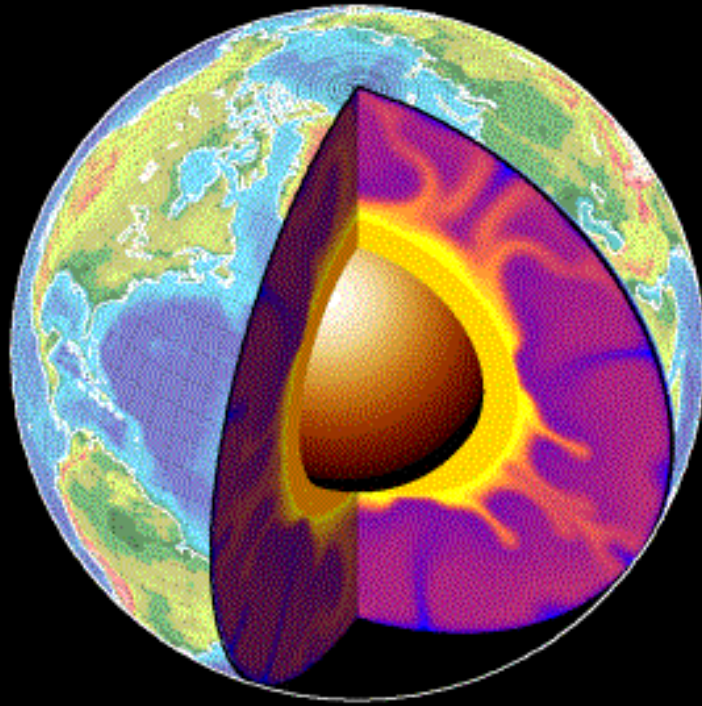




CHAPTER 1.5

Gravity and Topography

Gravity and topography

Why topography is important for gravity surveys

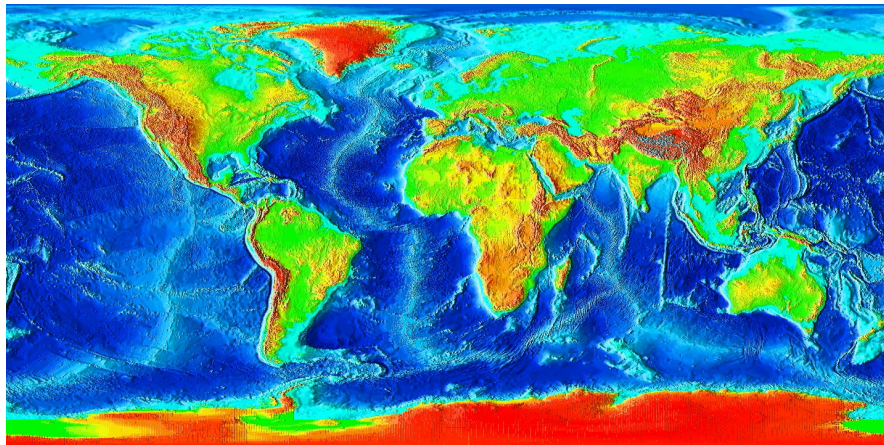
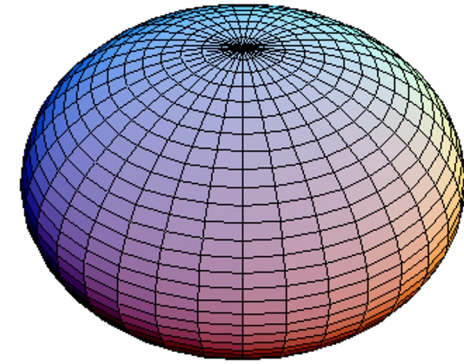
International reference ellipsoid – mathematical representation of the shape of the Earth

GRS80 – expected gravity for IRE

→ accounts for variations due to latitude

(shape and rotation)

→ gives gravity of “average” Earth



But the Earth has lots of topography!

This will affect surface gravity:

- 1) distance to centre of Earth changes
- 2) amount of mass changes

→ need to apply **2 corrections** to “move” observations to a common elevation

Measurement of Gravity

- gravity anomalies are generally less than a few mgals
 - $<0.01\%$ of Earth's gravity field ($\sim 981,000$ mgal)
- need very sensitive instruments to 'see' these anomalies
- instruments are called **gravimeters**
 - measure the **vertical** component of gravity at a site
- two types:
 - absolute gravimeters and relative gravimeters

Absolute gravimeters

- measure the total gravity field at a site

PROS:

- get the absolute (total) value of gravity
- value is accurate (0.00001 – 0.01 mgal)

CONS:

- instruments are very expensive and bulky
- time-consuming – must stay at a site for 24 hrs or more

Free-fall devices

- measure the acceleration of a falling object
- drop the object and measure its position as a function of time with a laser interferometer

after time t :

$$d = \frac{gt^2}{2}$$

- plot position as a function of time to get the acceleration
- object is inside a vacuum chamber



Near
Victoria BC



Tofino site

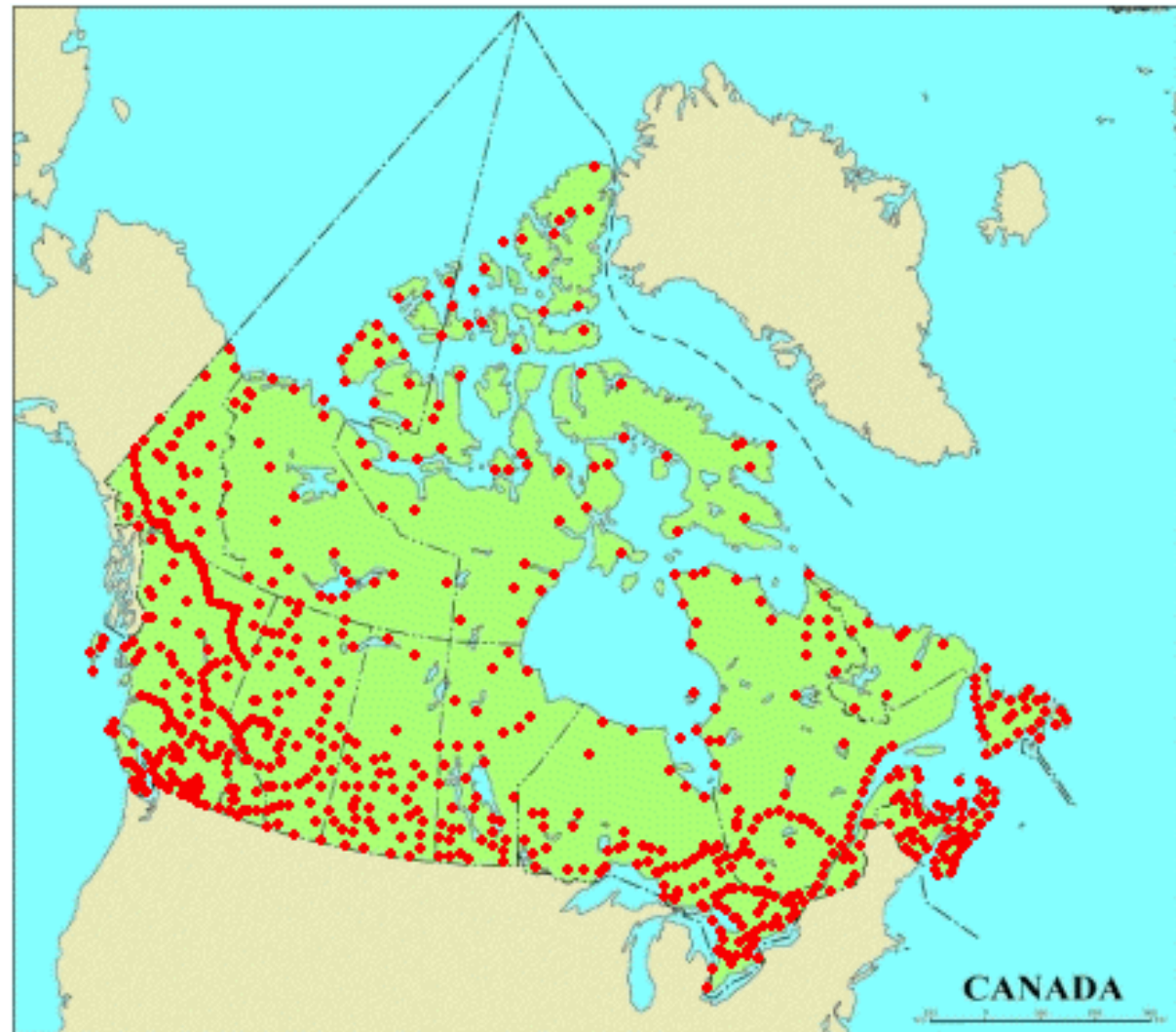


Sproat Lake field site



Canadian Gravity Standardization Net (CGSN)

Sites where absolute gravity has been determined in Canada.



(http://www.geod.nrcan.gc.ca/products-produits/ggns_e.php)

8

Relative gravimeters

- to investigate the subsurface density structure, we usually only need to know the **relative gravity** at a site (how gravity changes between sites)

PROS:

- quick to measure – can cover multiple stations in a day
- values are fairly accurate (0.00001 – 0.1 mgal)

CONS:

- relative measurement – need to tie observations to a reference station to know the absolute gravity

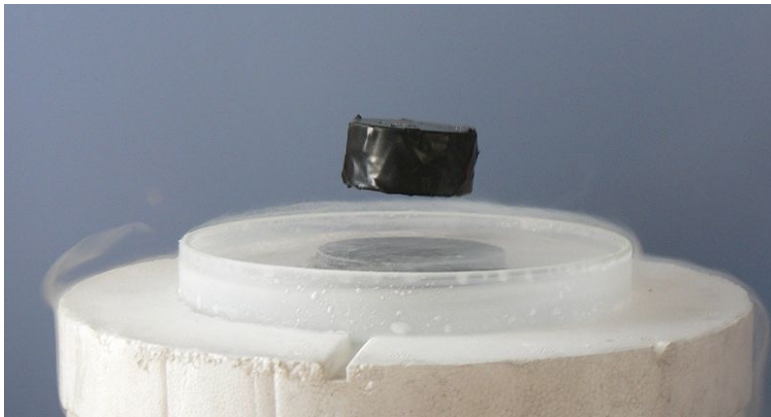
This concept is actually useful in gravity



The mechanism is similar to superconducting gravimeter where a mass is balanced through electromagnetic force.

