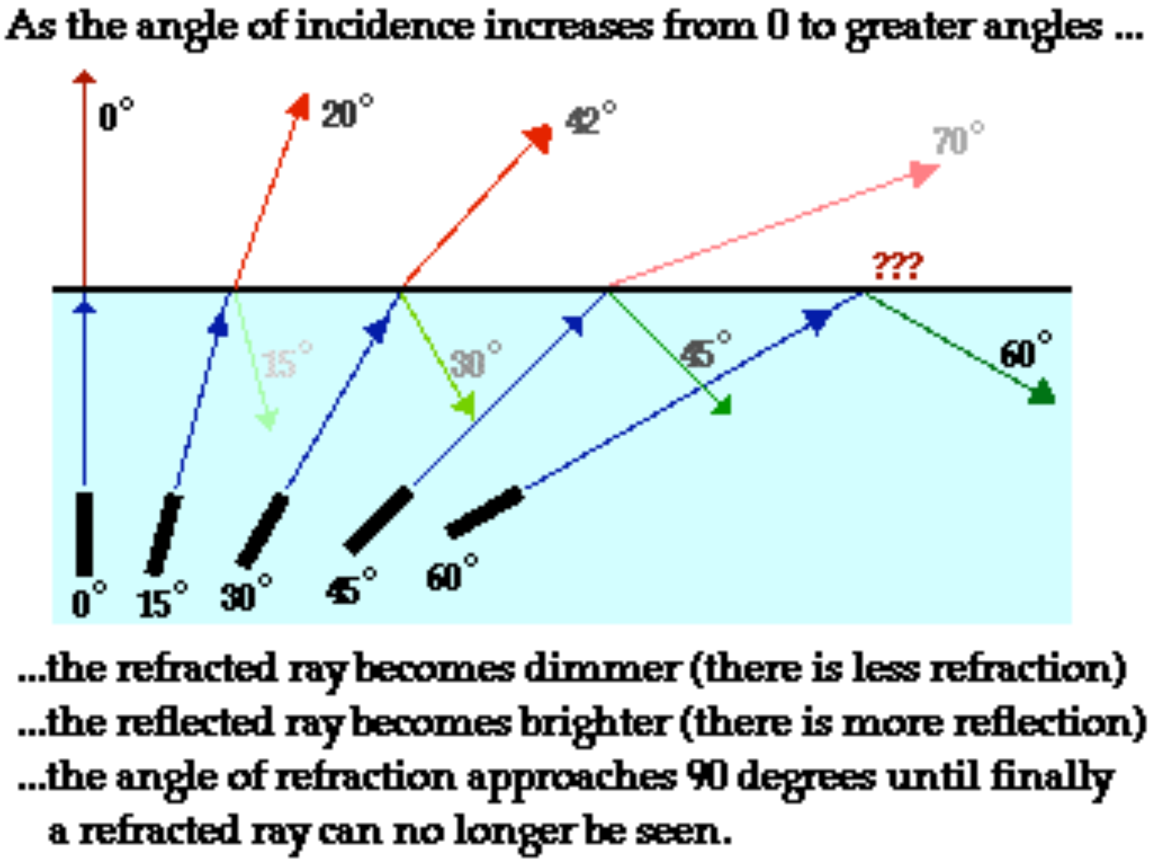
Surface Waves (Phenomenology)

Surface waves are related to critical reflections. To understand surface waves, we must first have some knowledge of critical waves and total internal reflections.



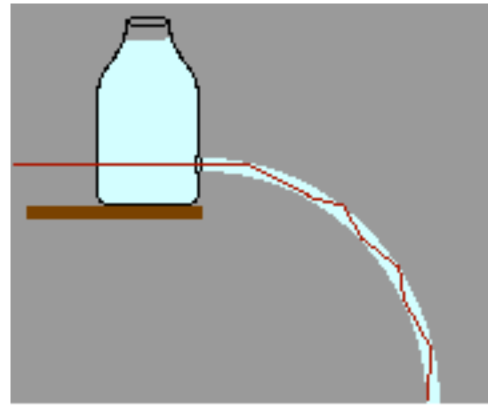
<u>Total Internal reflection</u> condition:

(1) second medium is has greater internal velocity than incoming

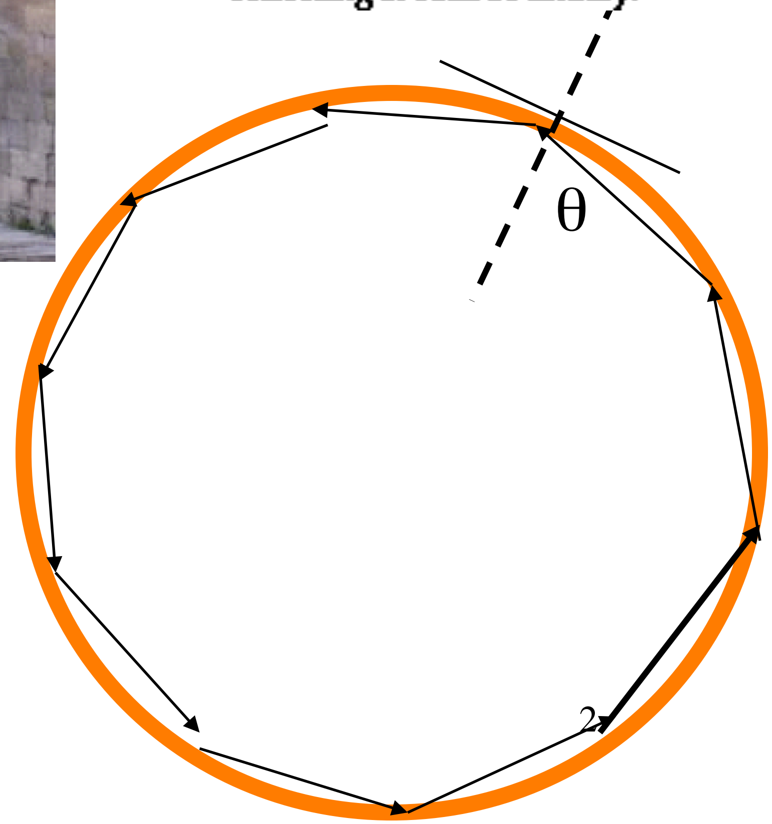
(2) angle of refraction (transmission) passes 90

Wispering wall (not total internal reflection, but has similar effect): Beijing, China

total internal reflection



The laser beam stays internal to the water, continuously reflecting at each boundary.

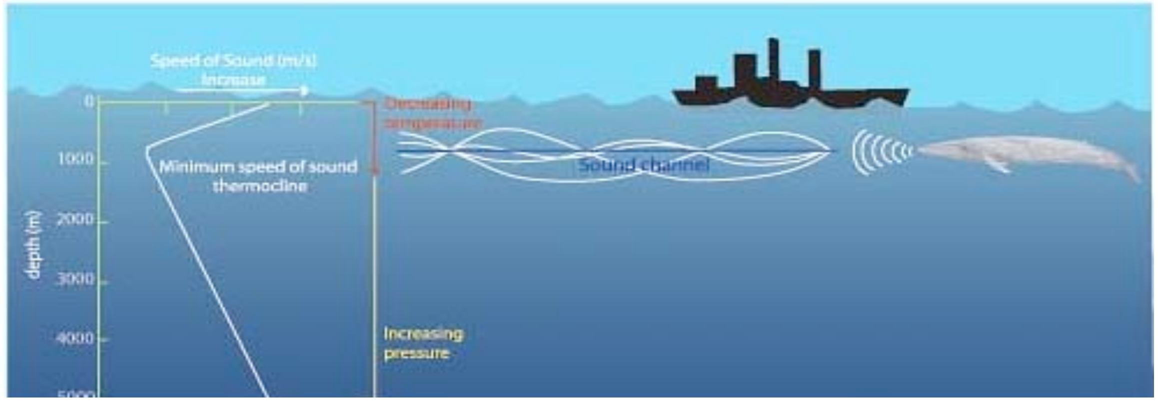


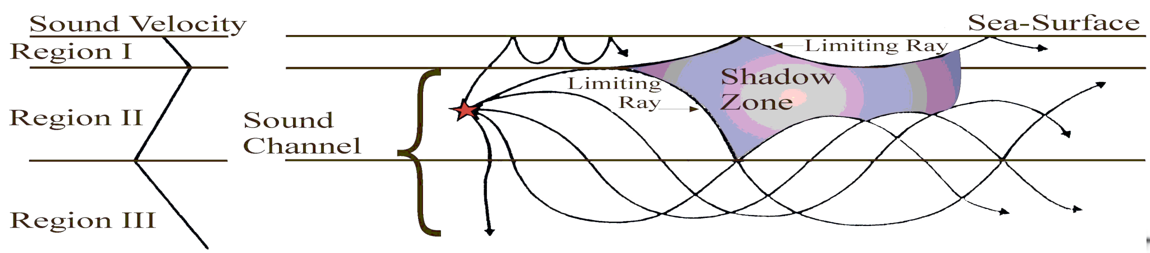
Nature of "wispering wall" or "wispering gallery": A sound wave is trapped in a carefully designed circular enclosure.

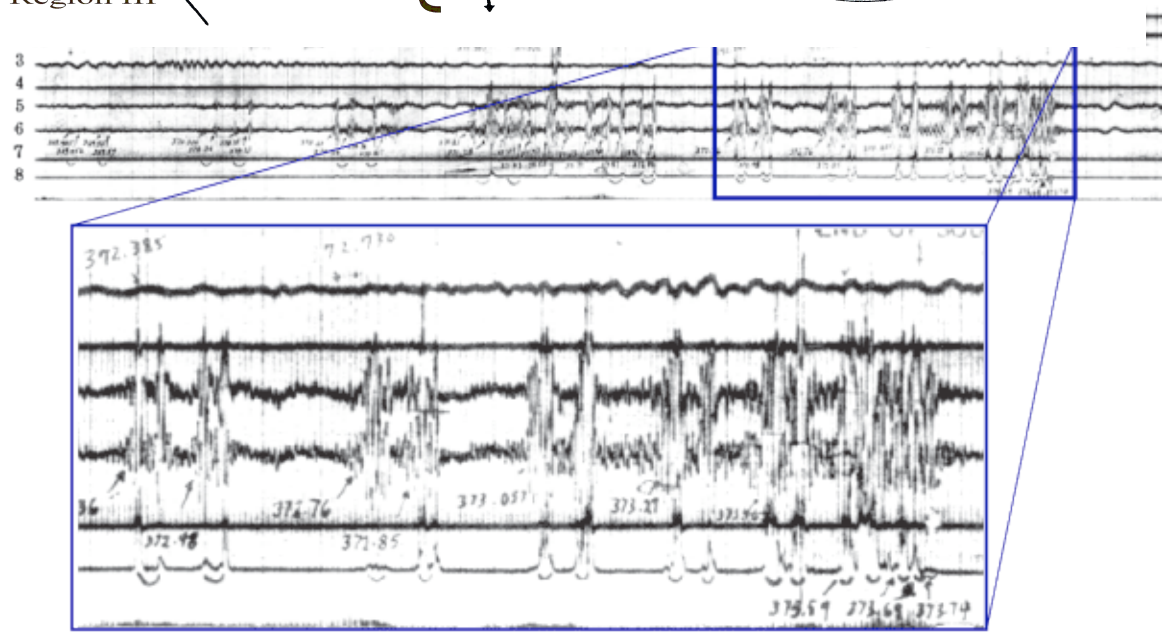
The Sound Fixing and Ranging Channel (SOFAR channel) and Guided Waves

A layer of low velocity zone inside ocean that ‘traps’ sound waves due to total internal reflection.

