

Largest city on Altiplano,
La Paz, Bolivia

Subduction along south
America, marked by
Flat (<30 deg) subduction
(*Haschke et al.*, Chap 16)

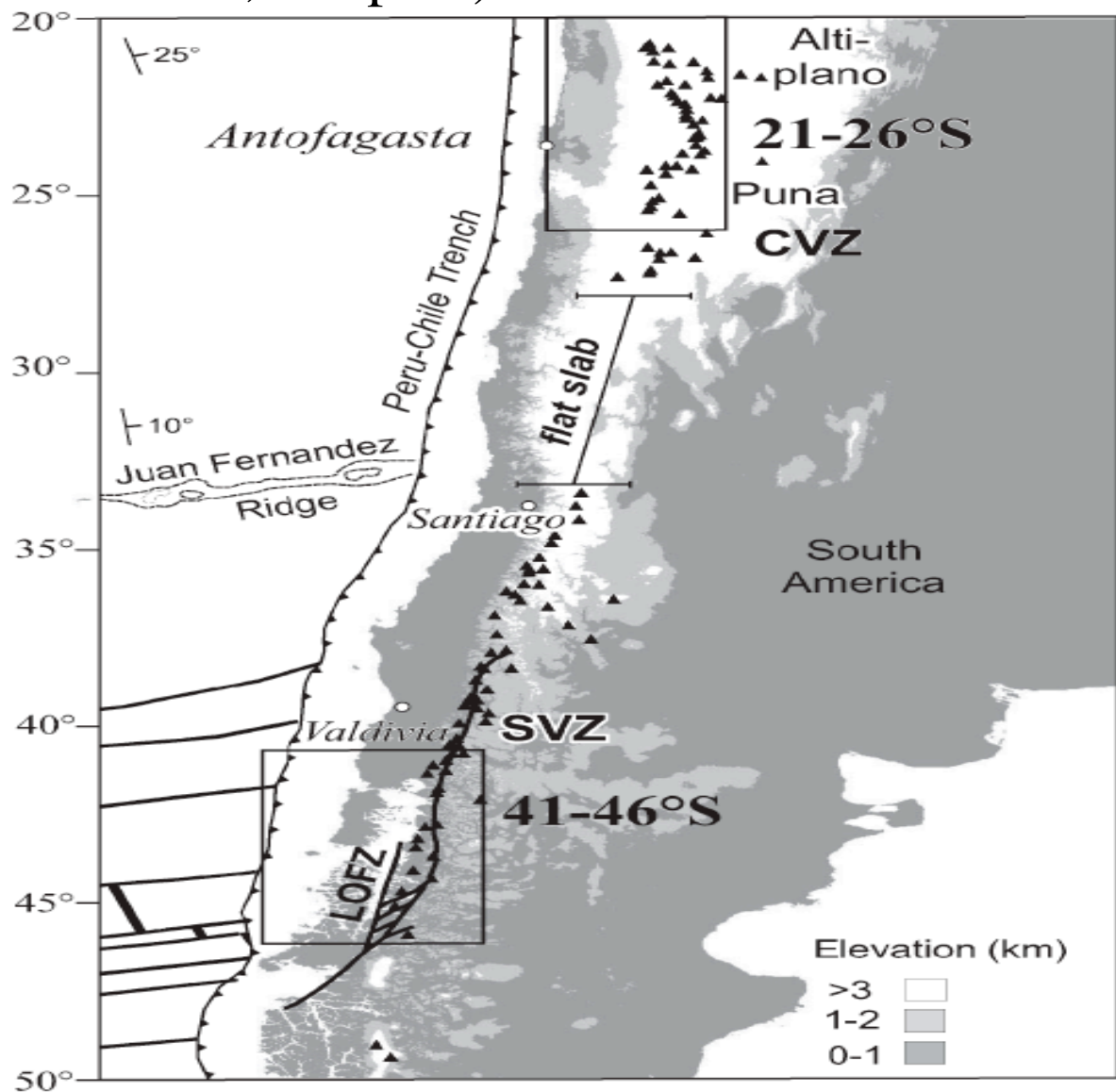
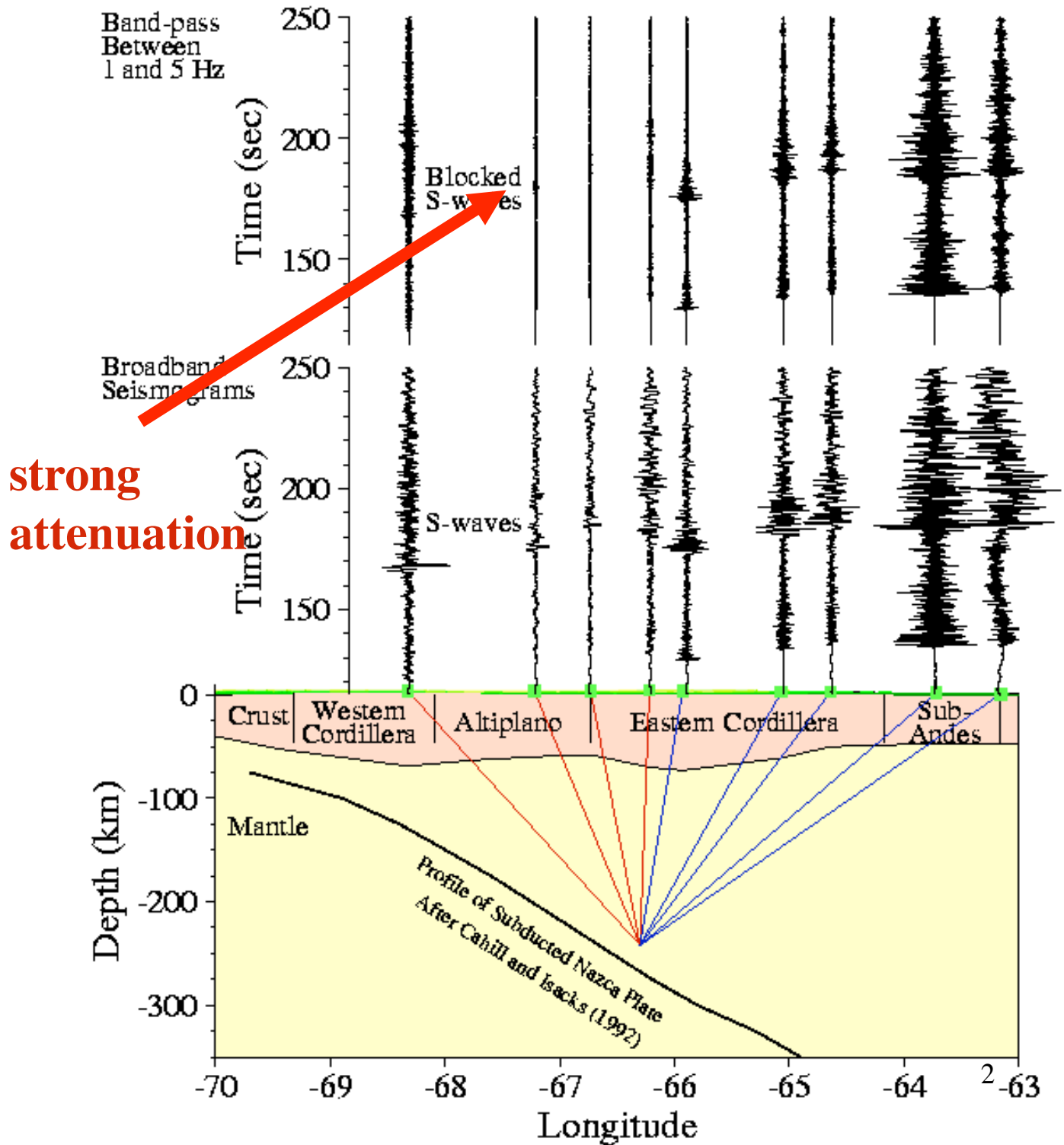