Superconducting gravimeter (relative)

- superconduction – zero electrical resistance (only at low temperatures)
- a magnetic field cannot penetrate a superconductor (Meissner effect)
- levitate a superconducting ball in a magnetic field
- voltage needed to suspend the ball is related to the strength of gravity
- accuracy of 0.00001 mgal, but not portable



Canadian superconducting gravimeter in Cantley, Quebec

(http://homepage.usask.ca/~jbm964/merriam_cur.html#CSGI)

Relative Gravimeters

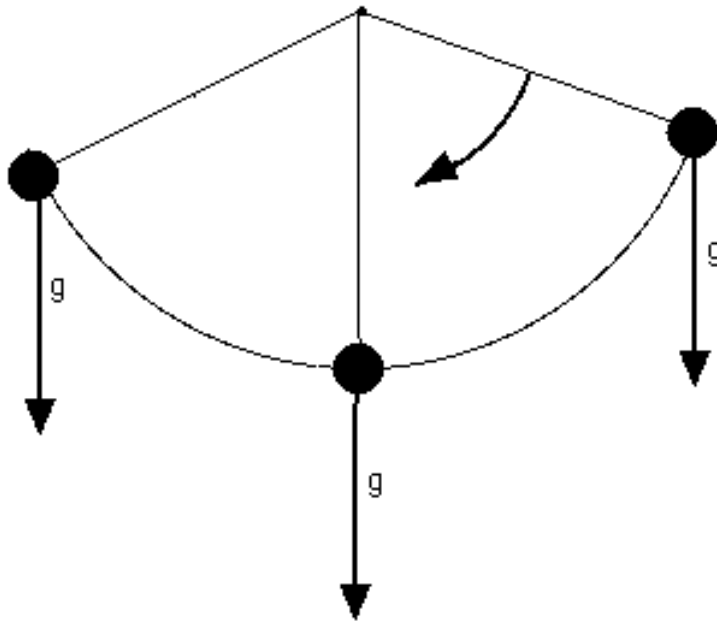
Superconducting (cryogenic) gravimeters: very low drift, very high accuracy (GWR Instruments)

- The ultra stable magnetic field, weak gradient and operation at cryogenic temperatures eliminate the sources of noise and drift commonly found in mechanical spring gravity meters.

- SG is the world's most sensitive and stable gravimeter.
- ($= 10^{-9} \text{ ms}^{-2} = 0.1 \text{ microgal}$) or better,
($= 10^{-11} \text{ ms}^{-2} = 1 \text{ nanogal}$).



Portable Pendulum Gravimeter



Period of a pendulum is:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- accuracy at one station is not that good due to difficulties in measuring L
- but relative accuracy is quite good (~ 0.1 mgal)
- measure period at two sites
- difference in period is proportional to the change in gravity between the two sites

Mass-on-a-spring gravimeter

Force of gravity on spring: $F=mg$

Hooke's Law: $F=kz$

- k is the spring constant

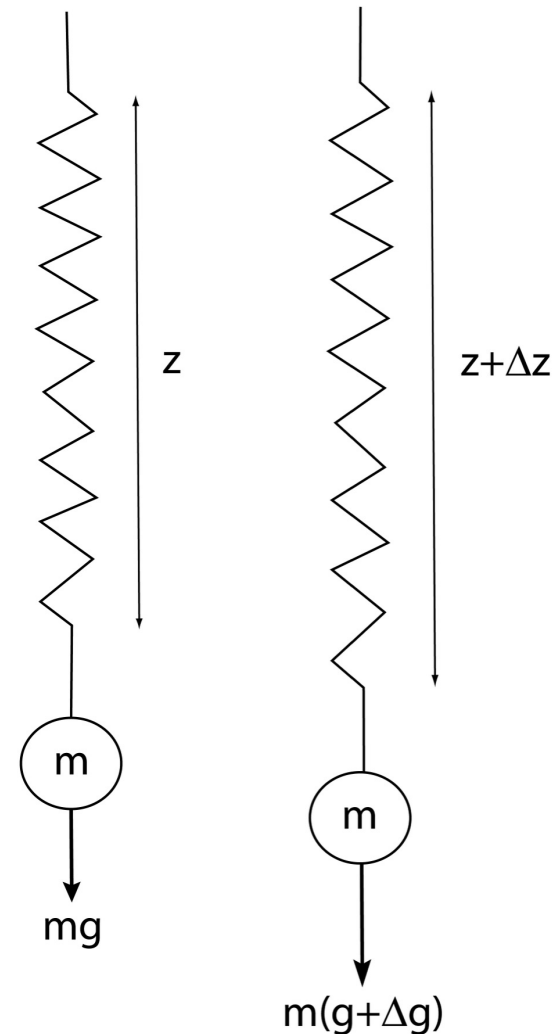
$$F=F \rightarrow kz = mg$$

Another site:

- gravity increases by Δg
- so spring length increases

$$k\Delta z = m\Delta g$$

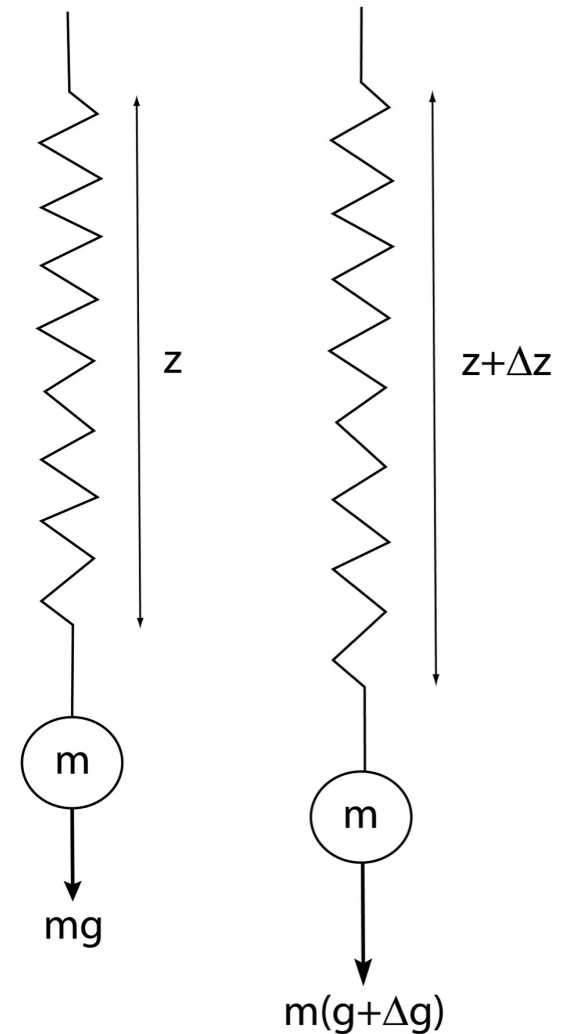
$$\text{or } \Delta g = k\Delta z / m$$



Mass-on-a-spring gravimeter

$$\Delta g = k \Delta z / m$$

- most common technique used to measure gravity
- instrument must be **leveled** before measurement
- Δz is very small – mechanically amplified using different techniques
- accuracy is ~ 0.01 mgal
- two main instruments: Lacoste-Romberg gravimeters and Worden gravimeters

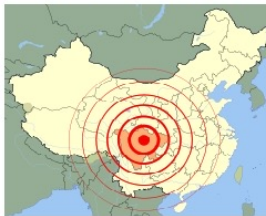


Relative Gravimeter

gPhone: made by Lacoste



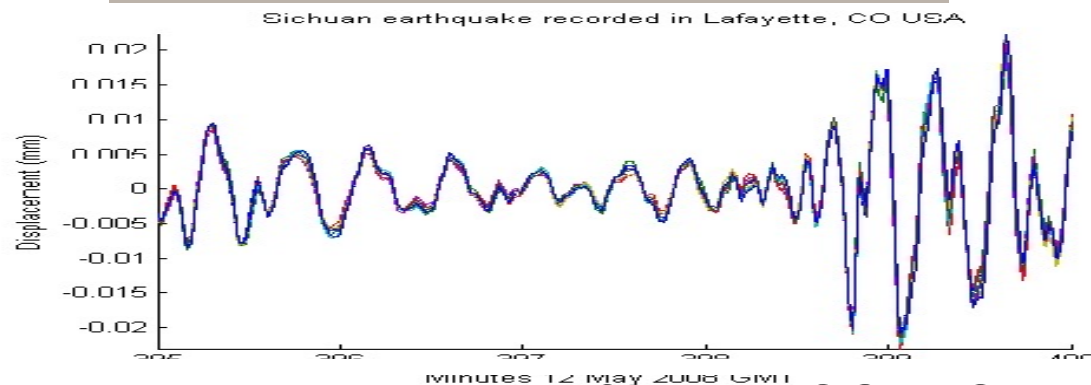
2008 Sichuan earthquake



Date May 12, 2008
Magnitude 8.0 M_s / 7.9 M_w
Depth: 19 kilometres (12 mi)
 31°01'16"N
 103°22'01"E?
Epicenter location: (Sichuan earthquake)
 (Wenchuan County in Sichuan province)

PERFORMANCE SPECIFICATIONS

RESOLUTION	0.1 μGal
PRECISION	1 μGal
SYSTEM NOISE	3 $\mu\text{Gal}/\sqrt{\text{Hz}}$
RANGE	7000 milliGals uncalibrated (worldwide)
FEEDBACK RANGE (DURING MEASUREMENT)	+/-50 milliGals
INSTRUMENT DRIFT	1.5 milliGals/month, typically <500 $\mu\text{Gals/month}$



Comparison of six gPhone recordings at Lafayette, CO, USA.
 From <http://www.lacosteromberg.com/sichuanEarthquake.htm>

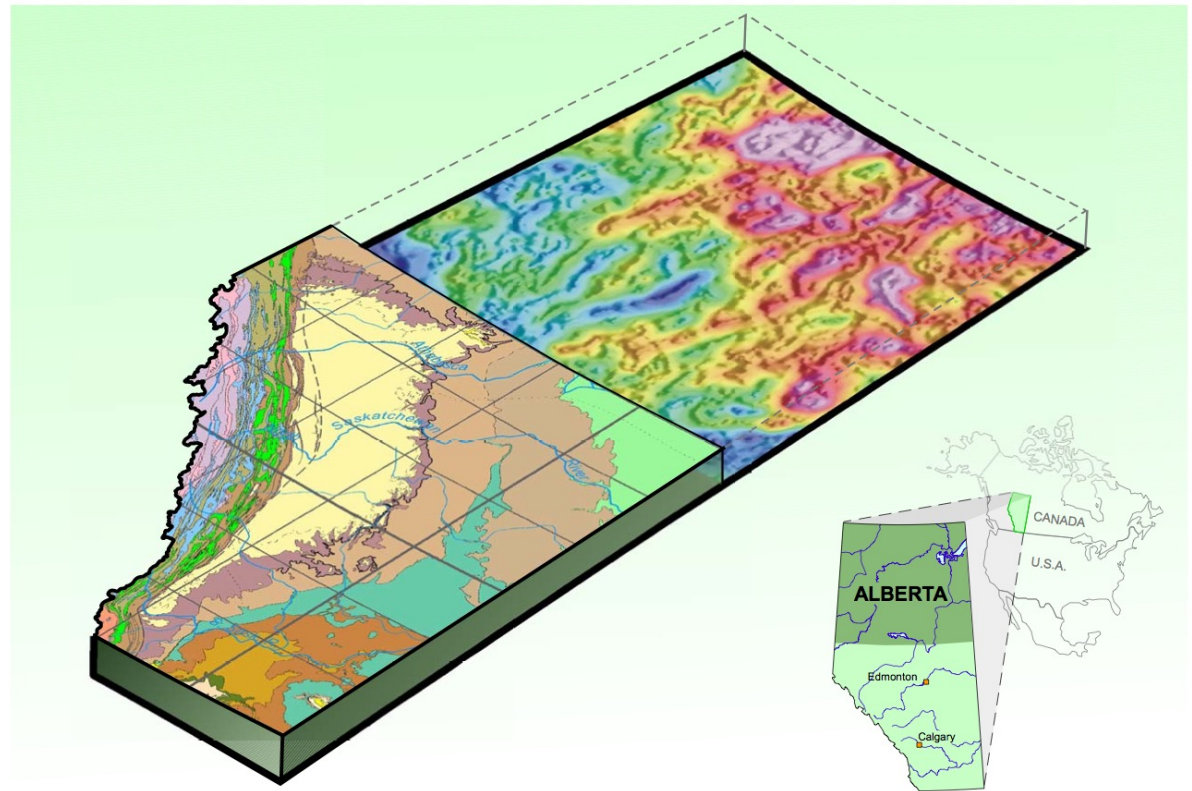
Airborne gravity surveys

- gravity measurements from a moving airplane or helicopter
- similar set-up to moving ship (gyroscopically stabilized platform)
- same set of corrections for movement
 - correction for steady velocity
 - motion caused by air turbulence
 - can be accurately corrected - measured by airplane instruments
- overall accuracy is ~ 1 mgal



