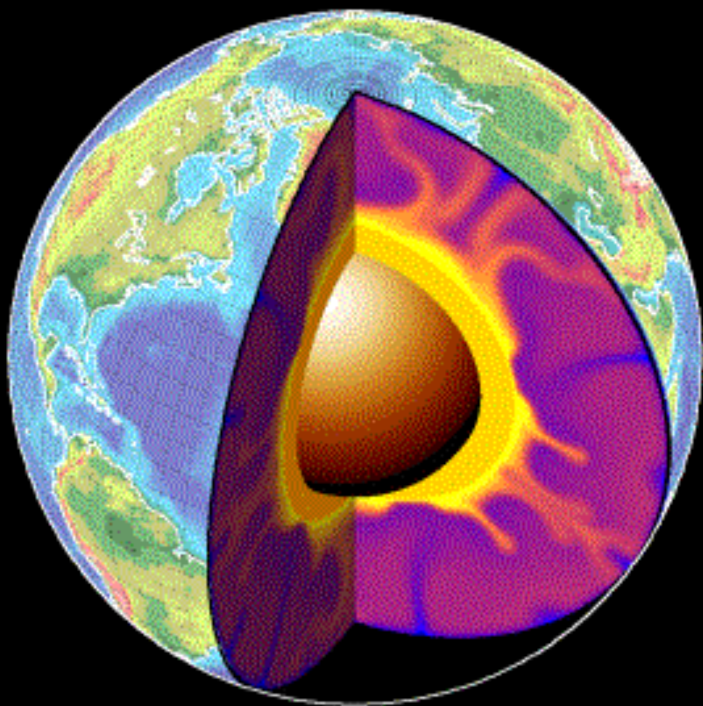


CHAPTER 1.2
Size and shape of the Earth

("Geodesy")

Part I. The non-flat Earth

Evidence (obvious today)

• mountain peaks are lit by the Sun after sunset

• ships disappear below the horizon when they sail across the ocean

• moon looks like a disk – is the Earth the same shape?

• shadow of the Earth on the Moon is circular during lunar eclipses

• stars and constellations vary as one travels south
  - constellations are higher above the horizon
  - some constellations disappear and new ones are visible

•astronauts and satellites have observed a spherical earth from space (just for good measure!)

Radius of the Earth

Approach # 1: Eratosthenes (275-195 BC, ‘founding father of geodesy’). Geodesy--- study of Earth shape, area, etc.

Experiment setup

- conducted on summer solstice
- in Aswan, Egypt, the sun was directly overhead as it illuminated the bottom of a deep well
- in Alexandria, the sun was at an angle of 7°14” (angle_in_deg *3.14159 rad/180 deg = 0.1262 radians)

Additional Information

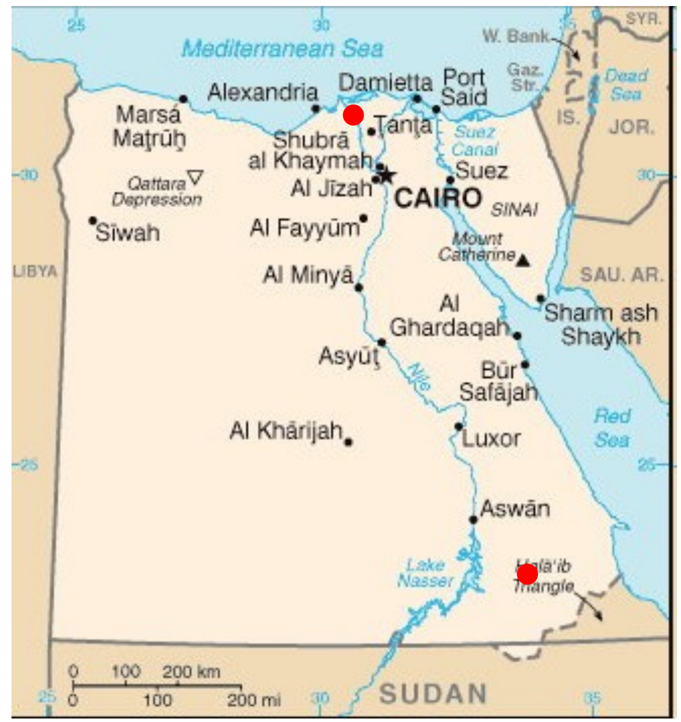
- Aswan (24 deg N) and Alexandria (30.2 deg N) were separated by 5000 stadia (925 km)

1 Stadia = length of U shaped stadium = 185 meters

5000 stadia = 5000 * 185 = 925000 m = 925 km

- both cities are on a north-south line

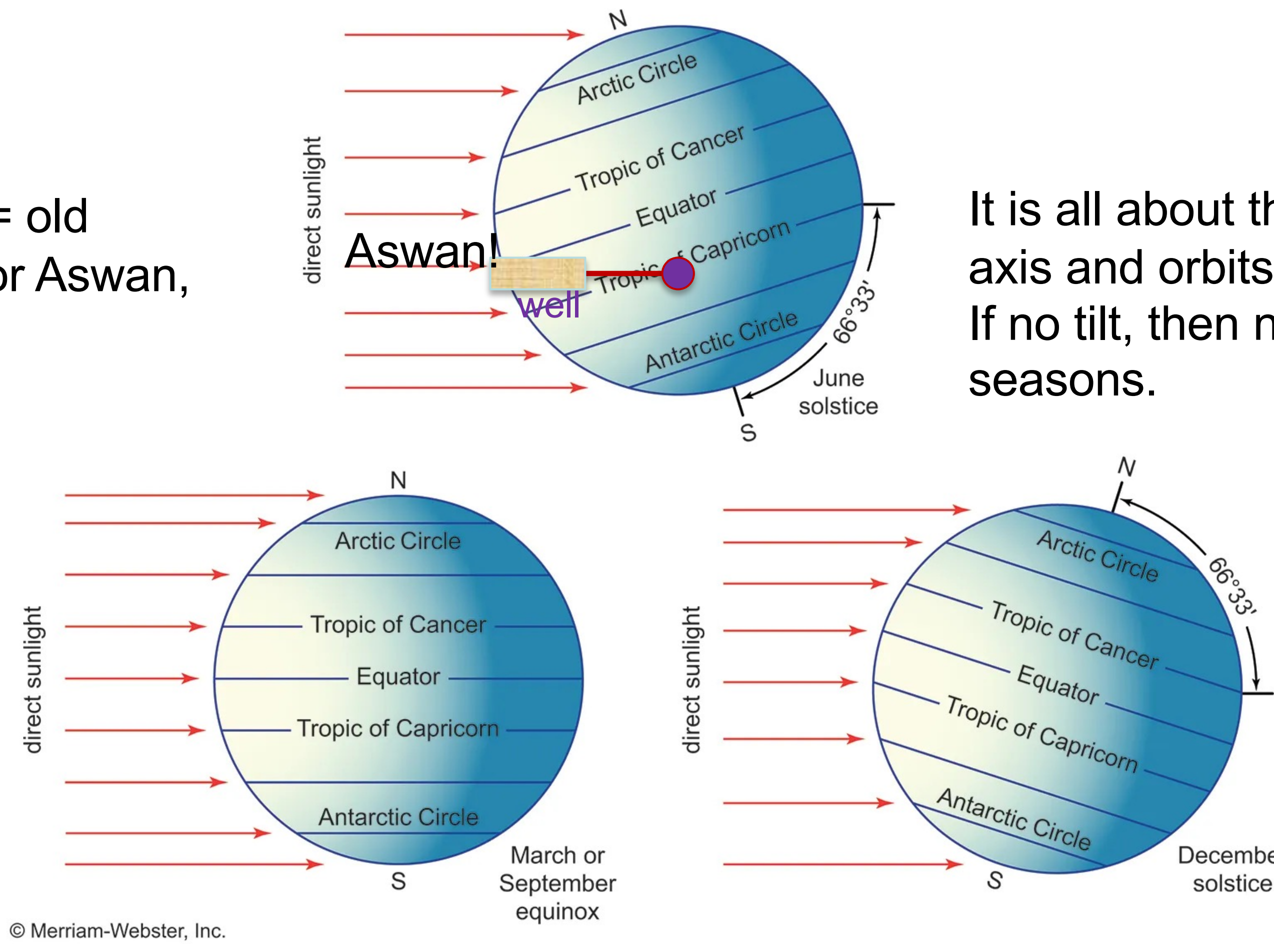
<u>Assumptions</u>: Great circle path (path that cuts Earth in half), purely vertical well in Syene (i.e., Aswan, Egypt; ~true). Sun light is parallel (plane wave assumption).