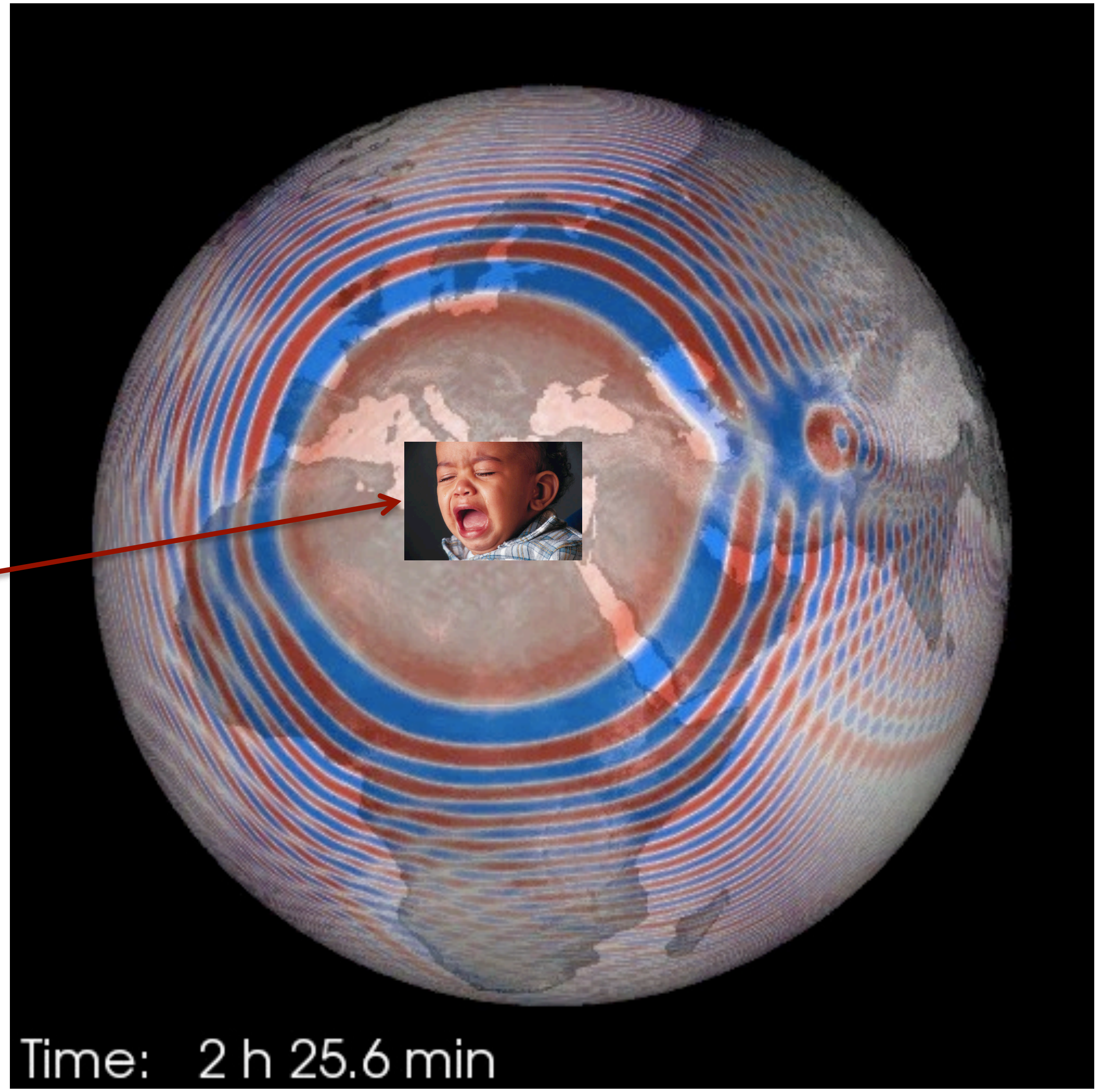
Mainly by Maurice Ewing (the ‘Ewing Medal’ at AGU) in the 1940’s







Seismic Surface Waves



Get me out of here,

faaaaaaassstttt….
.!!

Seismic Surface Waves Facts

We have discussed P and S waves, as well as interactions of SH, or P-SV waves near the free surface. As we all know that surface waves are extremely important for studying the crustal and upper mantle structure, as well as source characteristics.

Surface Wave Characteristics:

(1) Dominant between 10-200 sec (energy decays as r^{-1}, with stationary depth distribution, but body wave r^{-2}).

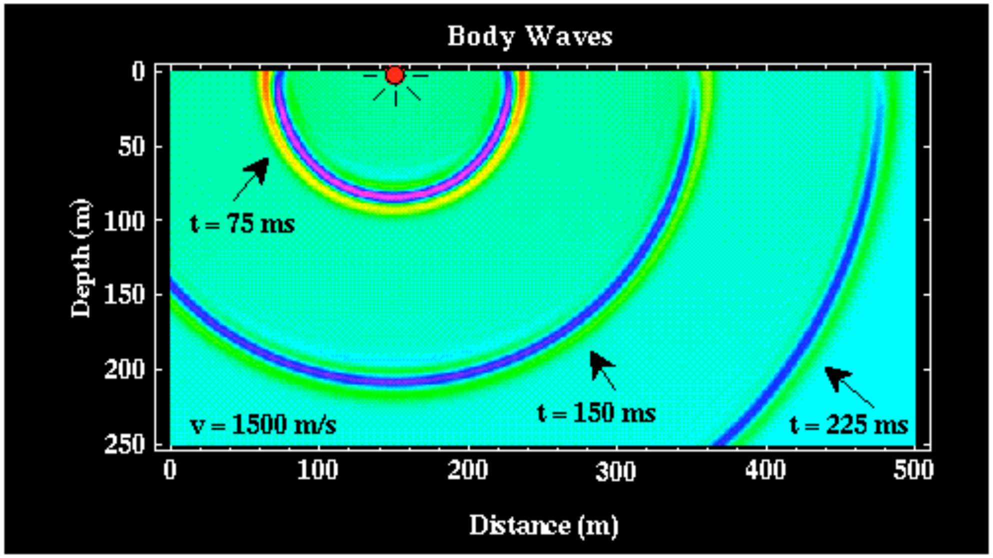
(2) Dispersive which gives distinct depth sensitivity

Types:

(1)Rayleigh: P-SV equivalent, exists in elastic homogeneous halfspace

(2) Love: SH equivalent, only exist if there is velocity gradient with depth (e.g., layer over halfspace)

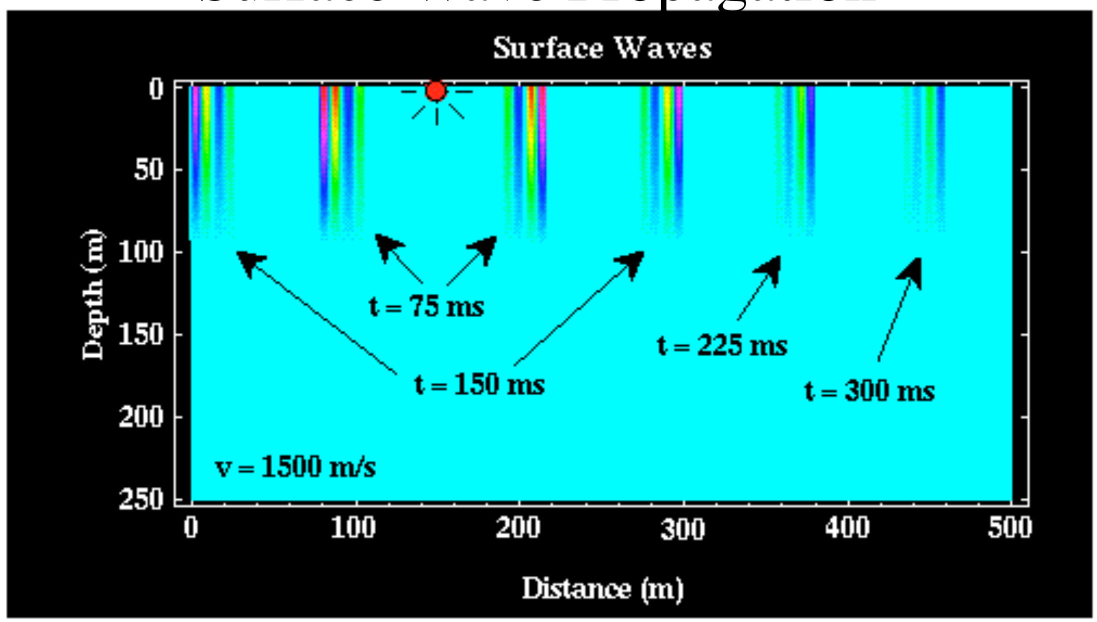
Body wave propagation



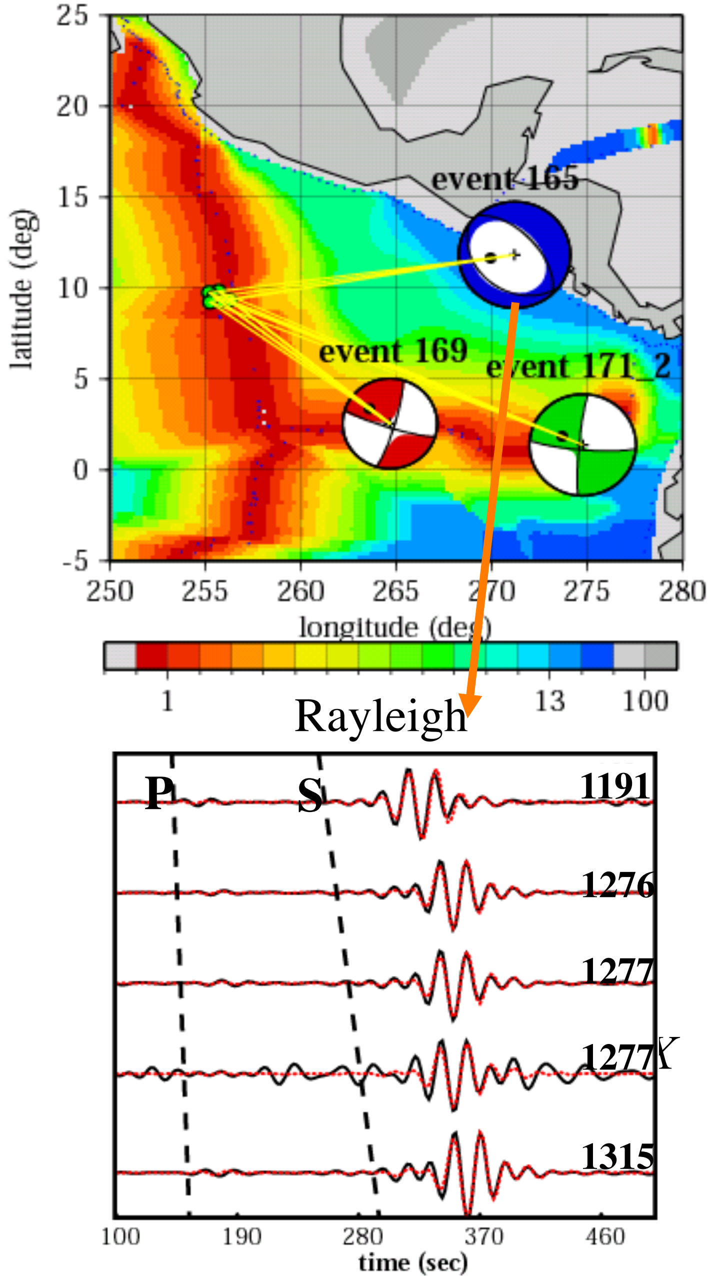
One person's noise is another person's signal. This is certainly true for what surface waves mean to an exploration geophysicist and to a global seismologist

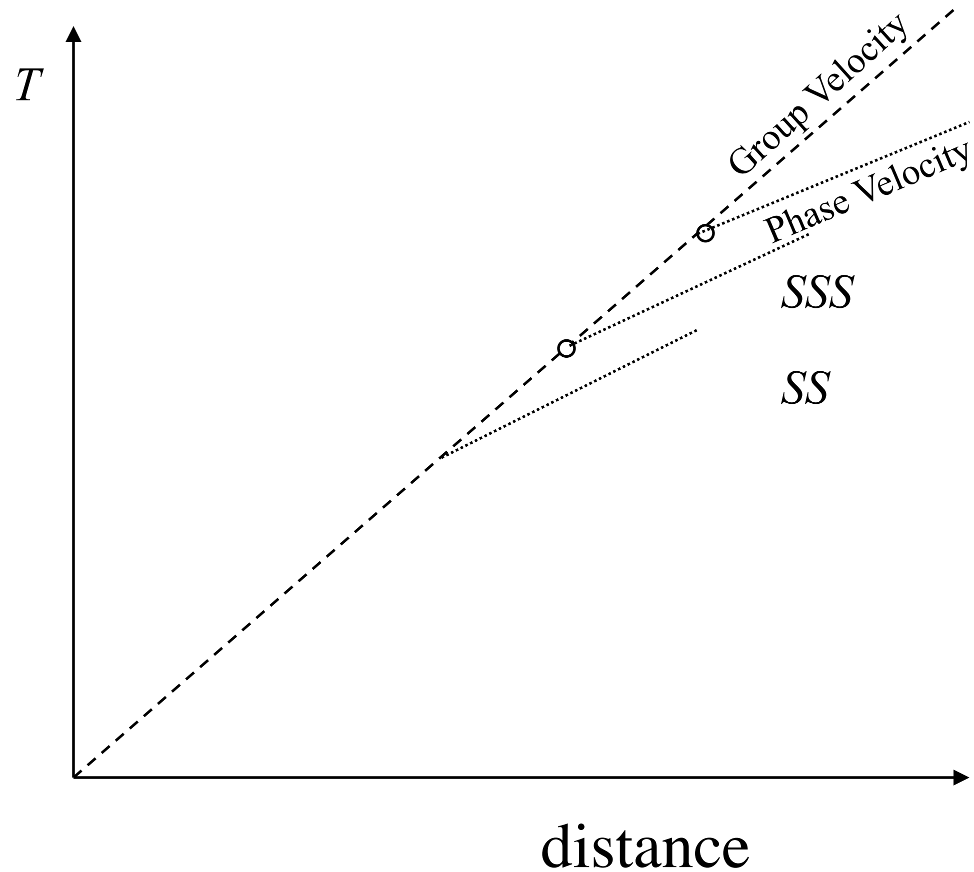
Energy decay in surface waves (as a function of r) is less than that of body wave (r^2)--- the main reason that we always find larger surface waves than body waves, especially at long distances.

