Moment Tensor Inversion

Symmetry of the ***Moment Tensor***: $\mathbf{M_{ij} = M_{ji}}$

Now assume the earthquake is a pure double couple, that means there is **no net volume** change, which means **trace must be 0**.

M11 + M22 + M33 = 0

That means independent elements can be represented by 5 independent parameters:

M11+M22

M11-M22

M12

M13

M23

M11=[(M11+M22)+(M11-M22)]/2

M22=[(M11+M22)-(M11-M22)]/2

M33=-(M11+M22)

This adds the double-couple constraint with no net volume change.

Formulation of source inverse problem

Resulting displacement $\mathbf{u}$ from a force couple

$$u_i(\mathbf{x},t) = G_{ij}(\mathbf{x},t;\mathbf{x_0},t_0) f_j(\mathbf{x_0},t_0) - G_{ij}(\mathbf{x},t;\mathbf{x_0} - \hat{\mathbf{x}}_k d,t_0) f_j(\mathbf{x_0},t_0)$$

Taylor expand second term:

$$G_{ij}(\mathbf{x},t;\mathbf{x_0} - \hat{\mathbf{x}}_k d,t_0) f_j(\mathbf{x_0},t_0) = G_{ij}(\mathbf{x},t;\mathbf{x_0},t_0) f_j(\mathbf{x_0},t_0) -$$

$$\frac{\partial G_{ij}(\mathbf{x},t;\mathbf{x_0},t_0)}{\partial x_k} f_j(\mathbf{x_0},t_0) * d$$

$$u_i(\mathbf{x},t) = \frac{\partial G_{ij}(\mathbf{x},t;\mathbf{x_0},t_0)}{\partial x_k} f_j(\mathbf{x_0},t) d$$

$$u_i(\mathbf{x},t) = \frac{\partial G_{ij}(\mathbf{x},t;\mathbf{x_0},t_0)}{\partial x_k} M_{jk}(\mathbf{x_0},t0)$$

Bingo! $\boldsymbol{A\ X = D}$

Choices of inversion parameters:

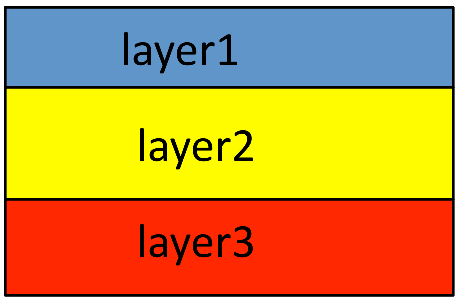
(1) Can invert for 5 moment tensor elements
(2) Heck, you can invert for 6, thereby removing the double-couple condition

Simple Scheme of Moment Tensor Inversion using Reflectivity

Known Quantities:
H, Vp, Vs, ρ, Qa, Qb

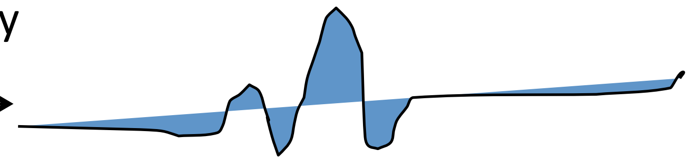
Need to solve: Mij

basis functions



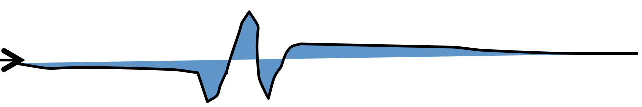
$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Use reflectivity →