From Googlemap

Tropic of Cancer (24 deg N)

Syene = old name for Aswan, Egypt



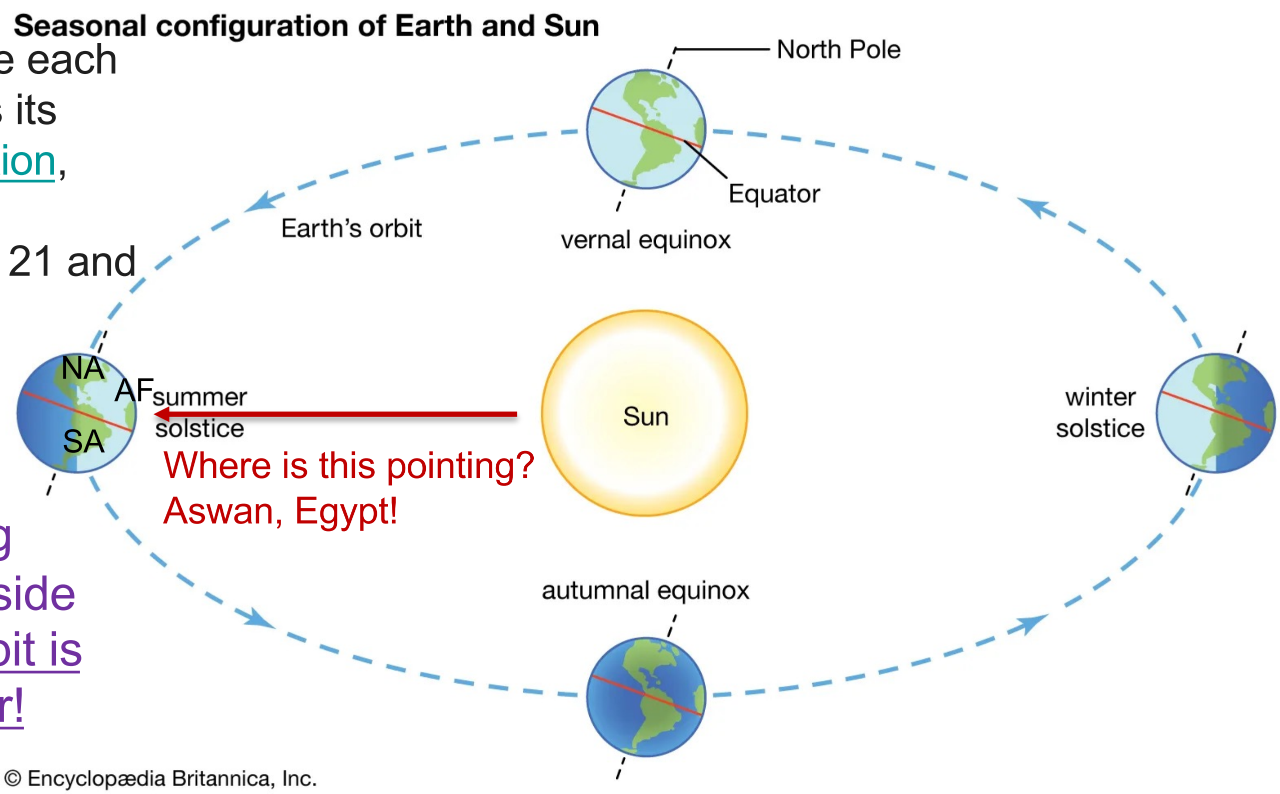
It is all about the Earth's tilted axis and orbits around the sun. If no tilt, then no solstice or seasons.

During summer solstice (in Northern Hemisphere), sun light is directly above Tropic of Cancer at 23 deg N. Aswan ~= 23 deg N

Tilt determines sunlight direction, which determines summer and winter, NOT Orbital distance (which barely changes and even shorter in winter!)

Idea of solstice & equinox

Solstice: the time or date (twice each year) at which the sun reaches its maximum or minimum declination, marked by the longest and shortest days (about June 21 and December 22).



Careful: You are viewing from above, not doing a side view of the orbit. The orbit is very very close to circular!

In northern Hemisphere, we have

Vernal equinox (about March 21): day and night of equal length, marking the start of spring
Summer solstice (June 20 or 21): longest day of the year, marking the start of summer
Autumnal equinox(about September 23): day and night of equal length, marking the start of fall
Winter solstice (December 21 or 22): shortest day of the year, marking the start of winter

Radius of the Earth

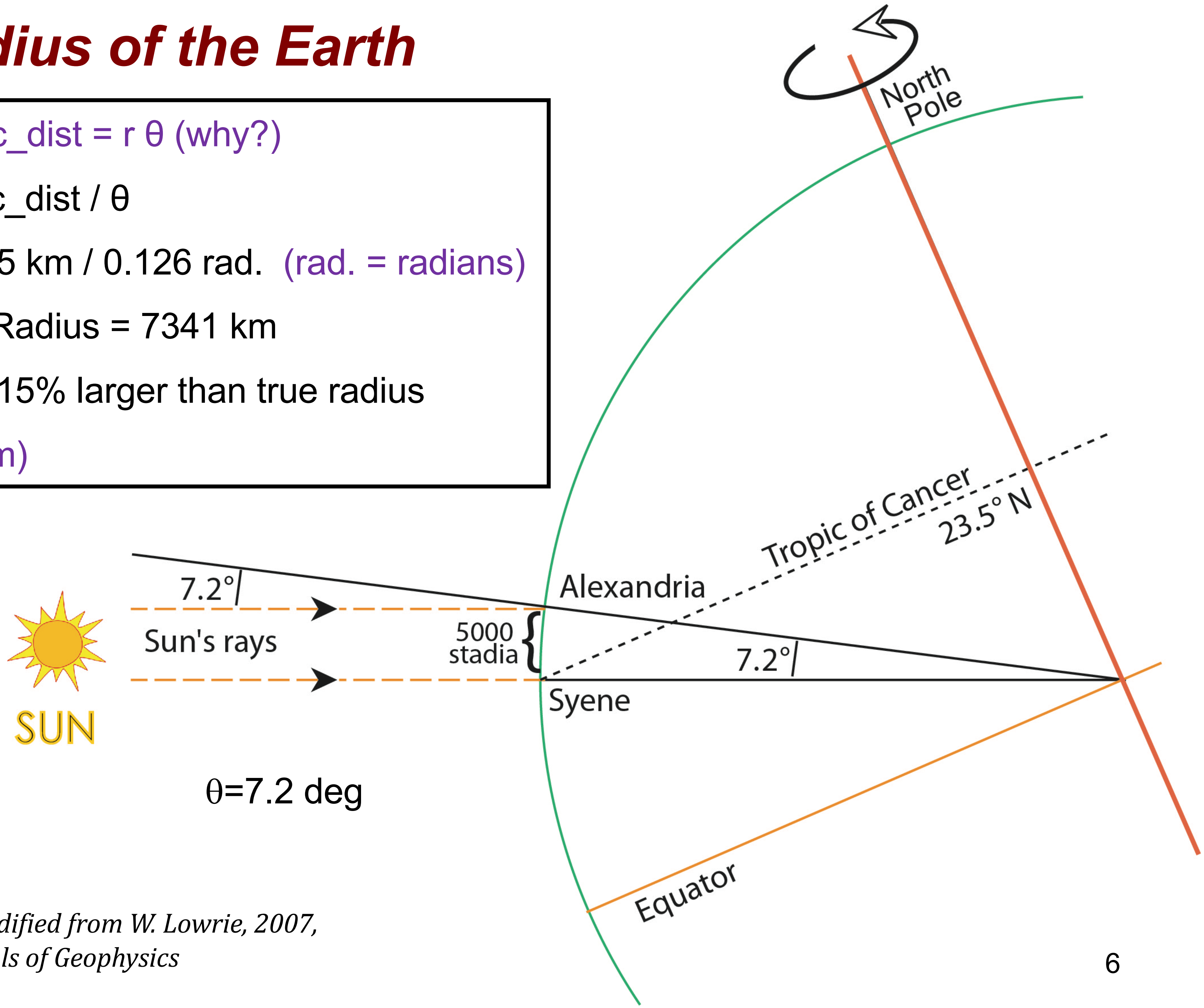
Since Arc_dist = r θ (why?)

→ r = Arc_dist / θ

= 925 km / 0.126 rad. (rad. = radians)