Example of airborne gravity data

(map by Dinu Pana)



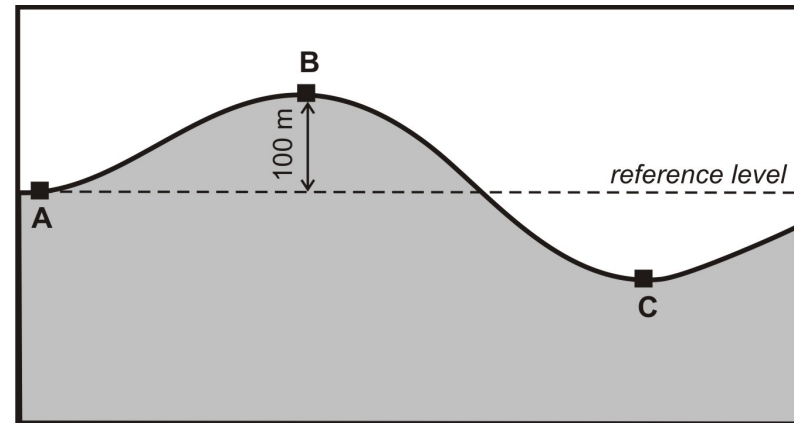
Alberta Energy and Utilities Board
Alberta Geological Survey

Upward continuation – mathematical technique to project data to a higher elevation (e.g. ground surface to 300 m)

Free air correction

Surface gravity depends on distance from the centre of the Earth:

$$g = \frac{GM_E}{r^2}$$



- going from Site A to Site B → gravity decreases as r increases
- can show (*with the help of calculus, not required*) that a change in elevation (Δh , in m) causes a change in gravity of:

$$\Delta g_{FA} = 0.3086 \Delta h \quad (\text{in mgal})$$

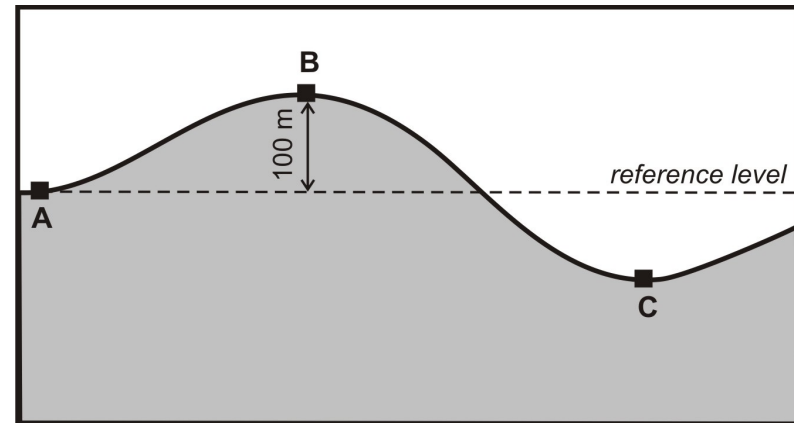
- going from Site A to Site B: $\Delta h = 100 \text{ m} \rightarrow \Delta g = 30.86 \text{ mgal}$ **(decrease)**

Datum: Some reference value was used in experiments, e.g., sea-level, average measurement, geoid. *Good choices of datum could make interpretation easier. This is called 'data reduction'.*

Free air correction

A correction to account for variations in g due to variations in distance from centre of Earth

→ you are “moving” all sites to the same elevation



1. Define a reference elevation: sea level, average elevation of study area, Site A...

2. For each observation site, determine Δh (in m)

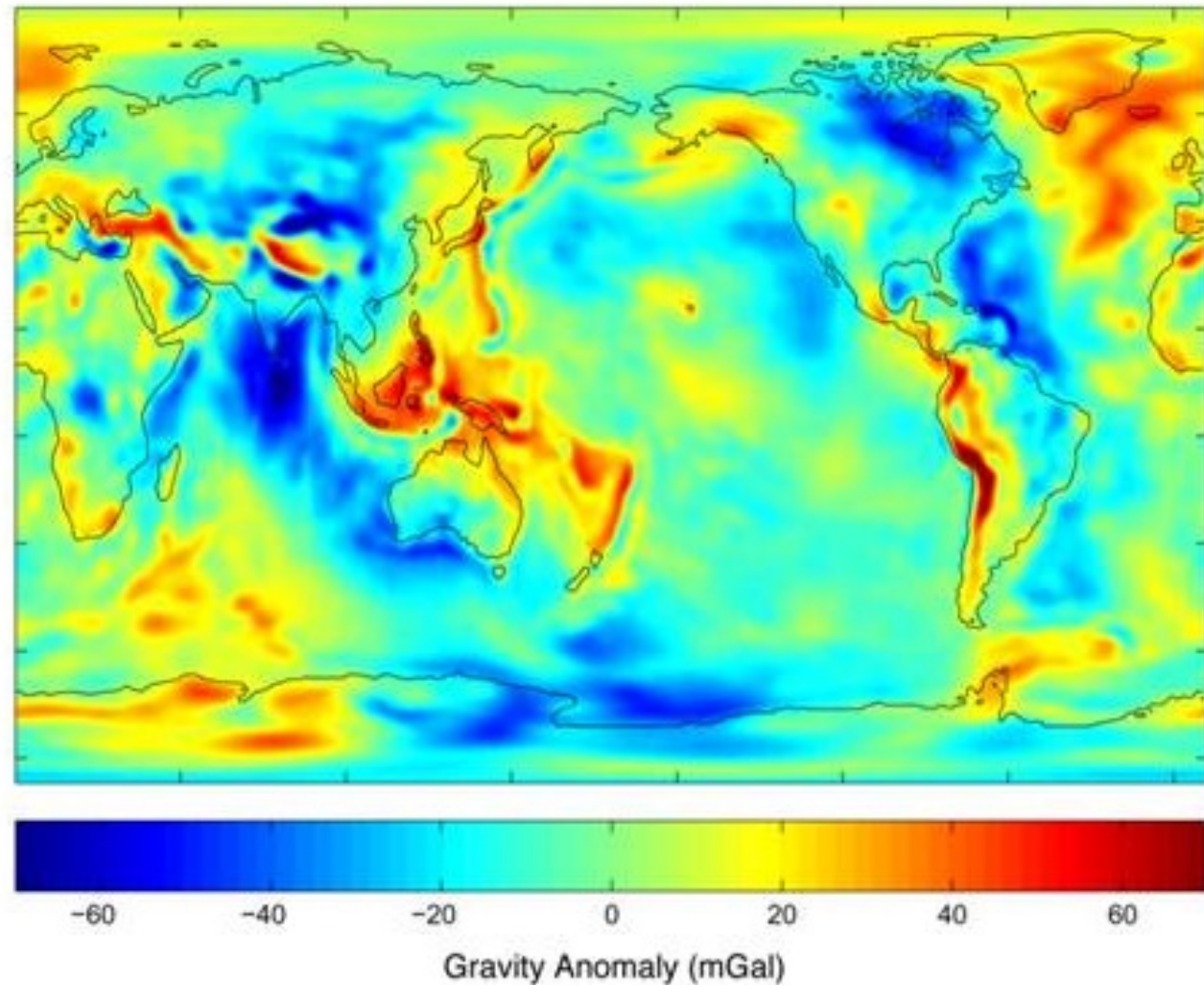
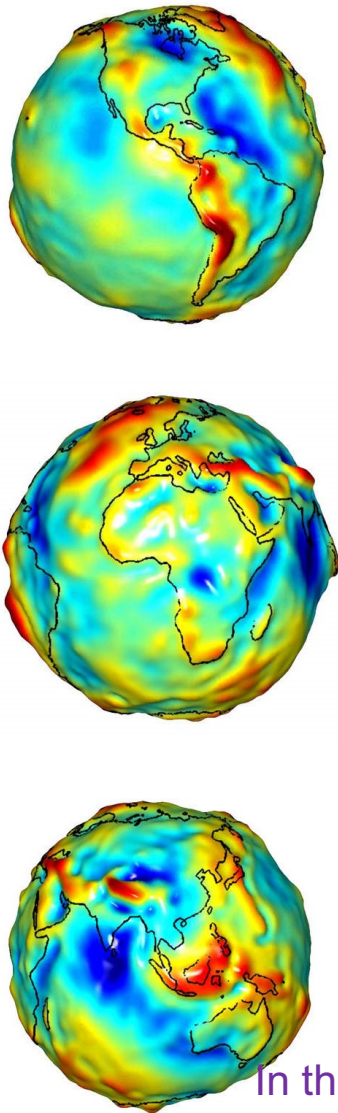
3. Free air correction is:

$$C_{FA} = \Delta g_{FA} = 0.3086 \Delta h \text{ (in mgal)}$$

- if site is **above** reference level, C_{FA} is **added** to gravity value
- if site is **below** reference level, C_{FA} is **subtracted** from gravity value

→ resulting gravity value is the **Free Air anomaly**

Free air gravity anomalies

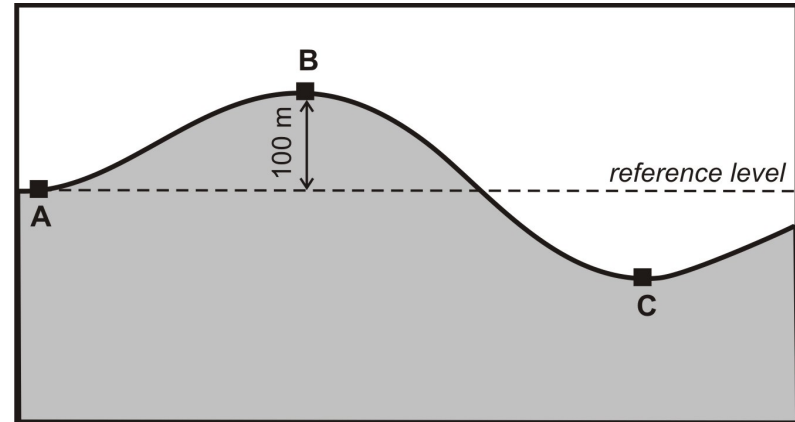


In this plot, there are lots of features similar to the geoid (red Pacific rim, blue Indian Ocean), but some differences exist. ²¹
(www.csr.utexas.edu/grace/)

Bouguer Correction

Free air correction – accounts for the effect of variations in r due to topography

$$g = \frac{GM_E}{r^2}$$



But topography also introduces variations in mass below a reference site...