$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Use reflectivity →

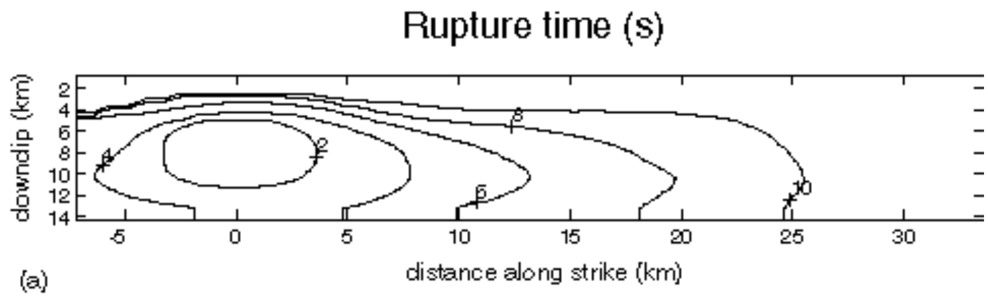


Do this for each element you need to solve, then you have 5 or 6 traces (or basis functions), they should be multiplied by the corresponding weights (or moment tensors). Your problem is to solve for the weights. This is Weights * basis = data. The data is the observed seismogram (a long vector)!

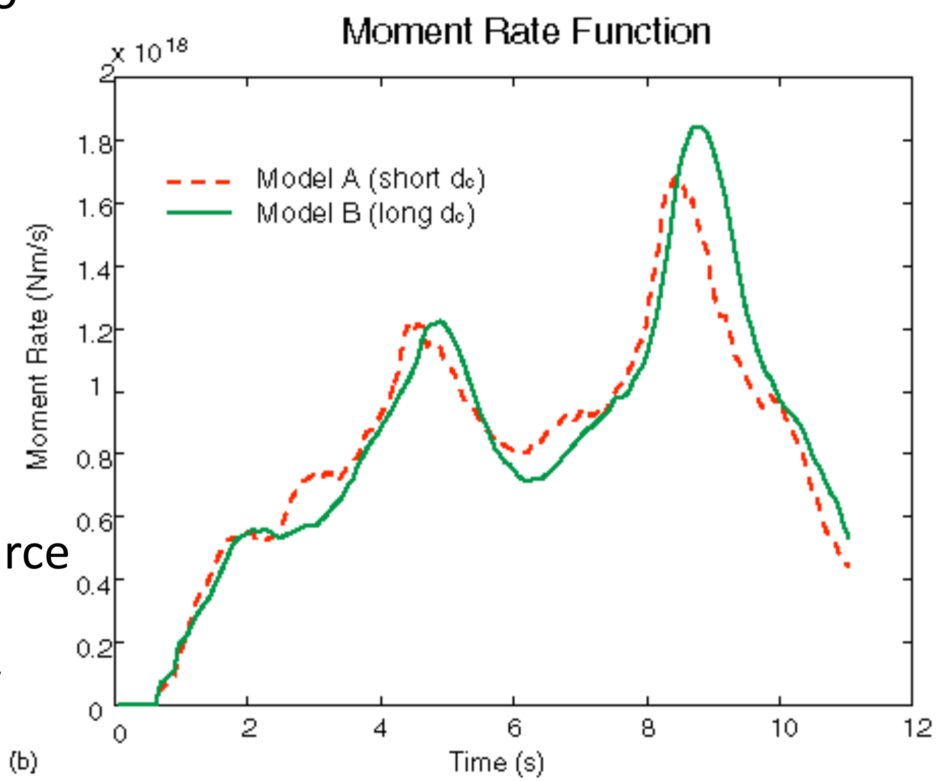
Earthquake Source Time Func

Time domain convolution:

$$y(n) = \sum_{i=-\infty}^{\infty} x(i)h(n-i) \stackrel{\Delta}{=} (x*h)(n)$$



For earthquakes, large ones can have a rupture process that last tens of seconds. To model it, people convolve a point source with a time function



Time Domain Inversion Recipe:

Step 1: Generate synthetic seismogram assuming source pattern and structure are known. Assume source duration is 1 sec.

Step 2: Shift the seismogram by 1 sec, name 1 seismogram 2.

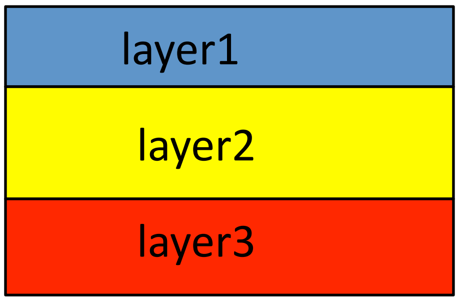
Step 3: Repeat Step 2 by say, 200 sec, if source duration is no longer than that.

