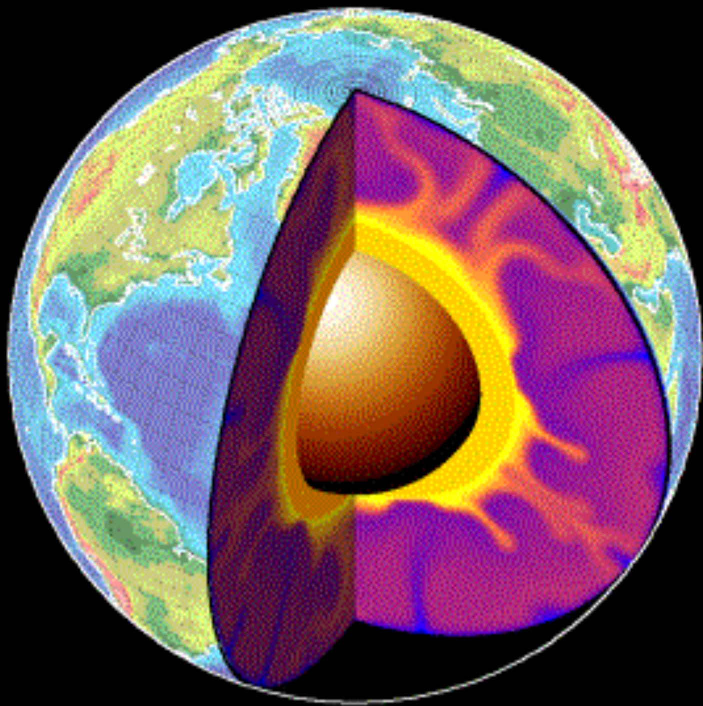


CHAPTER 1.4

Earth's density structure and surface gravity

Earth's density structure and surface gravity

What is the average density of the Earth?

$$\rho_E = \frac{M_E}{V_E}$$

Mass of Earth: 5.97×10^{24} kg

For a spherical Earth:

(6371 km radius)

$$V_E = \frac{4}{3}\pi R_E^{\;3} = 1.08 \times 10^{21} m^3$$

$$\boxed{\rho_E = 5528 kg/m^3}$$

Density of typical crustal rocks

Granite	ρ = 2520 – 2750 kg/m^3
Basalt	ρ = 2700 – 3200 kg/m^3
Gneiss	ρ = 2610 – 2990 kg/m^3
Shale	ρ = 2060 – 2660 kg/m^3
Limestone	ρ = 2600 – 2800 kg/m^3
Sandstone	ρ = 2050 – 2550 kg/m^3

→ density must increase with depth in the Earth

Average *g* for the Earth

Calculate the average gravitational acceleration at the surface of the Earth:

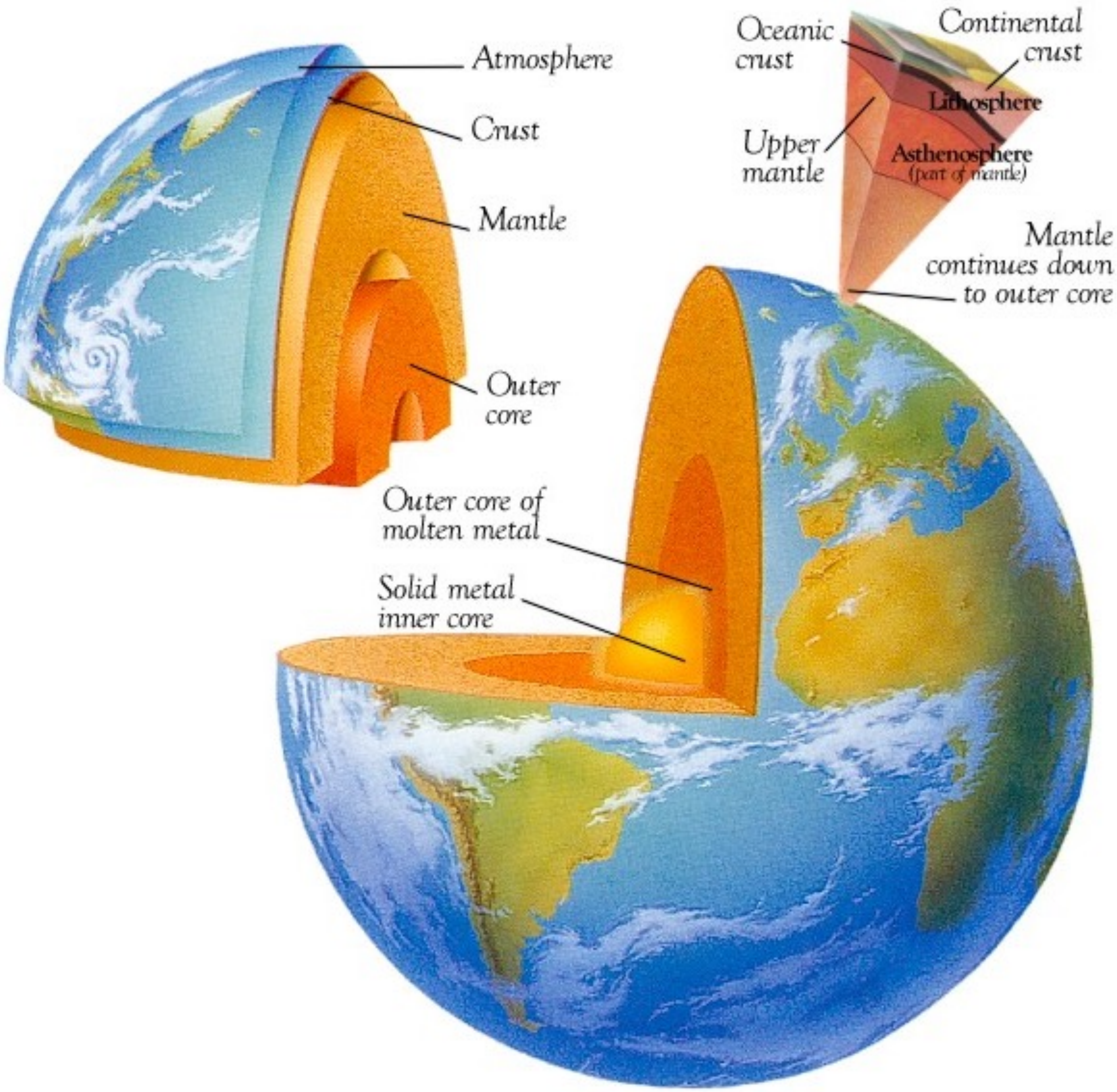
Mass (M_E) = 5.97214 x 10^{24} kg
(*~Venus, 1000% of Mars, 2000% of Mercury, 600% of Moon*)

G = 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}

Avg. radius (r) = 6371 km

$$g = \frac{GM_E}{r^2}$$ → g = 9.817 m/s^2 (near surface)

Radial density structure of the Earth



To first order, the Earth is radially symmetric

→ series of spherical layers of (nearly) constant density

But the variation in density with depth can not be determined from surface gravity measurements…

Radial density structure of the Earth

• can constrain density structure using observations from seismic studies, with the additional constraint of total M_E from gravity data

Preliminary Reference Earth Model (PREM)

• radial Earth model (ρ, V_P, V_S, etc.)

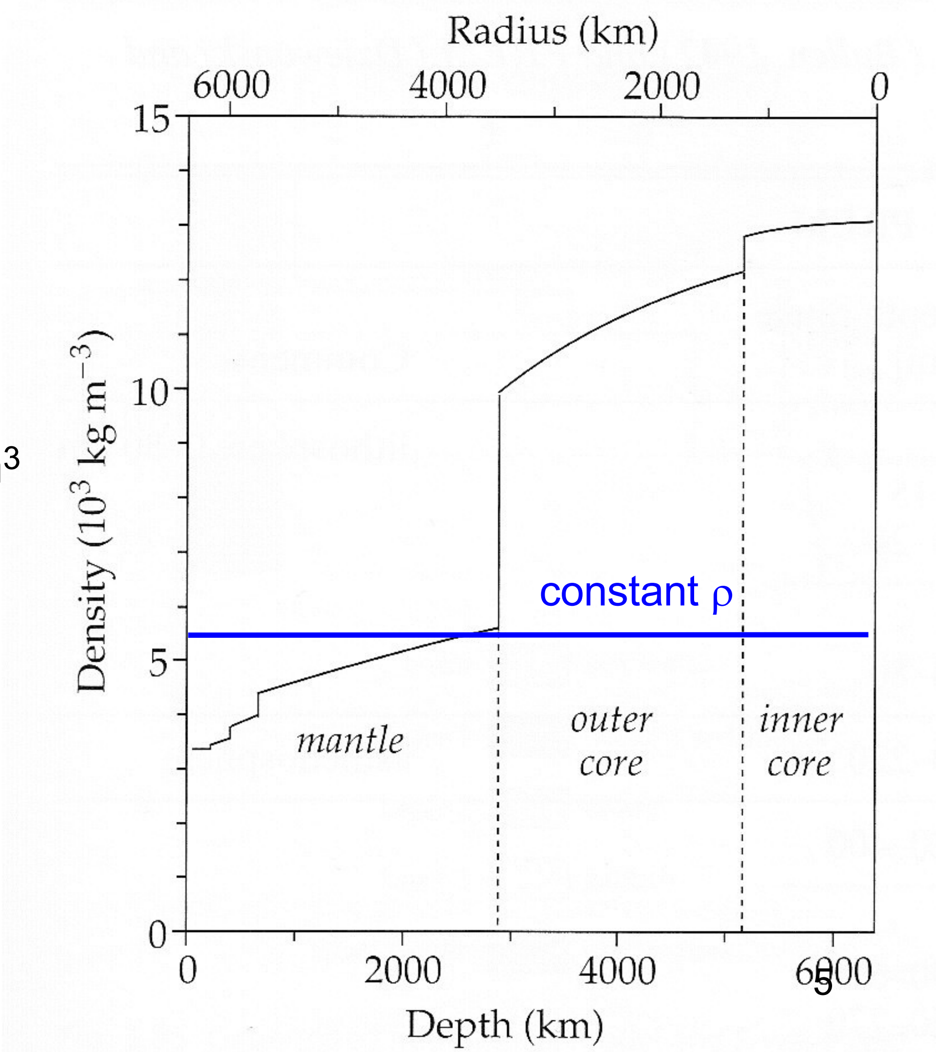
• density increases with depth

– average density: ~5500 kg/m^3

• discontinuities at compositional boundaries (Moho, CMB, ICB) or where there are mineral phase transitions (mantle TZ)