Radius = 7341 km

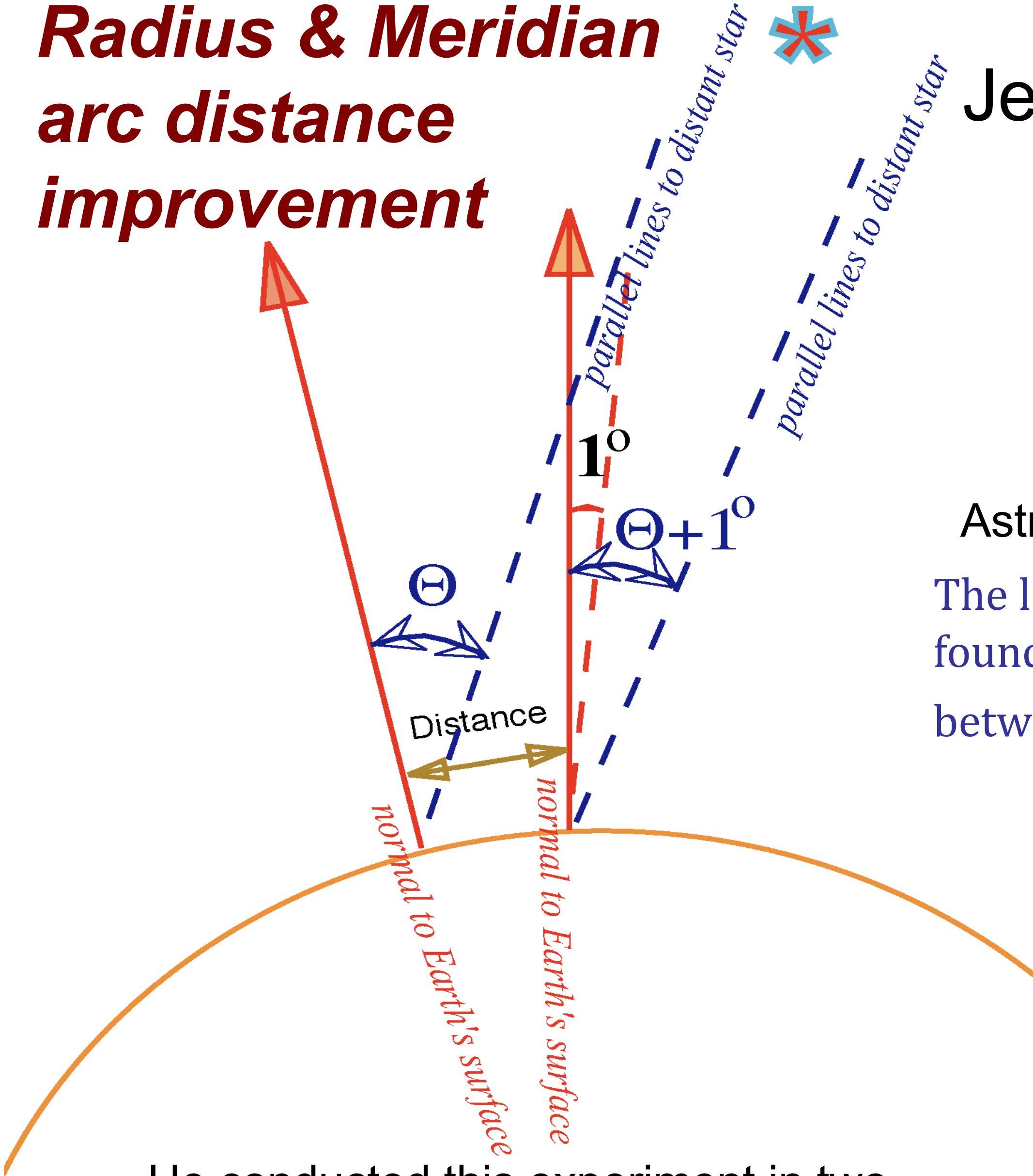
→ Only ~15% larger than true radius (~6371 km)



θ=7.2 deg

Figure is modified from W. Lowrie, 2007, Fundamentals of Geophysics

Radius & Meridian arc distance improvement



Jean Picard

Astronomer (1620-1682)

Jean-Luc Picard
(An inspired Fictional Char., Star Track: The next generation)

Sr. Patrick Stewart

The length of a degree of meridian arc was found with measuring the distance between two points on the Earth.

Known for his impeccable precision down to micrometer scales, the first use of cross-hairs on a telescope. His arc resulted in a radius of 6372 km (vs. 6371 km)

He conducted this experiment in two different positions near Paris.

Figure is modified from W. Lowrie, 2007, Fundamentals of Geophysics

Part II: the non-spherical Earth

After knowing the Earth is round, question starts to emerge:

Is the Earth a perfect sphere?

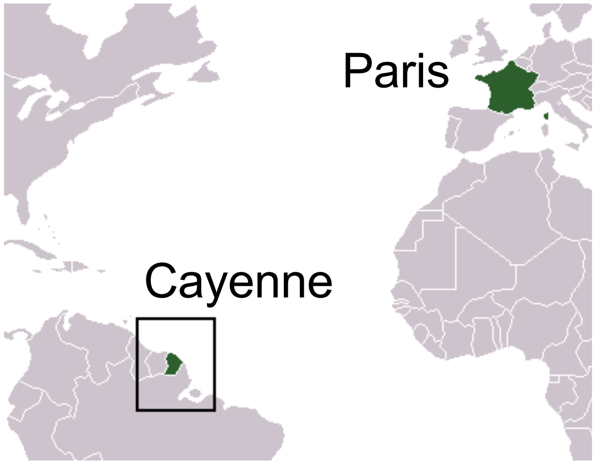
Professor Calculus
("Adventures of Tintin")

From Wiki

Questions emerge due to the 'length of day' problem

Non-spherical Earth

Jean Richer (1672):



-pendulum clock in Cayenne (French Guiana, or Guyanna, 5 deg N ~= equator),

French Guyanna lost 2.5 minutes/day relative to a pendulum clock in Paris

$$T_{Equator} = T_{Paris} + 2.5\,\mathrm{min}.$$

→ pendulum is swinging more slowly in Cayenne

Richer suggested that this was because Cayenne is further from the center of the Earth than Paris.

For truly physical explanation, we have to mention **Sr. Isaac Newton (the Greatest 'Gedanken' experimentalist). The real reason is below:**

For a pendulum, the period (T) is: $T = 2\pi\sqrt{\frac{L}{g}}$

L = length of pendulum, g = gravitational acceleration (~ 9.8 m/s^2)

In **Cayenne**, T is larger → g must be smaller (note: this is a bit raw)

Ultimately, why is the Earth non-spherical?
Answer: Earth does not stand still (rotation!)

Qualitative take: Kepler's Laws of Orbital Motion

Tycho Brahe made important contributions by devising the most precise instrument available before the invention of the telescope for observing the heavens.

Johannes Kepler made detailed studies of the apparent motions of the planets over many years, and was able to formulate three empirical laws. **Both of them also believe that the Earth rotates!**

Tycho Brahe (1546-1601)

Johannes Kepler (1571-1630)

1st Law: The orbit of every planet is an ellipse with the Sun at one of the two foci. (**Conservation of Energy**)

2nd Law: A line joining a planet and the Sun sweeps out equal areas during equal intervals of time. (**Conservation of angular momentum**)

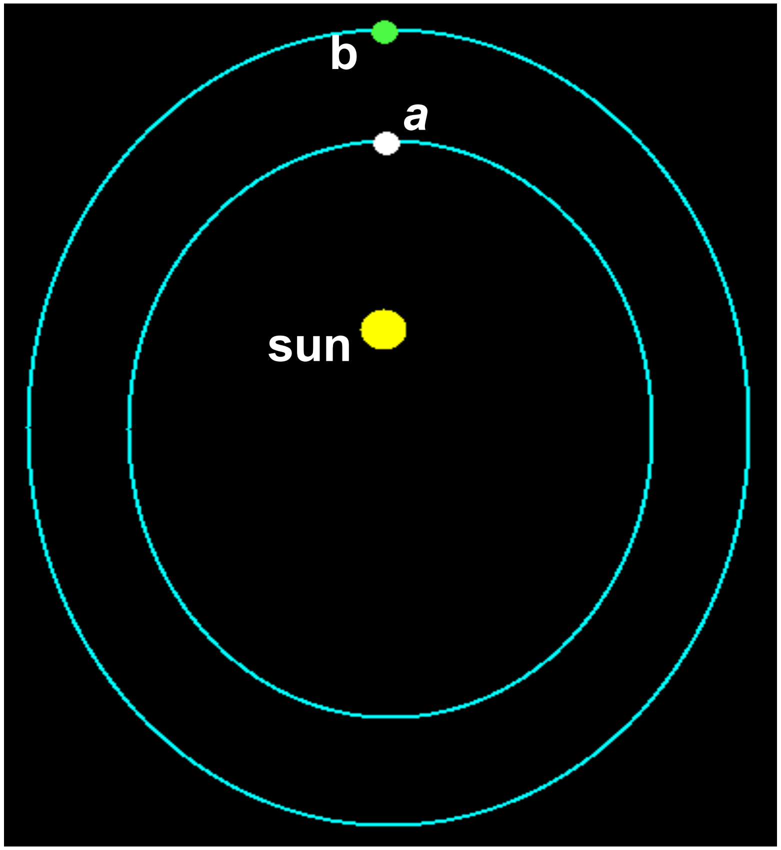
3^{rd} Kepler's Law of Orbital Motion

The squares of the periods of the planets are proportional to the cubes of their semi-major axes:

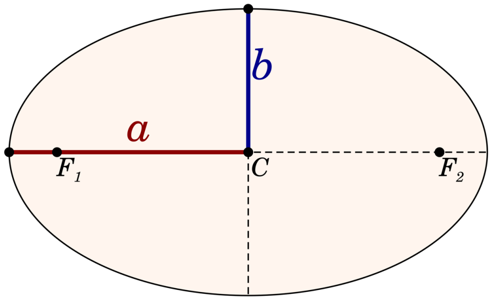
$$\frac{T_a^2}{T_b^2} = \frac{R_a^3}{R_b^3}$$

T_a=orbital period of planet **a**,
T_b = orbital period of planet **b**

This law is also called Law of Periods



How to find ***R***? In the plot below, it is 'a', which is the longest distance between two points that connects to the center of the ellipse.



Semi-major axis = a
Semi-minor axis = b

Note:
There is an 'absolute' form of Kepler's 3^{rd}

Newton's version of Kepler's 3rd Law:

Can be derived by equating gravitation force with centripetal force (can you do it?): $T^2 = r^3[4\pi^2/(GM)]$ Can you derive this?