Surface Wave Propagation



Surface Wave Observations





Surface waves come after most body wave arrivals, why?

(1) Lower speeds close to surface

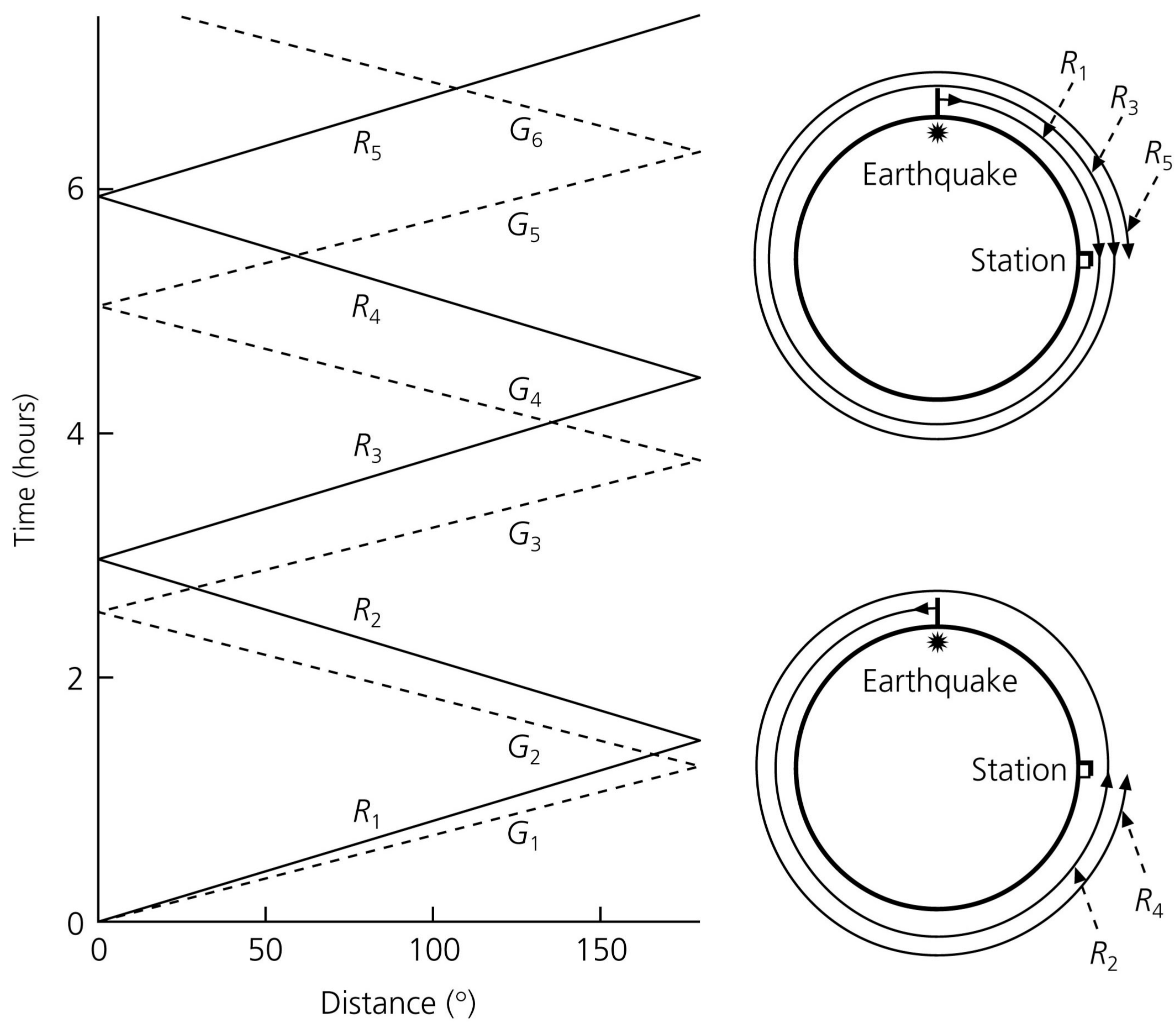
(2) Lower speeds than S

Naming conventions:

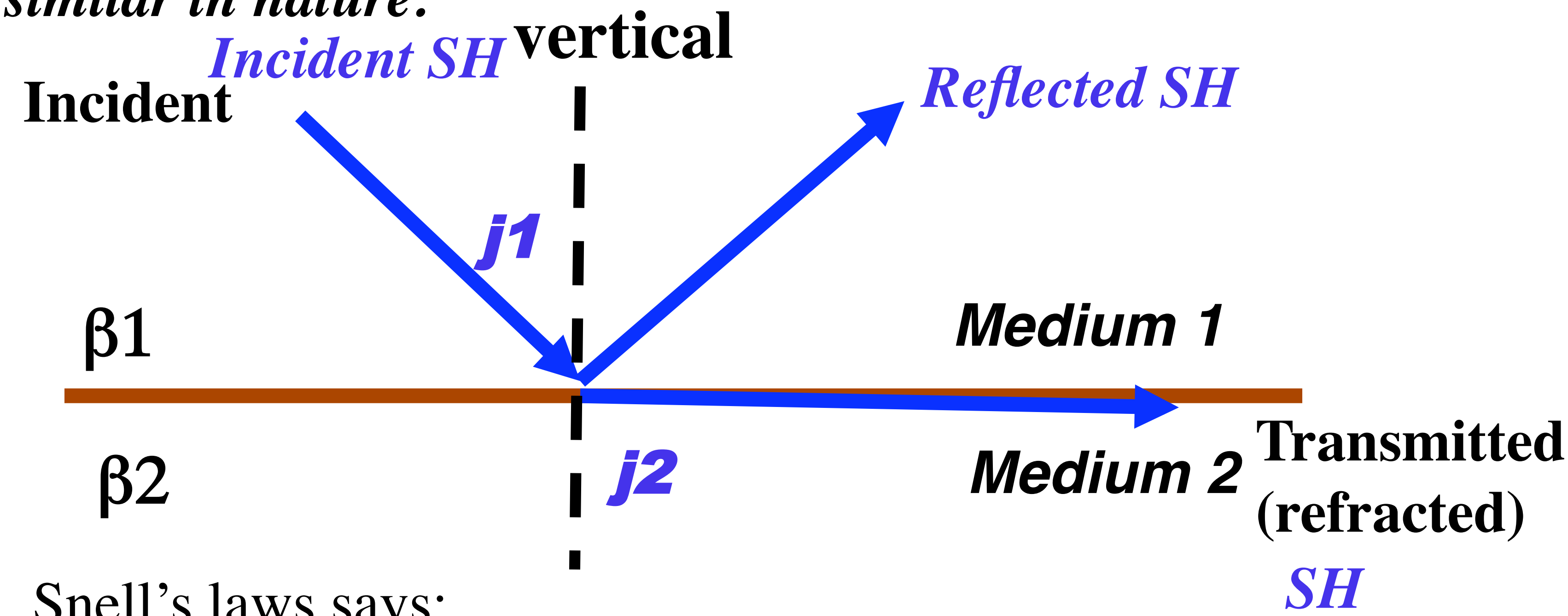
1. G for ***Love***, R for ***Rayleigh***.
2. Odd numbers are ***minor arcs***, even numbers are ***major arcs*** in the opposite direction.

At 180 degrees, we have an ***antipode***, the surface wave amplitude is greatly amplified due to interactions of surface waves coming from all directions around the globe.

Figure 2.7-4: Six-hour stacked IDA record section.



Evanescent wave: ***post-critical, not surface wave per se, but similar in nature.***



Snell's laws says:

$$\frac{\sin j1}{\beta 1} = \frac{\sin j2}{\beta 2} \mapsto \frac{\sin jc}{\beta 1} = \frac{1}{\beta 2} \mapsto \sin jc = \frac{\beta 1}{\beta 2}$$

Recall: $Cx = \frac{1}{p} = \frac{\beta 1}{\sin jc} = \frac{\beta 2}{\sin(90)}$

If ***j1> jc, sin(j1) > sin(jc)*** → **Cx** $< \beta 2$ (again, Cx is the "apparent horizontal speed", also called "phase velocity", constant for a homogeneous layered Earth).

In medium 2,

$$u_y(x,z,t) = B' e^{i(\omega t - k_x x - k_x r_{\beta 2} z)}$$

where $$r_{\beta 2} = \frac{k_z}{k_x} = \left(\frac{c_x^{\ 2} - \beta_2^{\ 2}}{\beta_2^{\ 2}}\right)^{\frac{1}{2}} = \left(\frac{c_x^{\ 2}}{\beta_2^{\ 2}} - 1\right)^{\frac{1}{2}}$$

For post-critical angles, **Cx** $< \beta 2$ → $r_{\beta 2}$ must be imaginary!

→ $$r_{\beta 2} = \pm i\left(1 - \frac{c_x^{\ 2}}{\beta_2^{\ 2}}\right)^{\frac{1}{2}}$$

Choose to define

$$r_{\beta 2} = -i r_{\beta 2}^{\ *} \qquad \text{where} \quad r_{\beta 2}^{\ *} = \sqrt{1 - c_x^{\ 2} / \beta_2^{\ 2}}$$

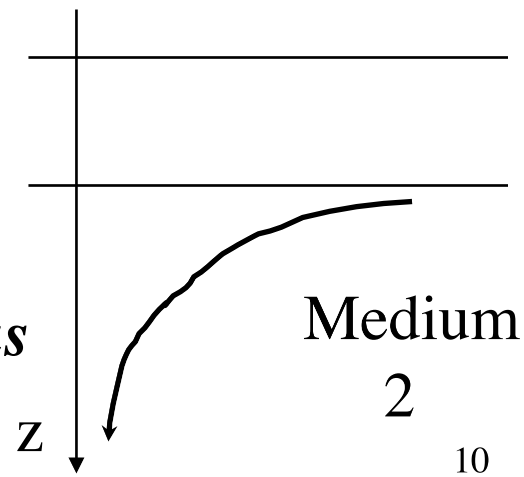
→ $$u_y(x,z,t) = B' e^{i(\omega t - k_x x + i k_x r_{\beta 2}^{\ *} z)}$$

The Z term (depth) in the complex exponential is no longer complex,
But an exponential decay!!