• constant composition → increasing density with depth (pressure)

(Dzeiwonski and Anderson, 1981
– figure from Lowrie, 2007)



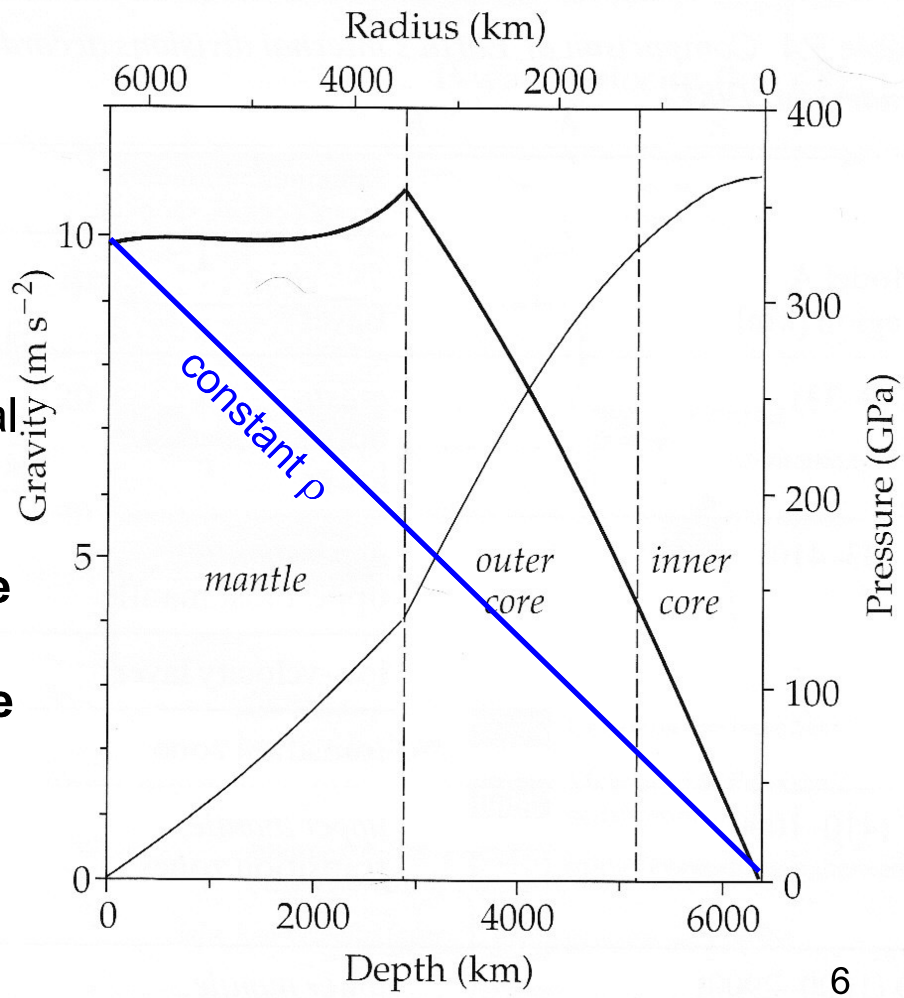
Radial density structure of the Earth

$$g = \frac{GM_E}{R_E^{\ 2}}$$

A few observations:

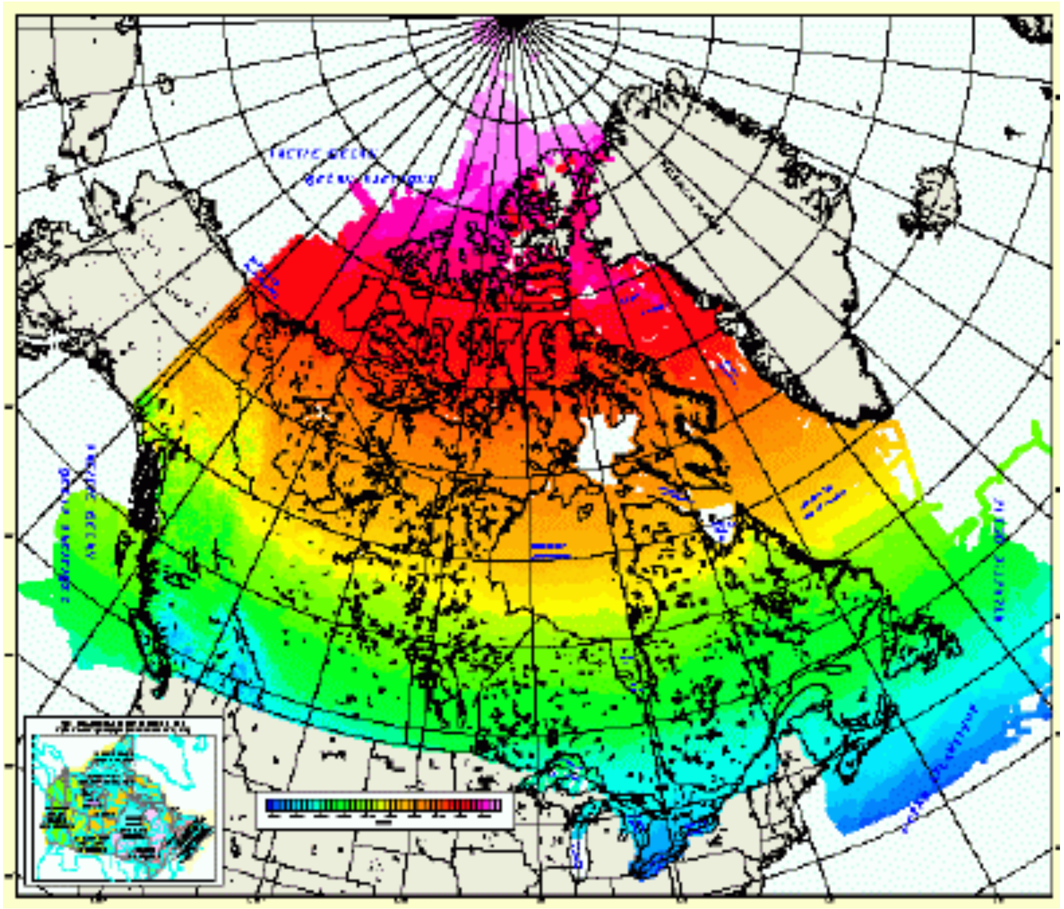
1. Pressure (thinner black line) generally increase from surface (0 km) to the center of Earth (6371 km), but it not exactly straight.
2. Under constant density, a spherical Earth should result in a gravity 'g' that follows the blue line.
3. In truth, the **thick black line is the true value of 'g'. Up to 2890 km (bottom of the mantle), the value hovers around 9.8 km/s, even increases (why?)**
4. **Below Core-mantle boundary (2890 km), g drops somewhat linearly (why?)**

Variations in pressure and gravity



Gravity variations over the Earth's surface

Observed gravity for Canada



http://gdcinfo.agg.nrcan.gc.ca/products/maps_e.html

Not a constant 9.81 m/s^2 !

<u>Variations in g due to:</u>

1. Earth is not a perfect sphere and it is rotating

   (latitudinal variations)

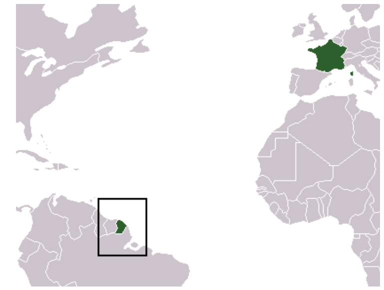
2. Earth has topography

   (elevation variations)

3. Near-surface geology

   (density variations)

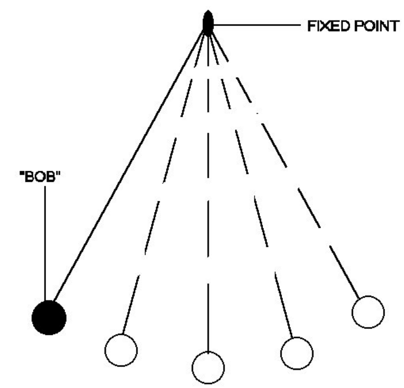
Variations in gravity with latitude



Back to section 1.1:

Jean Richer (1672):

- pendulum clock in Cayenne, French Guyanna lost 2.5 seconds/day relative to a pendulum clock in Paris



$$T = 2\pi\sqrt{\frac{L}{g}}$$

In Cayenne, T is larger
→ g must be weaker

Isaac Newton (1687):

- argued that the Earth must be an **oblate spheroid** because it is **rotating**

INTERNATIONAL REFERENCE ELLIPSOID (IRE)