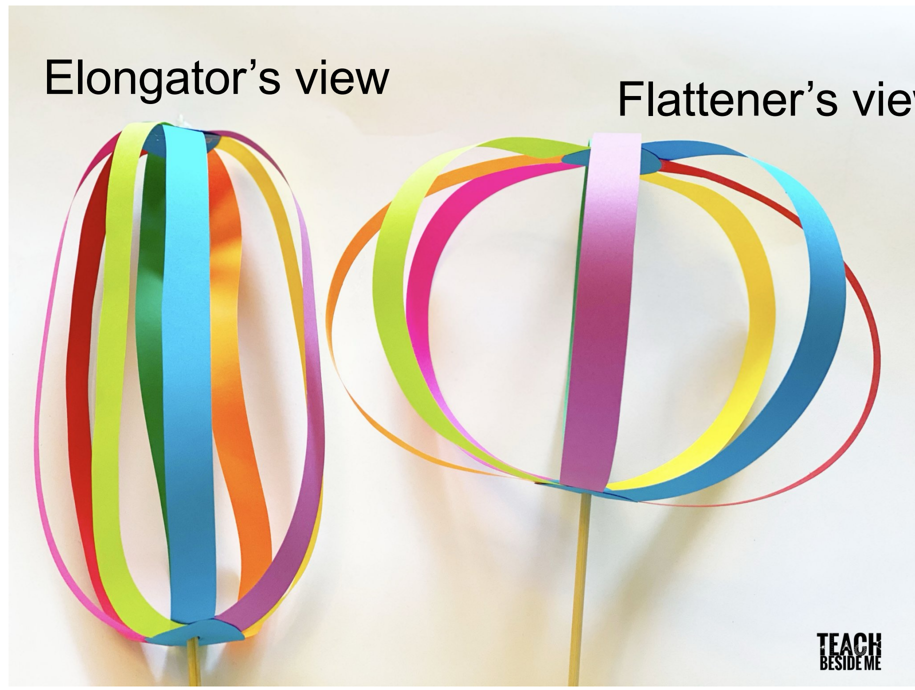
T = period of the rotating body (e.g., planet or satellite)
r = radius of a circle or semi-major axis of an ellipse
G = universal gravitation constant
M = mass of the non-rotating body (e.g., sun or star)

Ex: The semi-major axis of a planet (mass =1.3 x 10^{22} kg) orbits around a star is 6 x 10^{12} m. The mass of the star is 2 x 10^{30} kg. What is the period (T) of the planet?

Answer; 8 x 10^9 sec

Ultimately, why is the Earth non-spherical?
<u>Answer</u>: Earth does not stand still (rotation!)

Twirligig Paper Spinner Toy



This is the best illustration of why Earth is bulging around the equator.

Newton's thought experiment: imagine two dikes are drilled into the Earth center as follows. Then fill them with water. Now start spinning the Earth along axis. Centrifugal/escape force will tend to move masses outwards, hence reducing pressure to center of the Earth. The vertical water column will drop down, reducing height. This becomes the shape/elevation of the polar regions.

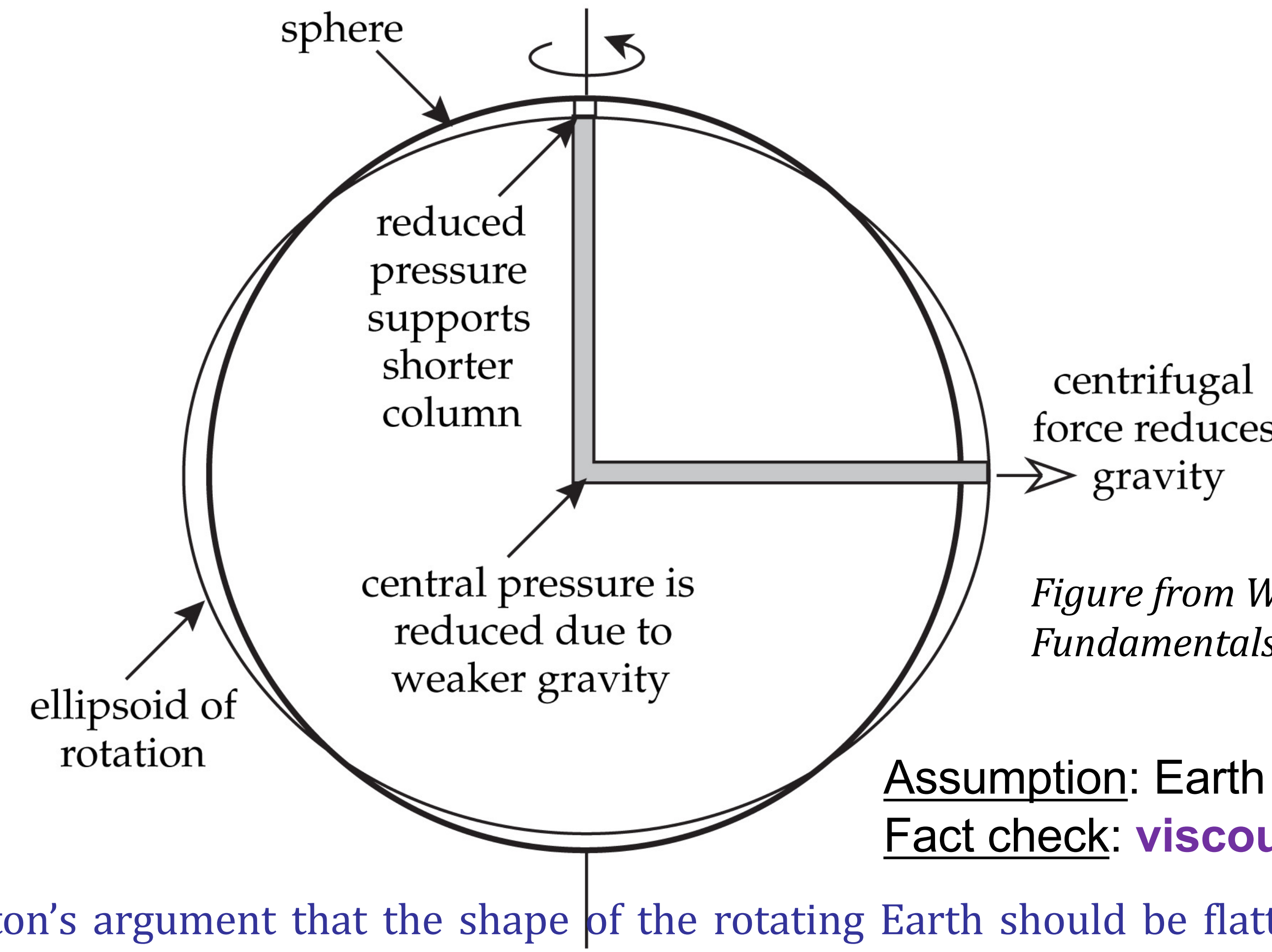


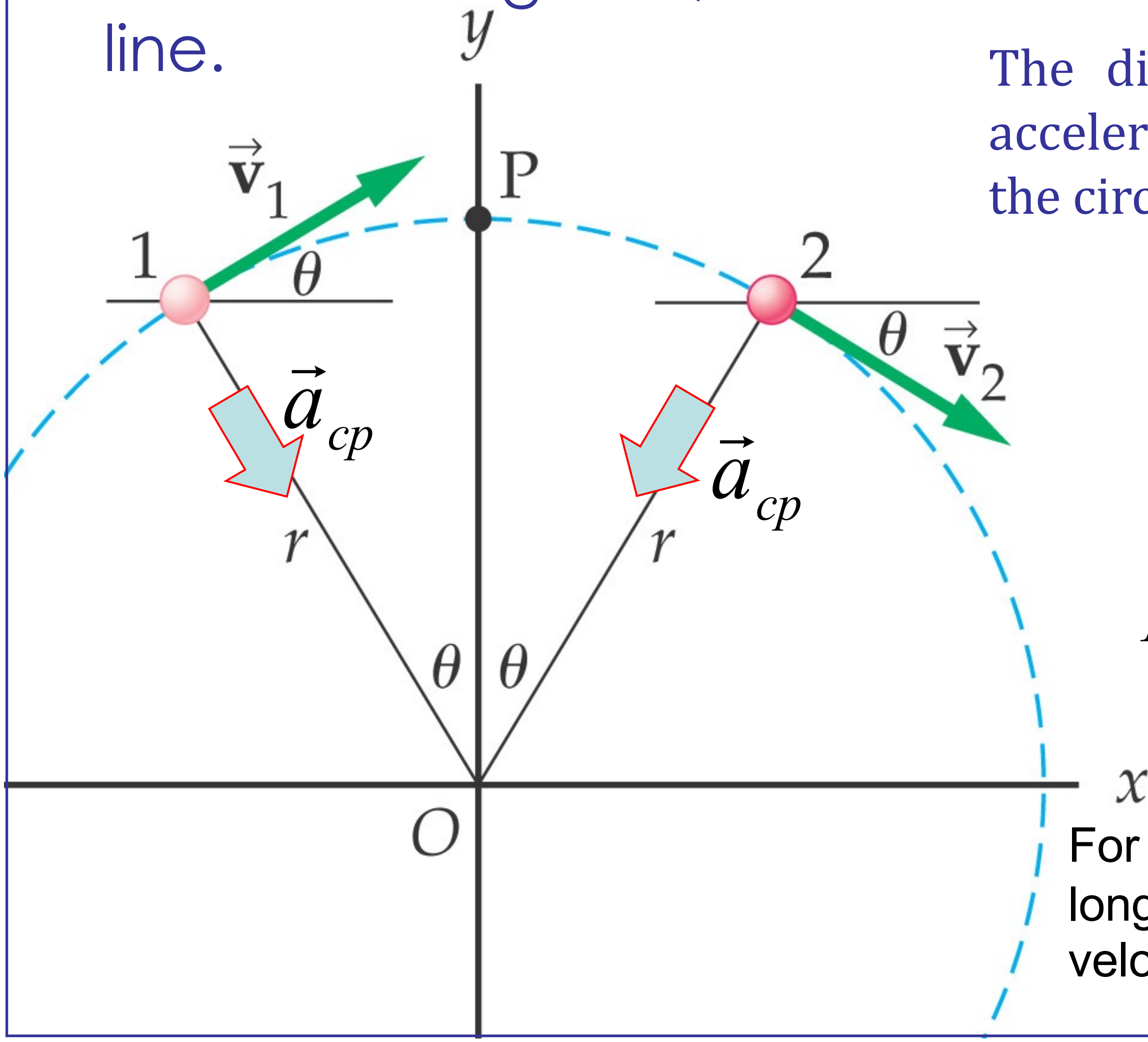
Figure from W. Lowrie (2007) Fundamentals of Geophysics

Assumption: Earth behaves like fluid.
Fact check: **viscous-elastic material**

Newton's argument that the shape of the rotating Earth should be flattened at the poles and bulge at the equator was based on hydrostatic equilibrium between polar and equatorial pressure columns.