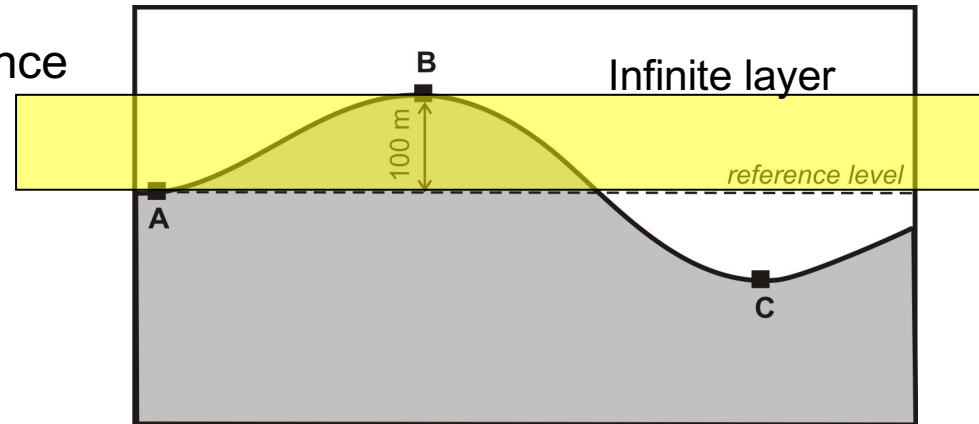
- **Site B** – gravity affected by extra mass of the hill (gravity is “too high”)
- **Site C** – Δg_{FA} moves C up to the reference level, but gravity will be “too low”
- in order to compare gravity measurements, we need to correct for this
→ **Bouguer correction**

Bouguer correction

Solution: approximate mass difference due to elevation as an infinite layer

For an infinite layer:

$$g = 2\pi G \rho \Delta h$$



G = Universal gravitational constant, ρ = density in SI unit, Δh = layer thickness in m.

Remark: no depth dependence

Therefore, the **Bouguer correction** is:

$$\Delta g_B = 2\pi G \Delta \rho \Delta h \quad (\text{in m/s}^2)$$

$$\Delta g_B = 2\pi G \Delta \rho \Delta h \times 10^5 \quad (\text{in mgal})$$

with: ρ in kg/m^3 and Δh in m

→ Δh is the difference in elevation between the site and reference level

Approximations: (1) infinite extent of layer & (2) uniform density

Bouguer correction

What is ρ of layer?

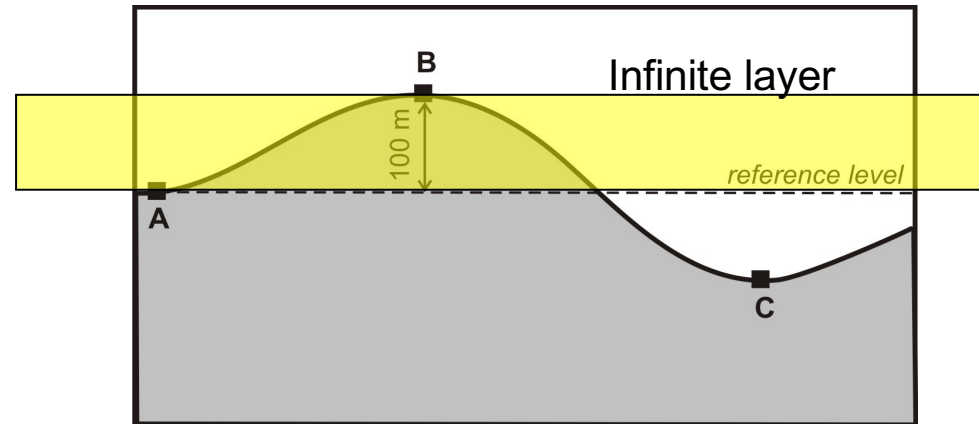
- use local geology
- or 2670 kg/m^3 (average crust)

With average crust:

$$\Delta g_B = 0.00004193 (2670) \Delta h$$

$\Delta g_B = 0.1119 \Delta h$

 (in mgal)



- if site is **above** reference level, C_B is **subtracted** from Free Air Anomaly
- if site is **below** reference level, C_{FA} is **added**

→ resulting gravity value is the **Bouguer anomaly**

Summary of elevation corrections

Choose a reference level

If site elevation is

Above reference level:

Below reference level:

Δg_{FA}

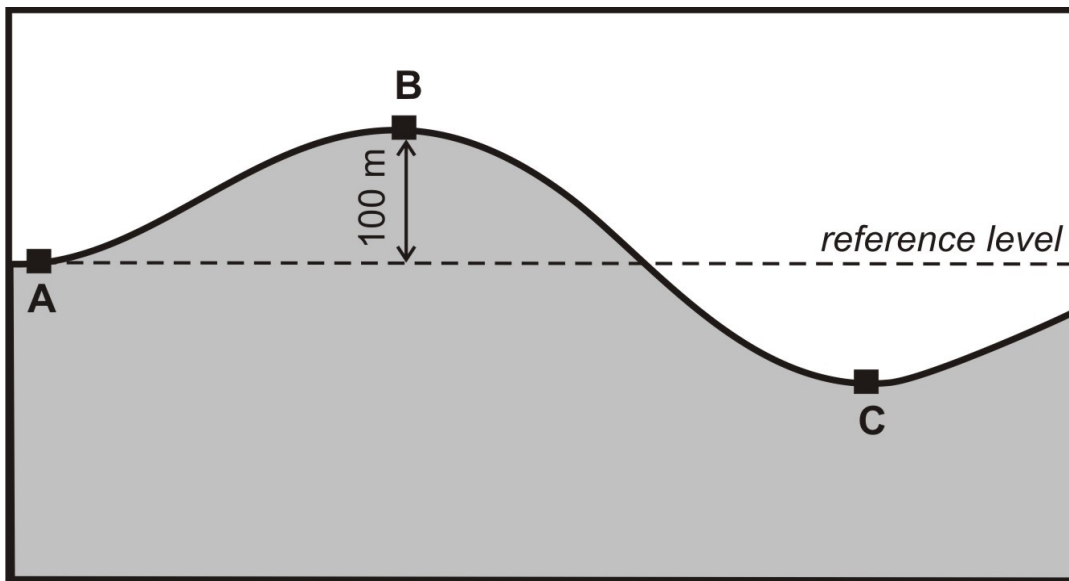
Add

Subtract

Δg_B

Subtract

Add



$$\Delta g_{FA} = 0.3086 \Delta h$$

$$\Delta g_B = 2\pi G \Delta \rho \Delta h \times 10^5$$

Sample calculation

g_n --- 'n' means 'nominal' or normal

Measurement in Edmonton: 981,172.647 mgal (9.8117... m/s²)

Latitude = 53.5° N

Elevation = 600 m above sea level

- want to compare this to measurements that were made at sea level

Calculate gravity anomaly (observed g – expected g)

$$g_n = g_e \left(1 + \alpha \sin^2 \lambda - \beta \sin^2 2\lambda \right)$$

$$g_n = 978031.846 \cdot \left(1 + 0.0053024 \sin^2(53.5^\circ) - 0.0000059 \sin^2(2 \times 53.5^\circ) \right)$$

GRS67 gives **981,377.634** mgal.

Gravity anomaly is:

$$\text{observed } g - \text{expected } g = -204.987 \text{ mgal}$$

→ difference from the average *spheroidal* rotating Earth with no topography (International Reference Ellipsoid)

Sample calculation

Gravity anomaly (wrt GRS67): -204.987 mgal

Now need to “move” Edmonton down to sea level

Calculate and apply Free Air correction

$$\Delta g_{\text{FA}} = 0.3086 \Delta h$$

$$\text{For } \Delta h = 600 \text{ m} \rightarrow \Delta g_{\text{FA}} = 185.160 \text{ mgal}$$

Edmonton is above reference level, so C_{FA} must be added
(moving closer to centre of Earth)

Free Air anomaly is $-204.987 + 185.160 = -19.827$ mgal

Sample calculation

Gravity anomaly (GRS67): -204.987 mgal

Free air anomaly: -19.827 mgal

Now need to get rid of the 600 m of rocks between Edmonton and sea level

Calculate and apply Bouguer correction (assume $\rho=2670$ kg/m³)

$$\Delta g_B = 0.1119 \Delta h$$

For $\Delta h = 600$ m $\rightarrow \Delta g_B = 67.140$ mgal

Edmonton is above reference level, so Δg_B must be subtracted
(getting rid of rocks)

Bouguer anomaly is $-19.827 - 67.140 = -86.967$ mgal

Bouguer gravity map of Alberta

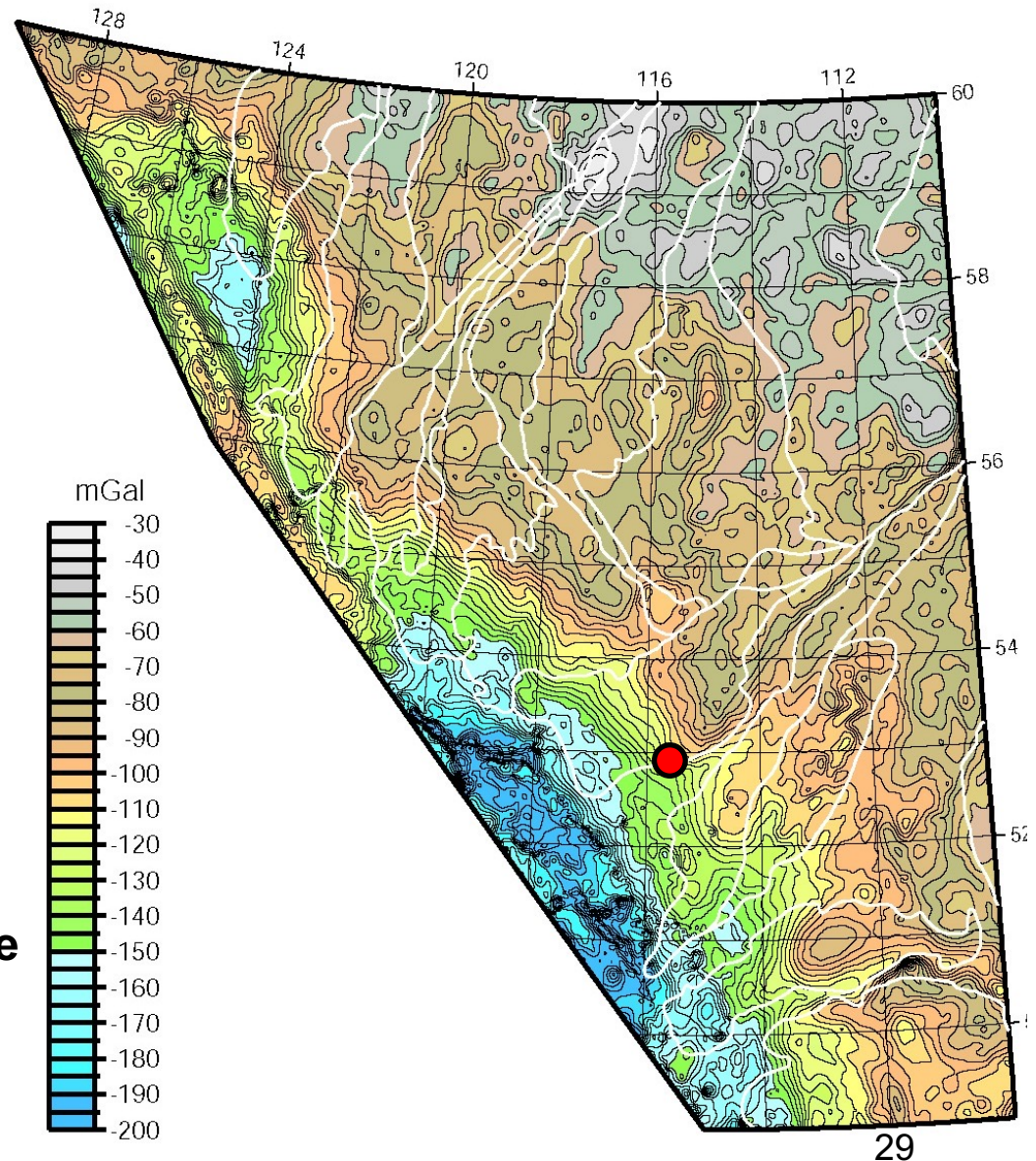
Gravity anomaly
(GRS67) :
-204.987 mgal

Free air anomaly :
-19.827 mgal

Bouguer anomaly :
-86.967 mgal
(sea level reference)