Equatorial radius = 6378 km

Polar radius = 6357 km

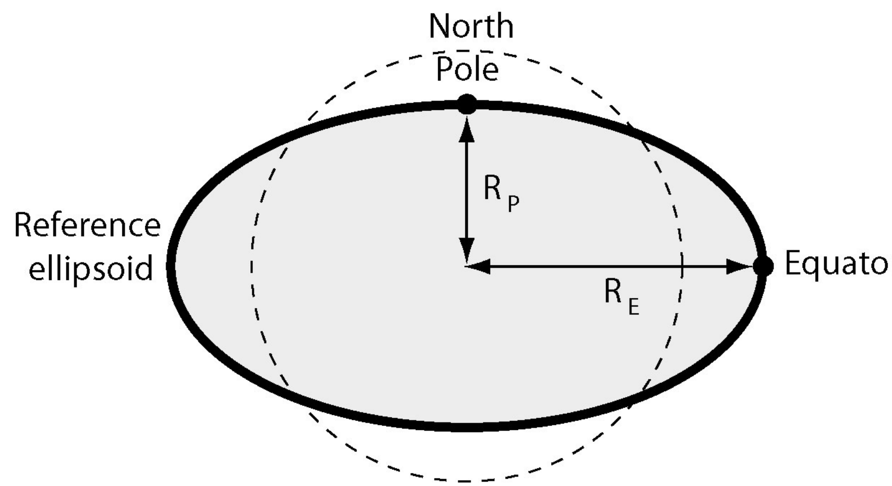
Variations in gravity with latitude

1. Shape of the Earth

Equatorial radius = 6378 km

Polar radius = 6357 km

$\boxed{g_E < g_P}$



Surface gravity depends on the distance from the centre of the Earth

- at equator, the radius is larger → surface gravity should be weaker

(inverse square law)

$$g(r) = \frac{GM_E}{r^2}$$

g_E = 979,540 mgal

g_P = 986,022 mgal

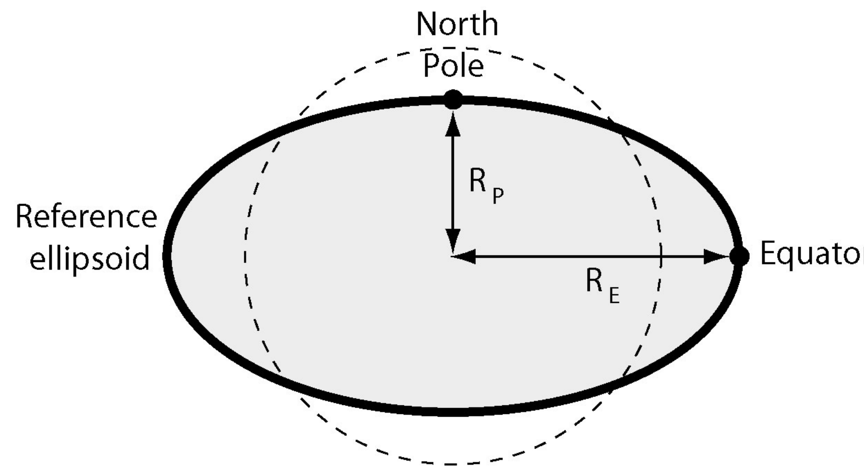
g_E is 6482 mgal smaller than g_P

Variations in gravity with latitude

2. Mass distribution

Equatorial radius = 6378 km

Polar radius = 6357 km



At the equator, there is more mass between the surface and the centre of the Earth.

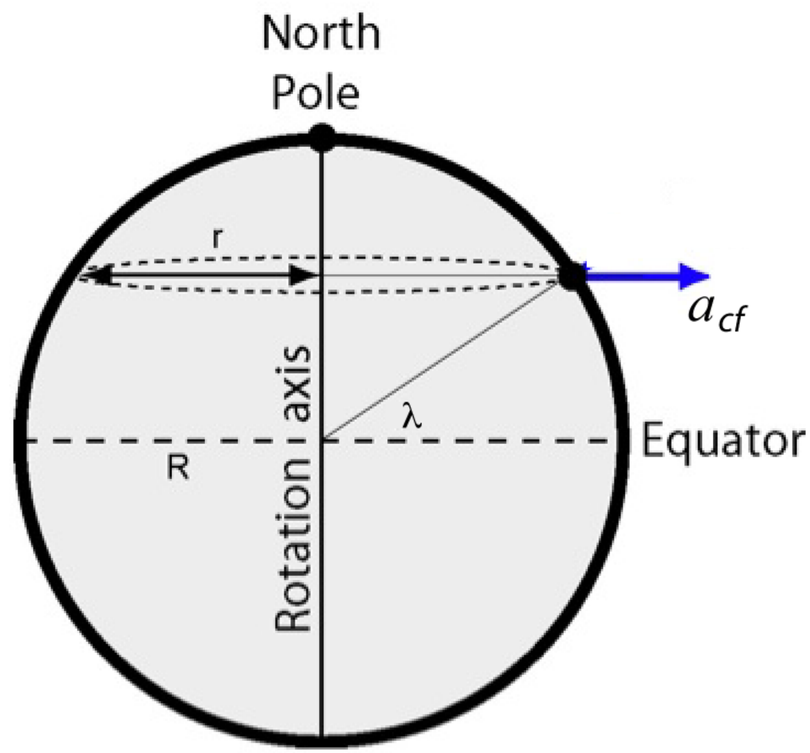
$$g(r) = \frac{GM_E}{r^2}$$

$$\boxed{g_E > g_P}$$

Through integration, it can be shown that this results in gravity that is **4800 mgal larger at the equator**

Variations in gravity with latitude

3. Rotation of the Earth



At equator, a person makes a rotation around a circle of radius 6378 km every day

At poles, person just turns in one place

Rotation around the Earth induced a **centrifugal force**

- acts outward

- part of the centrifugal force will act against gravity which is pulling the person toward the centre of the Earth

Expect that centrifugal force will result in a reduced (net) gravity

$$\boxed{g_E < g_P}$$

Variations in gravity with latitude

3. Rotation of the Earth

At latitude λ, an observer travels around a circle of radius:

$$r = R_E \cos \lambda$$

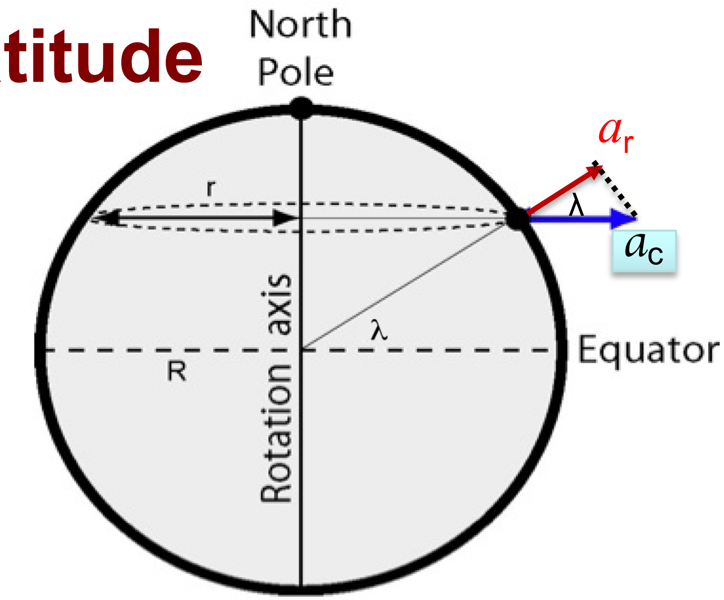
$$(R_E = 6378 \text{ km})$$

In one day, the Earth rotates through 360° (2π radians)

$$\omega = \frac{2\pi}{24 \times 60 \times 60} = 7.27 \times 10^{-5} (rad/\sec)$$

Centrifugal acceleration acts outward from rotation axis (*in this case, to the right*)

$$\boxed{a_c = r\,\omega^2}$$



The "outward" component of a_c is:

$$a_r = a_c \cos \lambda$$
$$= r\,\omega^2 \cos \lambda$$

$$\boxed{a_r = R_E\,\omega^2 \cos^2 \lambda}$$

Plugging in…

a_r = 0 mgal at North Pole (why?)

a_r = 3370 mgal at equator

→ gravity g is **3370 less** at equator

Variations in gravity with latitude

Overall effect of the three factors:

• shape:	$g_P - g_E = +6482$ mgal	$g_E < g_P$
• mass:	$g_P - g_E = -4800$ mgal	$g_E > g_P$
• rotation:	$g_P - g_E = +3370$ mgal	$g_E < g_P$
TOTAL:	$\mathbf{g_P - g_E = +5052}$ **mgal**	

→ g is **larger** at the poles!

Observations:

Gravity at equator = 978,032 mgal

Gravity at poles = 983,218 mgal

Gravity difference: g_P-g_E = 5186 mgal

Gravitational Potential of the Earth - we define its shape by an equipotential surface

Earth not a sphere - Gravity more complex

MacCullagh's Formula (1849) - accounts for oblate spheroid gravity potential (note, *not a formula for g*)

$$U_G = -G\frac{M_E}{r} - G\frac{(C-A)}{r^3}\cdot\frac{(3\cos^2\theta - 1)}{2}$$

where C and A are the principal moments of inertia of the Earth through the pole and the equator, respectively. θ is the angle as measured from the pole.