So far, we mainly presented kinematic view or Geodetic view, now we will explore more physical explanations (thanks to Huygens & Newton!)

Circular Motion: An object moving in a circle must have a force acting on it; otherwise it would move in a straight line.



The direction of the force and the acceleration is towards the center of the circle:

Angular velocity

$\omega = \Delta\theta/\Delta t$ rad/sec

$v = r\,\omega$ Why? m/sec

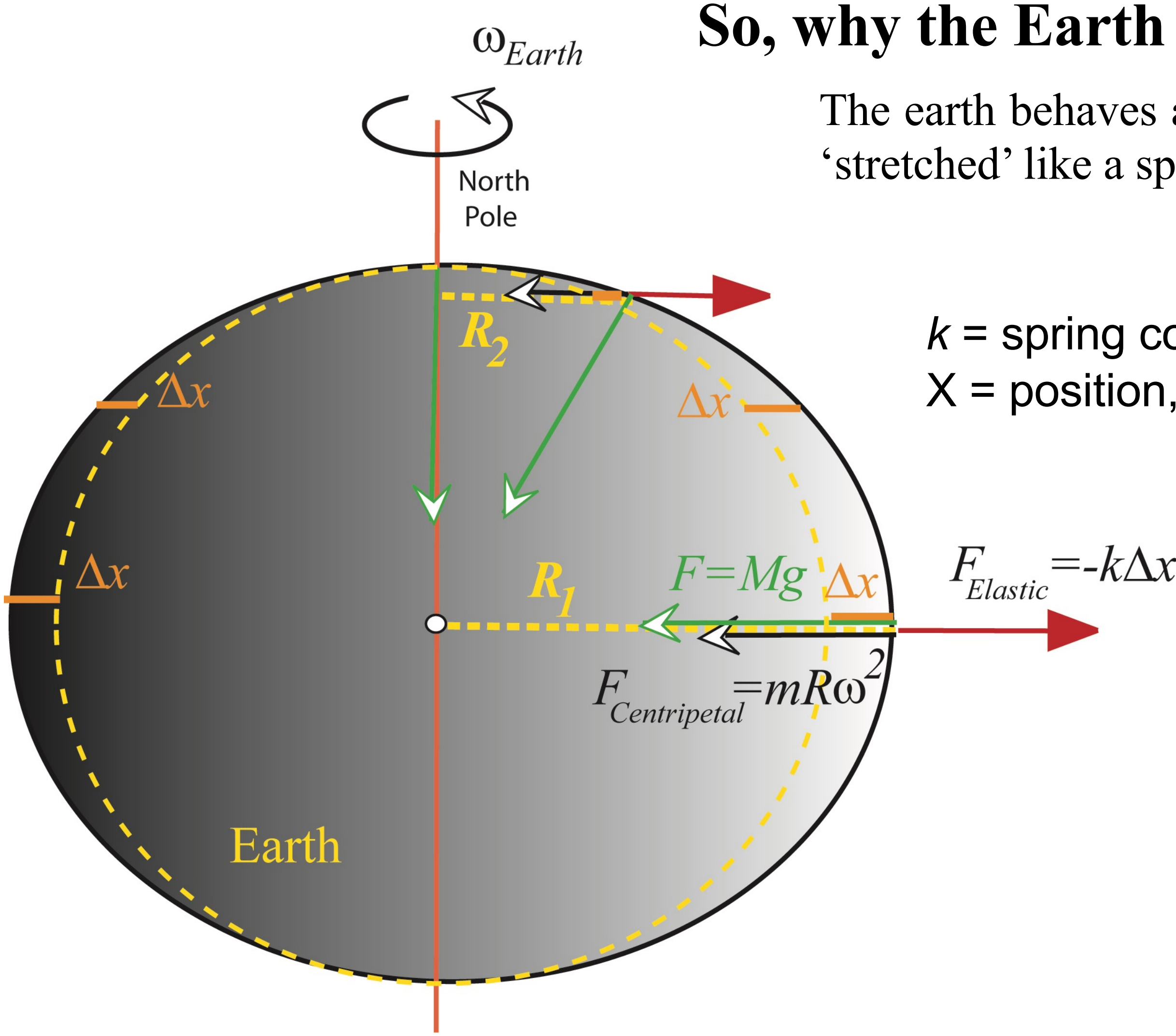
$$F_{cp} = ma_{cp} = m\frac{v^2}{r} = m\omega^2 r$$

newtons

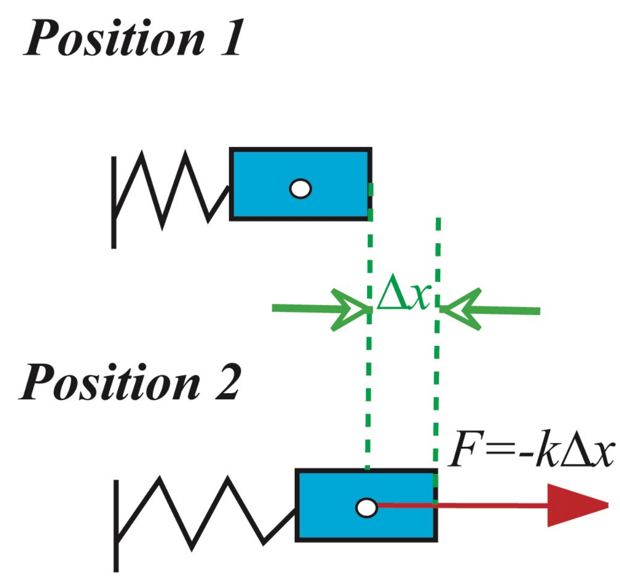
For the same rate of rotation, the longer the string, the greater the linear velocity

So, why the Earth is ellipsoidal?

The earth behaves as an elastic body and can be 'stretched' like a spring while rotating.



k = spring constant (N/m)
X = position, Δx = change of length



The Hooke's law can be applied to describe the amount of the stretching distance. At the same time the stretching force depends on the force of the rotational motion $\boldsymbol{F=ma_{cp}}$ (mass times centripetal acceleration). Such force is created by the gravitational attraction.