Bouguer Corrected Map/Anomaly =
Observation after corrections of Free
Air & Bouguer (*not just Bouguer*)

(Pilkington et al., 2000)

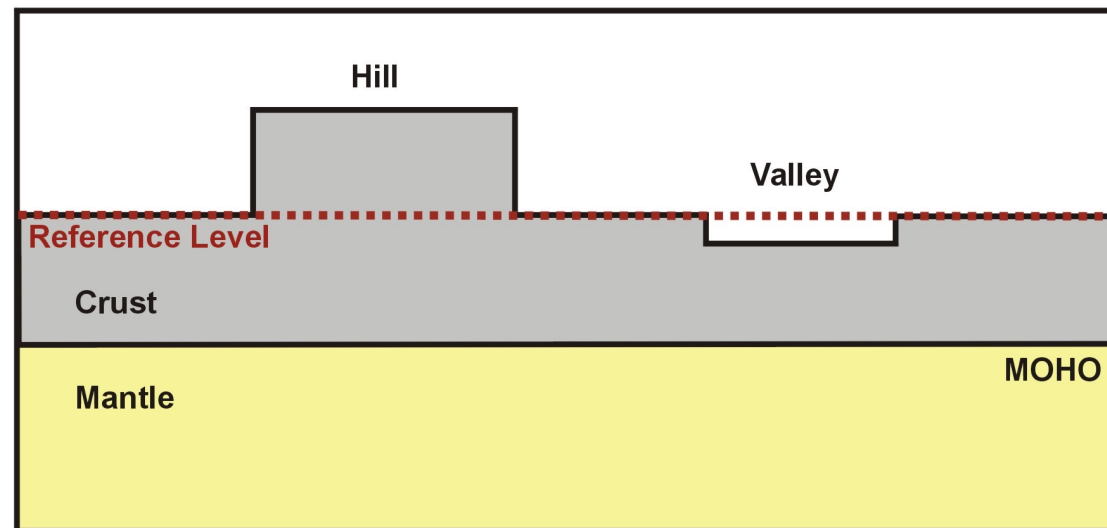
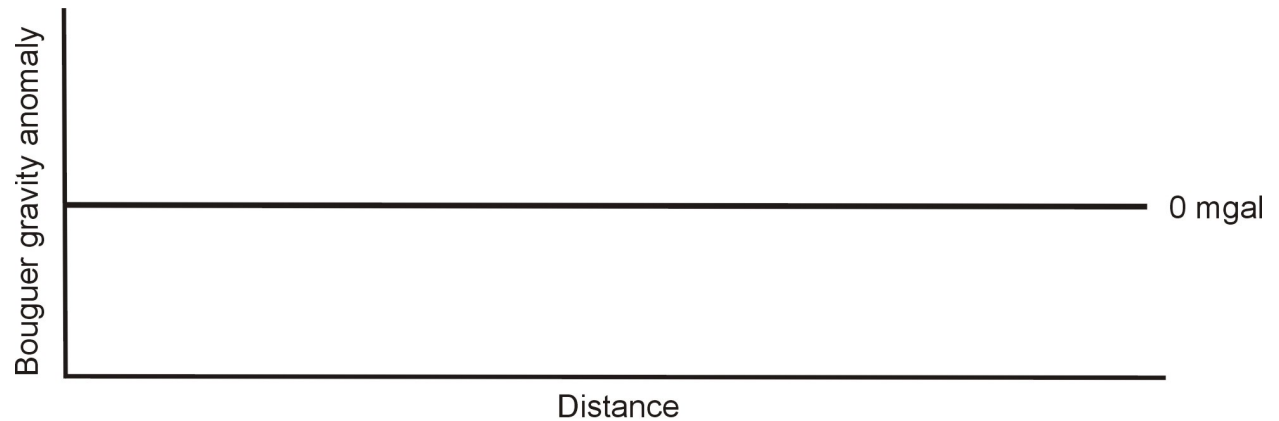


Importance of Bouguer gravity anomalies

Bouguer anomalies reflect variations in gravity from local geology (density variations)
→ crust and uppermost mantle

CASE 1

**Homogeneous
crust with
topography**

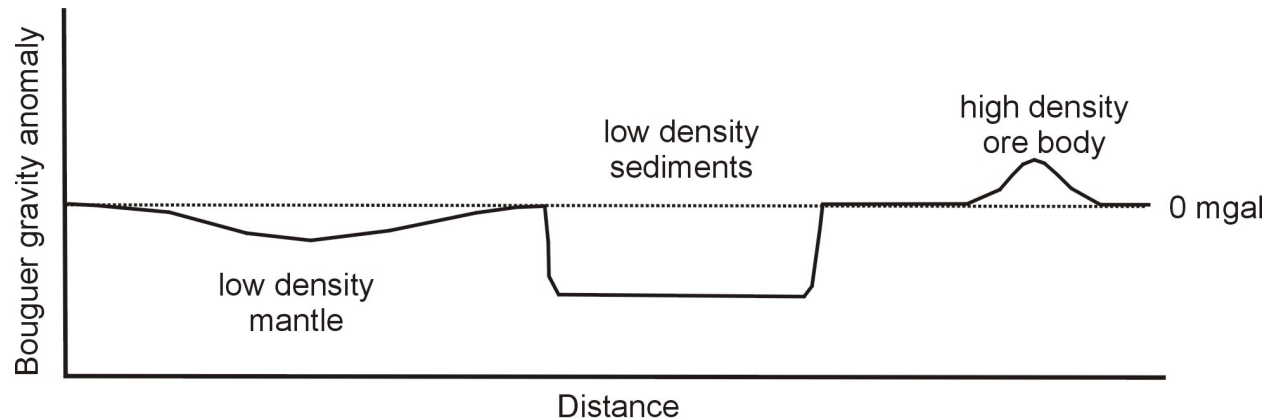


Importance of Bouguer gravity anomalies

Bouguer anomalies reflect variations in gravity from local geology (density variations)
→ crust and uppermost mantle

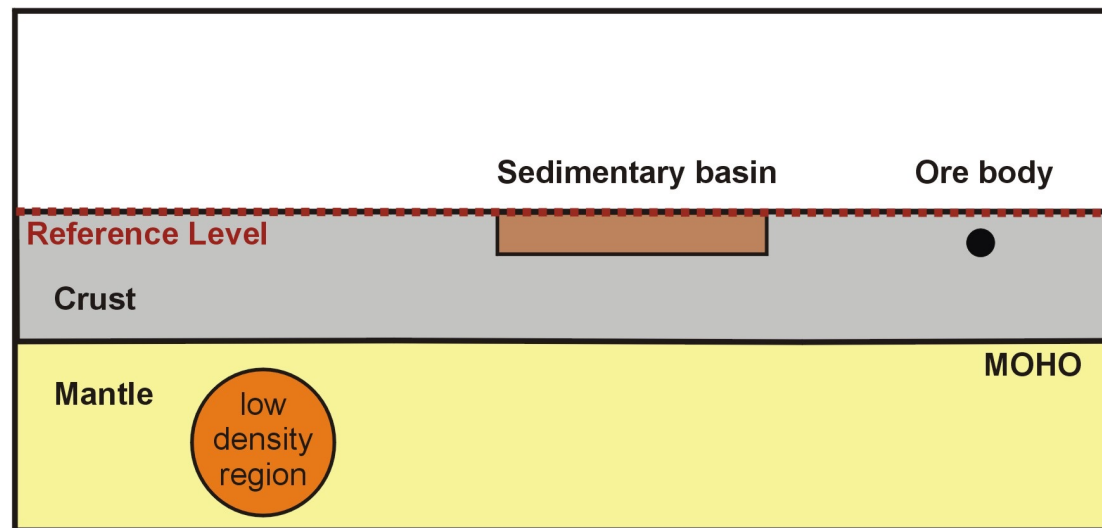
CASE 2

Variations in density in crust and mantle



Two observations:

1. Width of gravity pulse is related to depth
2. Gravity is an integrated effect (i.e., hard to pin point)

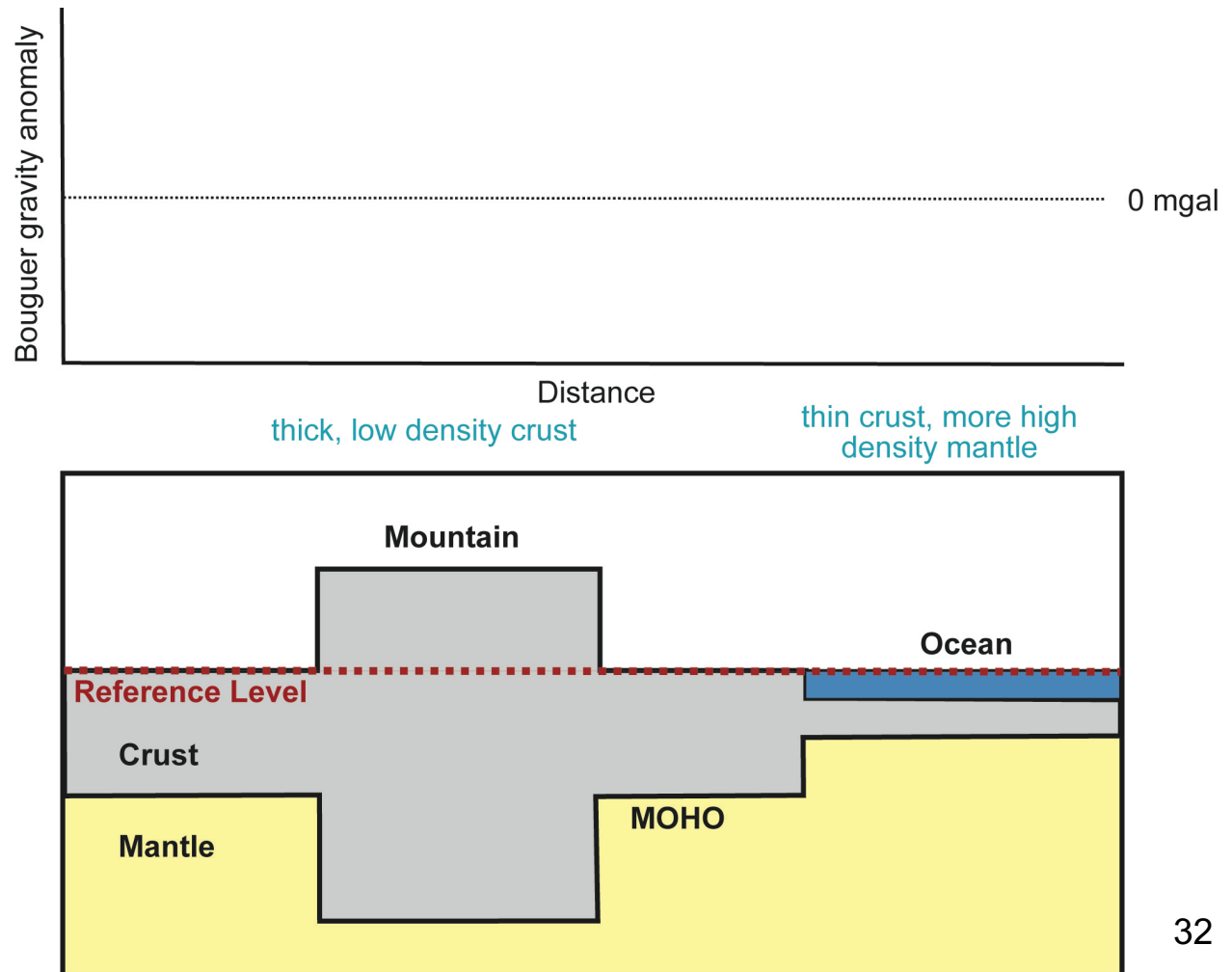


Importance of Bouguer gravity anomalies

Bouguer anomalies reflect variations in gravity from local geology (density variations)
→ crust and uppermost mantle

CASE 3

**Homogeneous
crust with
variable
thickness**

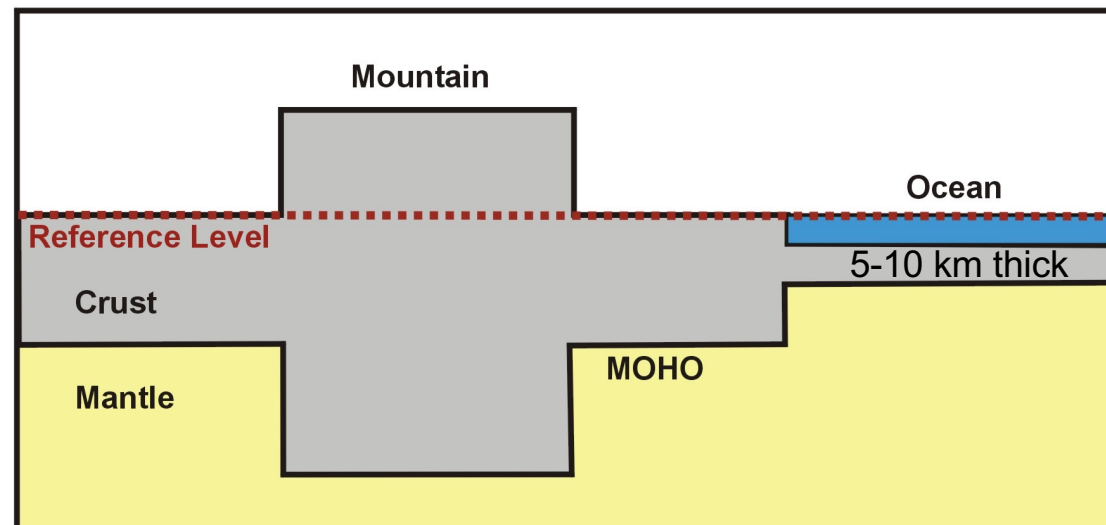
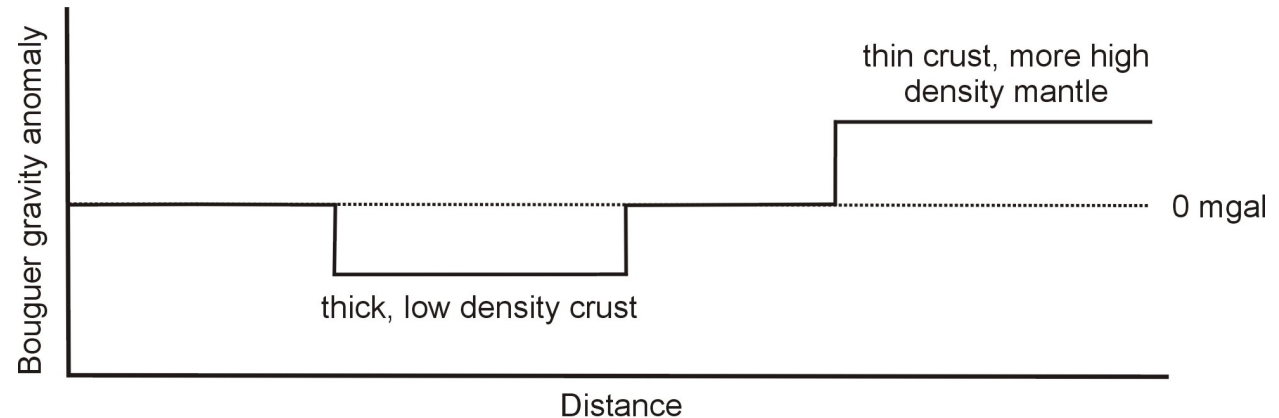


Importance of Bouguer gravity anomalies

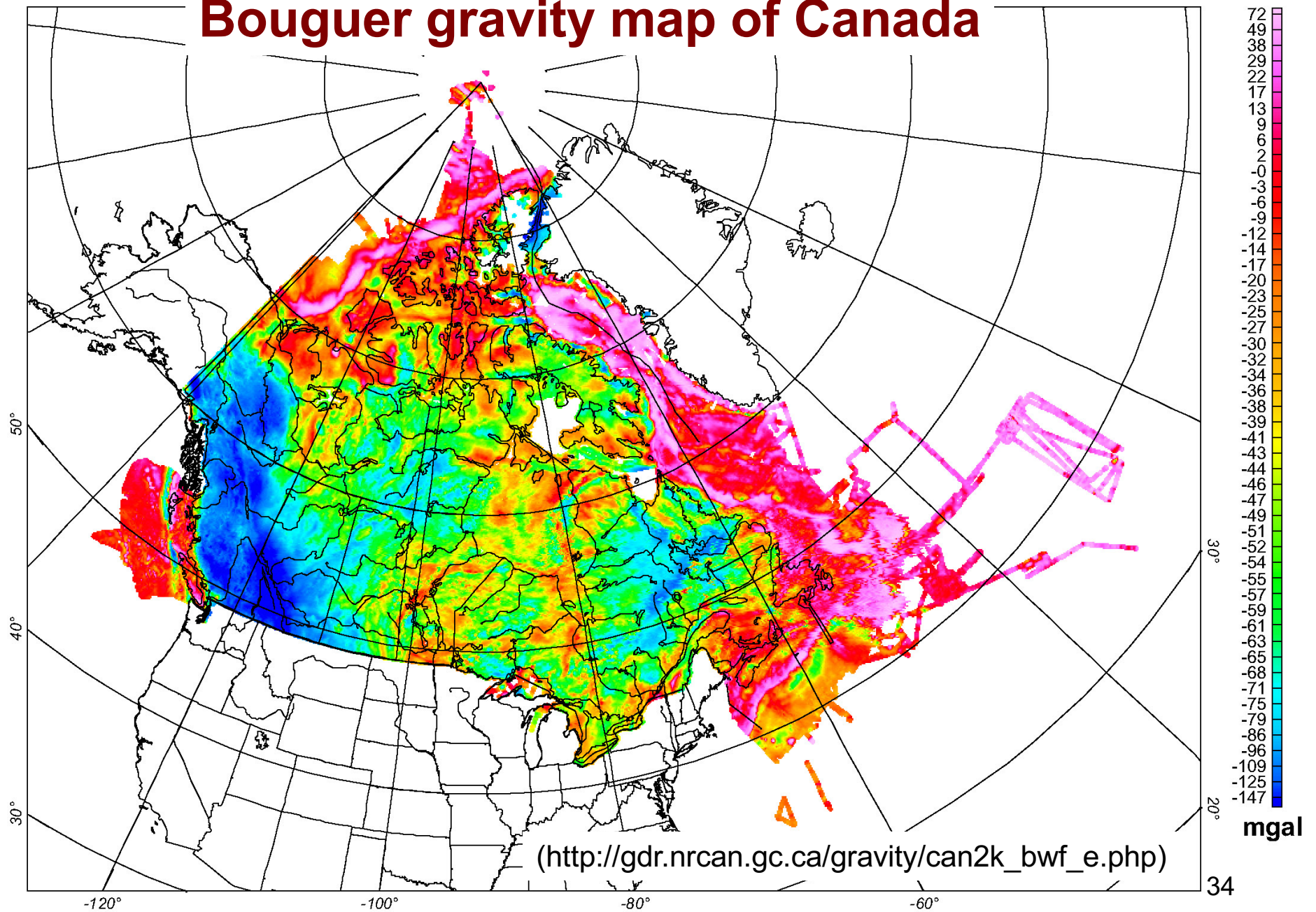
Bouguer anomalies reflect variations in gravity from local geology (density variations)
→ crust and uppermost mantle