Step 4: These shifted seismograms are now the basis function, solve for the weight to each synthetic to mimic the observed.

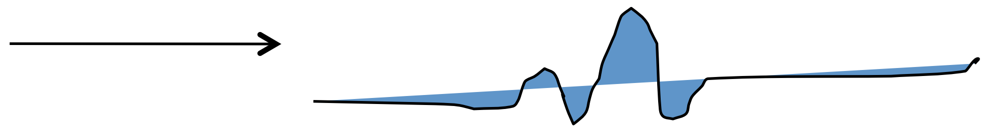
Q inversion using Reflectivity

Known Quantities:
H, Vp, Vs, ρ, Qa, Qb(0)

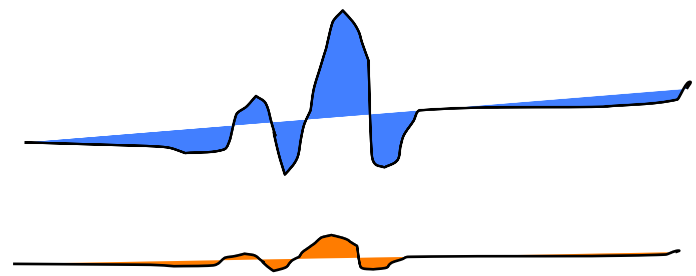
Wish to solve Qb(final)



Step 1: Assume Qb(0), use reflectivity



Step 2: add 0.01*Qb(0) to Qb(0) (1%), calculate synthetic seismogram.



Step 3: calculate difference seismogram by differencing step2 seismogram and step1 seismogram for each time point.

Step 4: divide the difference seismogram by 0.01, this gives a numerical derivative of dS/dQb (dS is the differential seismogram)

Step 5: Based on linear equation

$$\frac{\partial S}{\partial Qb} * \Delta Qb = D - S$$

D – S is the point by point subtraction of Observed Seismogram with synthetic seismogram computed by the starting Qb(0) model. Solve for ΔQb, add to Qb(0).

A few notes about seismic anisotropy

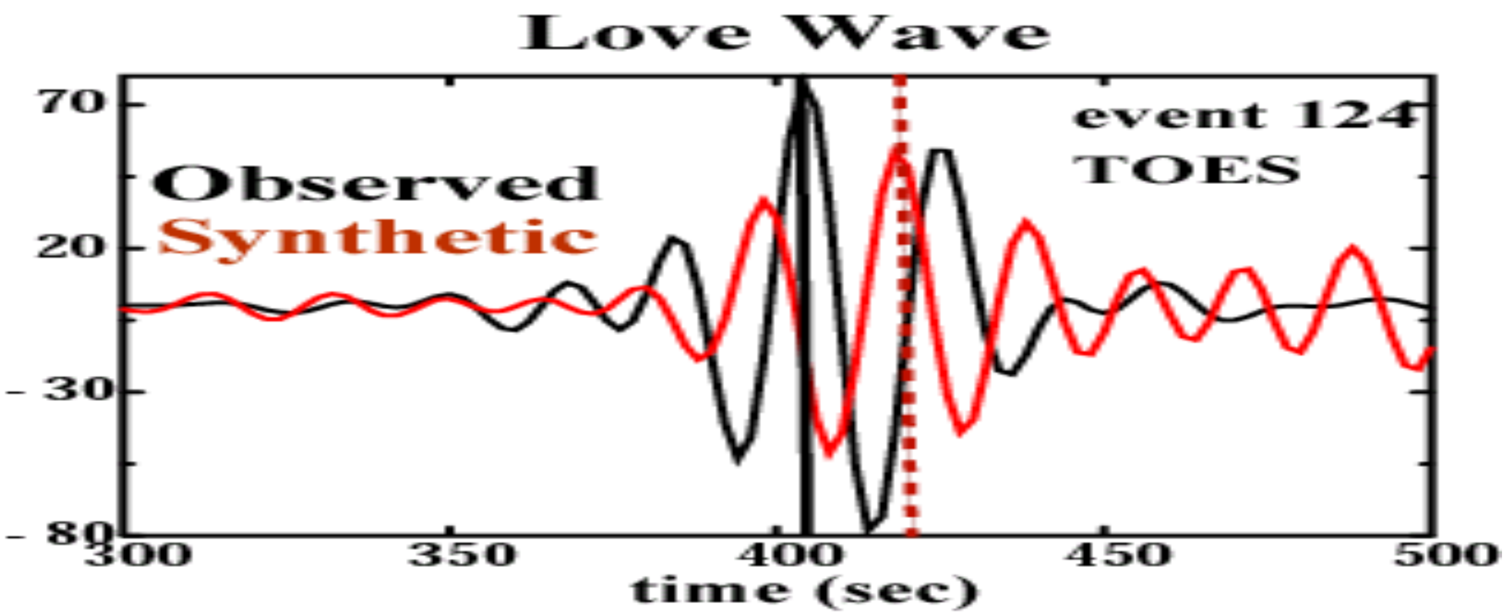
Chronology of *Seismic Structure Analysis*:

(1) ravel time observations, building travel time curves (Jeffreys & Bullen tables), obtaining major divisions inside the earth, core, mantle… Also, obtaining subsurface structure (roughly) from reflectors.

(2) Study of 1-D earth structure (inversion) PREM (Preliminary Reference Earth Model, by Dziewonski & Anderson, the "Half Million Dollar Paper!" (*1981*)

(3) Study of 3D velocity perturbations relative to a 1D reference model, say PREM (*1982, 1984 and onwards*).

(4) Study of anisotropy (*Mid 1980s*): Love-vs-Rayleigh wave analysis, shear wave splitting for S and ScS waves (**polarization anisotropy**), Pn wave speed anisotropy (**azimuthal**), anisotropic inversions for radial anisotropy (polarization + directional)



Gu et al., 2004

Love and Rayleigh Wave Anisotropy:

Black trace: Love waves on the transverse comp.
Red trace: Synthetic Love waves using a earth
Model obtained from the Rayleigh wave of the same source-receiver pair.

Indication: How particles move will result in different arrival time, if material speed is different in different directions.

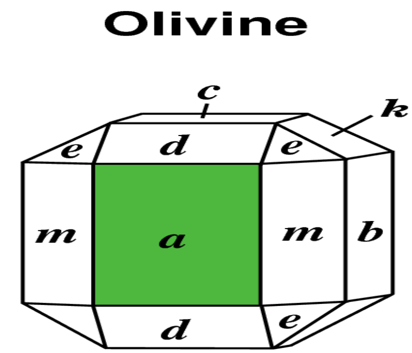
Seismic anisotropy:

(1) presence of anisotropic fabric or minerals:

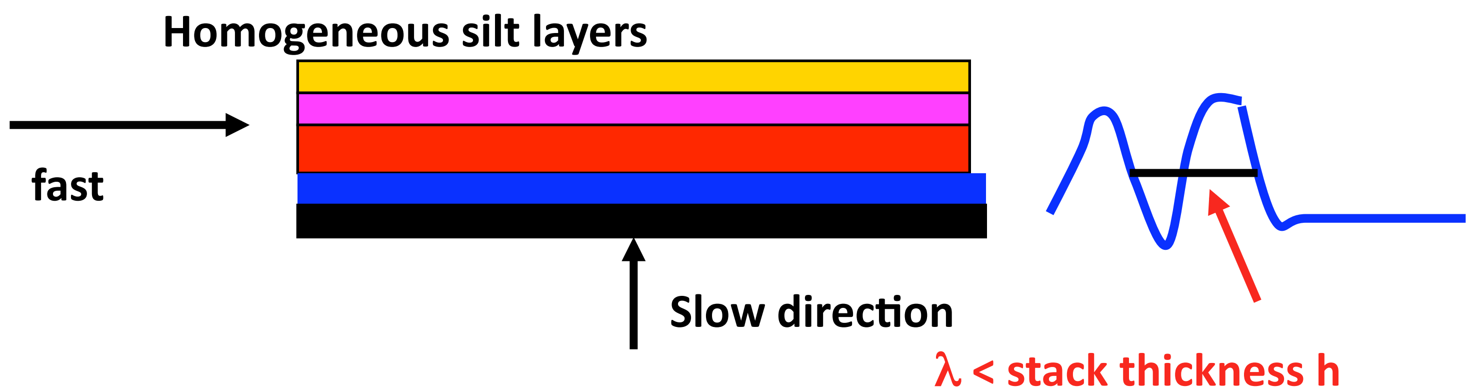
----- Lattice Preferred Orientation (LPO)