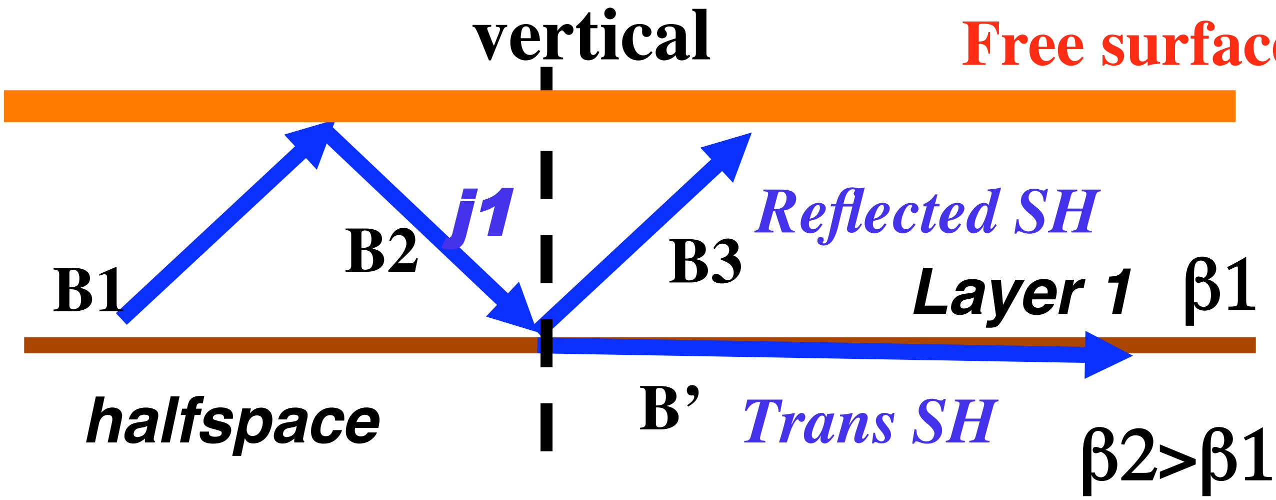
$$u_y(x,z,t) = B' e^{i(\omega t - k_x x + i k_x r_{\beta 2}^{\ *} z)}$$

Where z term is $$e^{+i * i k_x r_{\beta 2} z} = e^{-k_x r_{\beta 2}^{\ *} z}$$

Which means a boundary wave (***inhomogeneous*** or ***evanescent*** wave) “trapped” near boundary but decays with depth! **This is the basis of surface waves.**



Love waves



Love Wave **necessary** condition : $\beta_1 < c_x < \beta_2$ (1)

Love Wave Equation:

$$\tan(\omega\zeta) = \frac{\mu_2\sqrt{1-c_x^{\,2}/\beta_2^{\,2}}}{\mu_1}\left(\frac{h}{c_x\zeta}\right) \quad \text{where} \quad \zeta = h/c_x\sqrt{c_x^{\,2}/\beta_1^{\,2}-1}$$

h=layer thickness, β_1 = S wave speed of layer, β_2=S speed of halfspace

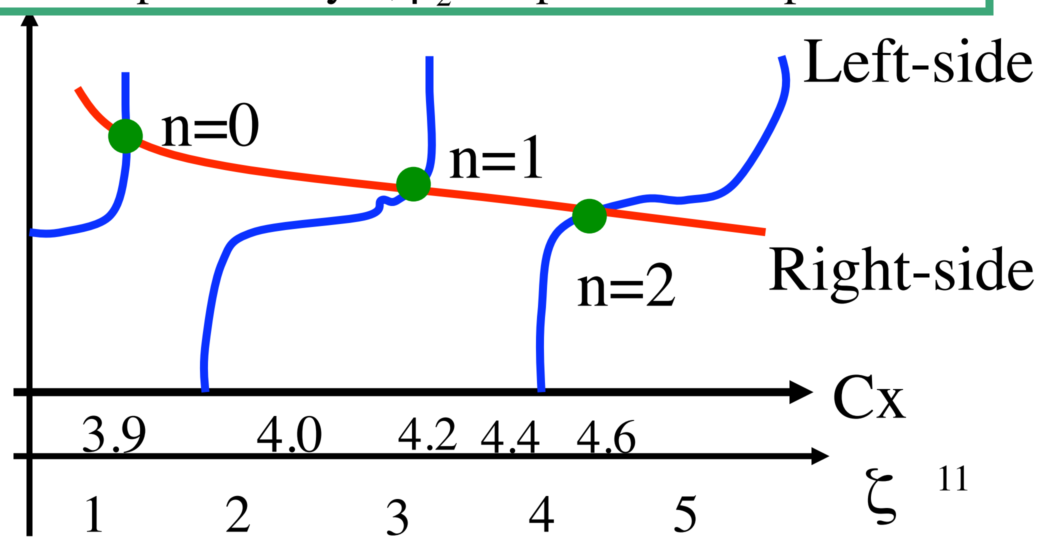
<u>Graphical Solution:</u>

Left-side: similar to tan(ζ)

Right-side: similar to 1/ζ

Sufficient Condition of Love:

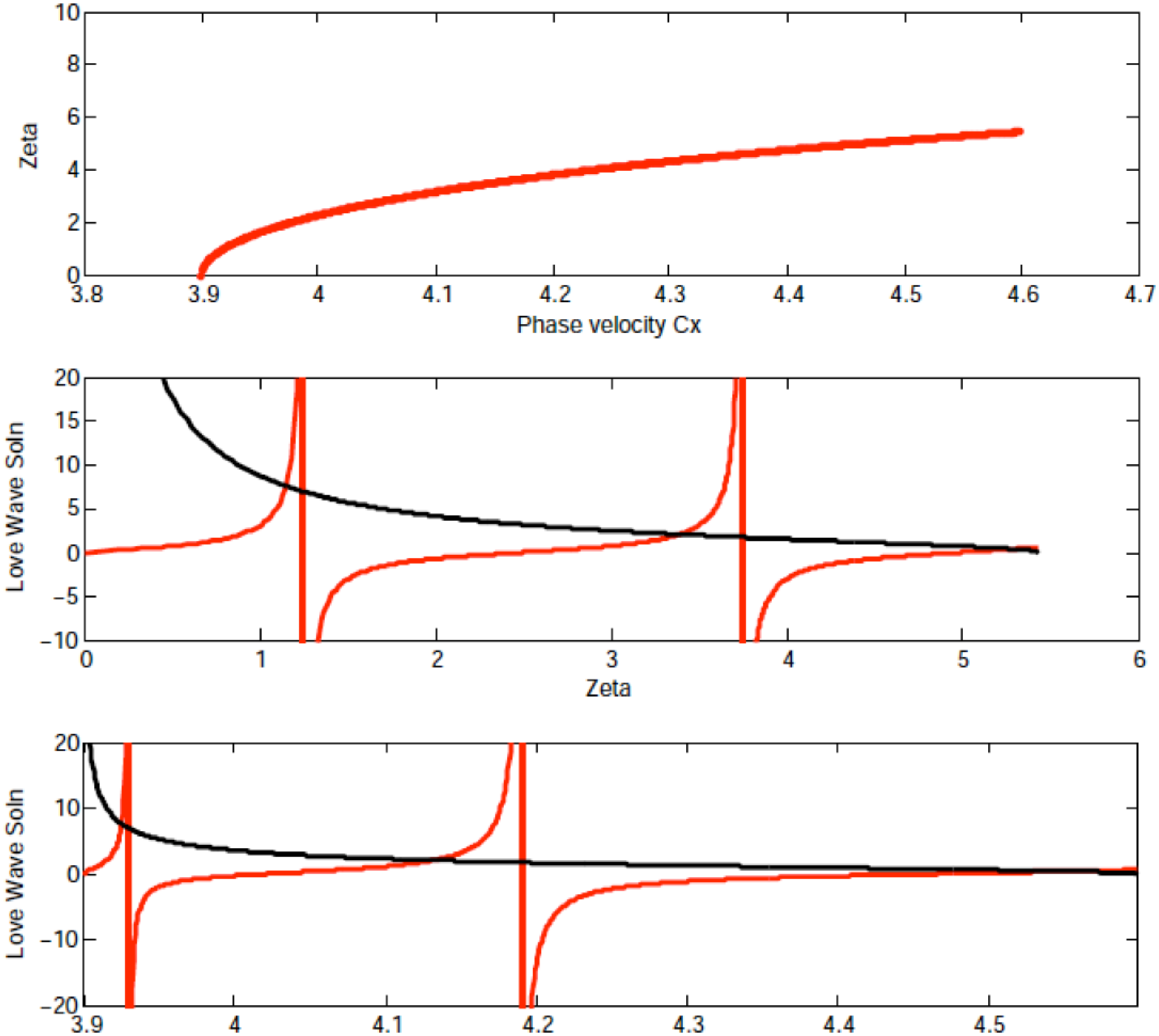
Graph intercept, which are
1. discret 2. Depends on ω 3. Cx must satisfy Eqn (1)



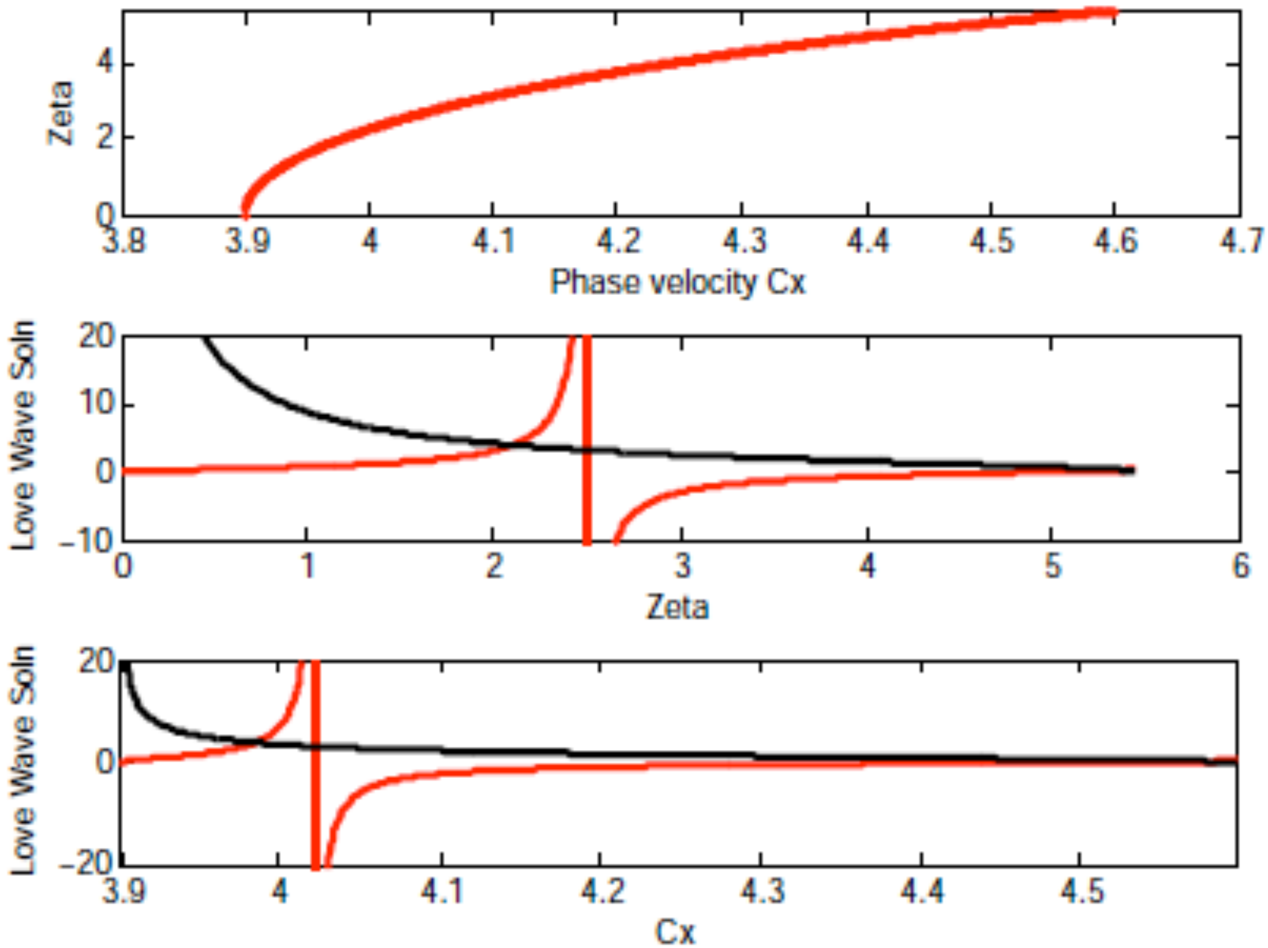
A simple code to plot Love Wave Solution (downloadable on course website)

```
% layer velocities v1 v2
 b1=3.9;
 b2=4.6;
% density in kilos of kg/m^3
 r1=2.8;
 r2=3.3;
 h=40;
 n=1000;
% mu values computed based on velocity and density
 mu1=b1*b1*1000000*r1*1000.;
 mu2=b2*b2*1000000*r2*1000.;
% frequency in Hz
 f=0.2;
w=f*2.0*3.1415926;
dcx = (b2-b1)/n;
% range of phase velocity cx where b1<cx<b2
 cx=b1:dcx:b2;
 zeta=[];
 lterm=[];
 rterm=[];
 for i=1:n+1
   zeta(i)=h.*(cx(i)*cx(i)/(b1*b1)-1)^(0.5)/cx(i);
   lterm(i)=tan(w*zeta(i));
   rterm(i)=(mu2/mu1)*h*(1.-cx(i)*cx(i)/(b2*b2))^(0.5)/(cx(i)*zeta(i))
 end
```