Validation of Earth's Ellipticity

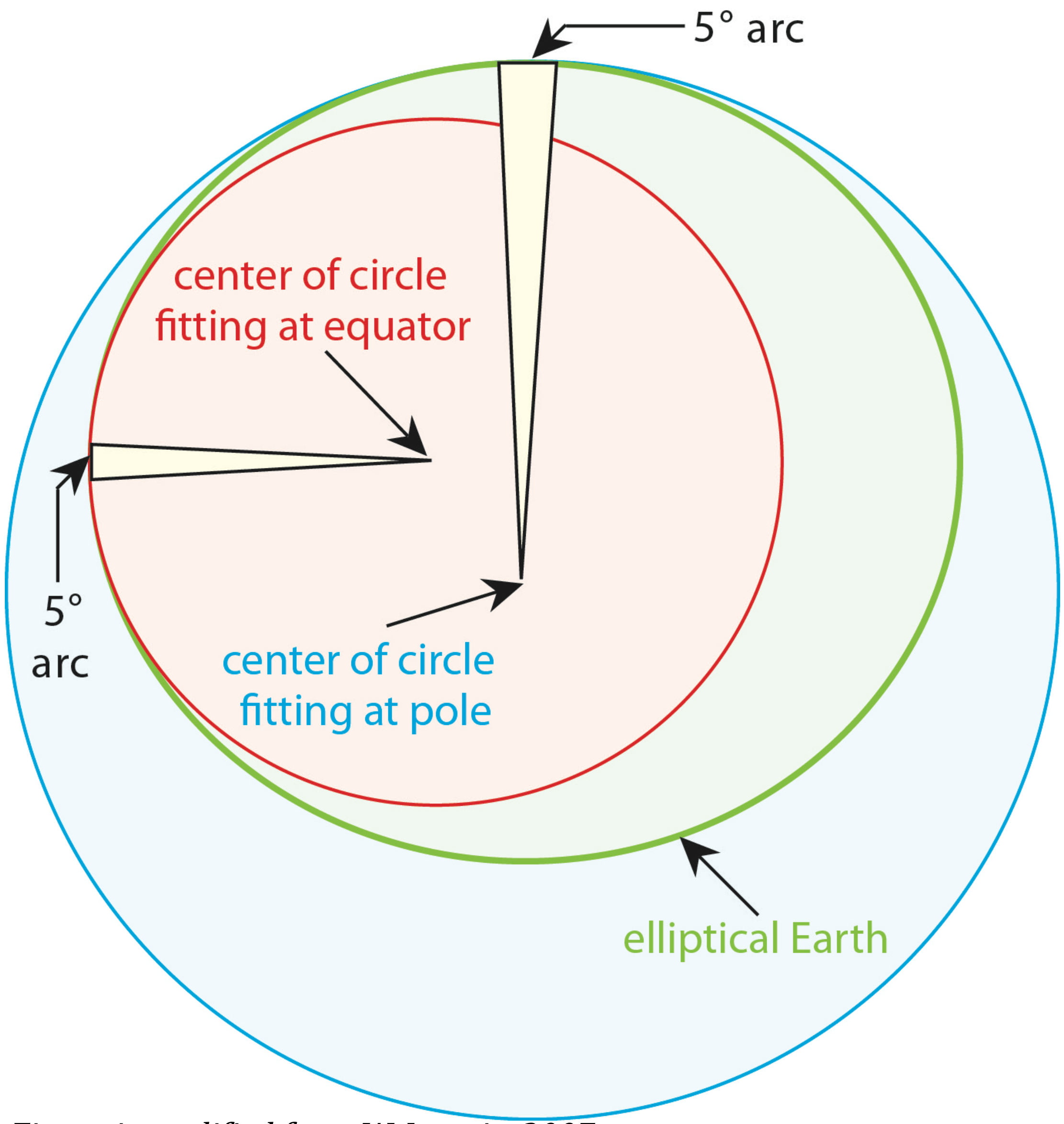


Figure is modified from W. Lowrie, 2007, Fundamentals of Geophysics

There were some errors in determining this and two camps emerged (**flatteners and elongators**).

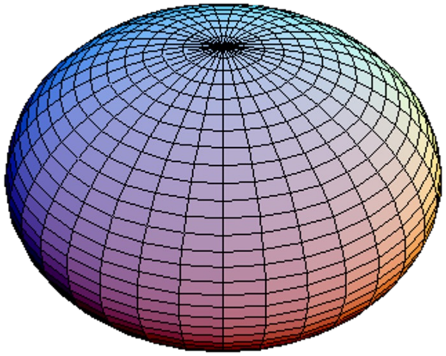
Isaac Newton (1643-1727):

- argued that the Earth must be an oblate spheroid because it is rotating

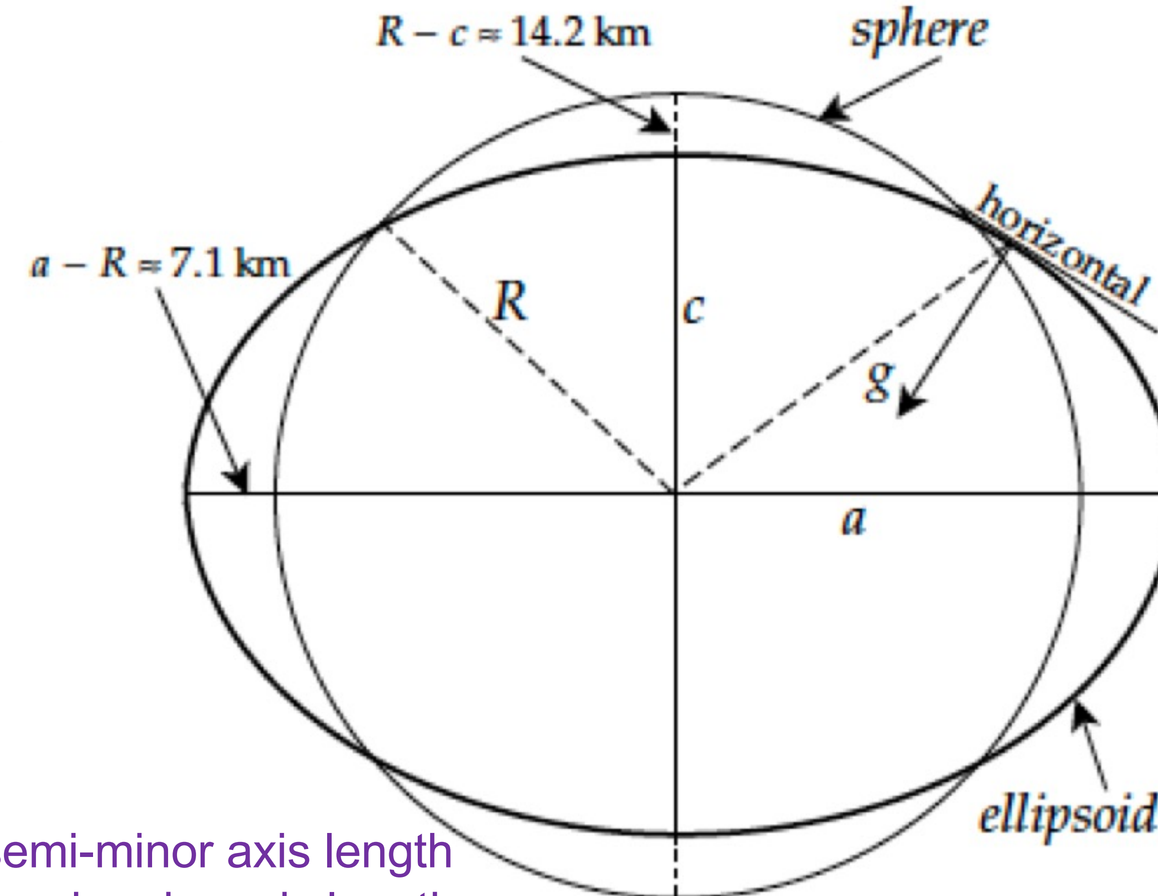
Who won after two surveys in Lapland, Finland (1736-37) and in Peru (equator, 1735-43) ?

'Flatteners' (bulging at the Equator) win

International Reference Ellipsoid



• mathematical description of the shape of the Earth, if its composition was uniform and there was no topography



Polar flattening

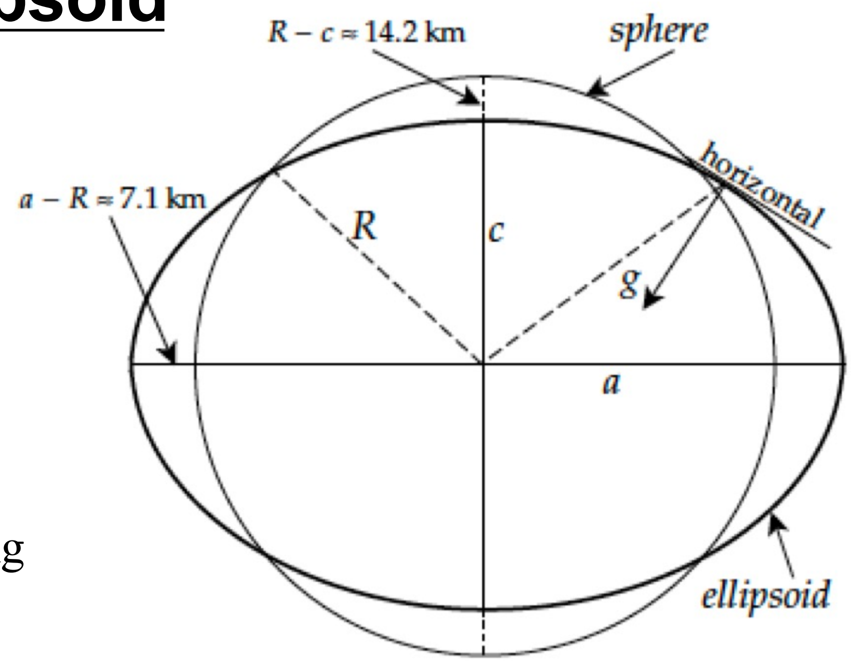
$$f = \frac{a - c}{a}$$

***f* = ellipticity**
(equals 0 for circle)

c = semi-minor axis length
a = semi-major axis length

International Reference Ellipsoid

Questions on the numbers here?



Latitudinal dependence of the radius of an oblate spheroid of ellipticity f (or polar flattening) is (simplified formula):

$$r = R_{equator}\left(1 - f\sin^2\lambda\right)$$

Ellipsoids have been revised with increasing volumes of higher quality data:

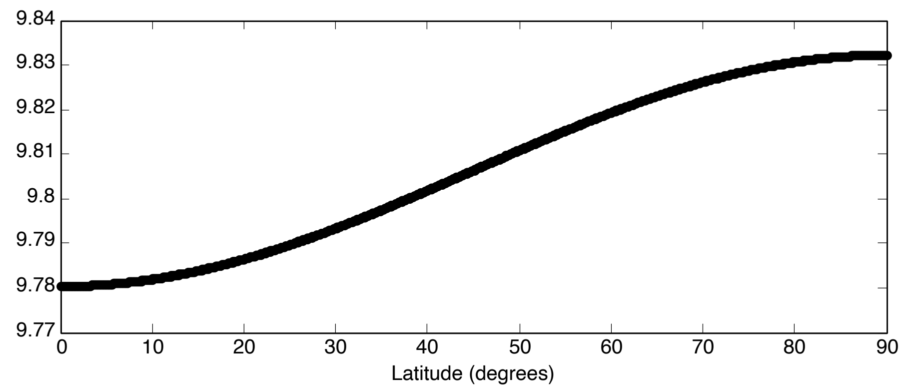
Ellipsoid reference	**Semi-major axis *a***	**Semi-minor axis *c***	**Inverse flattening *1/f =a/(a-c)***
GRS 67	6,378,160.0 m	6356,774.5 m	298.24717
GRS 80	6,378,137.0 m	6,356,752.3 m	298.25722
IERS 2003	6,378,136.6 m	6,356,751.9 m	298.25641

c = semi-minor
a = semi-major

GRS67 equation

The theoretical variation in gravity with latitude given by the Geodetic Reference System for 1967 (GRS67) equation:

$$g(\theta) = 978031.846 * \left(1 + 0.0053024 \sin^2 \theta - 0.0000059 \sin^2 2\theta\right)$$