1. 1st order term describes gravity for a perfect sphere

2. 2nd order term describes additional corrections for the oblateness (deviates from sphere 10-20 km)

NO need to memorize or calculate (not on exams, even if given on the formula sheet)

Variations in gravity with latitude

- rotation of a non-spherical Earth produces surface g variations of ~5000 mgal
  - variation is a function of latitude → can be predicted

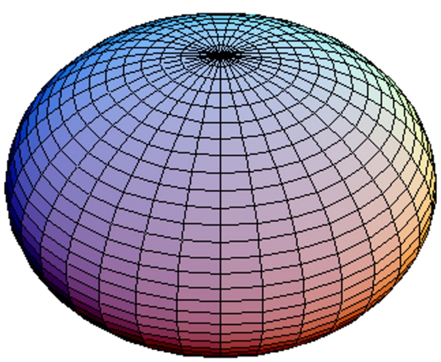
- **Geodetic Reference System 1967 (GRS67) equation**
  - gives expected (theoretical) value of gravity at latitude θ for an Earth of mass M_E and shape defined by International Reference Ellipsoid

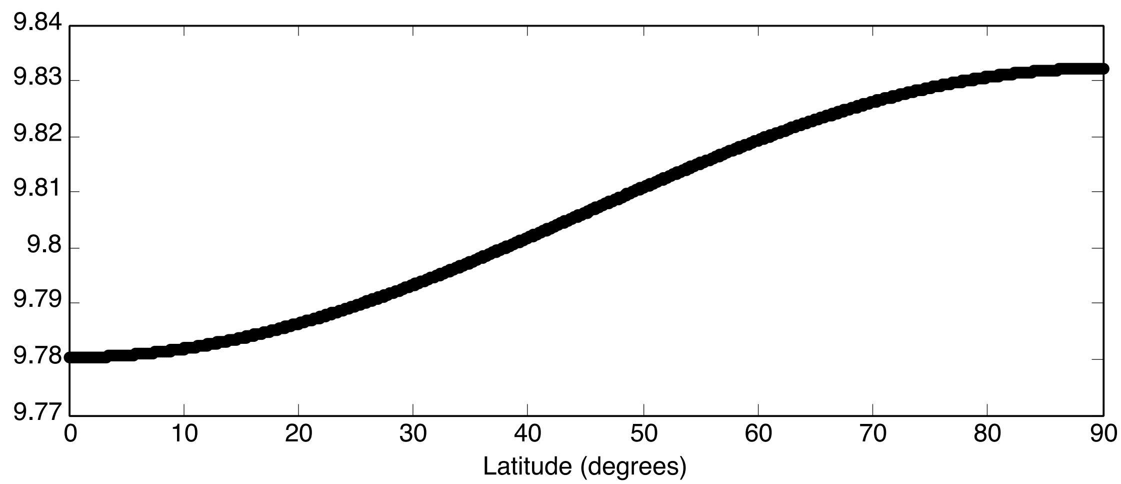
$$g_n = 978031.846 \cdot \left(1 + 0.0053024 \sin^2 \lambda - 0.0000059 \sin^2 2\lambda\right)$$

(in mgal)

Again, lambda is the latitude

I.R.E.





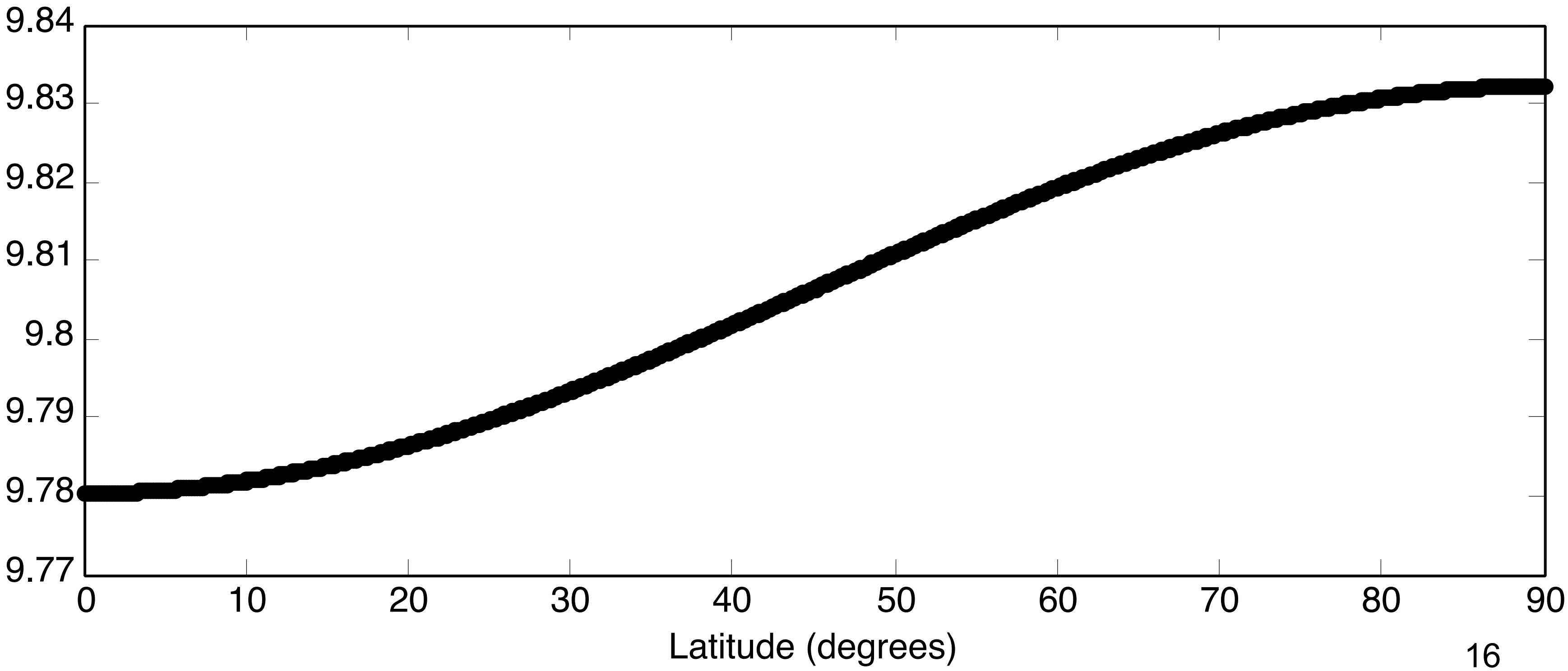
This is similar to the relationship we gave earlier (GRS67), but contains the **2*lambda term**.

Variations in gravity with latitude

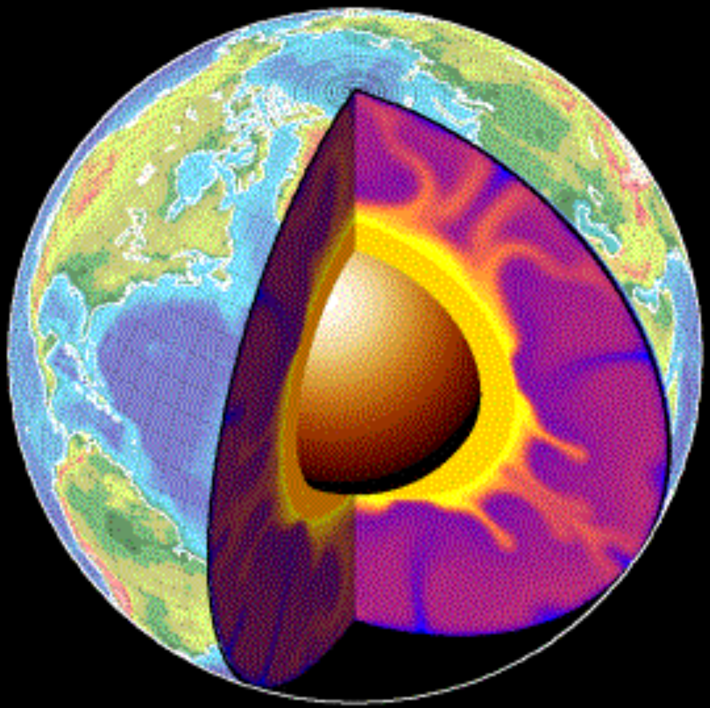
Expected value of gravity in Edmonton (53.5ºN)

$$g_n = 978031.846 \cdot \left(1 + 0.0053024 \sin^2\left(53.5^\circ\right) - 0.0000059 \sin^2\left(2 \times 53.5^\circ\right)\right)$$

GRS67 gives: g = 981377.634 mgal = 9.8138 m/s^2



This is similar to the relationship we gave earlier, but contains the 2*theta term.