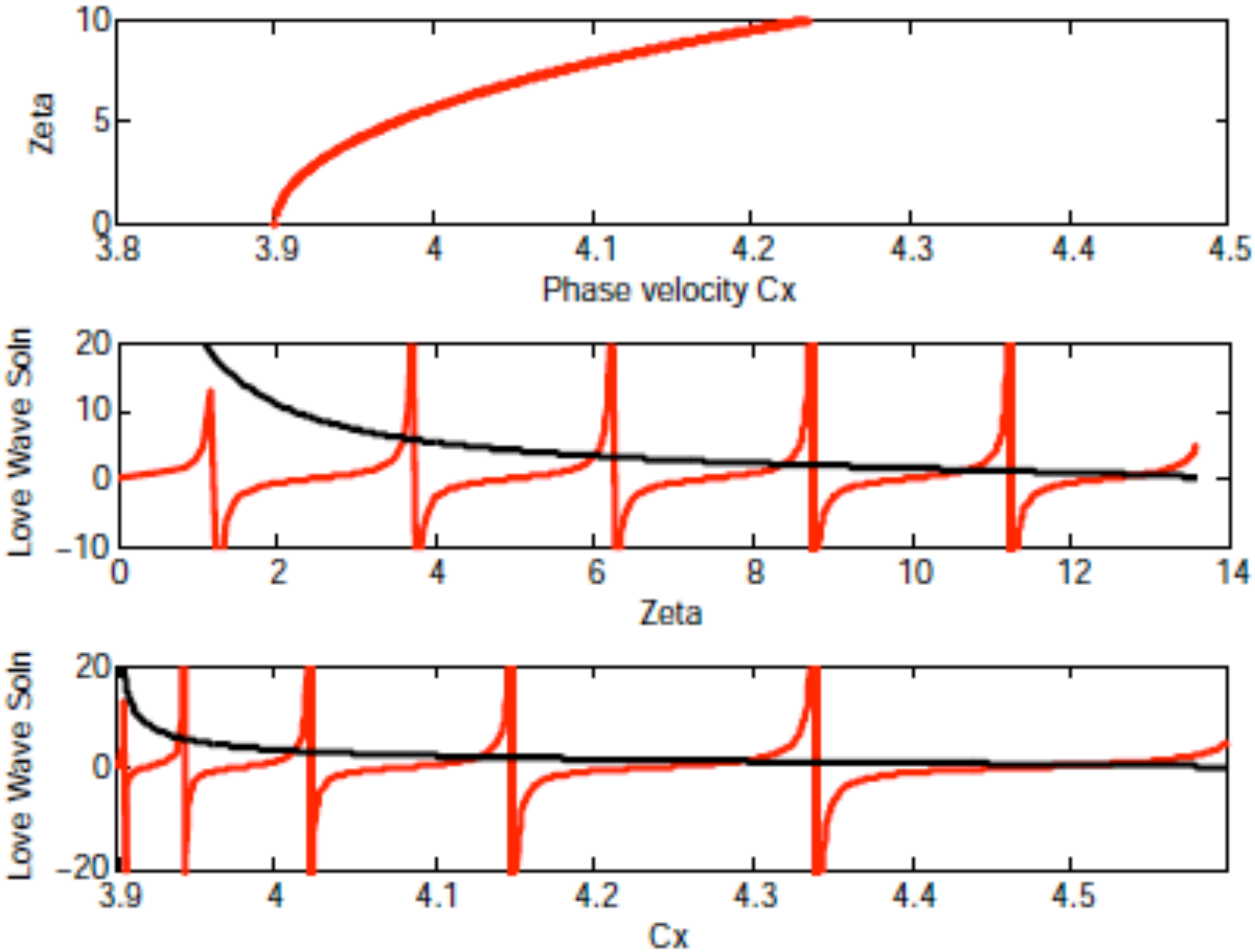
Program output without modifications

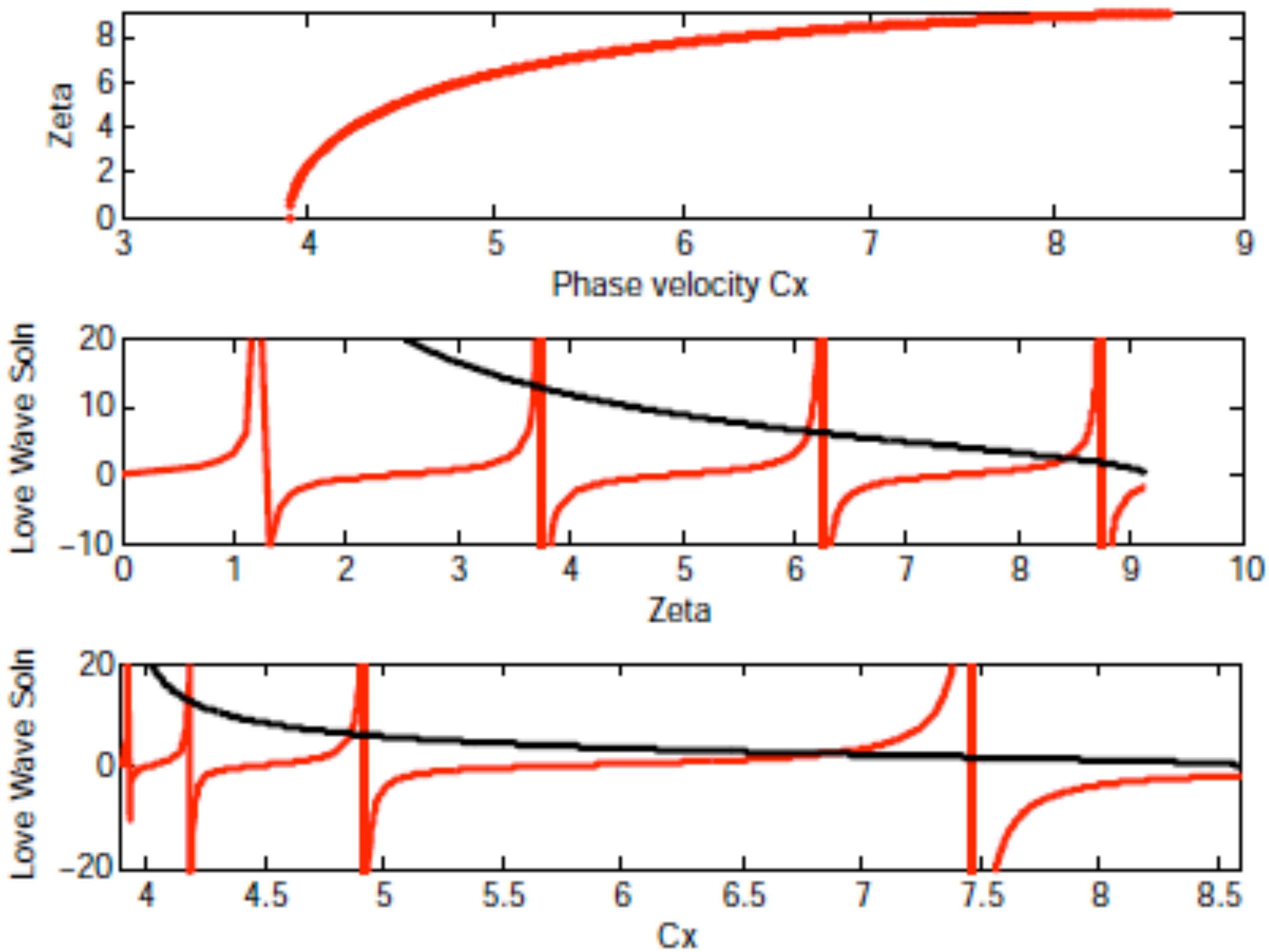


Decreasing frequency from 0.2 Hz to 0.1 Hz



More solutions when thickness increases from 40 to 100 km





When V2 increases, the number of solutions also increase

Surface Wave Modes:

See Love Wave Equation -→ $\tan(\omega h / c_x \sqrt{c_x^2/\beta_1^2 - 1}) = \dfrac{\mu_2\sqrt{1 - c_x^2/\beta_2^2}}{\mu_1\sqrt{c_x^2/\beta_1^2 - 1}}$

(1) Solutions exists when the two curves meet, and Love wave speed is between the two shear speeds.

(2) When increase T (period), we decrease ω, which leads to bigger **Cx**. ----> **fewer** number of solutions in allowable velocity range.

(3) Cutoff angular frequency:

assume ζ_{max} for a given velocity range ($\beta 1$ and $\beta 2$),

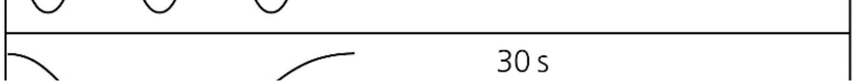
When does it occur? $\mathbf{c_x = \beta_2}$

Let's look at the spacing between two tangent curves, which is determined by

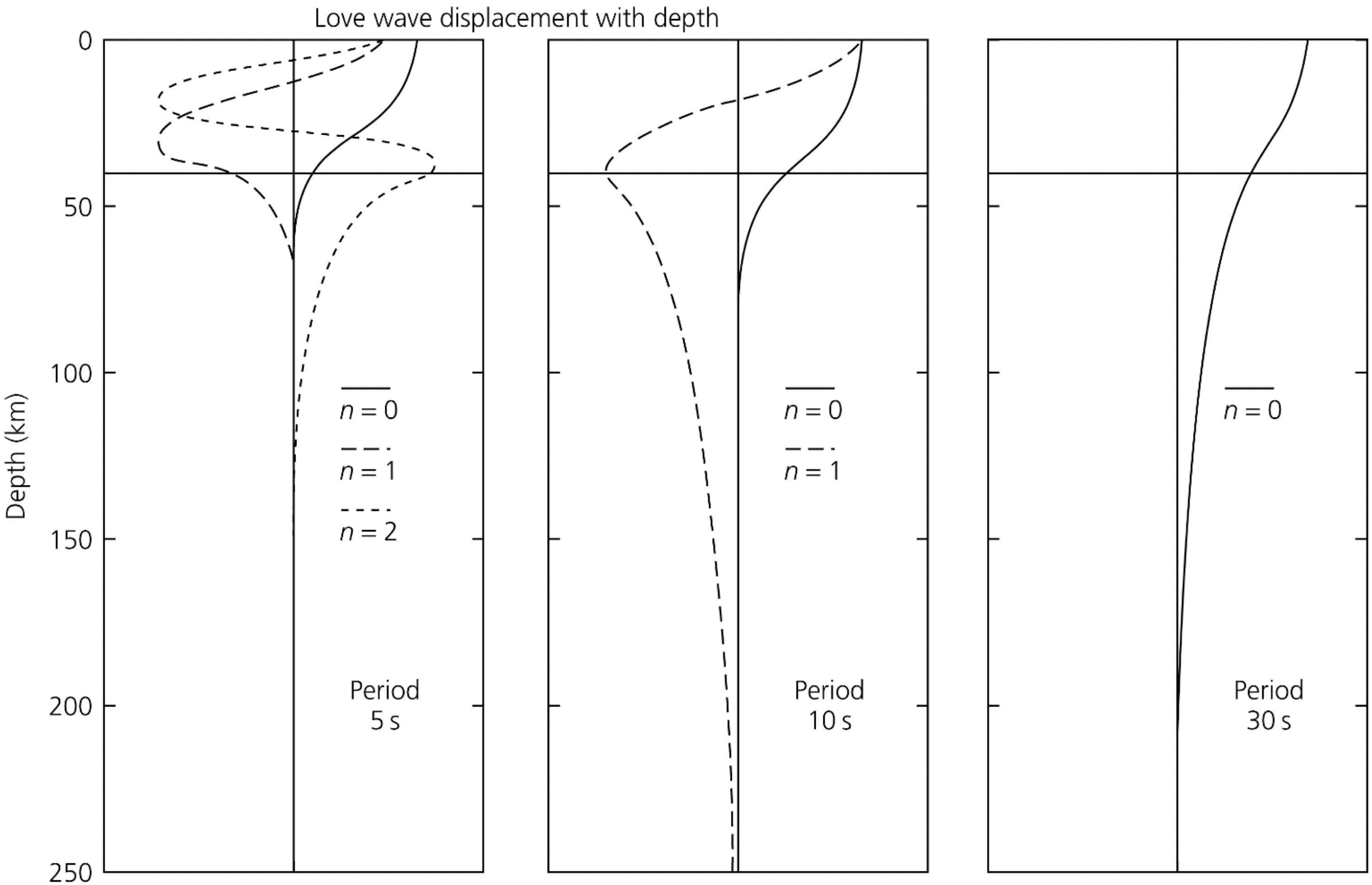
$$\tan(\omega\zeta) = 0 \quad \text{----->} \quad \omega\zeta = n\pi$$

$$\omega = \omega_{cn} = \frac{n\pi}{h/\beta_2\sqrt{\beta_2^2/\beta_1^2 - 1}} = \frac{n\pi}{h\sqrt{1/\beta_1^2 - 1/\beta_2^2}}$$

