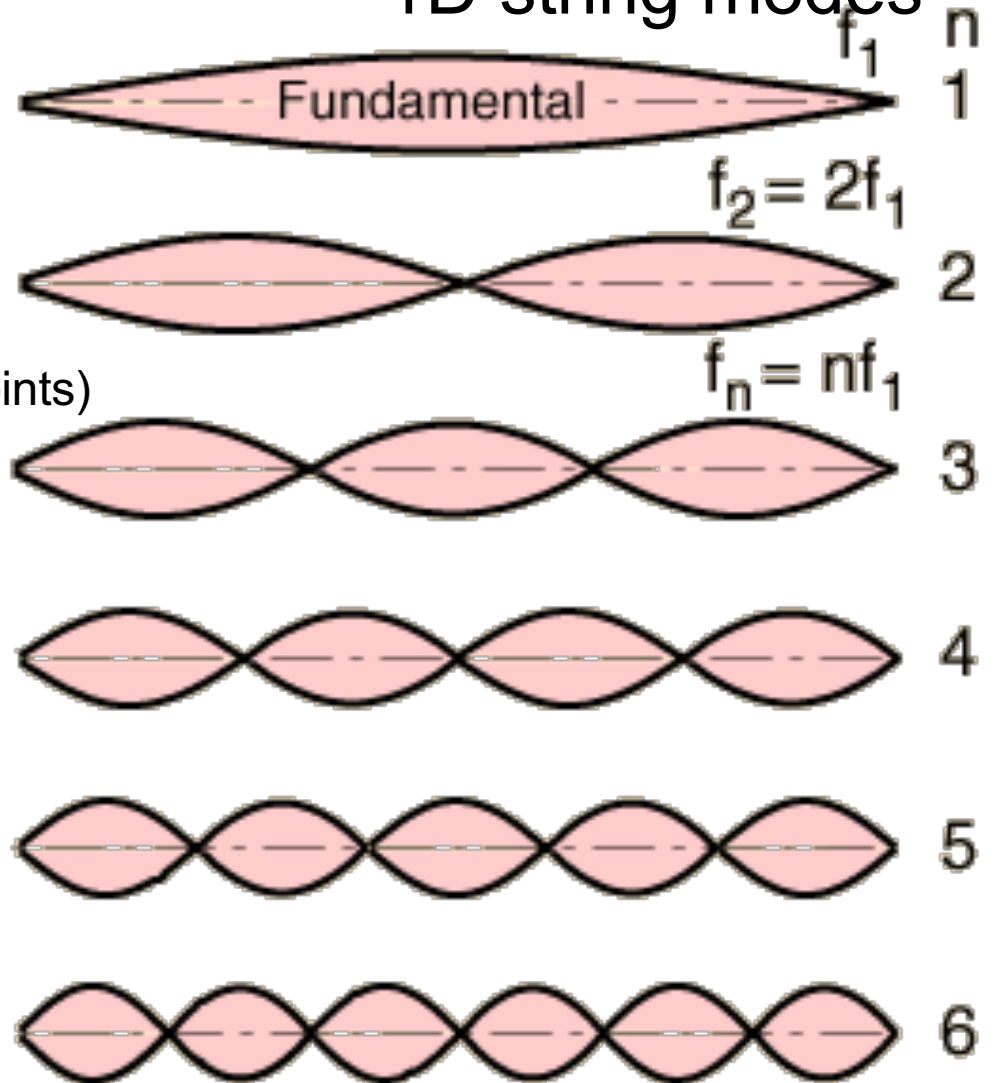
From <https://salfordacoustics.co.uk/sound-waves/standing-waves>

Normal Modes & Earth's Free Oscillations

n = mode number (start with 1)
= number of half cycles
= number of peaks

n-1 = number of nodes (stationary points)