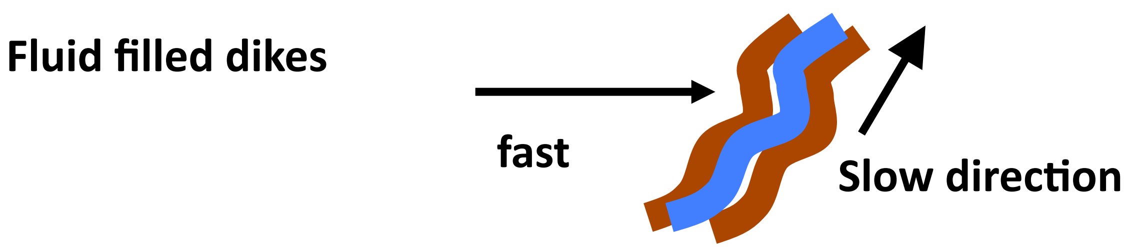
Flow direction is parallel to direction of fast shear or compression velocity

----- Anisotropic effect induced at the refraction surface (behavior of head waves)

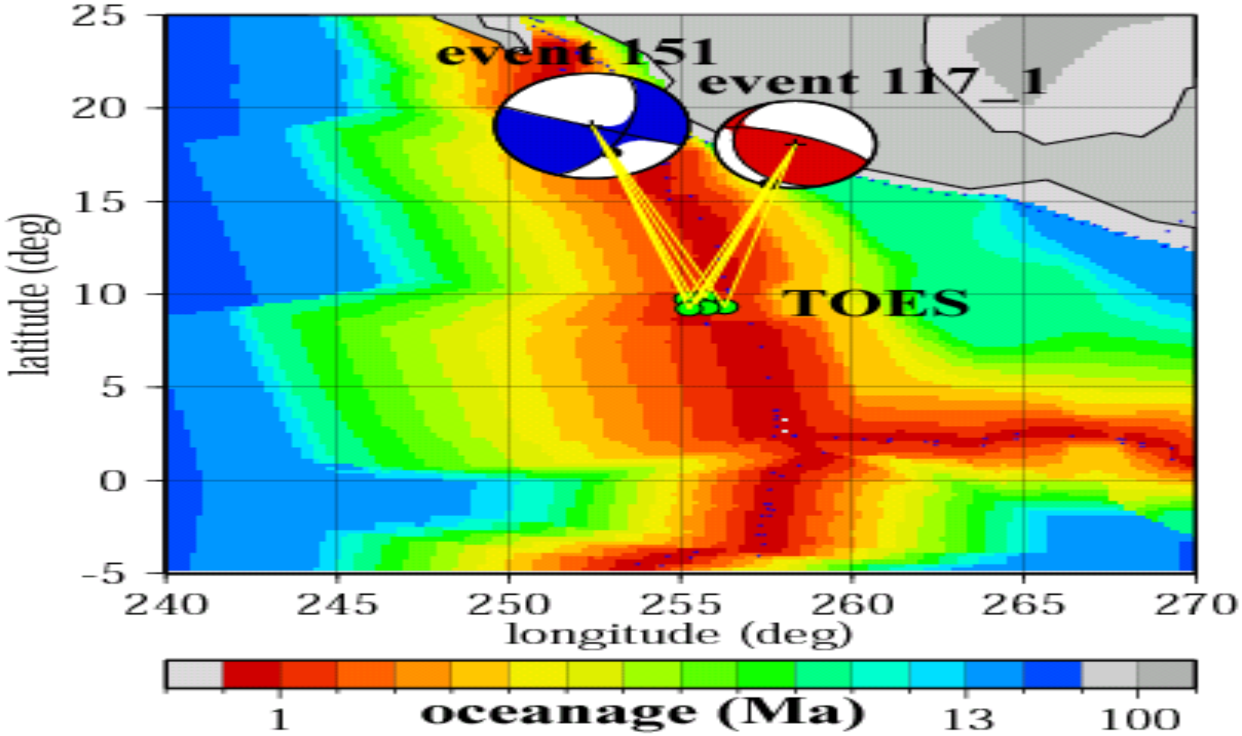


(2) shape preferred orientation (SPO)