ω_{cn} is the ***cutoff frequency*** for the $\boldsymbol{n}$th higher mode



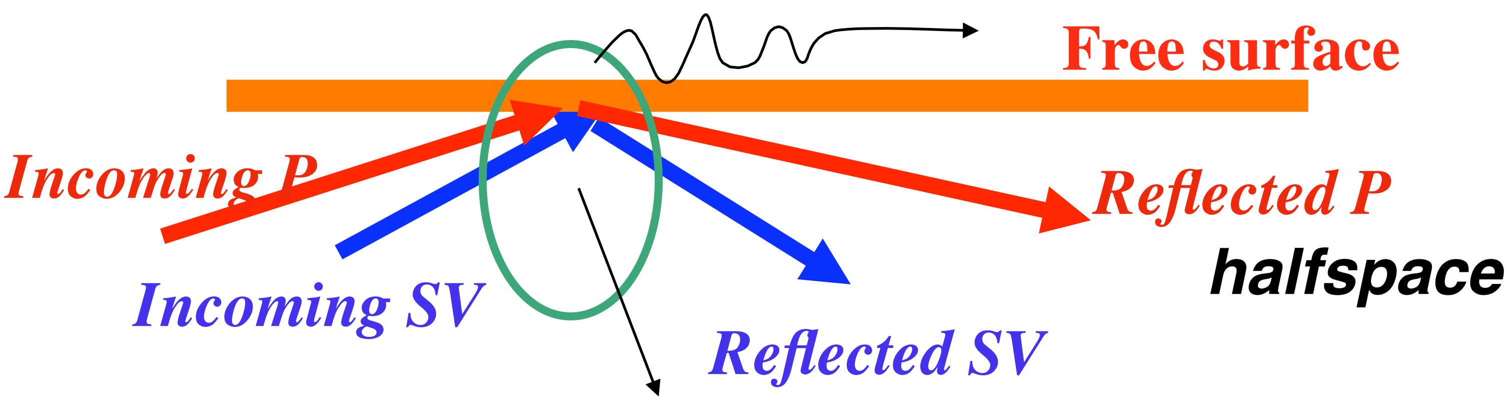
Displacement as a function of depth



Two regimes are different, the layer(-) and halfspace (+). Assume fixed x, plot as a function of z. Layer: oscillation, halfspace: decay

Rayleigh waves in a homogeneous halfspace

Definition: **"Rayleigh waves are a combination of P and SV waves that can exist at the top of homogeneous halfspace"**



Jeff said: "**There was no Rayleigh, but when an incoming P and SV interact at the surface, there became one…"**

Assume a Poisson Solid (more or less Earth material), then $\lambda = \mu$, first, what are the speeds of Vs and Vp?

Poisson Solid ---> **$Cx \sim 0.92\ \beta$** (Cx = Rayleigh/horizontal speed)

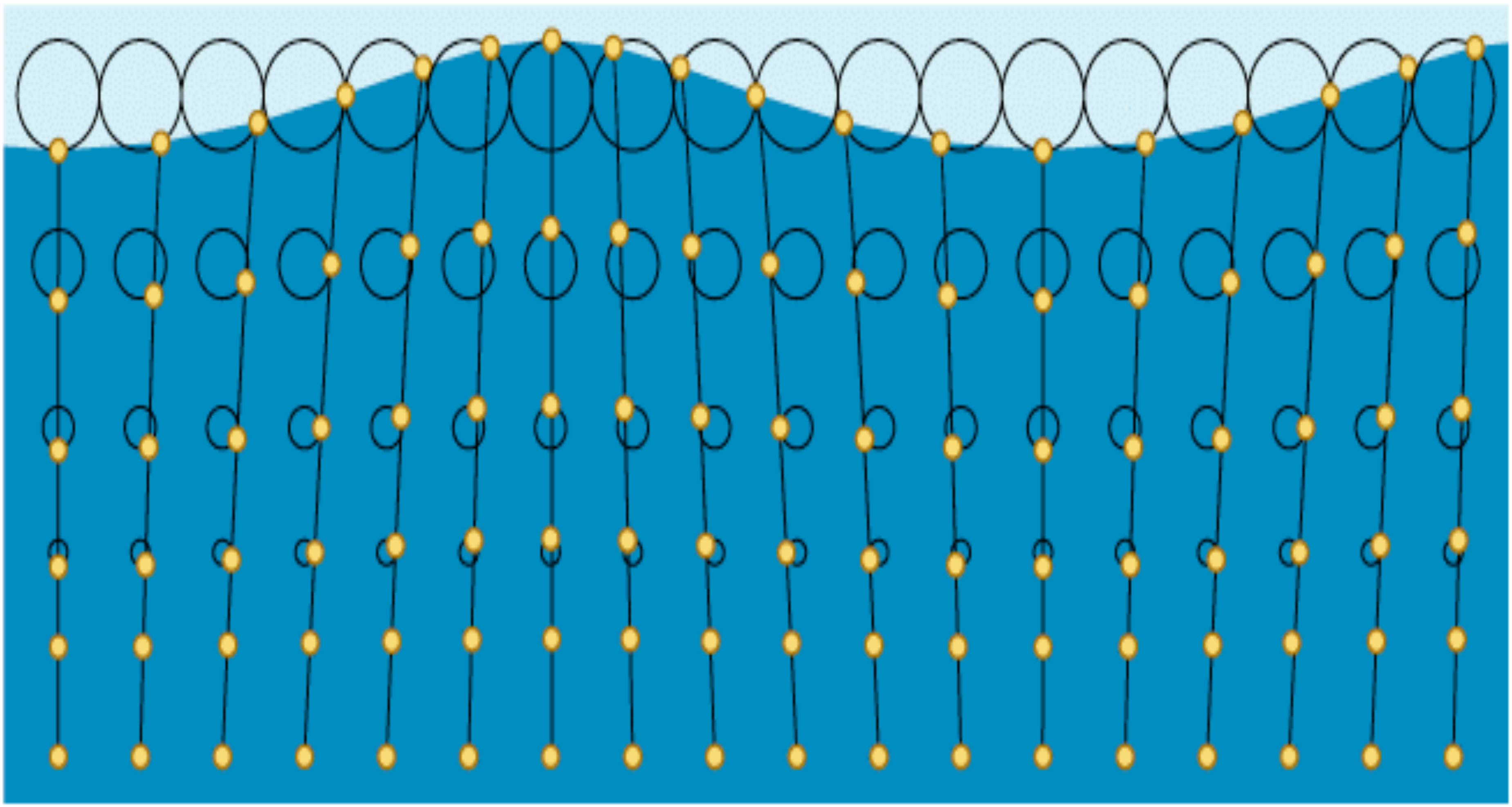
<u>Questions:</u> 1. Which is usually faster, Rayleigh or Love??

2. Is answer above always right?

3. Similar to P-SV body wave, which means it should be observed on which component?

4. Do Rayleigh and Love have the same depth sensitivity?

Water wave polarization: Similar to Rayleigh waves, but Opposite particle motions!



Dispersive Properties of Surface waves (Love & Rayleigh):

The following traces show a seismogram (containing body waves too) in both time domain (filtered at different frequencies) and in frequency domain (velocity differs at different frequencies).

Figure 2.8-4: Example of Love wave group velocity dispersion through bandpass filtering.

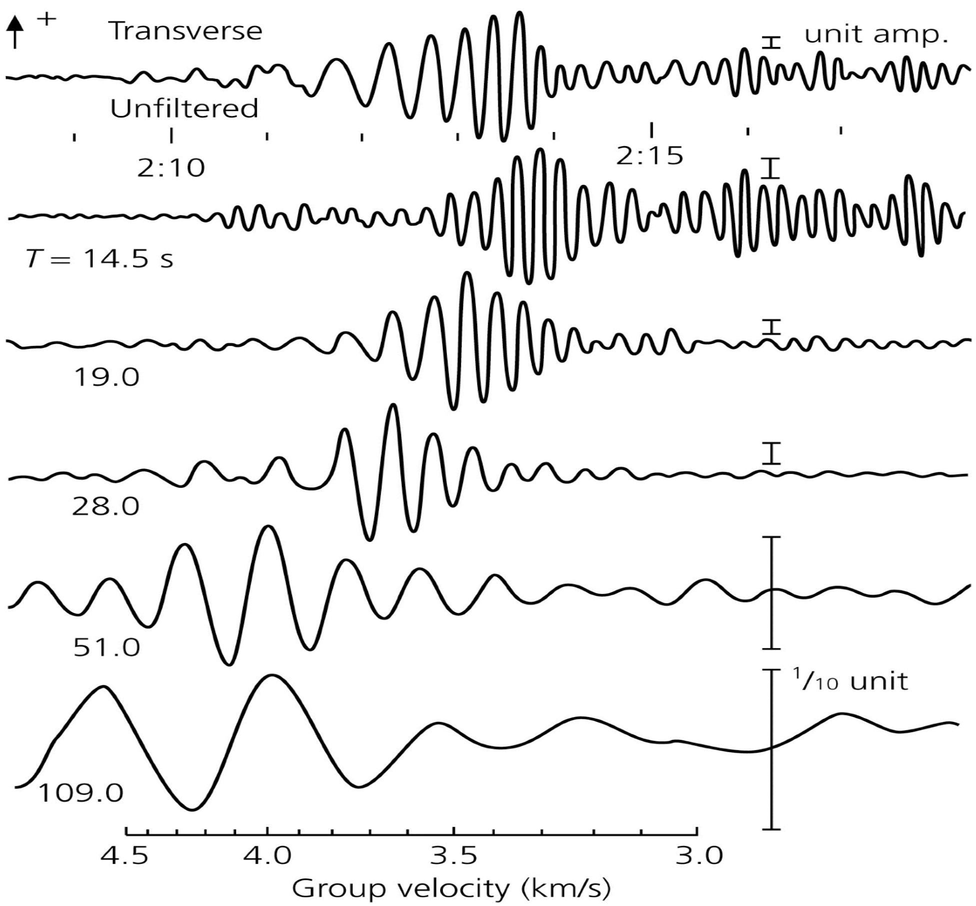
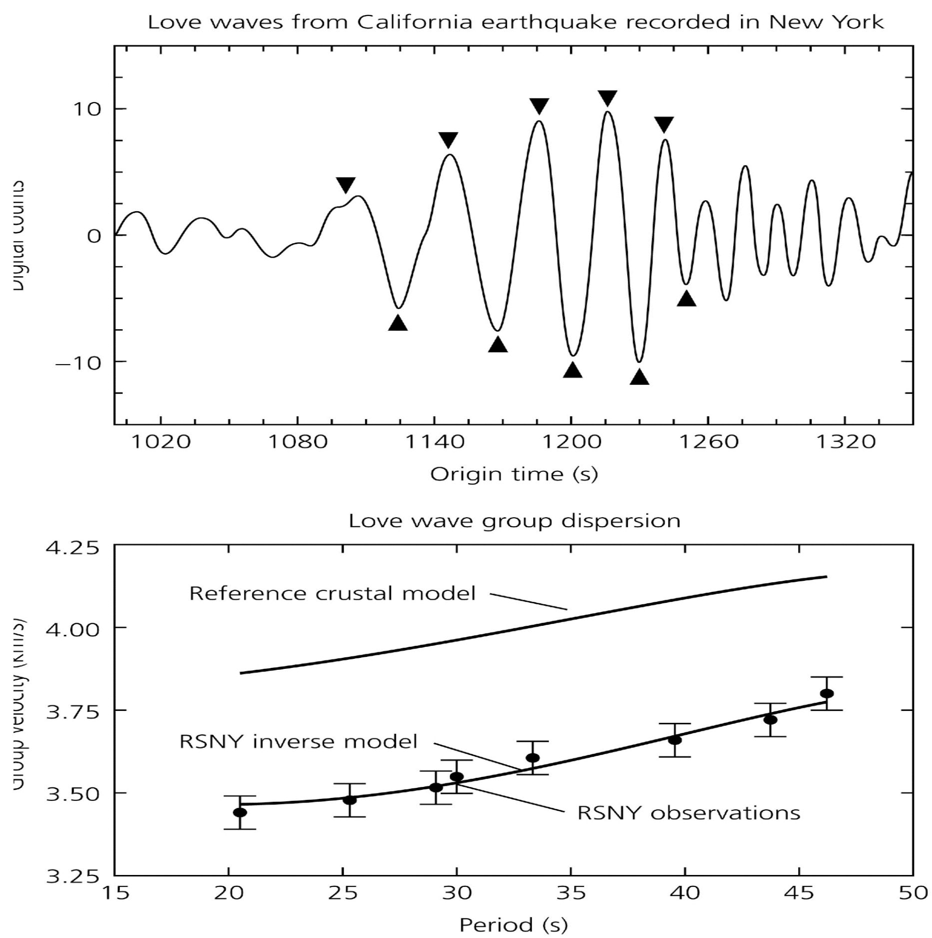
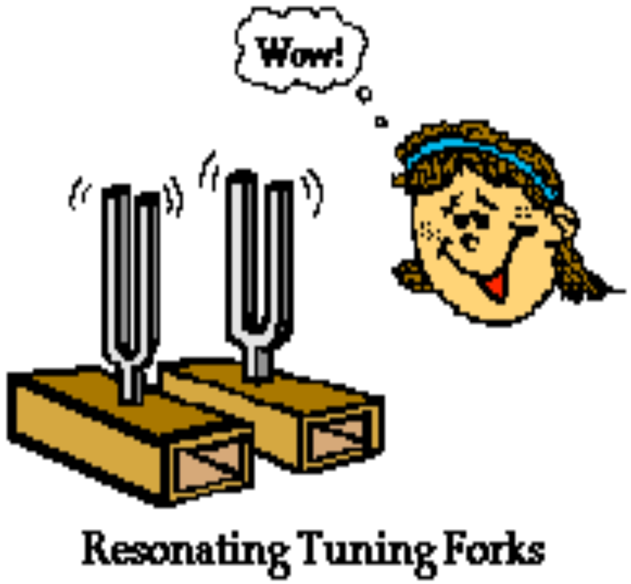