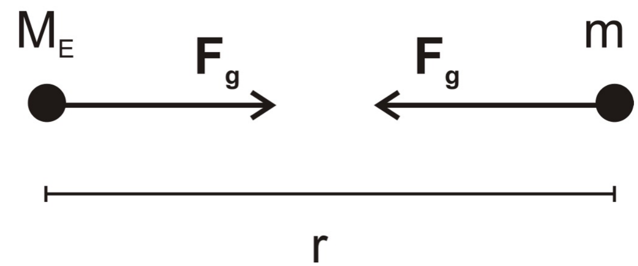
CHAPTER 1.3-1.4
Gravitational Potential, Geoid, Shape, density

The Geoid

Gravitational acceleration (g): vector

Point mass (M_E):

$$g = \frac{F_g}{m} = \frac{GM_E}{r^2}$$



Gravitational potential (U): scalar quantity, represents the ***work done per unit mass*** to move an object from infinity to a specific point in a gravitational field, representing the field's strength at that point independent of the object's mass

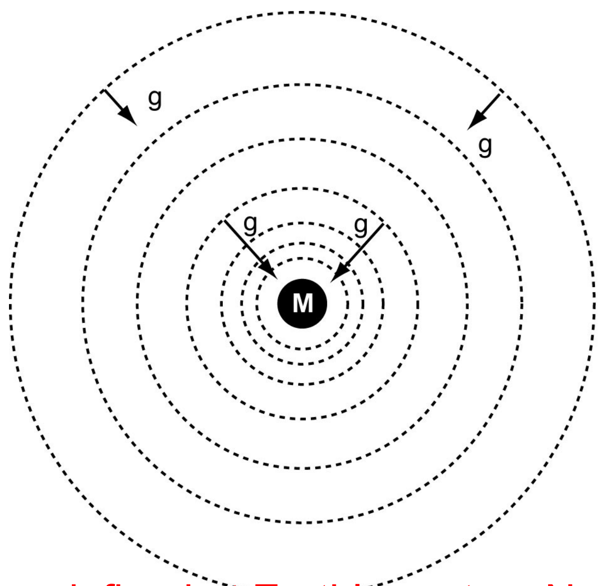
Point mass (M): *Work = F_G* Δx*

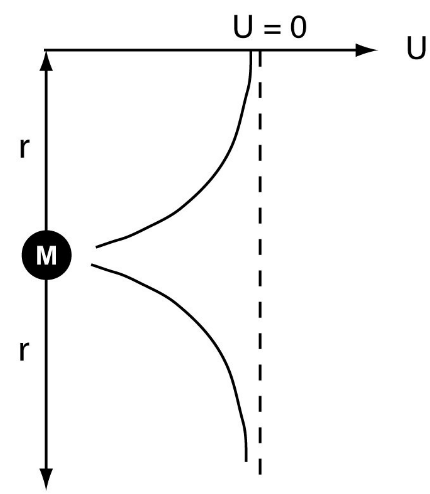
$$U = -\frac{GM}{r}$$

equipotential lines = constant U

- no work is needed to move along equipotential lines

- work is needed (change in energy) to move to different U





Gravitation potential is 0 at infinity and undefined at Earth’s center. Note g is always perpendicular to equipotential surfaces.

Gravitation Potential vs. Gravitation Potential Energy

Gravitation Potential

Def: Work done by moving a unit mass from infinity to a specific point in space against gravity field.

Extreme value: U=0 at infinite distance

Sign: U < 0 at all positions

Value: Most negative at object center, not defined actually!

Formula:

$$U = -\frac{GM}{r}$$

SI Unit = Joules/kilogram

Gravitation Potential Energy

Def: Energy stored in an object that is being held in a vertical position against gravitation force, i.e., mass & height dependent

Extreme value: P=infinite at infinite distance

Sign: P > 0 at all positions

Value: 0 at surface of the object

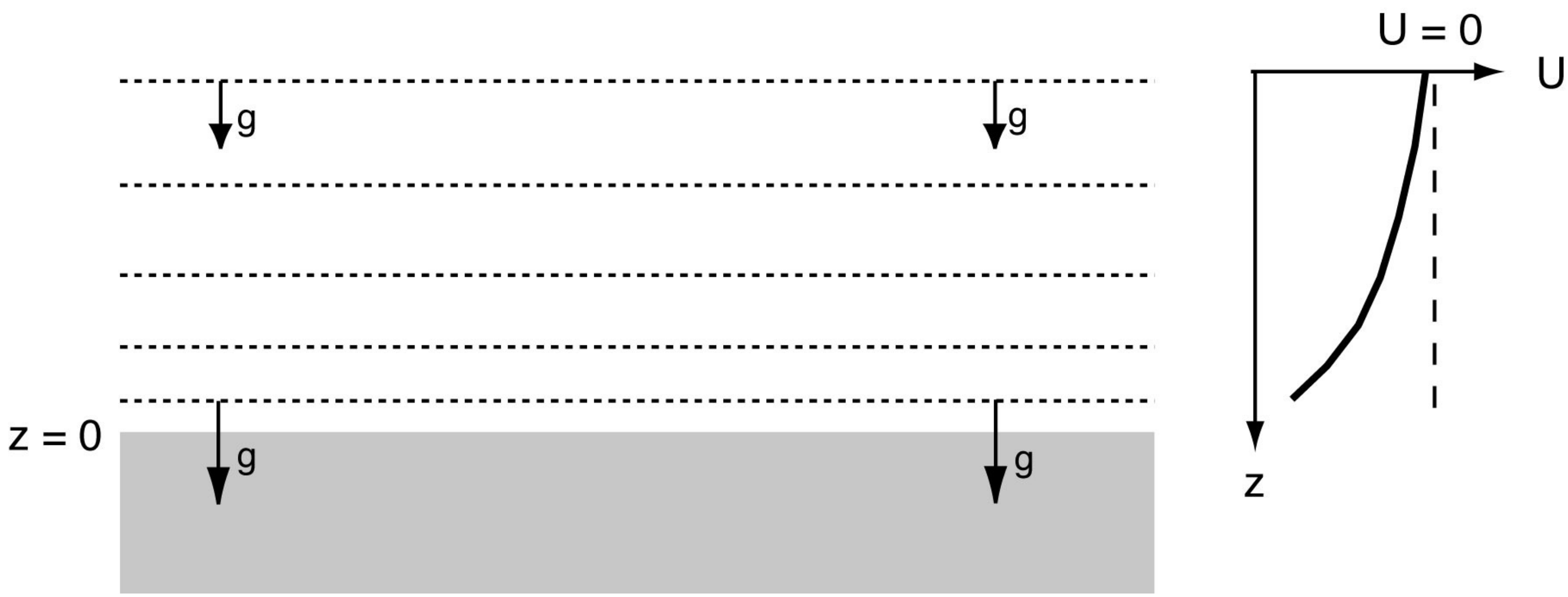
Formula:

$$PE = mgh$$

m = mass of object
g = gravitational acceleration
h = height of object

SI Unit = J (Joules)

Gravitational potential for a homogeneous flat Earth



• ***m*** moves down → falling
- gravitational potential energy converted to kinetic energy. The worked needed to move a mass downward is negative.

• ***m*** moves up → need to do some work to lift m against gravitational force.

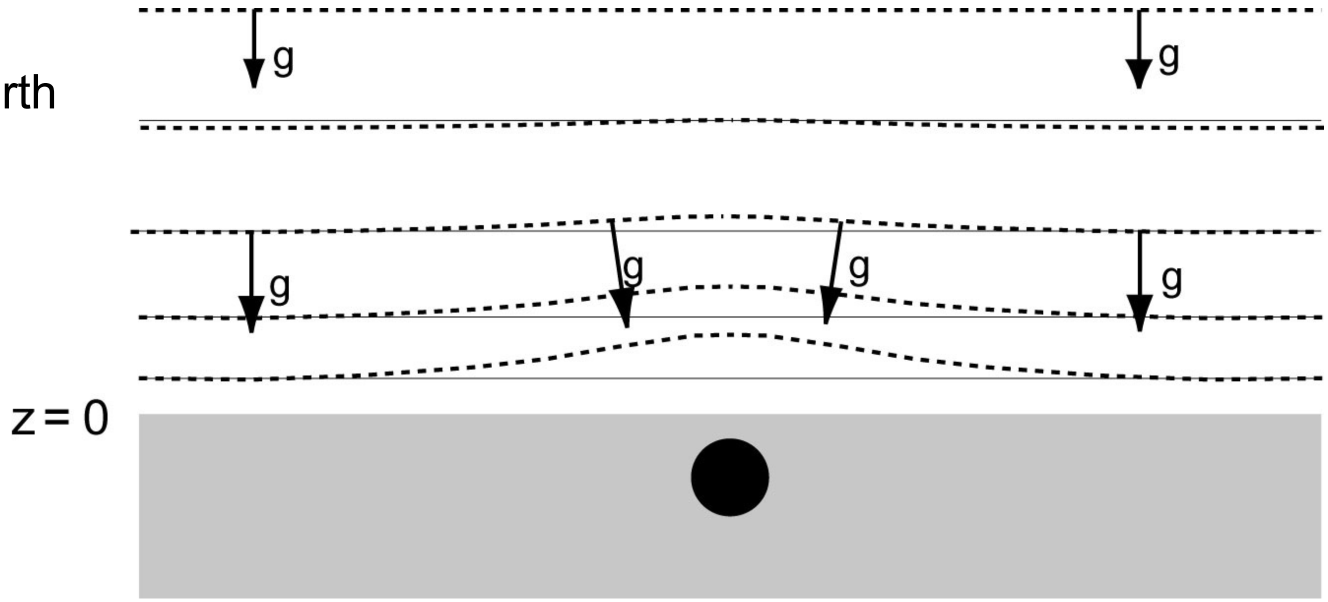
Homogeneous Earth → horizontal equipotential lines

$$g = -\Delta U/\Delta z$$

• g points down (direction of decreasing U)
• g decreases in magnitude with increasing elevation

High density mass below the surface

• homogeneous Earth
+ point mass



Equipotential lines bend upward on top of point mass
(U from point mass + U from Earth)

$$g = -\Delta U/\Delta z$$

• g always orthogonal to equipotential lines (surfaces)

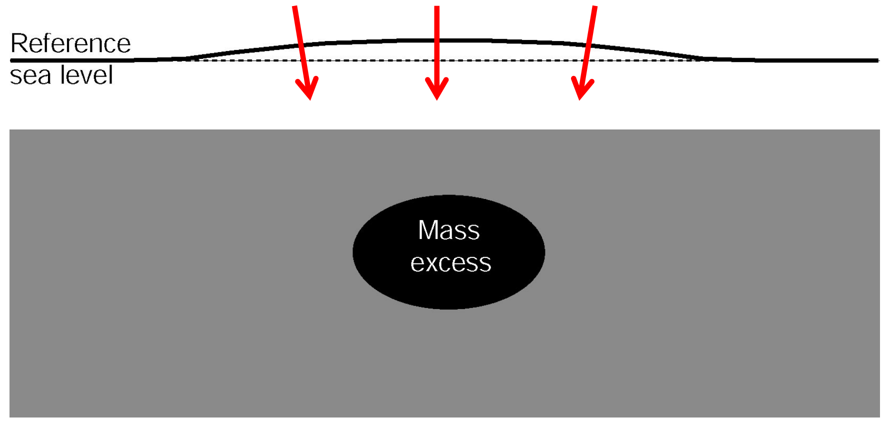
• magnitude of g depends on the gradient of U
→ high gradient = large g