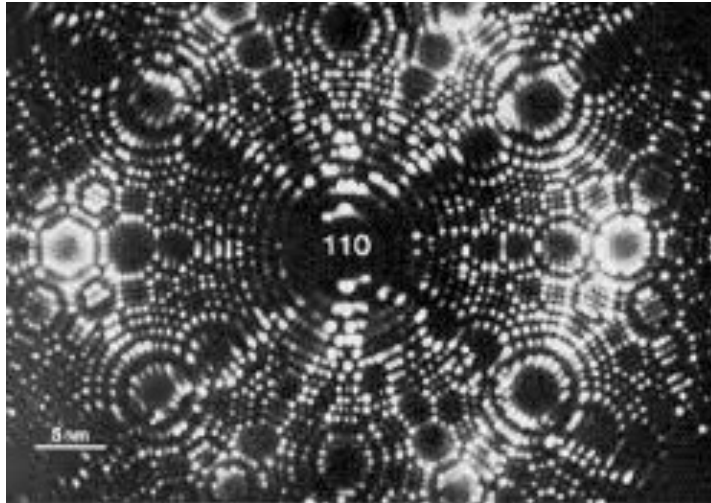
1D string modes



**Check this for animation
of Mode-Ray
Duality**

<http://physics.usask.ca/~hirose/ep225/animation/standing1/anim-stwave1.htm>

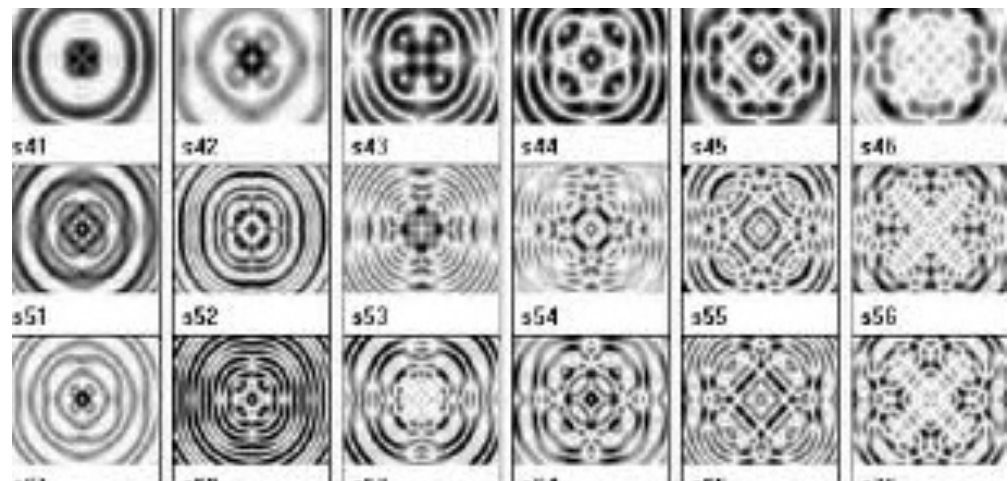
Standing Waves in 2-3 D



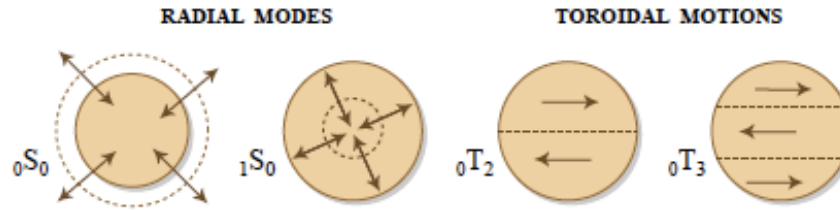
Left: Vertically shaking a cylinder of water with a constant frequency.

Figure taken from
<http://www.blazelabs.com/f-p-wave.asp>

Below: Sand patterns on a “shake table”. Set vibration to a frequency and wait until sand patterns form. Like strings, this requires multiple tries to excite interesting-looking modes.. Mode summation = simulating movements by adding all the small patterns with variable weights.