Figure 2.8-3: Example of calculating Love wave group velocity dispersion.



Phase and Group Velocity



Resonance occurs when two waves of similar frequency but different phase interfere.

Consider two harmonic waves:

$$u(x,t) = \cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$$

Can write in some average ω **as**

$$\omega_1 = \omega + \delta\omega \qquad \omega_2 = \omega - \delta\omega, \qquad \omega >> \delta\omega$$

$$k_1 = k + \delta k \qquad k_2 = k - \delta k, \qquad k >> \delta k$$

Substitute into u(x,t)

$$u(x,t) = \cos(\omega t + \delta\omega t - kx - \delta kx) + \cos(\omega t - \delta\omega t - kx + \delta kx)$$

$$= \cos(\omega t - kx)\cos(\delta\omega t - \delta kx) - \sin(\omega t - kx)\sin(\delta\omega t - \delta kx)$$

$$+\cos(\omega t - kx)\cos[-(\delta\omega t - \delta kx)] - \sin(\omega t - kx)\sin[-(\delta\omega t - \delta kx)]$$

$$= 2\cos(\omega t - kx)\cos(\delta\omega t - \delta kx)$$

Two waves: **carrier wave** (**ω, κ**) **envelope wave** (**δω, δκ**)

Group velocity:

U= δω/δκ

$$U = \frac{d\omega}{dk} = \frac{d(ck)}{dk}$$

$$= c + k\frac{dc}{dk} = c - \lambda\frac{dc}{d\lambda}$$

Phase velocity:

c=ω/κ

Two types of dispersion:

(1) Geometric--- waves spreading

(2) Physical---frequency dependence

Figure 2.8-1: Demonstration of group and phase velocities for the sum of two sine waves.

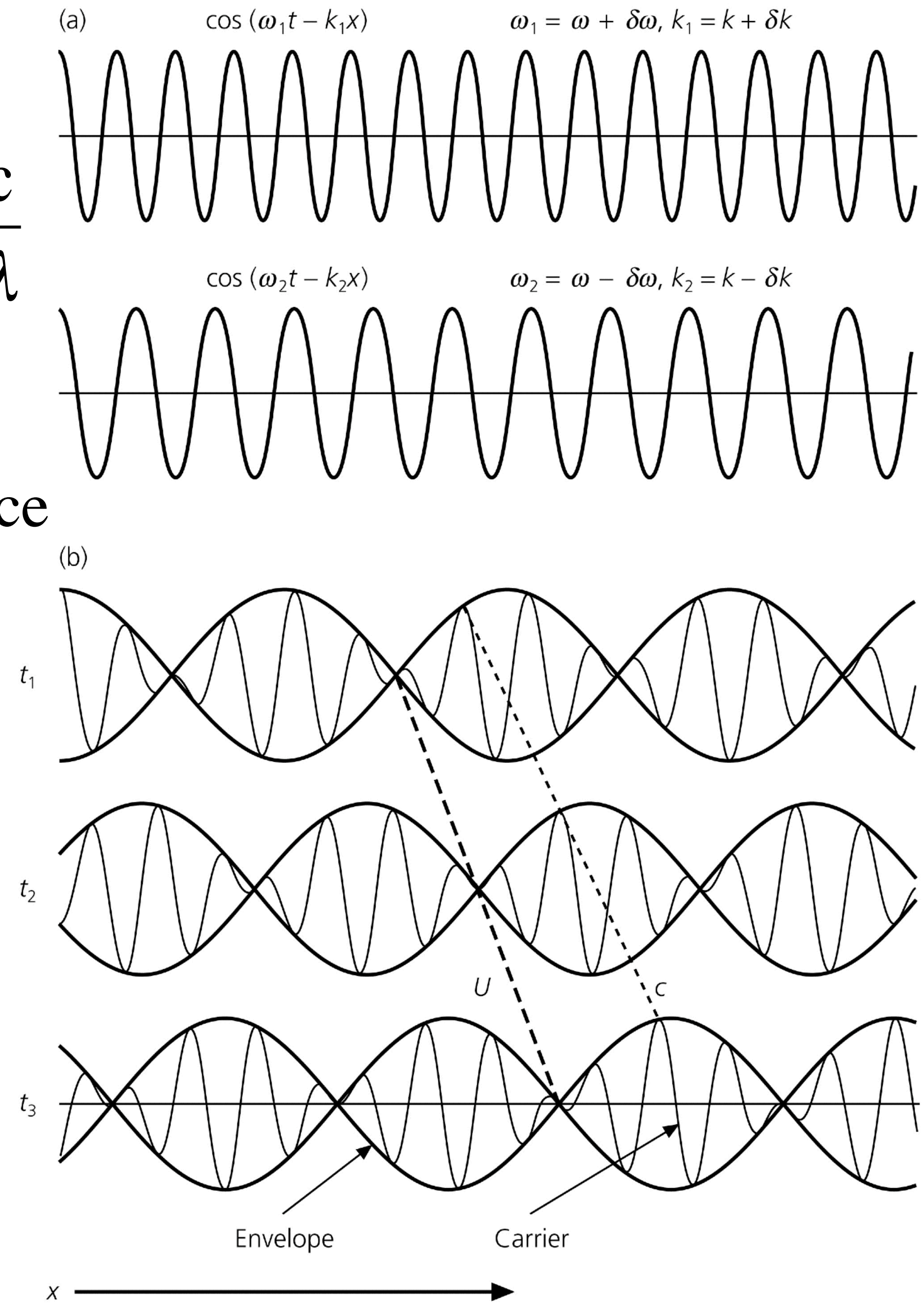


Figure 2.8-2: Fundamental mode Love wave group and phase velocities.

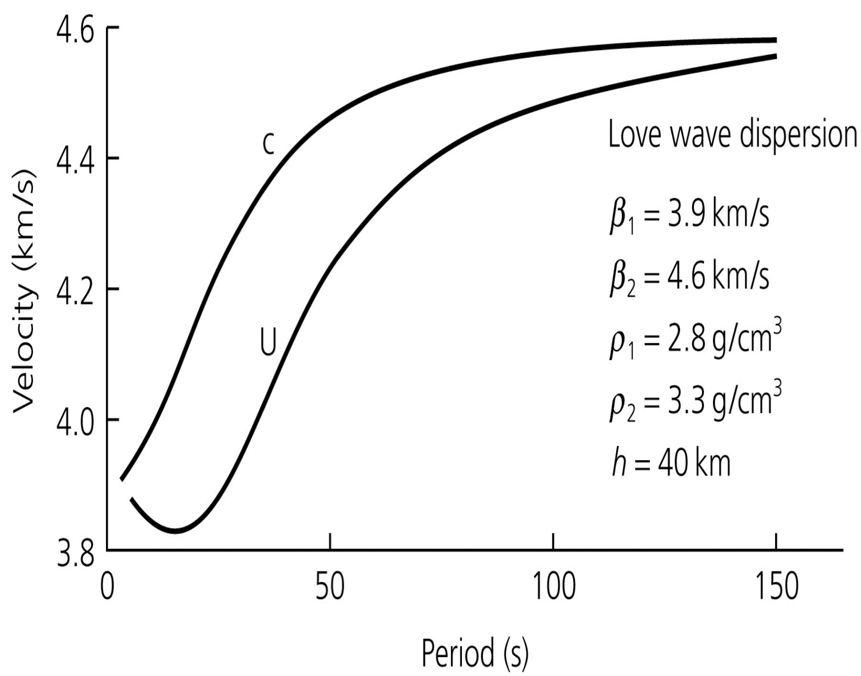
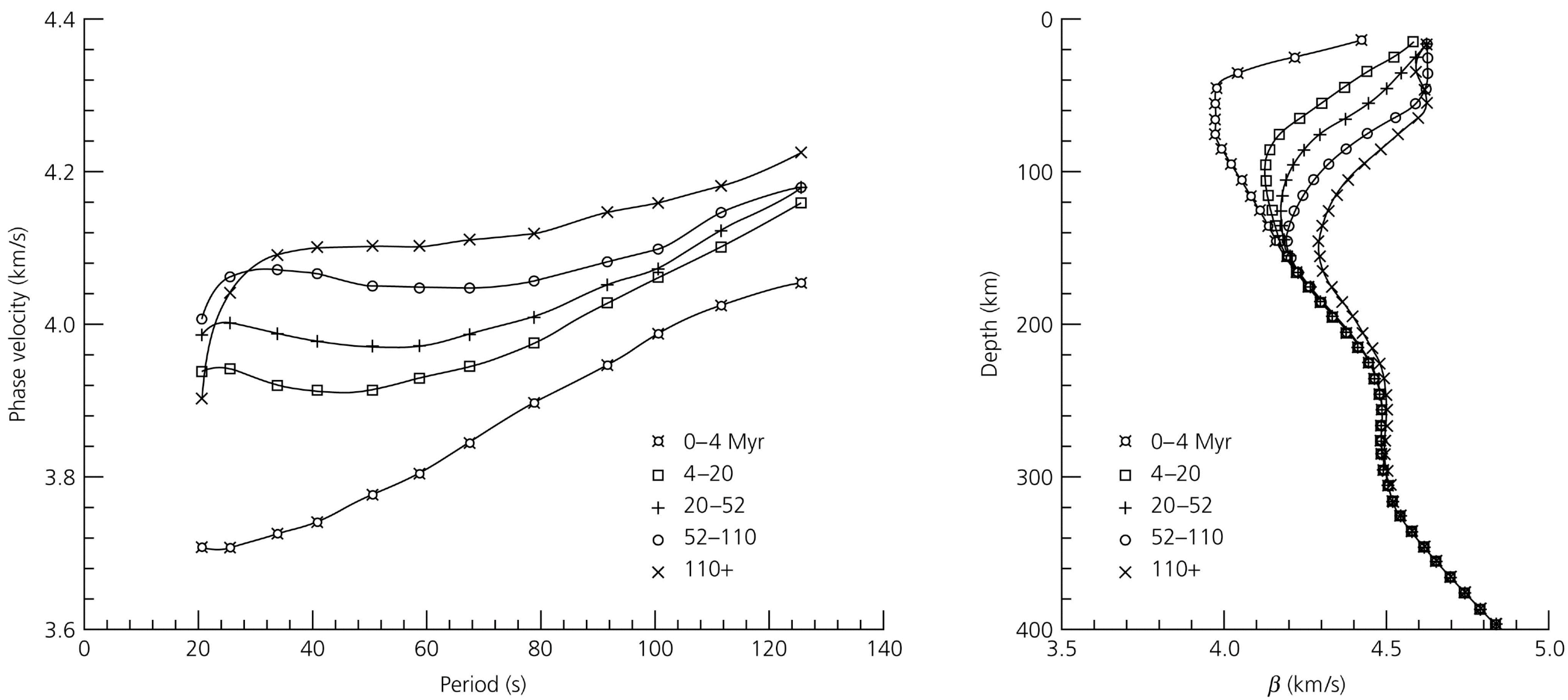