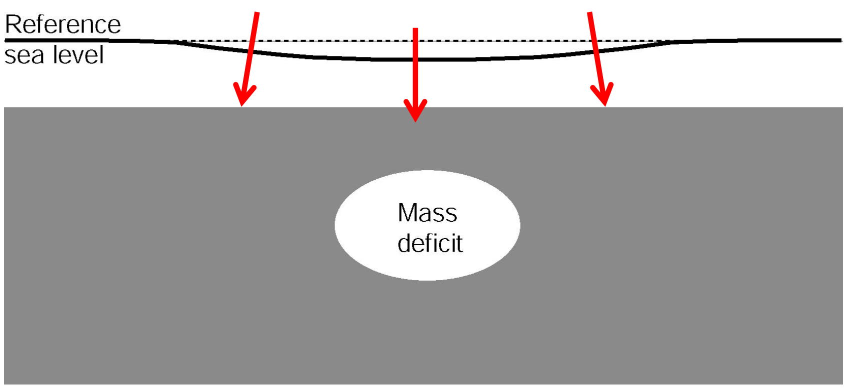
Sea Level and Equipotential Surfaces

Region of high density
(excess mass)

- sea surface will bulge upwards (excess mass “pulls” water toward it)

→ gravity is still orthogonal to the equipotential surface





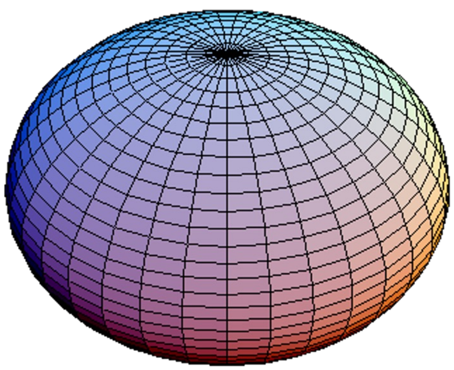
Region of low density
(mass deficit)

- sea surface will have a slight depression

Sea Level and Equipotential Surfaces

Water is not rigid
- will flow downhill until it reaches the lowest elevation

→ surface of the ocean is an equipotential surface
(if there are no currents, winds, tides, etc.)



International reference ellipsoid
• mathematical representation of sea level
- for a rotating, non-spherical Earth with no lateral variations in density structure

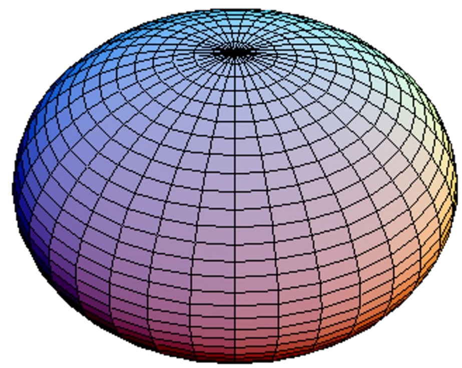
• but Bouguer gravity anomalies indicate that internal density varies laterally…

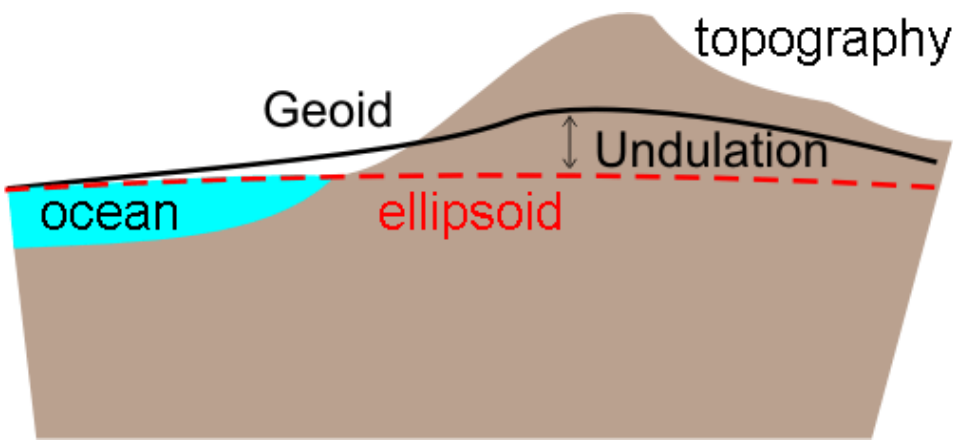
The Geoid

• **equipotential surface that coincides with mean sea level**

- actual sea level for the oceans (measure directly)
- level of water if continents had canals cut through them (derive from gravity measurements)

• if there were no lateral density variations (or topography), the geoid would exactly coincide with the International Reference Ellipsoid





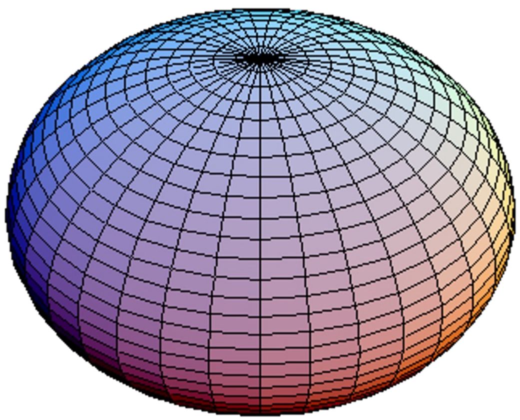
• differences between geoid and I.R.E. are **geoid undulations**

(positive undulation means geoid is above I.R.E.)

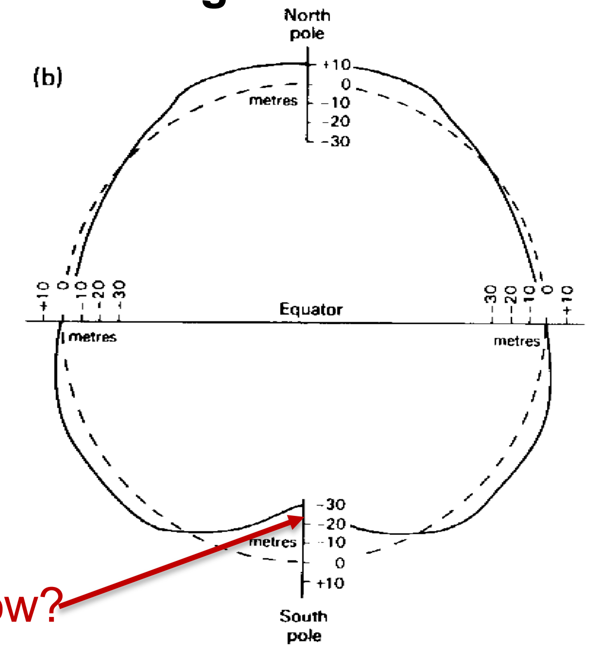
• reflect variations in mass distribution within the Earth

The geoid and the shape of the Earth

International Reference Ellipsoid



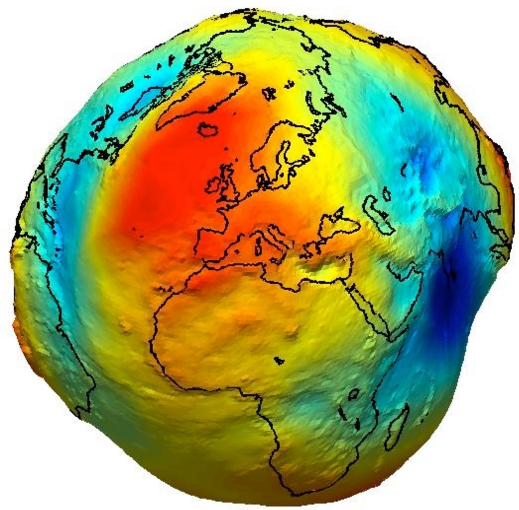
Latitudinally averaged geoid



Why geoid low?