CASE 3

**Homogeneous
crust with
variable
thickness**



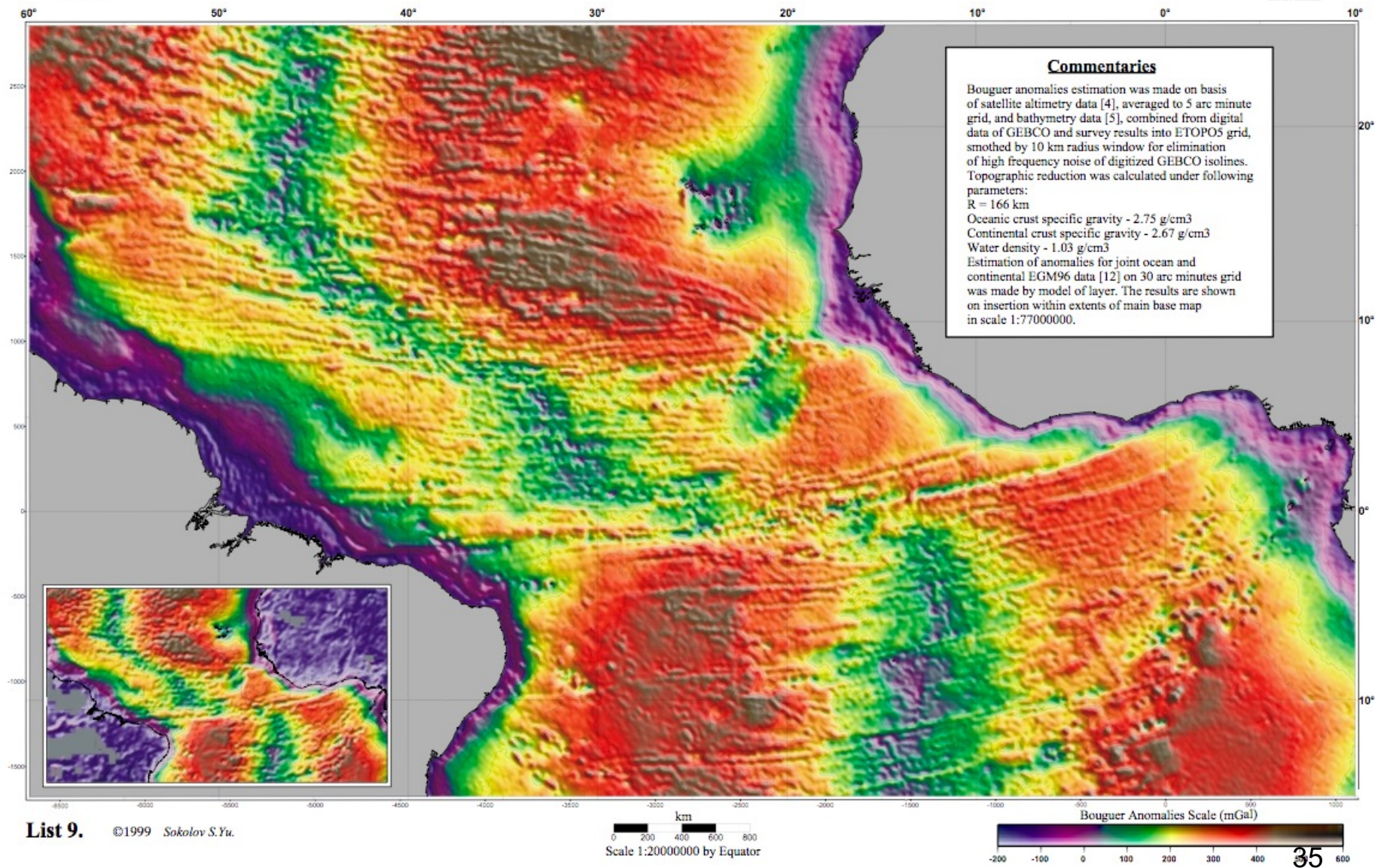
Bouguer gravity map of Canada





Bouguer Anomalies Calculated from Altimetry and Bathymetry Data on 5'x5' Grid.

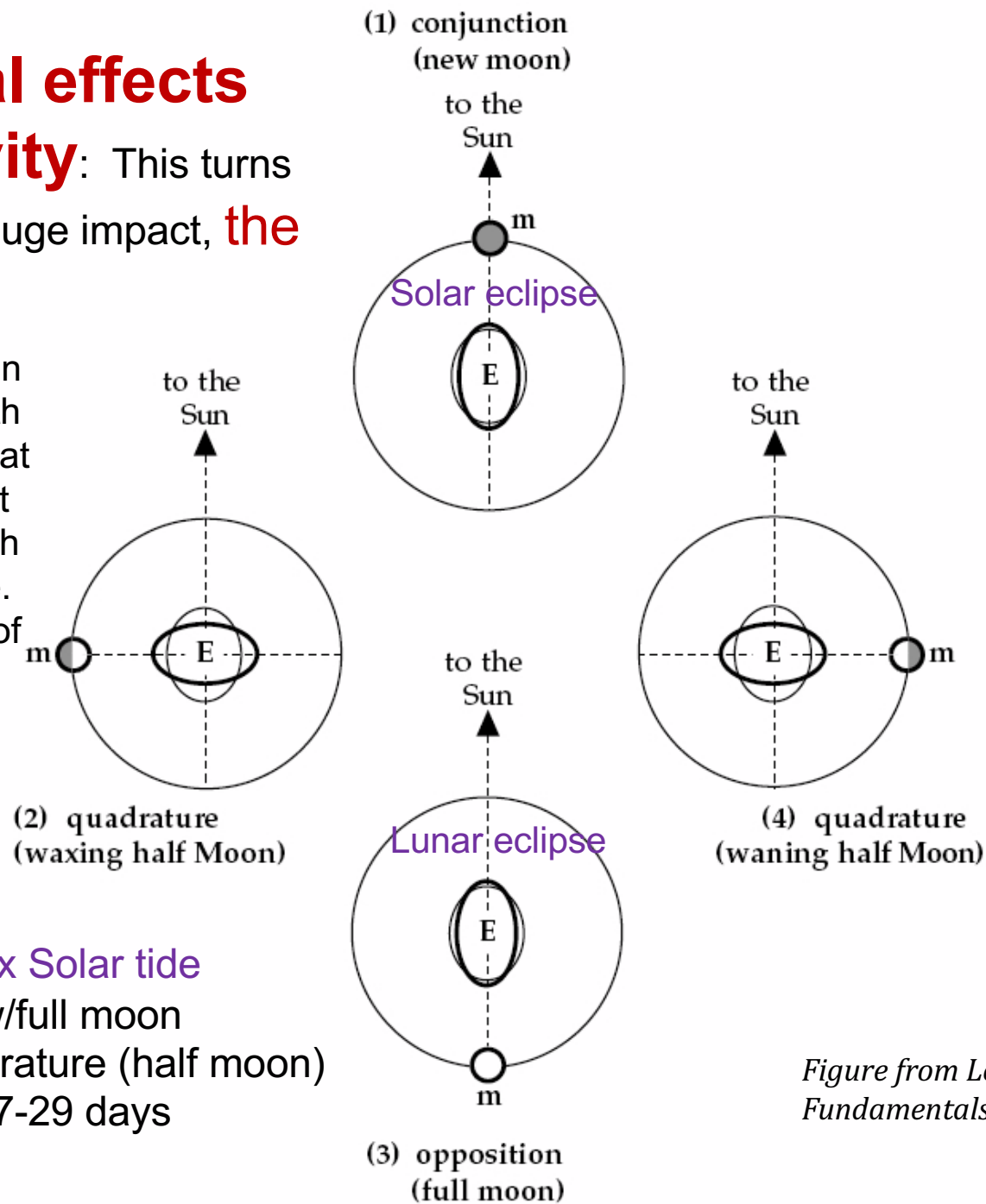
INSTITUT
GEODEZII I
KARTOGRAFI
RUSSKOY AKADEMII
NAUK
LABORATORY OF GEOMORPHOLOGY
AND OCEAN FLOOR TECTONICS



External effects

on gravity: This turns out to be a huge impact, **the tides!**

Tidal locking: moon keeping phase with Earth and rotates at the same rate as it orbits around Earth due to tidal torque.
Result: dark side of moon



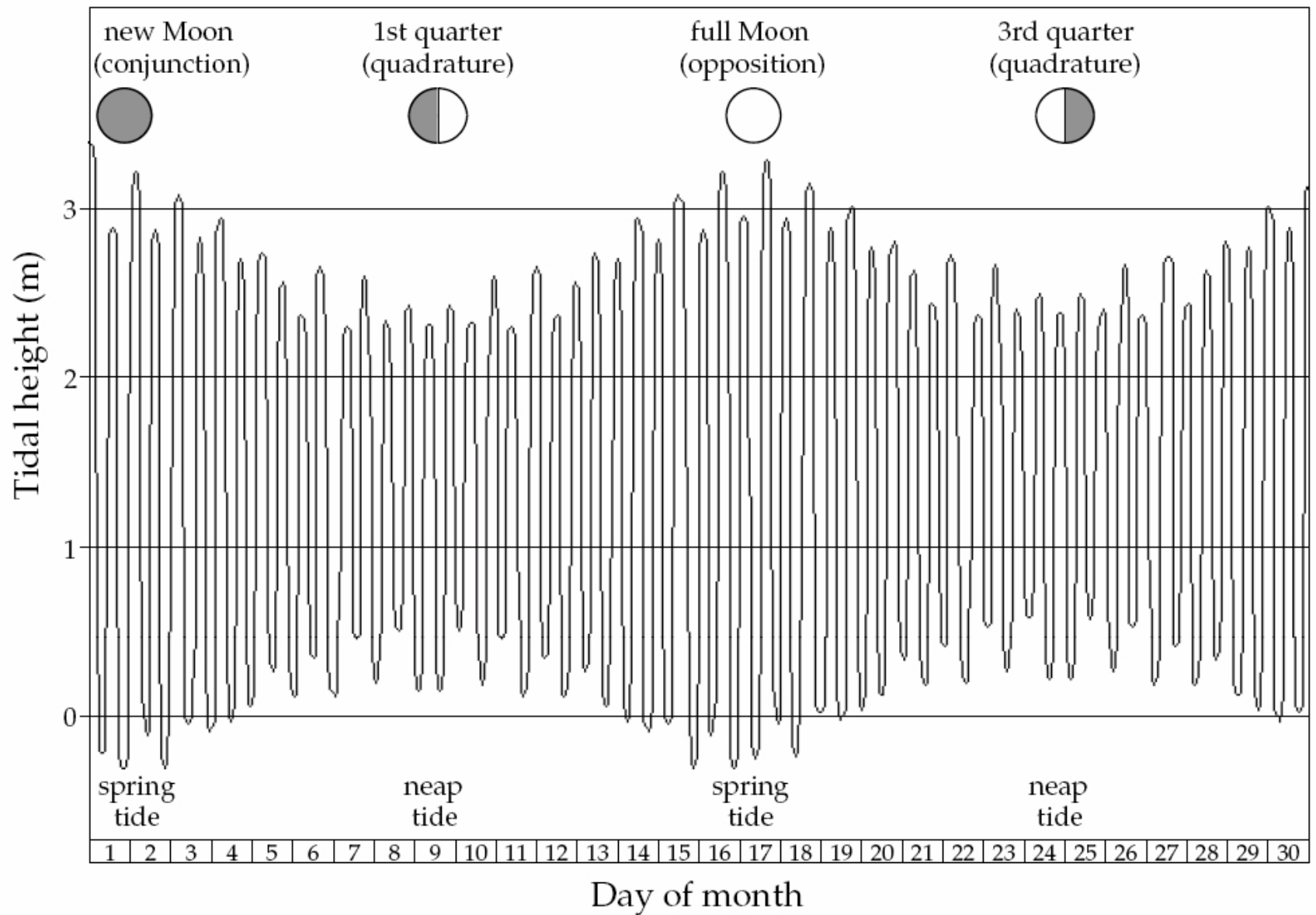
Lunar tide $\approx 2 \times$ Solar tide

High tide = new/full moon

Low tide = quadrature (half moon)