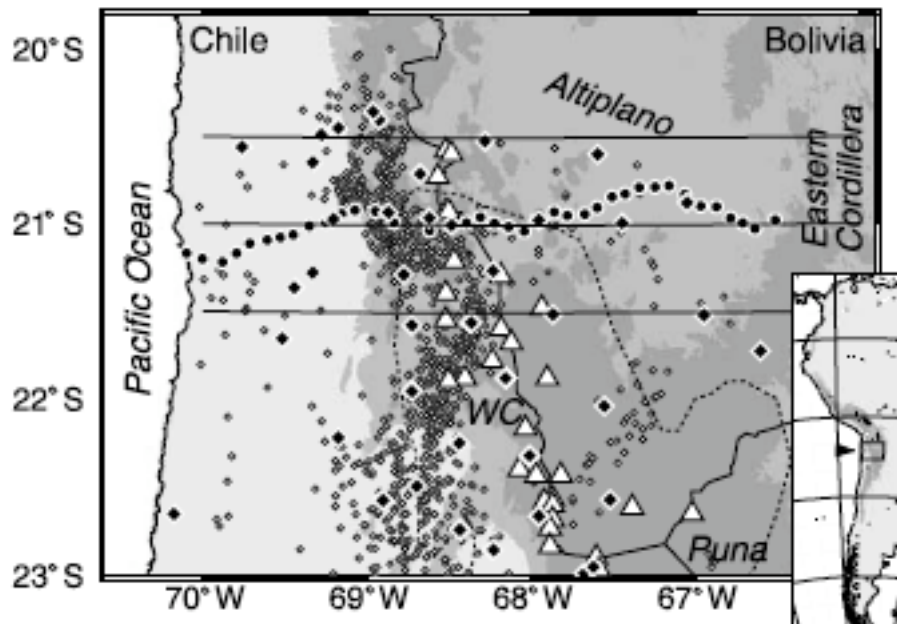
Fig. 16.1. Distribution of present-day volcanism (*triangles*) in the Andes between 20° and 50° S. Topography is shaded at 1 000 and 3 000 m elevation. Obliquity is relative to the Peru-Chile trench axis after Sommoza (1998). *Rectangles* indicate studied arc segments in north and south Chile

Attenuation and Anelasticity

Illustration of S-wave blockage to Altiplano Stations



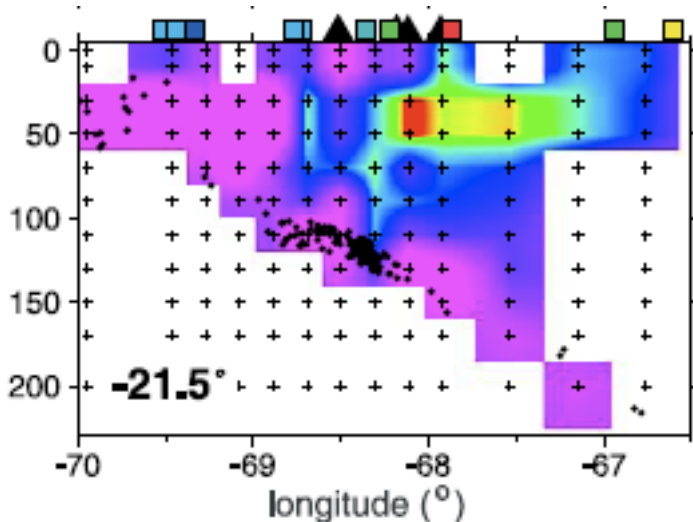
Station and Earthquake setting



Stations: squares

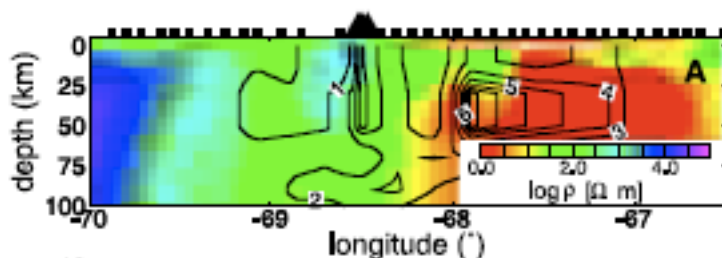
Earthquakes:
solid dots

Using local earthquakes to image the Q structure of the magma chamber beneath Arc volcanos.