(Figure 5.3b from Fowler)

Geoid

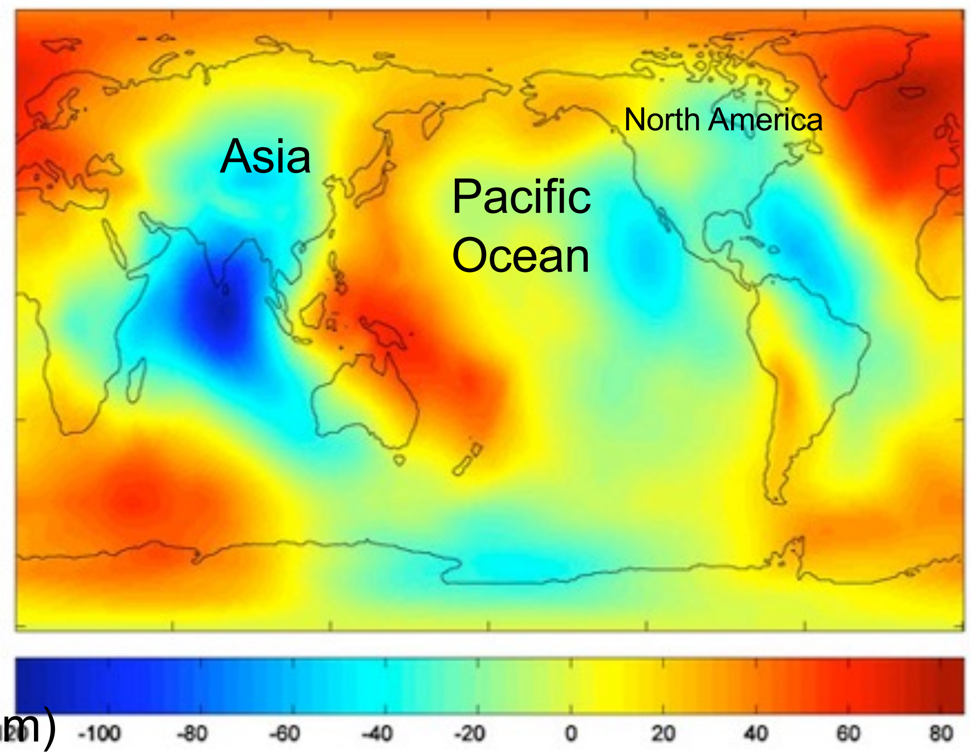


(www.csr.utexas.edu/grace/)

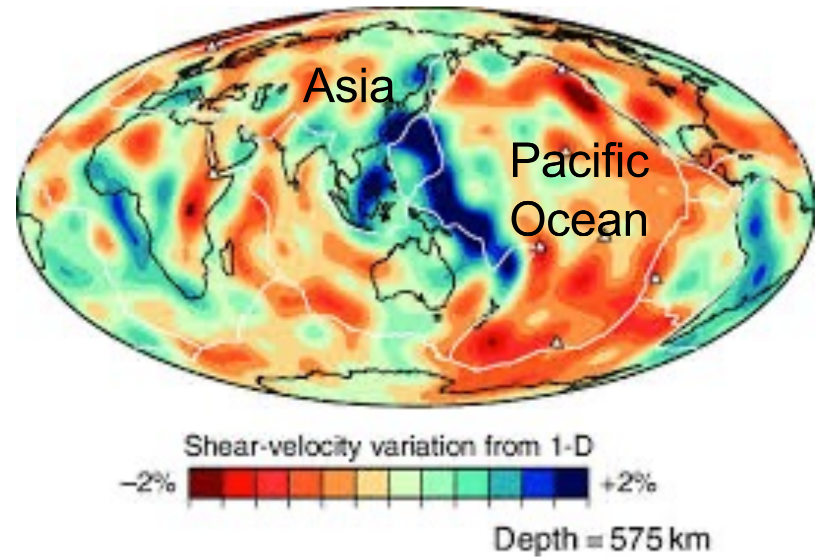
In Edmonton, the geoid (undulation) is: -21.383 m

http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl

Geoid undulations



Wave speed at 550 km depth



Most important fact about Seismic Velocity above:

1. Subducted ocean lithosphere (also called 'slab') are faster and more water rich
2. Low temperatures within slab, causing higher relative density to surrounding mantle
3. ***If we color reverse the above map***, it seems to correlate with geoid undulations (left). Why?

- undulations from -106 m to +85 m
- gravitational acceleration is orthogonal to the geoid
- geoid provides insight into mass distribution in the Earth

(http://www.csr.utexas.edu/grace/ … see also Fig 5.3 in Fowler)

Geoid undulations and Earth structure

• latest refinements to the geoid derived from satellite gravity observations

• e.g., GRACE mission (Gravity Recovery And Climate Experiment)

• two satellites: ~220 km apart, ~16 Earth orbits/day

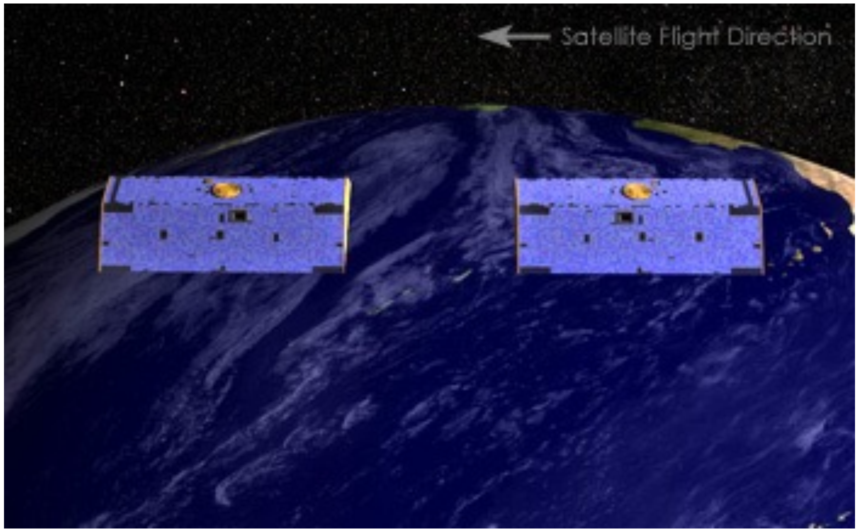
• started at ~500 km altitude…have lost 30 km of altitude

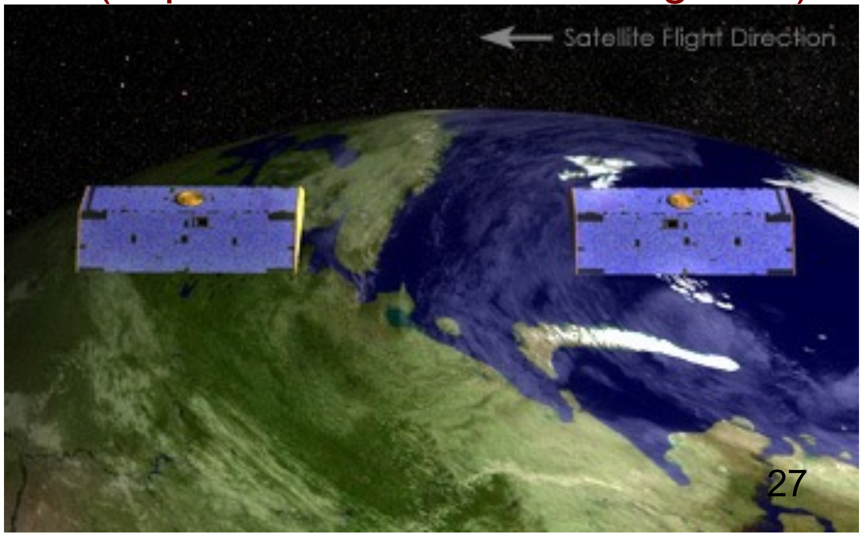
• change in gravity field = change in distance between satellites

• microwave-ranging system to measure distance between them (0.01 mm accuracy → ~1 mgal)

In this case, the distance between these two vehicles are normal until the first one experienced greater attraction from incoming land, thus speeding up but vehicle 2 still goes at its normal speed. Distance increases.

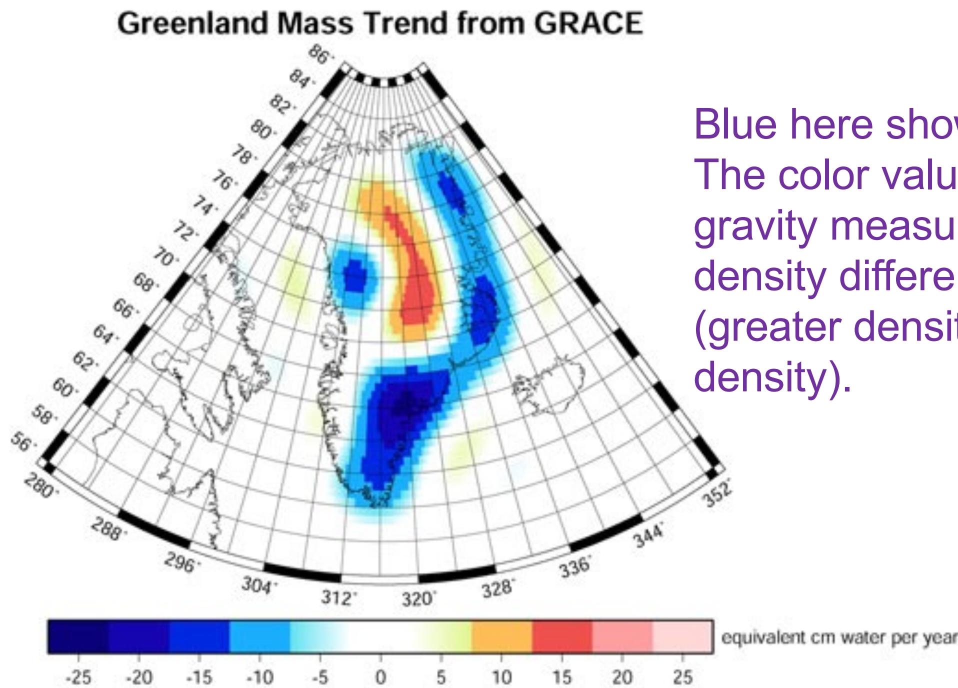
(http://www.csr.utexas.edu/grace/)





Ice sheet loss in Greenland

GRACE satellite data between 2003 and 2005 → net ice loss of ~200 gigatons

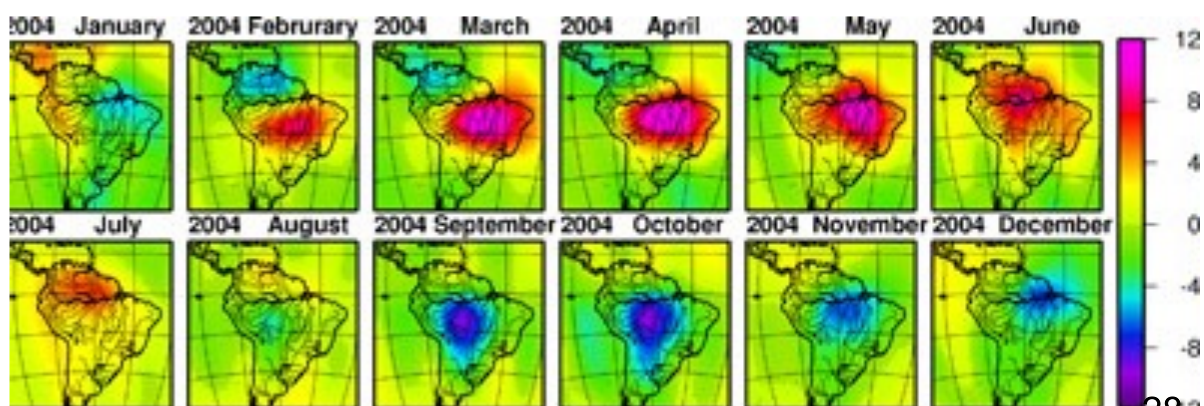


Blue here shows ice, red shows water. The color value is converted from a gravity measure of 'g' that reflect the density difference between water (greater density) and ice (smaller density).