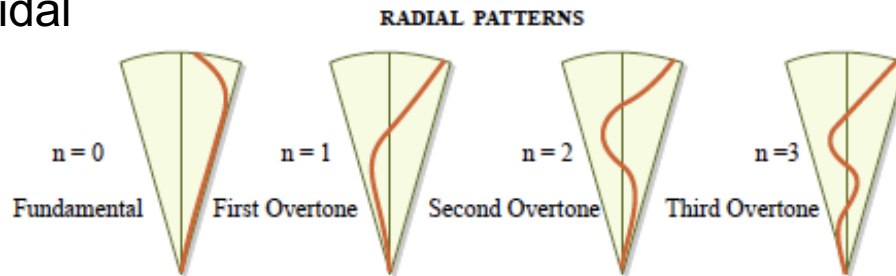
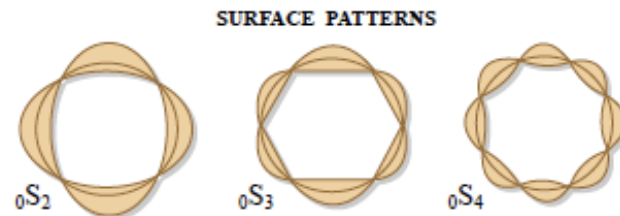


Normal modes (standing waves) of the Earth

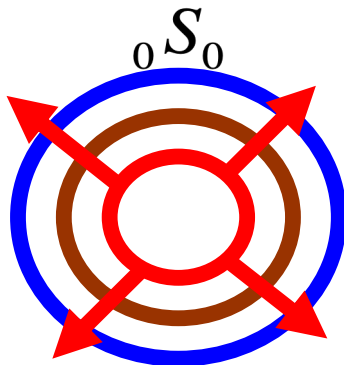


Toroidal mode:
produces SH waves
and Love Waves

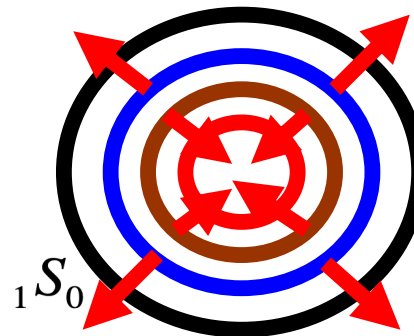
Spheroidal mode:
produces P, SV and
Rayleigh waves
(radial mode is one
type of spheroidal
mode)