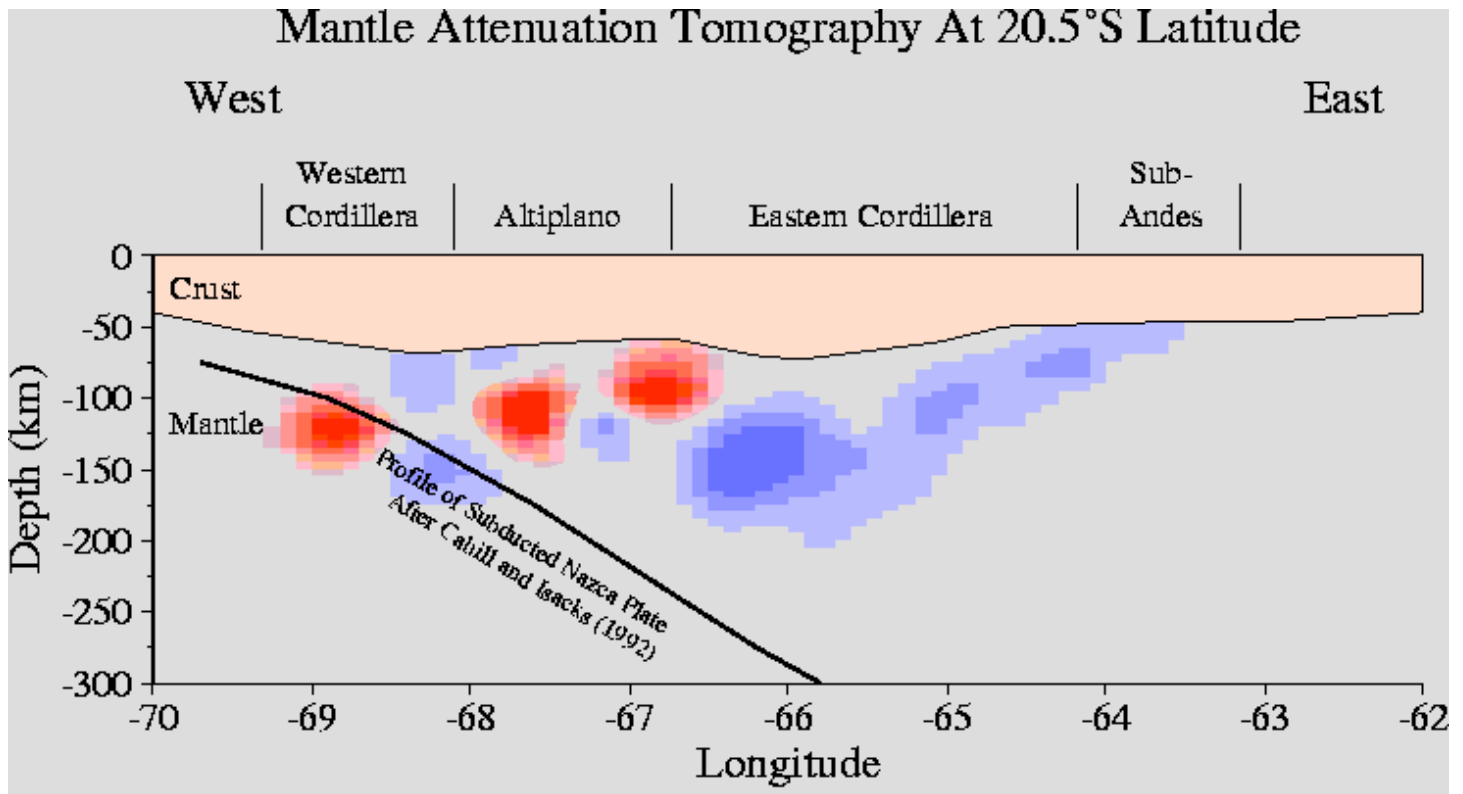
1/Q from inversions

* shows a red (high attenuation or low Q) zone, possibly magma chamber with melt.



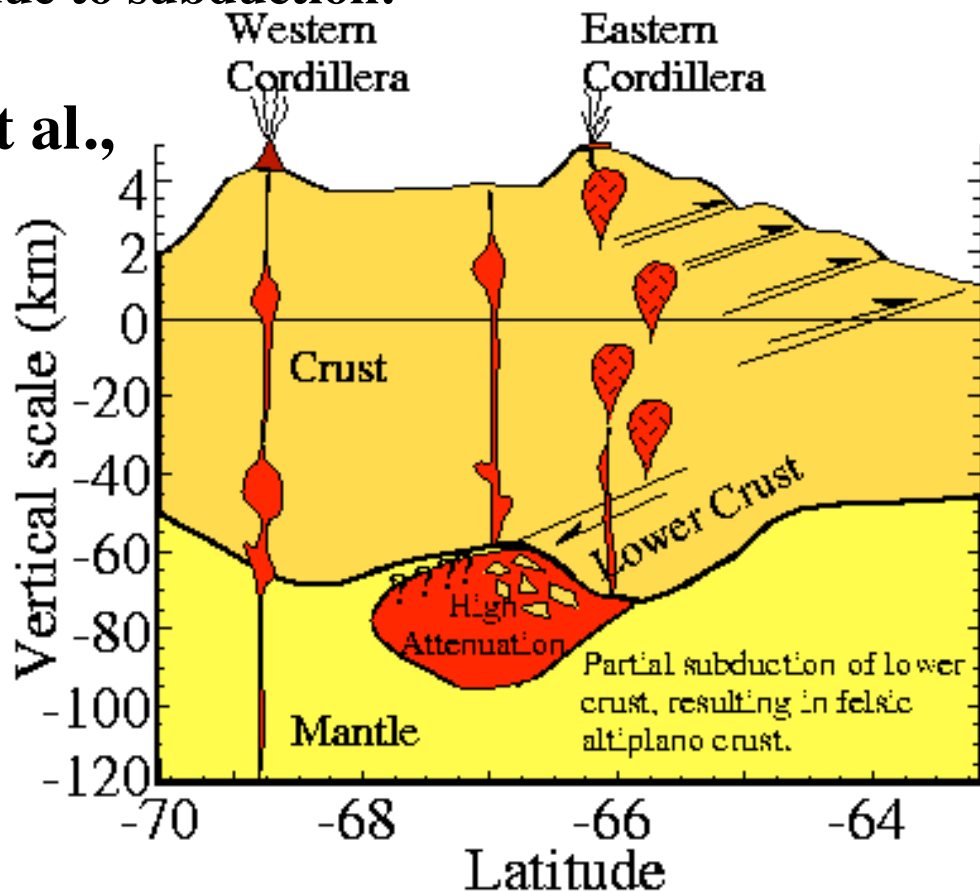
MT image of the same region which shows high conductivity zone (means fluid is present)

Haberland et al., GRL 2003

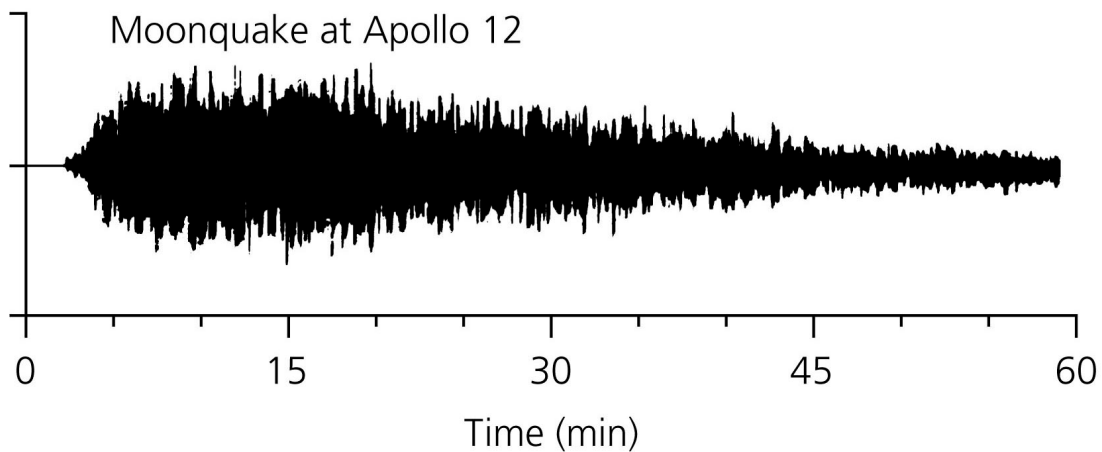
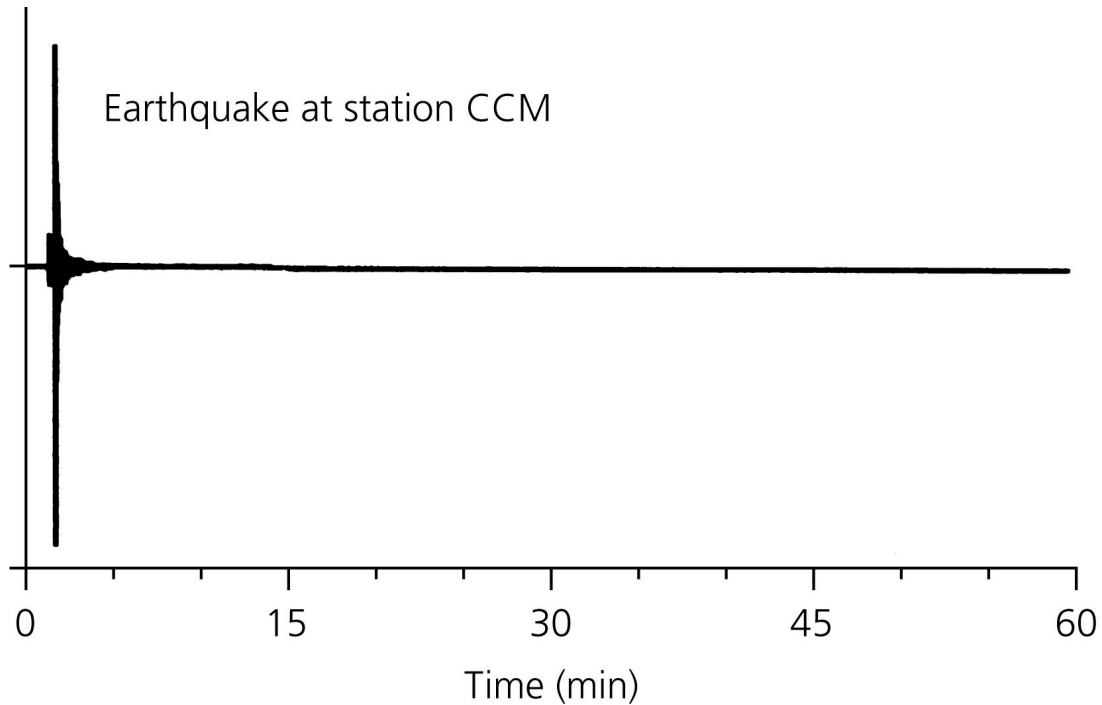