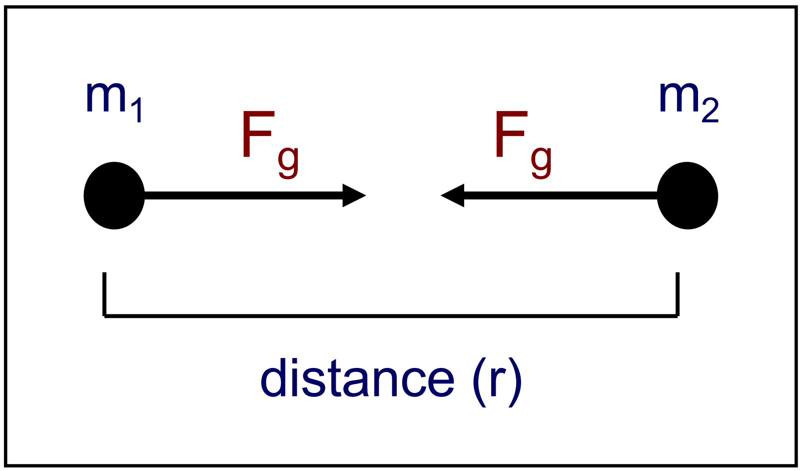
For CCIS (my office): latitude = 53.52589ºN

FROM GRS67, g = 981,379.88 mgal = 9.8138 m/s^2

Newtonian Gravitation

Gravity: force of attraction between objects with mass

Consider two objects with mass m_1 and m_2:



Newton's theory of gravitation:

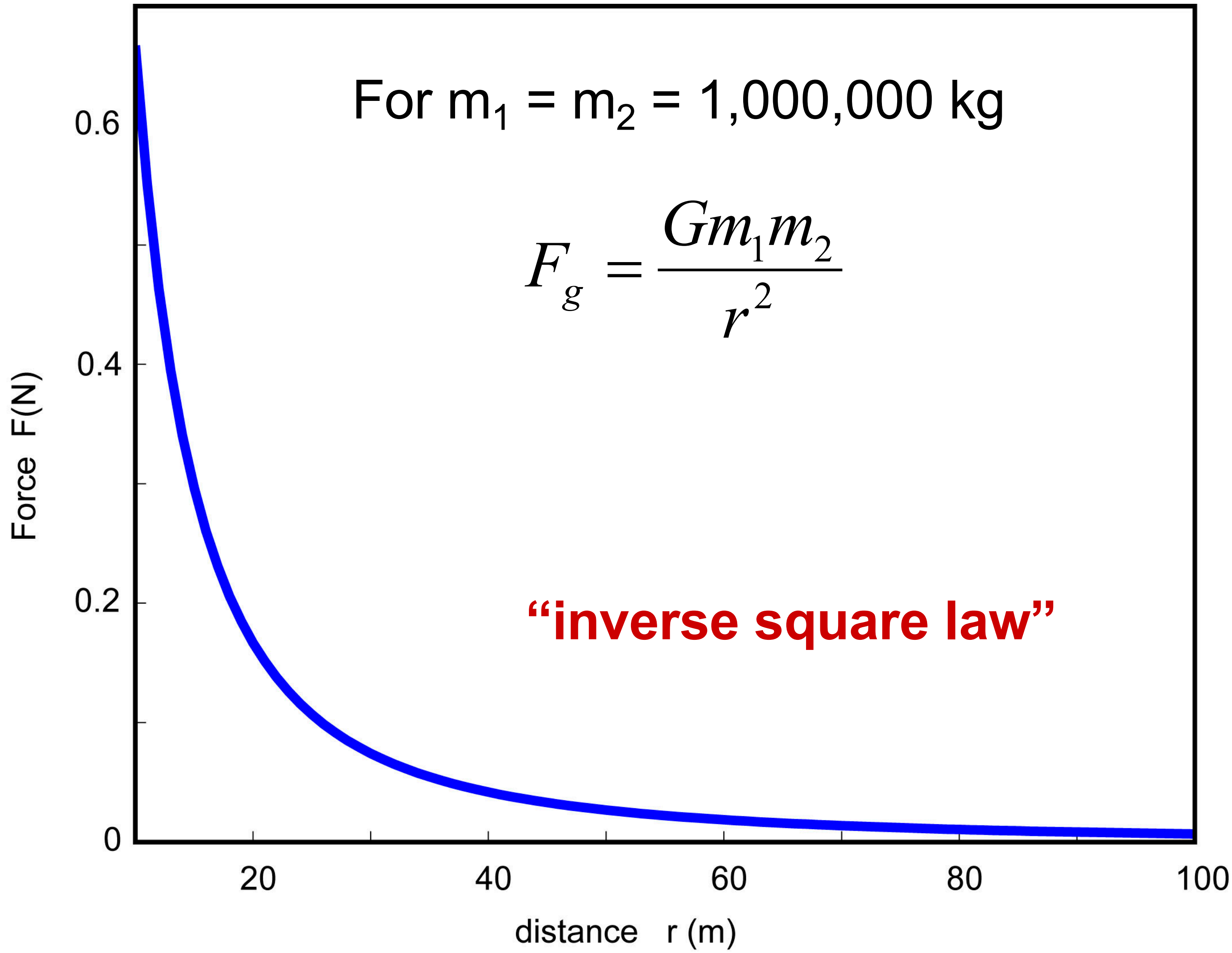
$$F_g = \frac{Gm_1m_2}{r^2}$$

G is the universal gravitational constant ("big G")

$G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

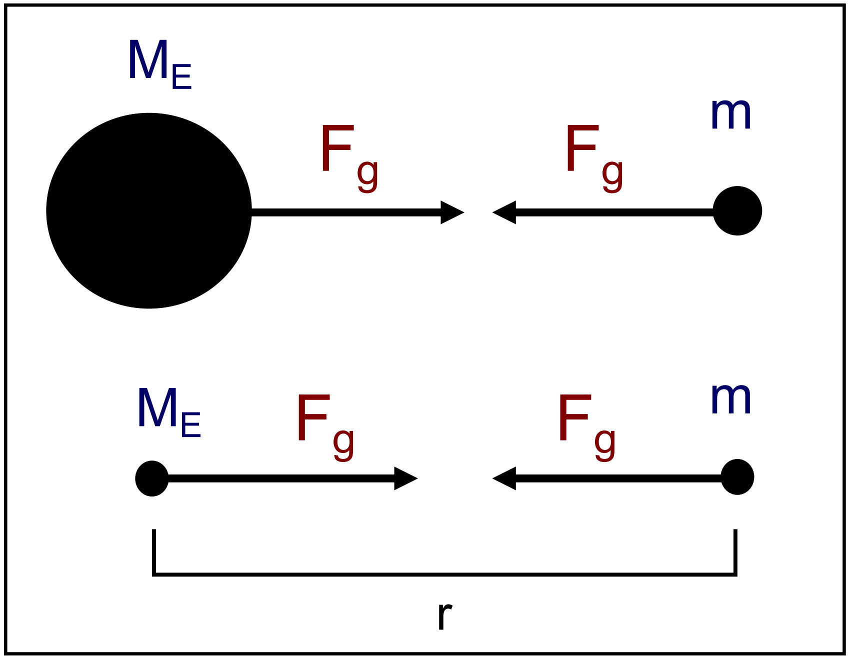
(or $N\ m^2\ kg^{-2}$)

Variation in F_g with distance



Consider the Earth (mass M_E) and a small object (mass m)

For a *spherical, non-rotating, homogeneous* Earth → the force of gravity (outside the Earth) will be the same as if all mass were at the centre.