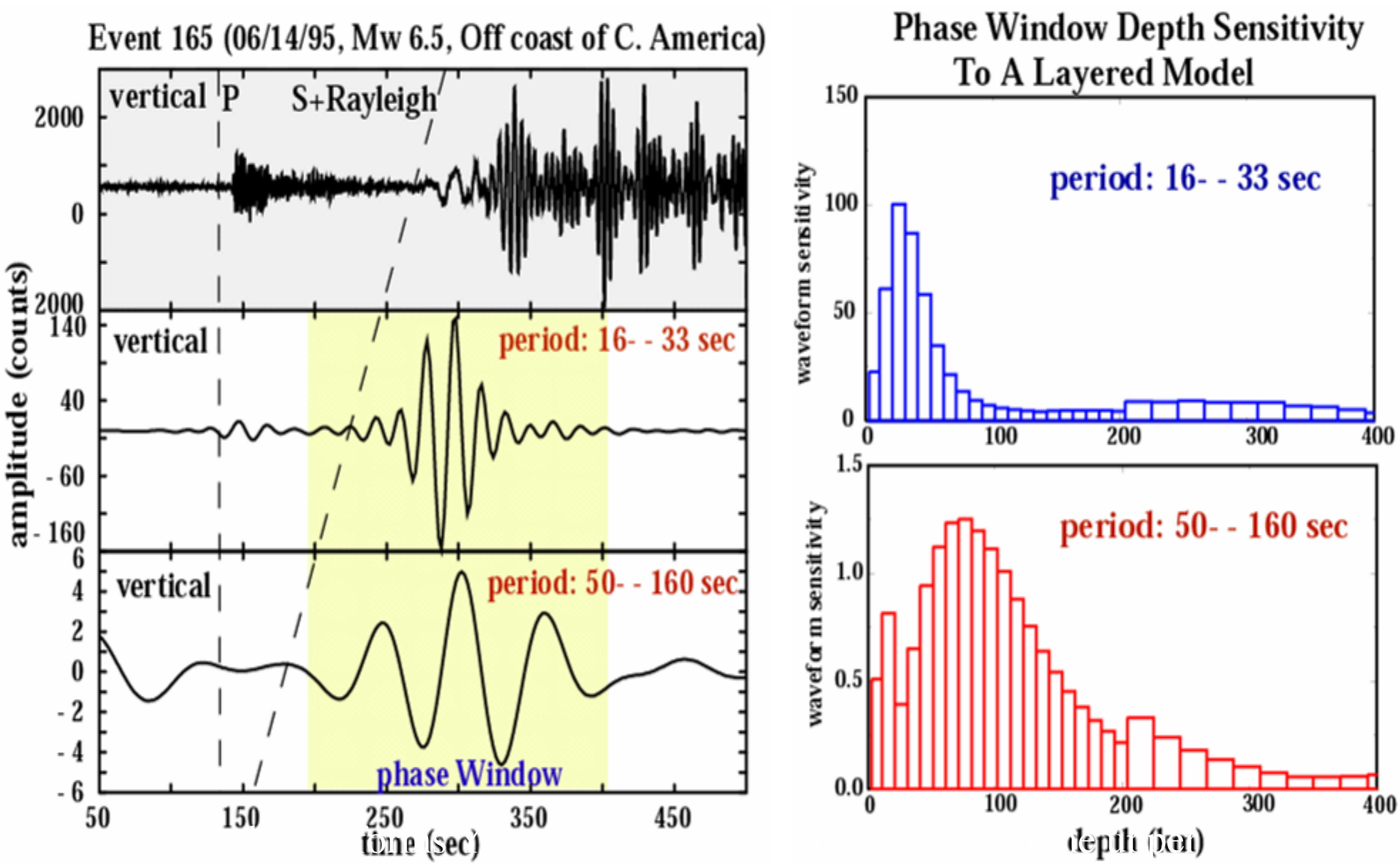
Figure 2.8-7: Rayleigh wave phase velocity dispersion as a function of oceanic plate age.

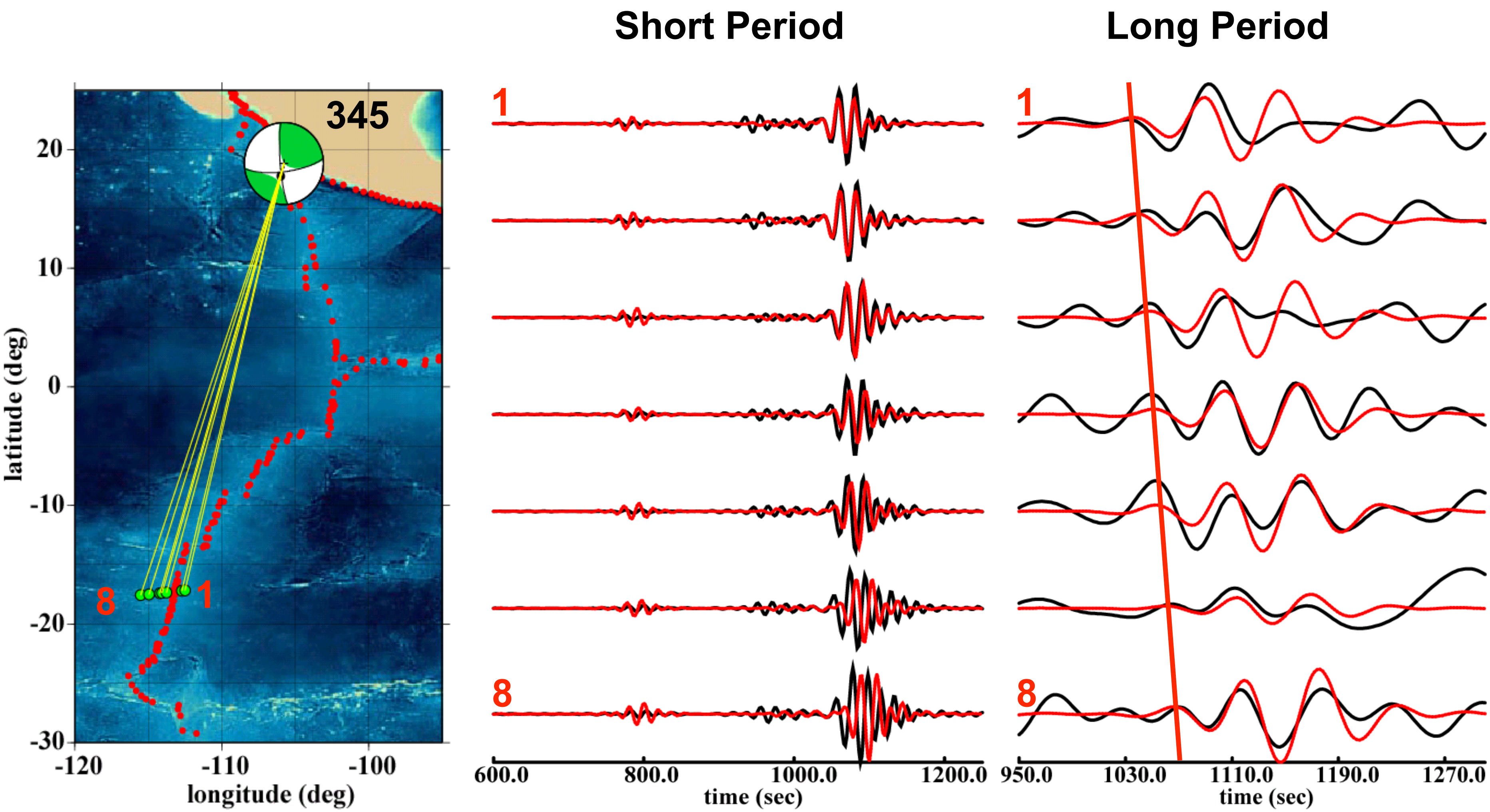


Obtained by an anisotropic modeling of shear speeds using the dispersion curves for waves that travel near the East Pacific Rise (*Nishimura and Forsyth*, 1989).

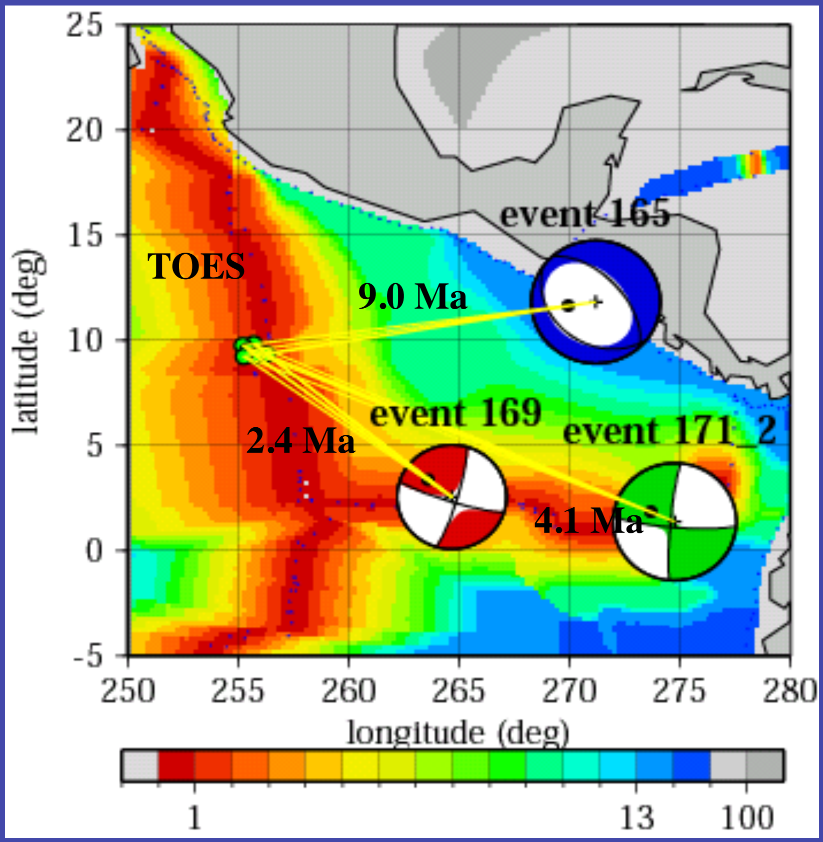
Moral: the closer to the ridge, the younger and slower!

Dispersion and frequency-dependent depth sensitivity





Effect of ridge (time-wise) is strong in the shallow mantle, but is greatly reduced below ~80 km (Gu et al., 2005).



- A low velocity zone is clearly needed to fit the data. All paths require models with a LVZ peaking at depths slightly beneath N&F 0-4 Ma.
- A much weaker LVZ is needed for the 9-Ma year path.