The key here is to correlate a decrease in Q with fluids in the crust and mantle. The fluid layer again represents melting due to subduction.

Myers et al.,
1995



Another interesting example of attenuation

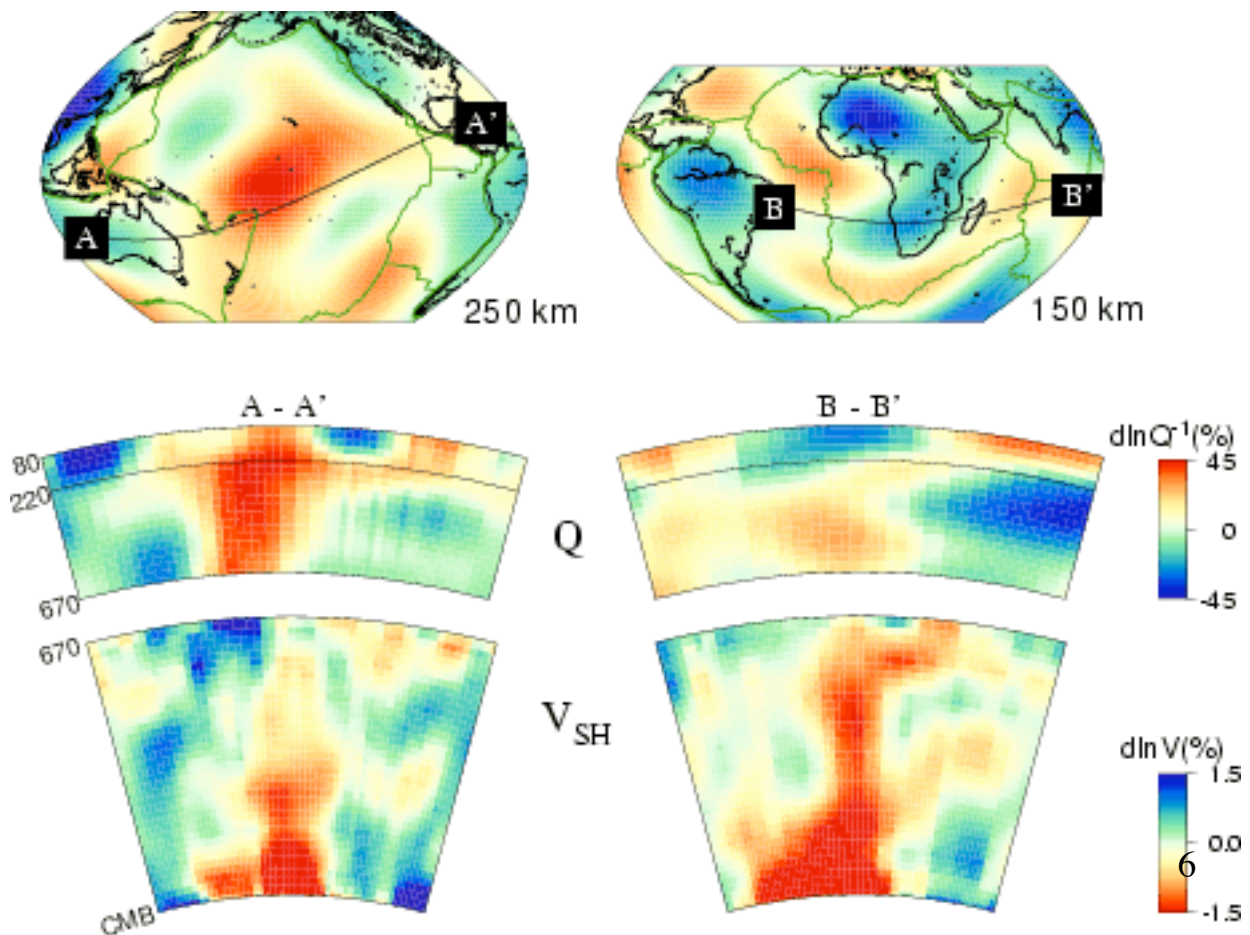


The moon is a lot less attenuative, thus producing many high frequency signals.
Problem: Can't find P, S and Surface waves ⁵