Consider mass m. Gravity is a force. This causes an acceleration according to Newton's third law of motion:

$$F_g = ma \xrightarrow{\textit{rearranging}} a = \frac{F_g}{m}$$

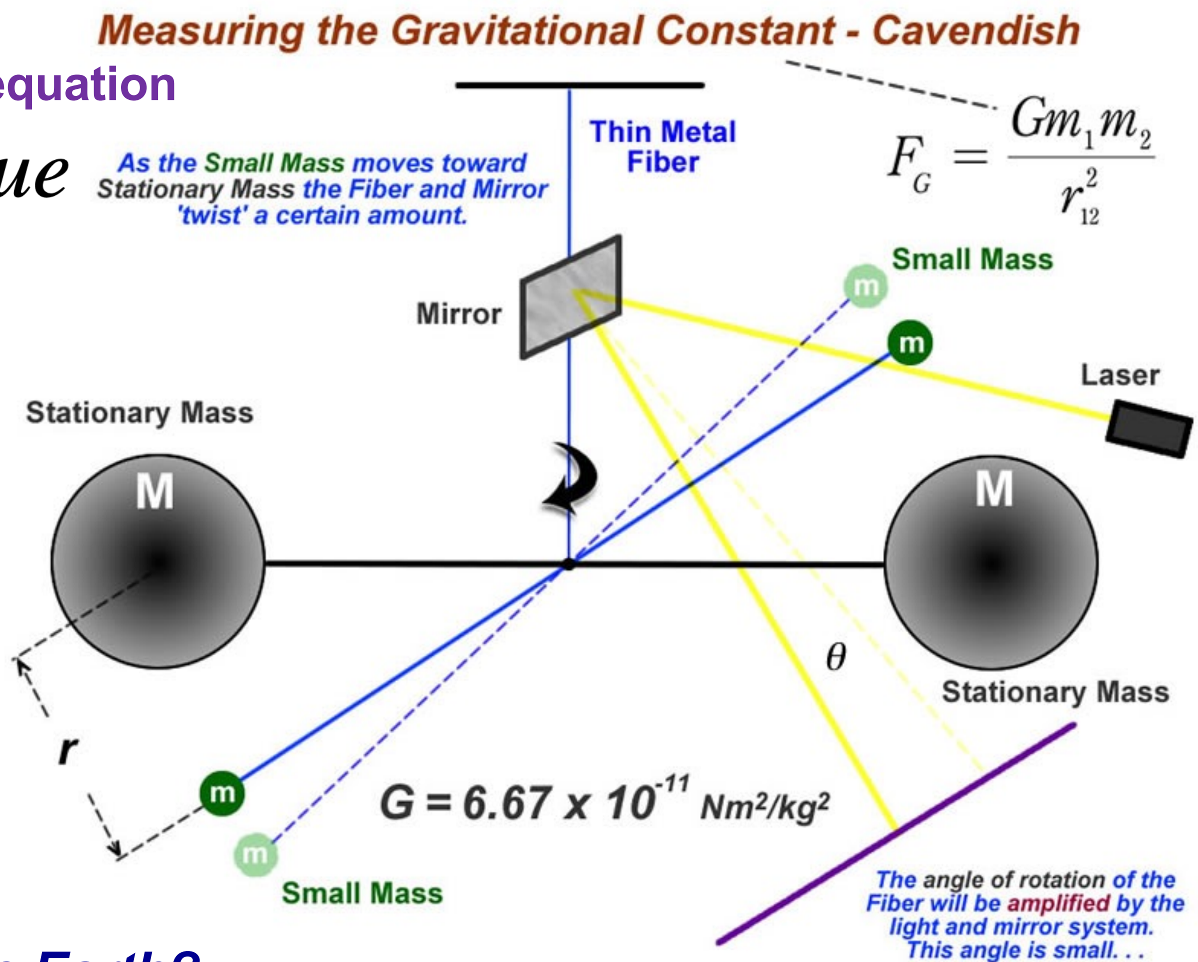
"Weighing the Earth" ----- Lord Cavendish

Measurement of Gravitational Constant/ Mass of the Earth (first performed in 1797-1798 by Lord Cavendish, performed using lead balls of 1.6 pounds and 348 pounds, the former are suspended on 6 ft bar and free to rotate, *accurate to within 1%*)

Lord Cavendish Used this equation

$$k\theta = LF = torque$$

θ = angle of rotation
k = rotational stiffness Coef.
L = length of wood bar
F = gravitational force



Why called Weighing the Earth?

http://sdsu-physics.org/

Lord Cavendish determined G, so what is the mass of the Earth?

Gravitational constant: $G = 6.672598 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

Henry Cavendish experiment, 1798

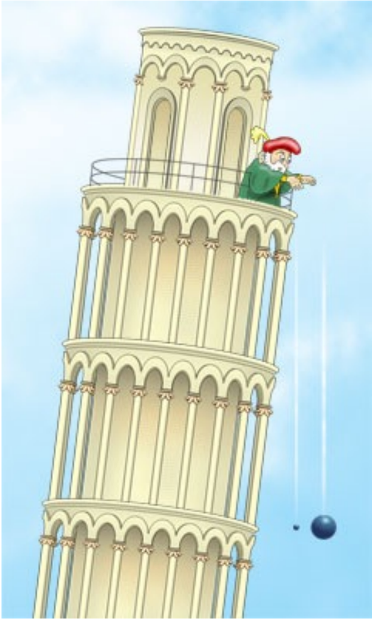
Average Radius:
$R_E = 6371$ km

Gravitational acceleration:

$g = 9.81\ m/s^2$

$$g = \frac{GM_E}{R_E{}^2} \longrightarrow M_E = \frac{gR_E{}^2}{G} = \boxed{5.97 \times 10^{24}\, kg}$$

Gravitational Acceleration



Acceleration of small object: $$a = \frac{F_g}{m}$$

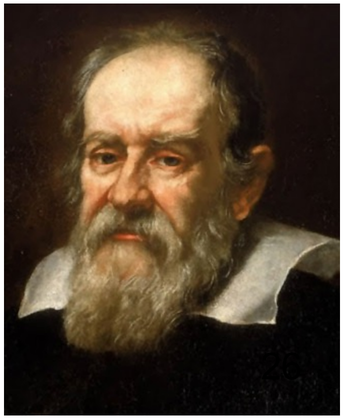
Newton's Law of Gravitation: $$F_g = \frac{GM_E m}{r^2}$$

Therefore: $$a = \frac{GM_E m}{r^2}\frac{1}{m} = \boxed{\frac{GM_E}{r^2} = g}$$

GRAVITATIONAL ACCELERATION (units are m/s^2)

<u>Note that:</u>

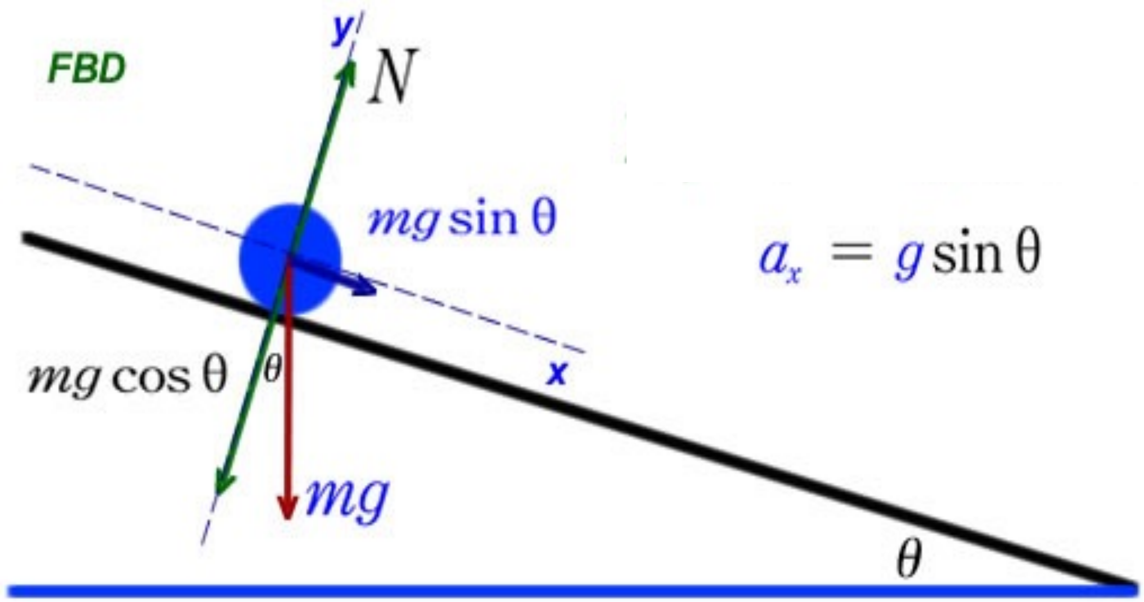
- g does not depend on mass of the small object (m)
- g decreases with distance from M_E (inverse square law)



Galileo Galilei (1564-1642)

Galileo Galilei was often credited with the first discovery of gravitation accelaration at the Earth's surface "g" through a kinematic experiment involving an inclined plane (to slow down the motion so that he could make a measurement, which is not easy to do in the leaning tower experiment)

Galileo was able to find little g using this simple experiment



motion equation

$$\Delta x = \frac{1}{2} a_x t^2$$

$$\frac{2\Delta x}{t^2} = a_x = g\sin\theta$$

$$g = \frac{2\Delta x}{t^2 \sin\theta} = 9.8 m/s^2$$

Galileo, simply had to record the angle of the incline and time down the incline plane.

He did this at many different angles in an effort to reduce friction and error in his timing.

Recording observations and experimental results

From http://sdsu-physics.org/

No need to memorize for this class

Units for gravity (milligals)

- average g at the Earth's surface is 9.81 m/s^2

- we are interested in tiny variations in g on the Earth's surface

→ convenient to use a smaller unit: **the milligal (mgal)**

$$\boxed{1 \text{ mgal} = 10^{-5} \text{ m/s}^2}$$

$$\begin{aligned} 9.81 \text{ m/s}^2 &= 981{,}000 \text{ mgal} \\ &= 981 \text{ Gal (after Galileo)} \end{aligned}$$

Rock density

- gravitational force depends on the mass of an object

- the mass of an object is given by: **$m = \rho \times V$**

  where V is its volume

  ρ is its **density** (mass per unit volume)

  → material property

- the units for density are kg/m^3

  - sometimes given in g/cm^3 ($1\ g/cm^3 = 1000\ kg/m^3$)

- density of Earth rocks range from ~1000 to >7000 $kg\ m^{-3}$