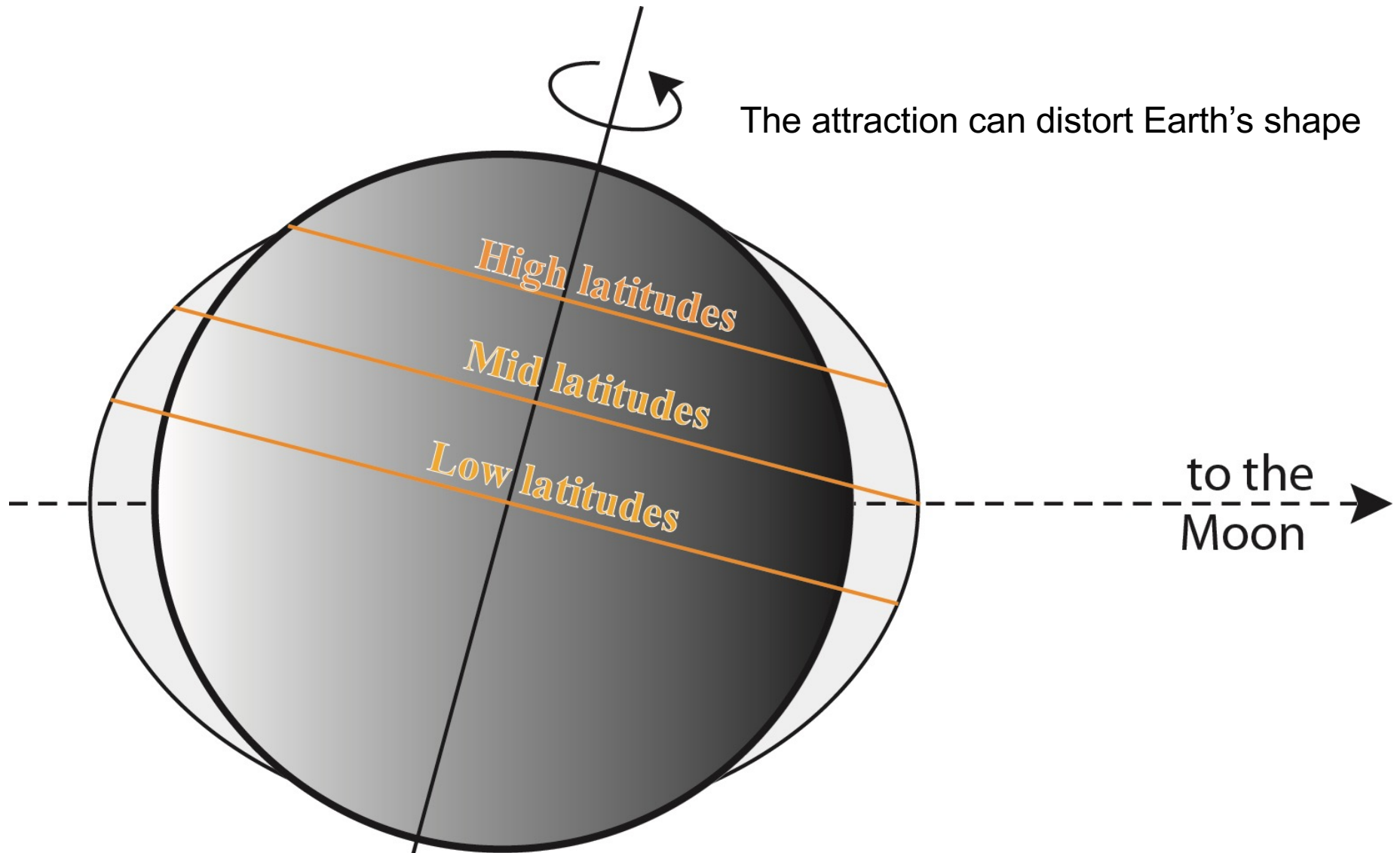
Periodicity ≈ 27 -29 days

Figure from Lowrie, 2007,
Fundamentals of Geophysics



Tidal forces change over a month, full/new moon causes larger tides

Figure from Lowrie, 2007, *Fundamentals of Geophysics*³⁷



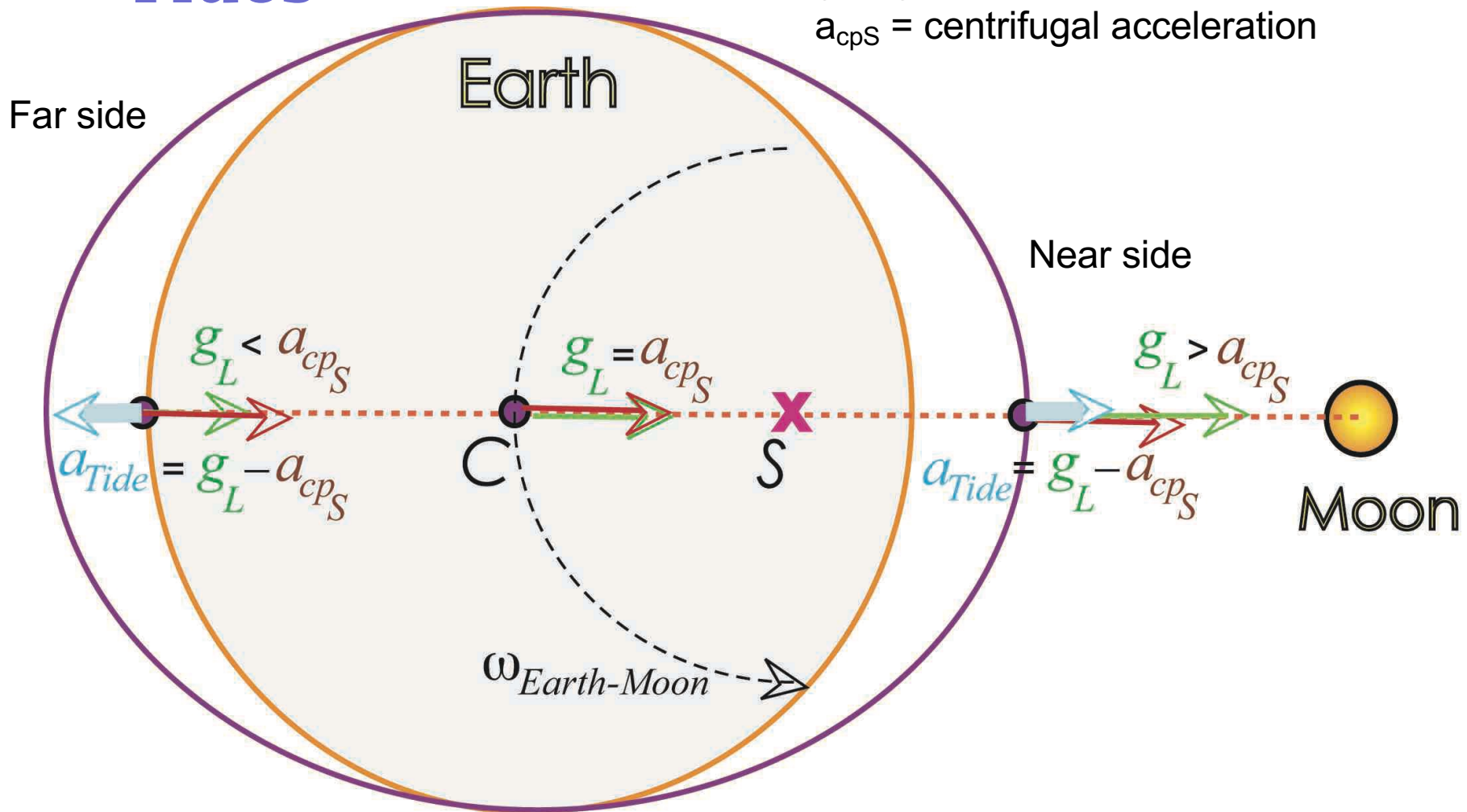
There is actually a bit of a 'tidal torque' due to the mis-alignment of Earth's rotation axis with the pulling force from the Moon. For this reason, equator is not in a same line as tidal force, resulting in a torque and distortion of the Earth.

Tides

S = center of gravity in the Earth-Moon system

g_L = gravitational acceleration

a_{cpS} = centrifugal acceleration



at the Earth's centre

a_{cpS} is g_L

It takes two to Waltz

Tidal acceleration a_{tide} at a given point on/inside Earth = difference between g and a

Revolution without rotation:

It takes two to Waltz

Ignoring rotation, how does Earth-Moon system *waltz*?

s= point of center
of mass between
Earth & Moon,
never moves

E=center of mass of the Earth

1 to 4: 4 sample points on Earth

Keys:

1. every point on Earth rotates in a circle during each month, i.e., same centrifugal force.
2. E also moves in a circle. At E, centrifugal acceleration = gravitation attraction of Moon.

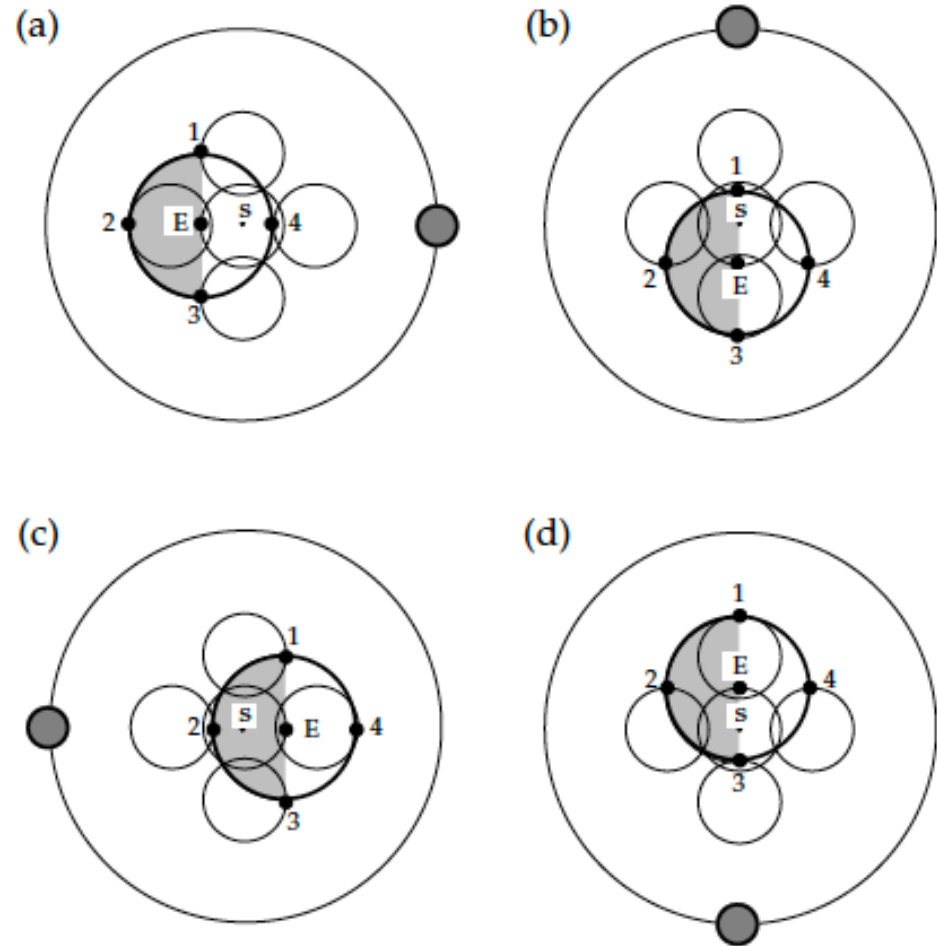


Fig. 2.10 Illustration of the “revolution without rotation” of the Earth–Moon pair about their common center of mass at S.