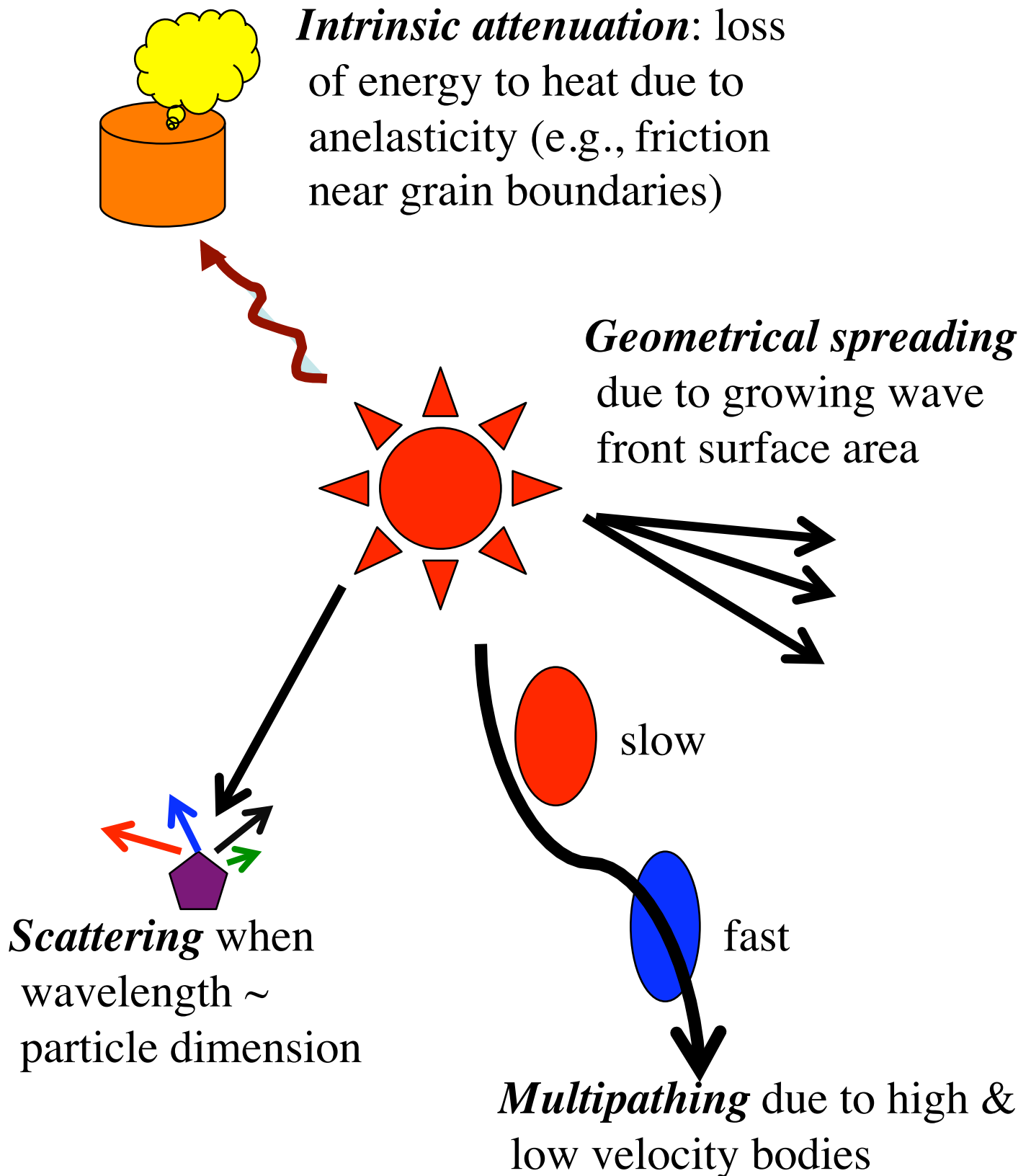
Global 3D Q tomography

Gung et al., 2002

Idea: Measure the amplitude difference between observed and predicted seismograms, for surface and body waves both, then invert for Q^{-1} . The big idea here is that there appears to be a correlation between the so called “Superplumes”, which are hot mantle upwellings, with a decrease of Q . This is additional evidence that they exist, and the fact they get to the surface of the earth may imply the earth only has 1 layer, not two layers, of convection.



Imagine a light source (seismic source):



Attenuation Mechanisms:

- (1) **Geometrical spreading:** wavefront spreading out while energy per square inch or becomes less.
- (2) **Multipathing:** waves seek alternative paths to the receiver. Some are dispersed and some are bundled, thereby affecting amplitudes.
- (3) **Scattering:** A way to partition energy of supposedly main arrivals into boundary or corner diffracted, scattered energy.

Key: very wavelength dependent.

(4) intrinsic attenuation: due to anelasticity

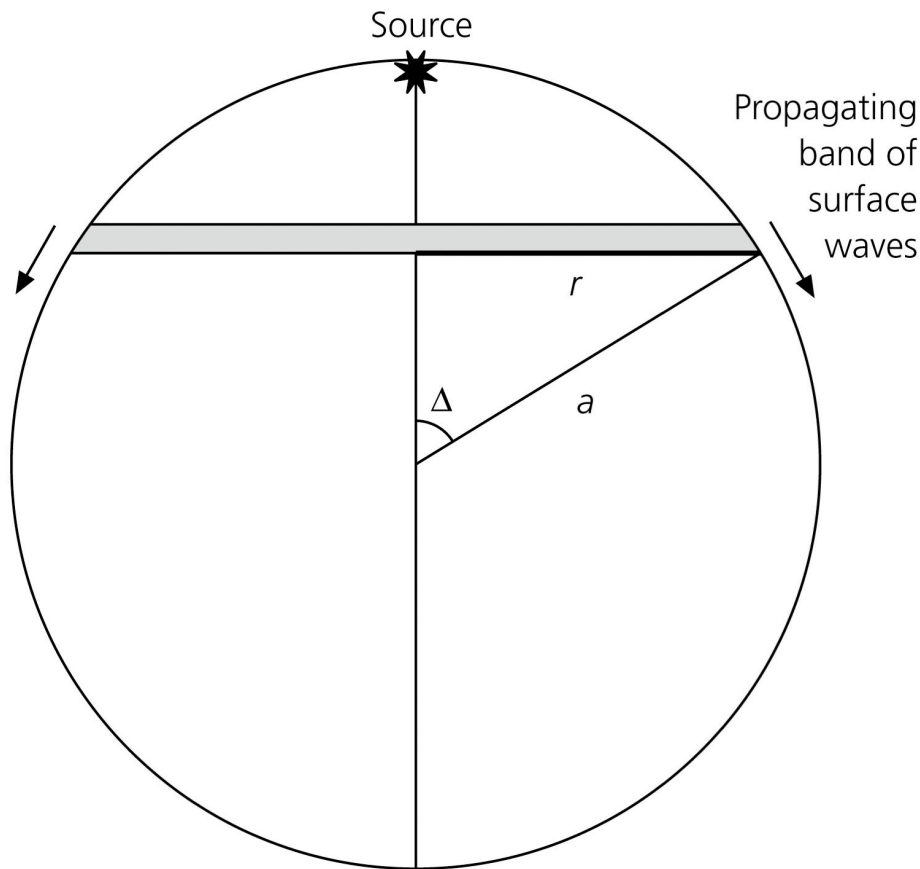
So far we have only concerned with purely elastic media, the real earth materials are always “lossy”, leading to reduced wave amplitudes, or *intrinsic attenuation*.

Mechanisms to **lose** energy:

- (1) Movements along mineral dislocations
- (2) Shear heating at grain boundaries

These are called “internal friction”.

(1) geometrical spreading



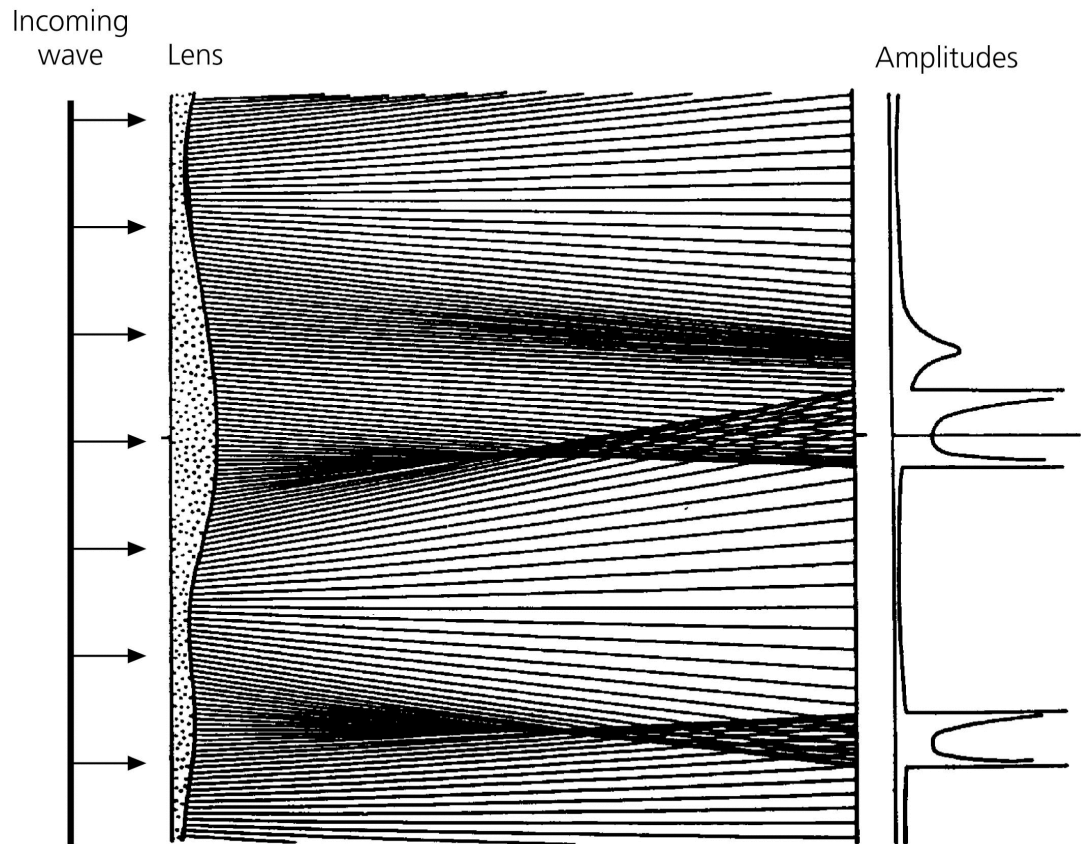
For laterally homogeneous earth, surface wave will spread out as a growing ring with circumference $2\pi r$, where r is distance from the source. Conservation of energy requires that energy per unit wave front decrease as $1/r$ and amplitude as $(1/r)^{1/2}$. Energy decays as