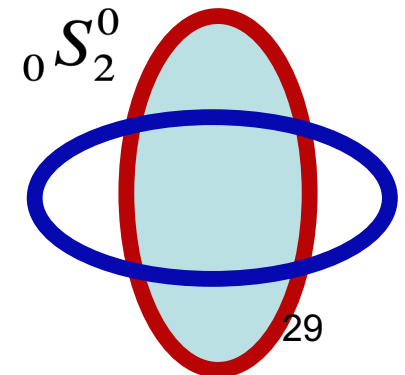
Breathing mode



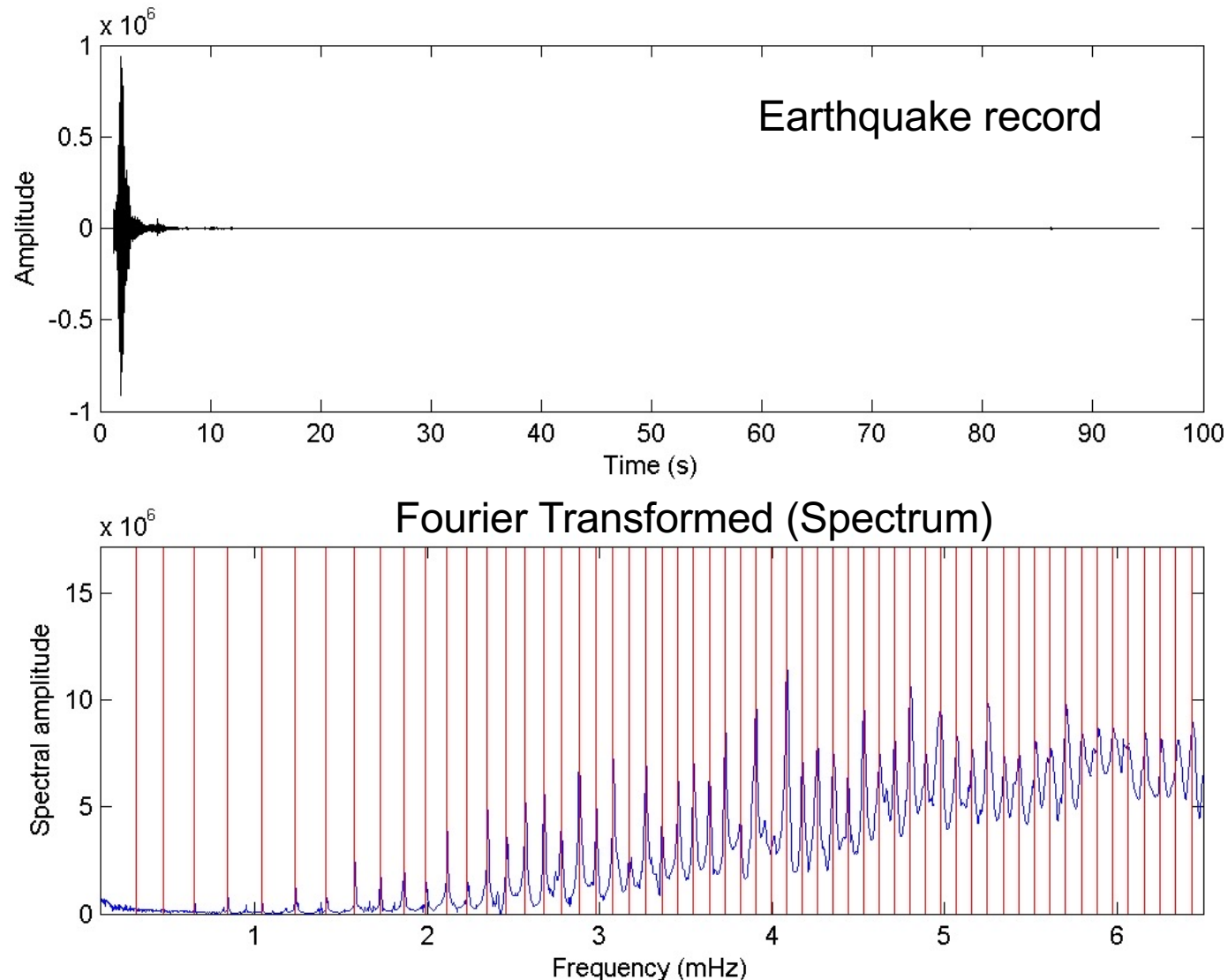
Breathing mode with a boundary



Football mode

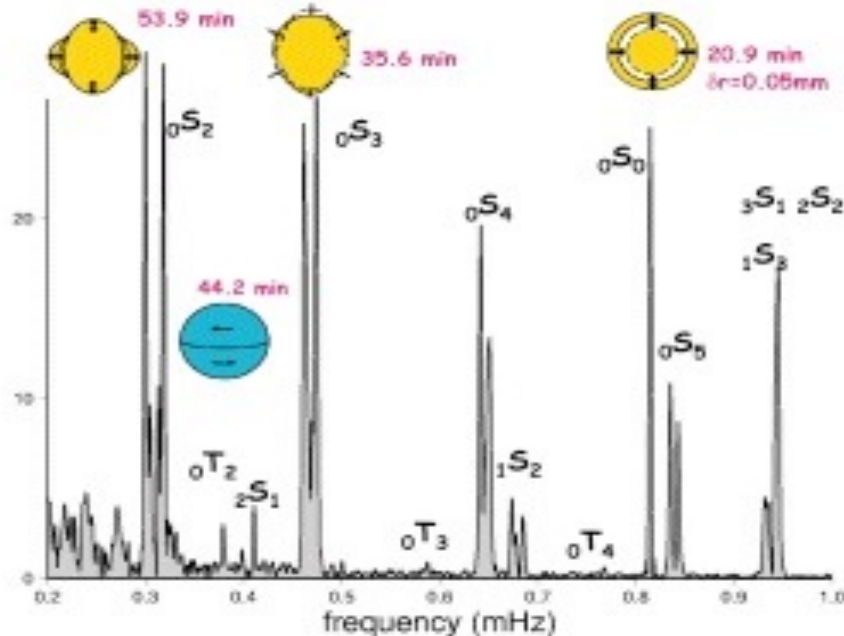


Sumatra Earthquake Normal Mode Observations (Dec 26, 2004)



During an earthquake, lots of vibration modes are excited and persist for some time. Typically, the useful earthquakes are greater than Magnitude 8.0.

Mode Observations & Usage



Top: Mode observations by Fourier transforms of the time-domain recordings. Modes are usually very long periods and only comes in certain discrete frequencies.

Note: only a few modes are excited.

Bottom plot: Mode Summations in the simulation of Love waves. This differs from other approaches, but get the same results for simulating traveling waves.

(Pro: Very fast if a mode catalogue exists, just pick and add.

Con: Mode catalogue is HUGE for high-frequency simulations!