(www.nasa.gov/vision/earth/lookingatearth/greenland_slide.html)

Amazon Basin drainage

(http://photojournal.jpl.nasa.gov/catalog/PIA13243)

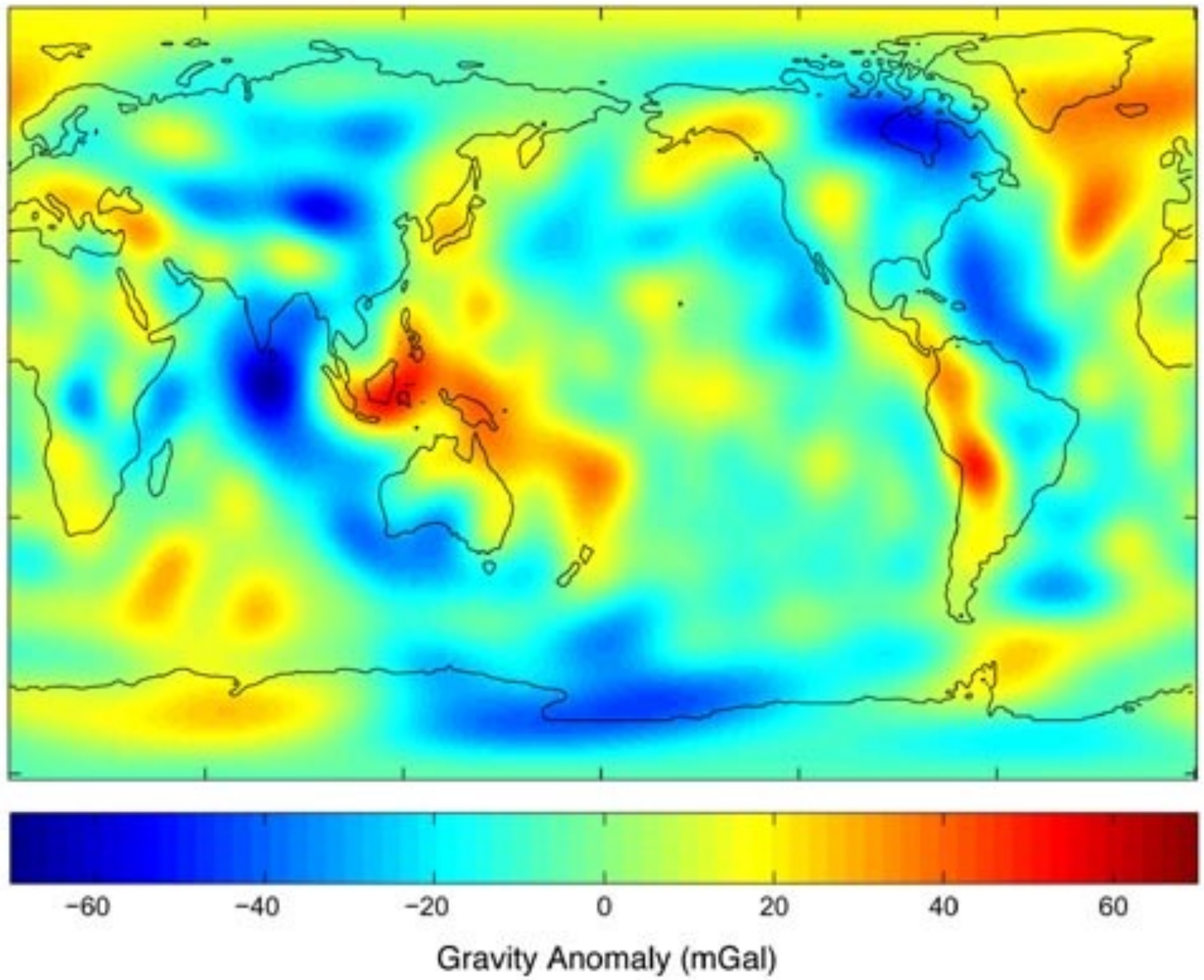


Again, here the red color shows the effective measure of g that is higher during the spring and summer months (due to larger density of water) that becomes smaller (drained) from September to December.

Gravity anomaly data from decades of previous satellite data

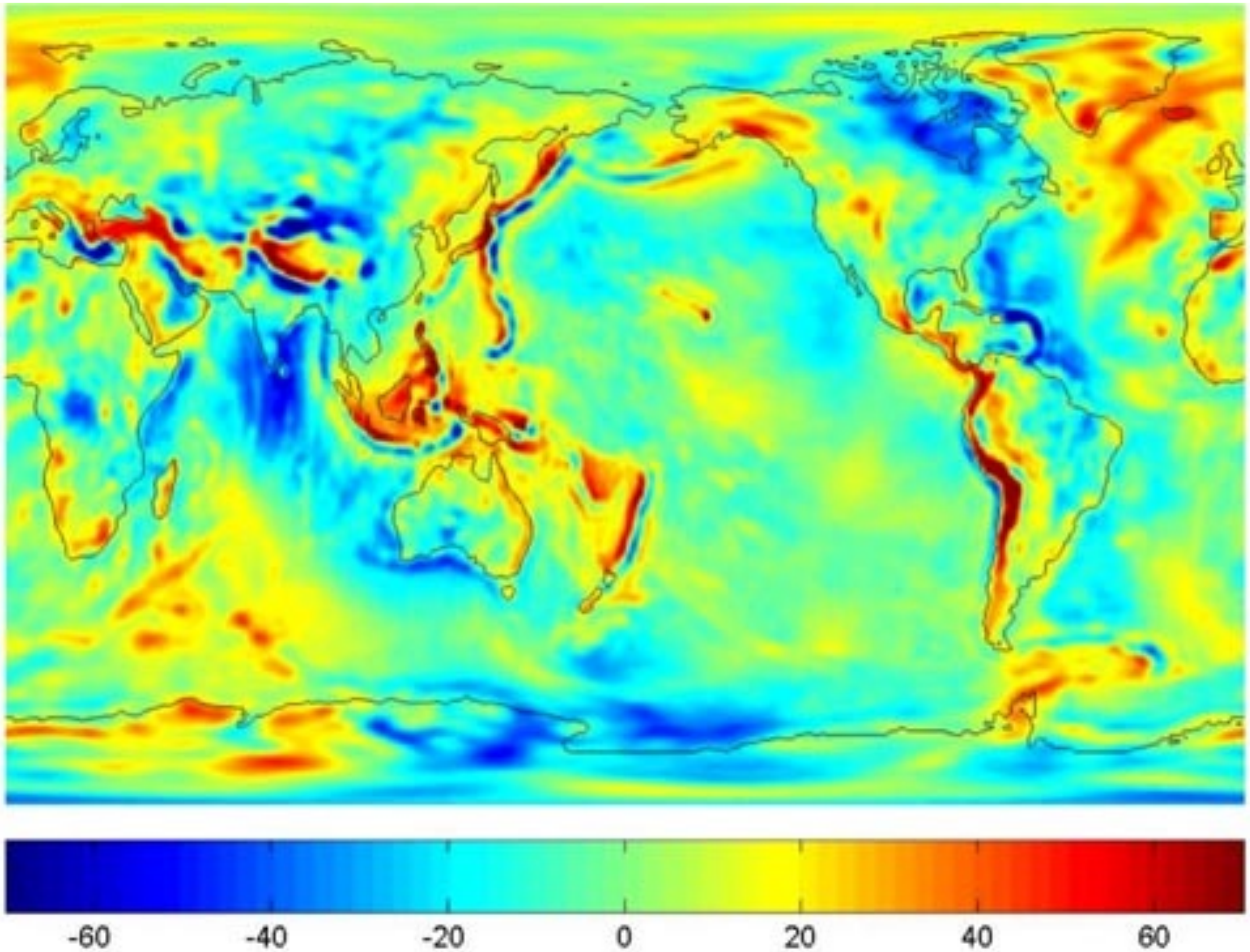
(free air anomalies)

(http://www.csr.utexas.edu/grace/gravity/)



This plot shows the pattern of g after the removal of effect of elevation, that is, some places are elevated compare to others and this effect has been considered. In essence, every location on Earth is technically leveled. The red means greater gravity than the reference value (datum). Note the red anomaly around Japan and Indonesia, which means greater than average density. Why?

Gravity anomalies after 1 year of GRACE observations



(http://www.csr.utexas.edu/grace/gravity/)

Much higher resolution (i.e., more details) are presented here than the earlier observations