$$\frac{1}{r} = \frac{1}{a \sin \Delta} \longrightarrow \frac{1}{a \sin \Delta}$$

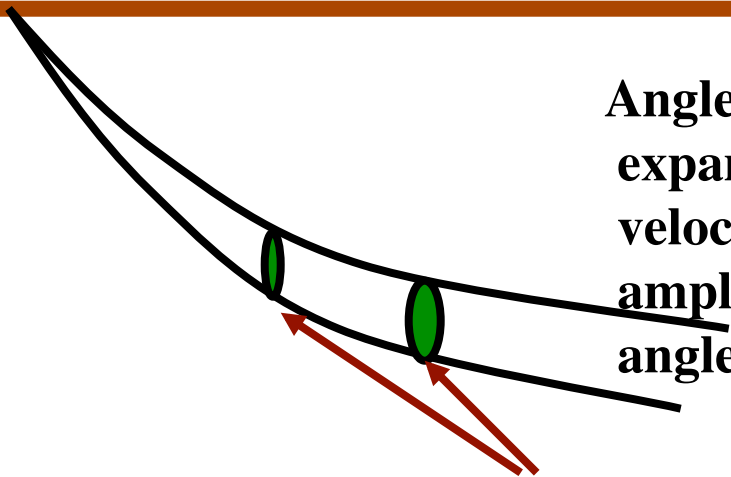
Where for $\Delta=0$ and 180 , this is maximum and $\Delta=90$ this is minimum (another way of understanding the antipodal behavior) Body waves amplitude decay $1/r$.

(2) Effect of Multipathing

Figure 3.7-5: Example of velocity heterogeneities affecting wave amplitudes.



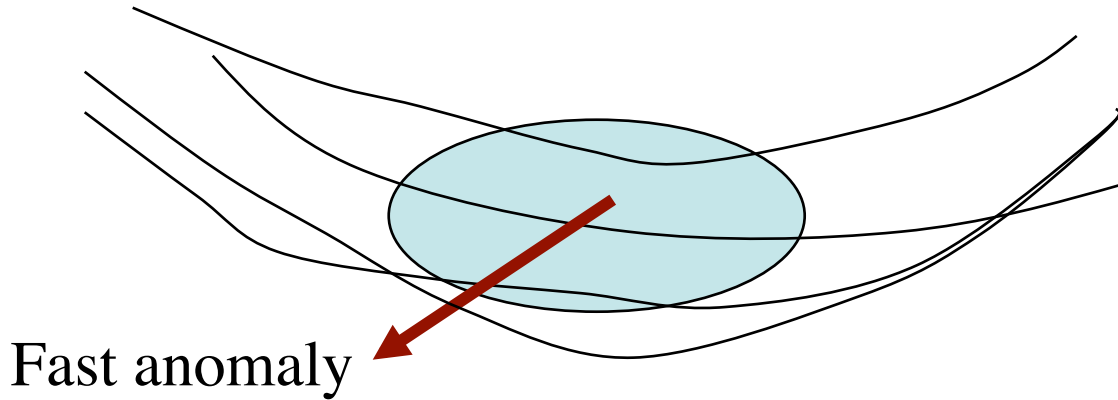
Idea of Ray Tubes in body waves



Angle dependent: ray bundle expands or contracts due to velocity structure. Also: amplitude varies with takeoff angle

Ray tube size affects amplitudes, smaller area means larger amplitude

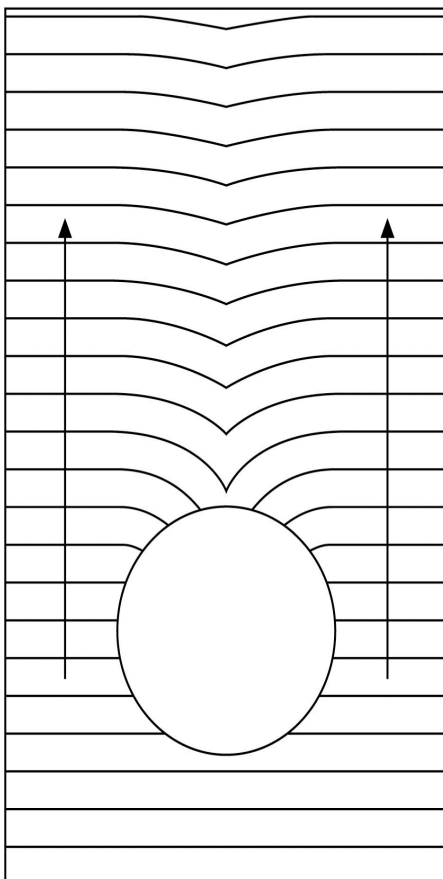
Wavelength: If heterogeneity bigger than λ , we will get ray theory result. If smaller than λ , then get diffraction result!



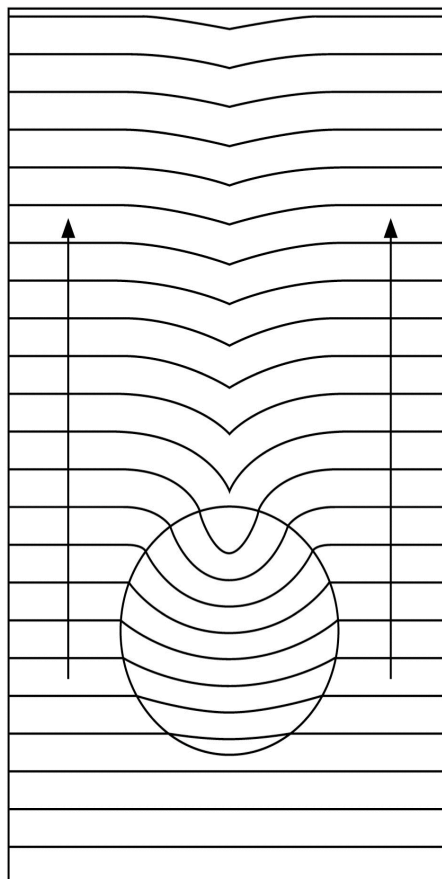
Terribly exaggerated plot of *multipathing*!