GU ET AL.: DISCONTINUITY IN THE PATTERN OF HETEROGENEITY

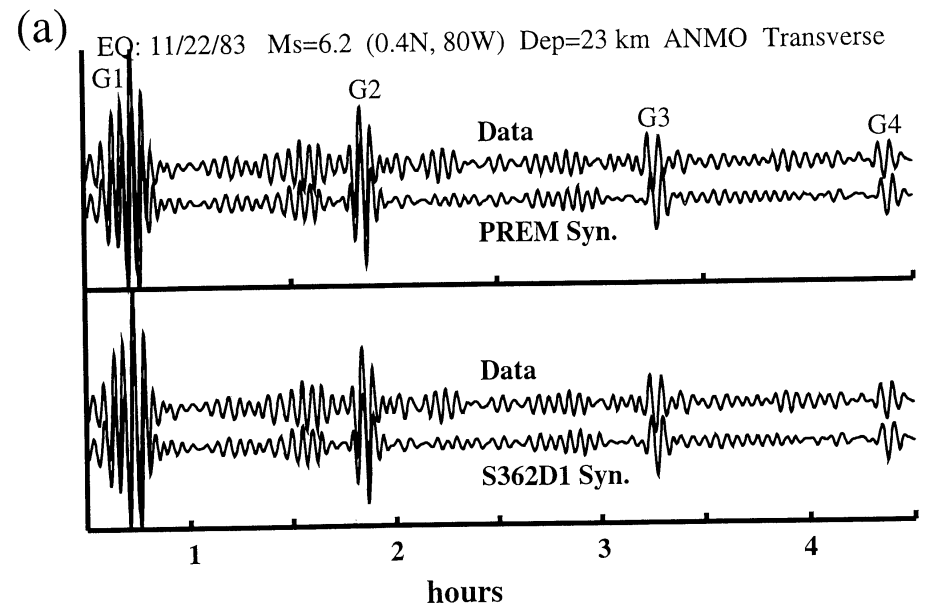


Figure 2.9-15: Example of singlets for a split spheroidal mode multiplet.

Singlets and multiplets:

Each azimuthal order represents a spectral Peak. Without **lateral velocity and density variations, also without consideration of earth's ellipticity**, they all should come at the same frequency in the spectrum. The fact they all have the *Same frequency* is called

“degeneracy” of a mode.

Each of them is called a ***“Singlet”***.

Their superposition forms one spectral peak called ***“multiplet”***.

But with those above factors present,

They will split (called ***“mode splitting”***).

By measuring the split of the singlets, we can learn about density + velocity changes

Why is it important? *Hint: what do travel times of a seismic phase (e.g., P) measure?*