Huygen's principle

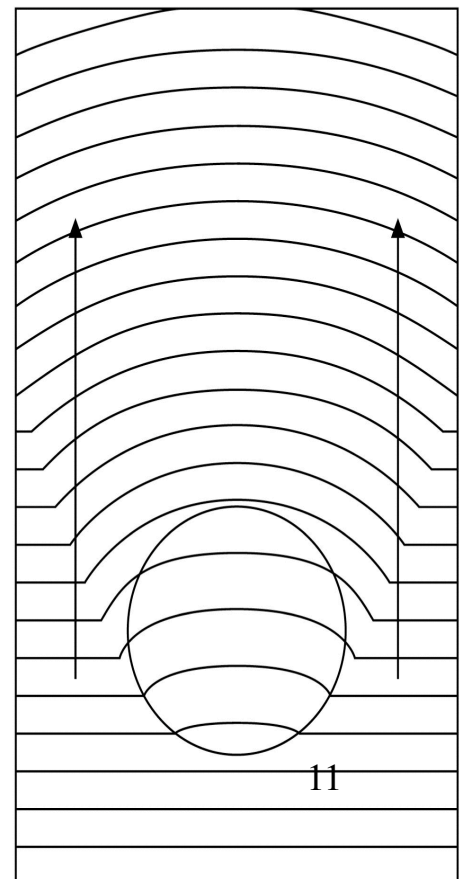
a. Spherical obstacle



b. Spherical slow anomaly

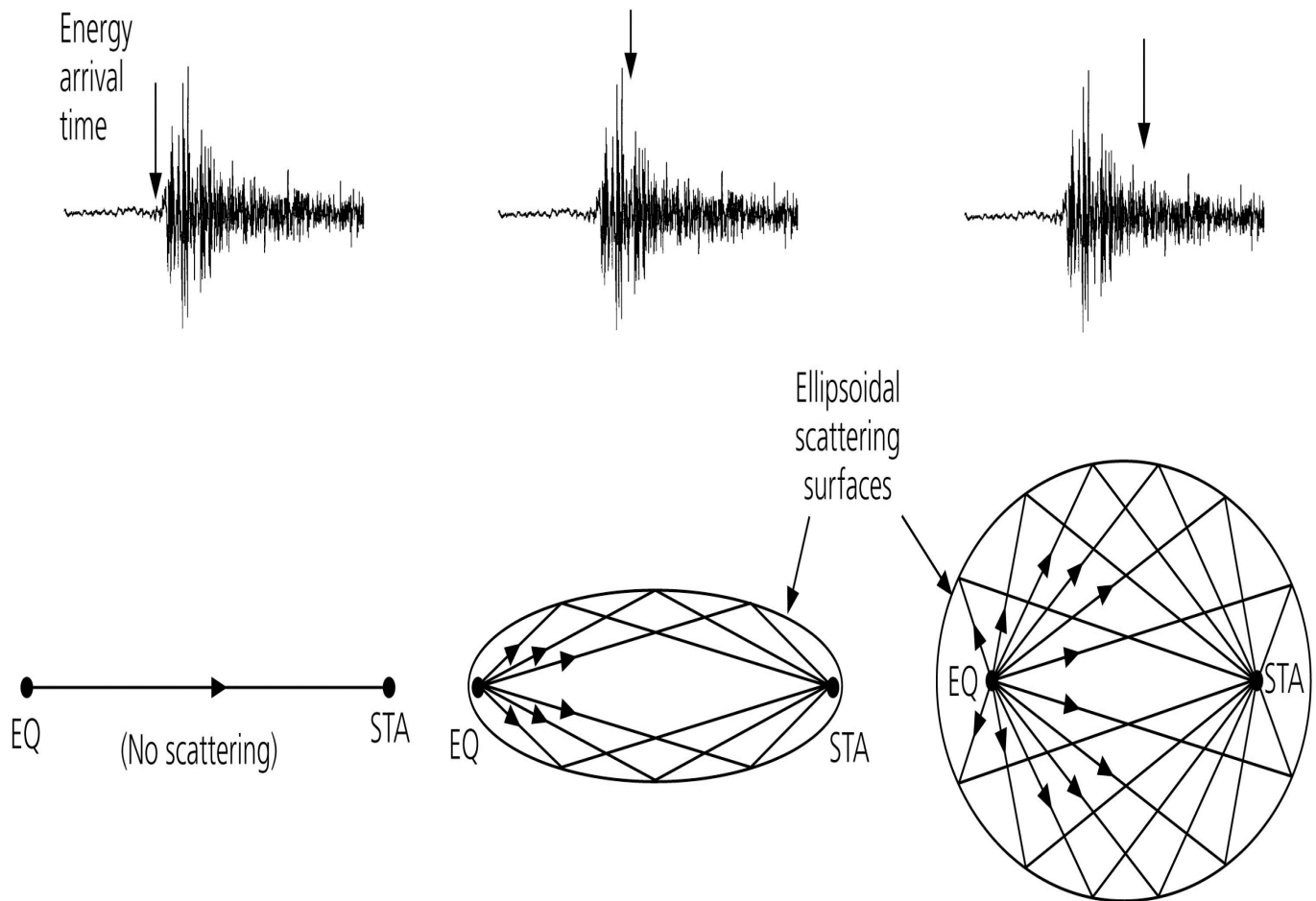


c. Spherical fast anomaly

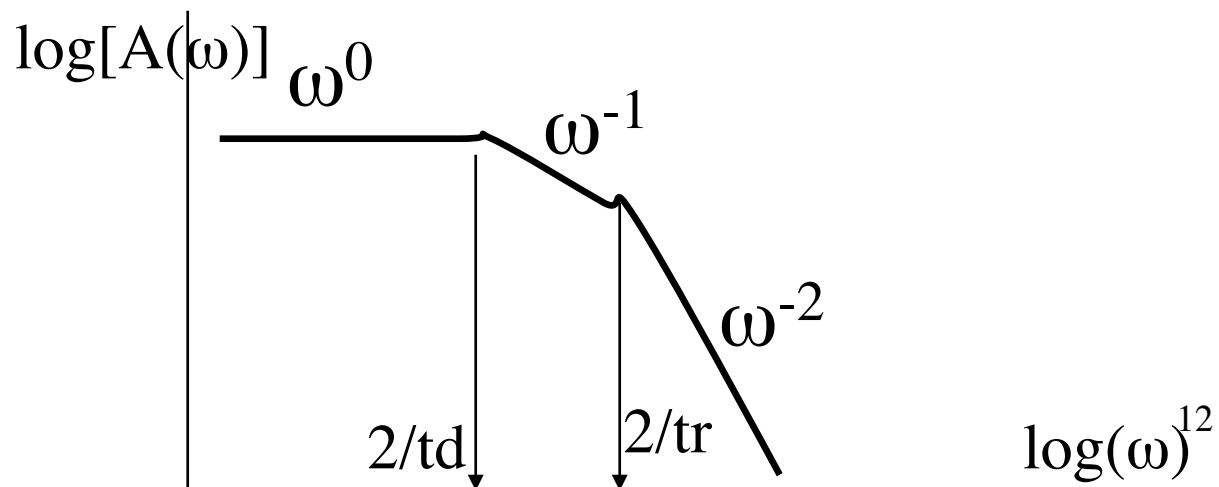


(3) Scattering

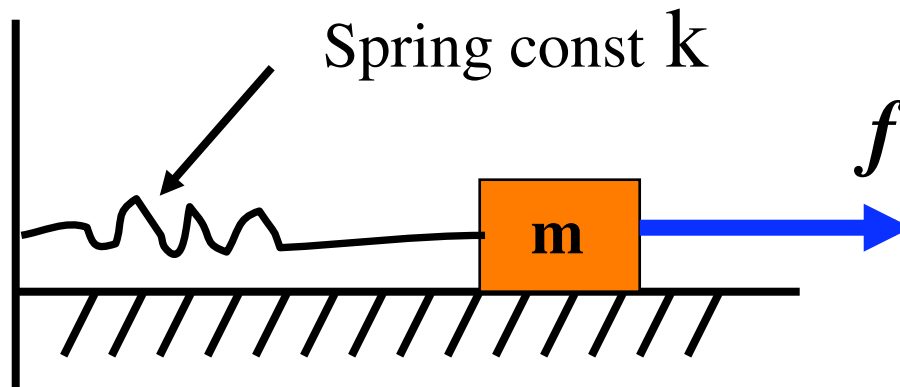
Figure 3.7-9: Development of a P-wave coda.



Wavelength effect demonstrated for P wave coda. People use source spectrum to analyze the coda and obtain information about Q and scatters about a given path



(4) Intrinsic Attenuation and Q



$$m \frac{\partial^2 u}{\partial t^2} + ku = 0 \quad \text{(Equation of Motion)} \\ \mathbf{F = ma)}$$

General solution to this equation

$$u = Ae^{i\omega_0 t} + Be^{-i\omega_0 t}$$

Lets take $u = A_0 e^{i\omega_0 t}$

Plug in the equation:

$$m(i\omega_0)^2 A_0 e^{i\omega_0 t} + kA_0 e^{i\omega_0 t} = 0 \\ -m\omega_0^2 + k = 0 \quad \longrightarrow \quad \omega_0 = \sqrt{\frac{k}{m}}$$

Suppose there is damping (attenuation), this becomes a ***damped harmonic oscillator*** (or a electronic circuit with a resistor): :

$$m \frac{\partial^2 u(t)}{\partial t^2} + \gamma m \frac{\partial u(t)}{\partial t} + ku(t) = 0 \quad 13$$

where γ is a damping factor.

Define $Q = \omega_0 / \gamma$ (Q = Quality factor)

$$\frac{\partial^2 u(t)}{\partial t^2} + \gamma \frac{\partial u(t)}{\partial t} + \frac{k}{m} u(t) = 0$$

$$\frac{\partial^2 u(t)}{\partial t^2} + \frac{\omega_0}{Q} \frac{\partial u(t)}{\partial t} + \frac{k}{m} u(t) = 0 \quad (1)$$

For this second order differential equation, let's assume it has a general solution of the form

$$u(t) = A_0 e^{ipt}$$

where the p is a complex number and we assume the measured displacement $u(t)$ to be the real part of this complex exponential.

Substitute $u(t)$ into equation (1)

$$\begin{aligned} -A_0 p^2 e^{ipt} + i \frac{\omega_0}{Q} A_0 p e^{ipt} + \frac{k}{m} A_0 e^{ipt} &= 0 \\ -p^2 + i \frac{\omega_0}{Q} p + \omega_0^2 &= 0 \end{aligned} \quad (2)$$

Since p is complex, we can write

$$p = a + ib, \quad p^2 = (a + ib) * (a + ib) = a^2 + 2iab - b^2$$

Substitute in, then Equation (3) becomes

$$-a^2 - 2iab + b^2 + i(a + ib)\frac{\omega_0}{Q} + \omega_0^2 = 0$$

Split this into real and imaginary parts:

Real: $-a^2 + b^2 - b\frac{\omega_0}{Q} + \omega_0^2 = 0 \quad (3)$

Imag: $-2iab + ia\frac{\omega_0}{Q} = 0 \rightarrow -2b + \frac{\omega_0}{Q} = 0$

$\rightarrow b = \frac{\omega_0}{2Q} \quad (4)$

Substitute (4) into (3) to solve for a :

$$-a^2 + \left(\frac{\omega_0}{2Q}\right)^2 - \frac{\omega_0}{2Q} \frac{\omega_0}{Q} + \omega_0^2 = 0$$

$$a^2 = \omega_0^2 + \frac{\omega_0^2}{4Q^2} - \frac{\omega_0^2}{2Q^2}$$

$$a^2 = \omega_0^2 \left(1 - \frac{1}{4Q^2}\right) \rightarrow a = \omega_0 \left(1 - \frac{1}{4Q^2}\right)^{\frac{1}{2}} = \omega$$

What have we done???

(1) We have defined an angular frequency that is not exactly the original frequency ω_0 , but a **MODIFIED** frequency ω based on Q !

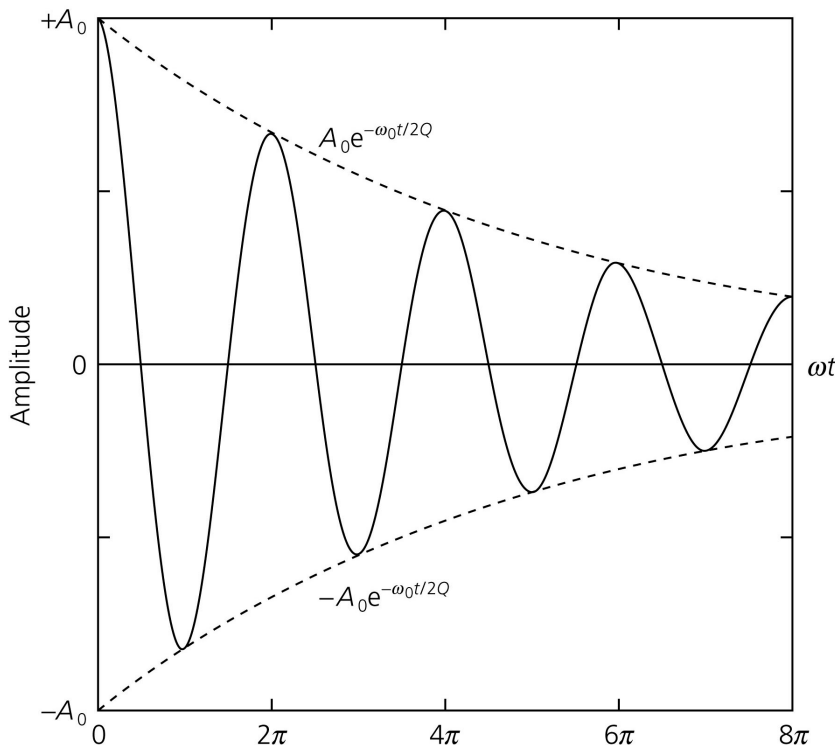
(2) Lets substitute the solutions into the original solution

$$\begin{aligned} u(t) &= A_0 e^{ipt} = A_0 e^{i(a+ib)t} = A_0 e^{iat} e^{-bt} \\ &= A_0 e^{i\omega t} e^{-bt} = A_0 e^{i\omega t} e^{-\omega_0 t / (2Q)} \end{aligned}$$

Plot: real part of the displacement

$$A_0 e^{-\omega_0 t / (2Q)} \cos(\omega t)$$

Figure 3.7-11: Wave amplitude for a damped harmonic oscillator.



**Dotted line
(envelope
function)**



$$A = A_0 e^{-\omega_0 t / (2Q)}$$