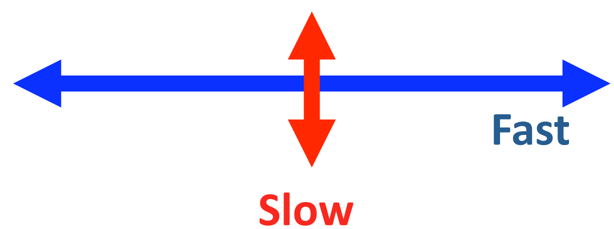


East Pacific Rise: the Fastest spreading ridge in the world (18 cm/year).



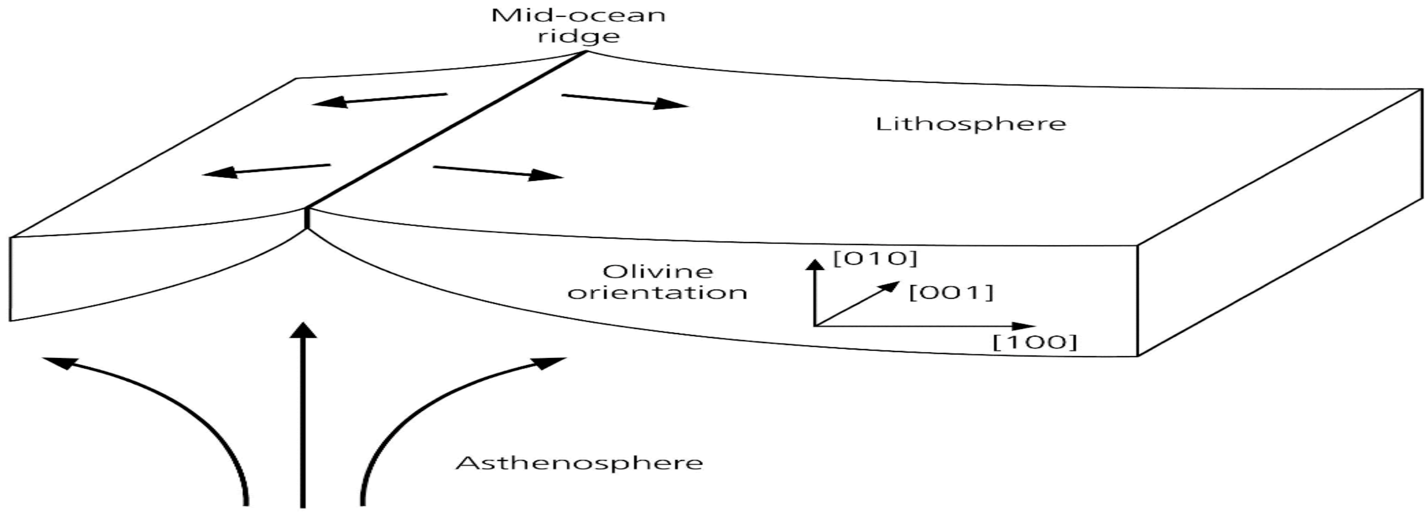
Waves going through a ocean ridge will "see" different speeds depending on how it goes across it.

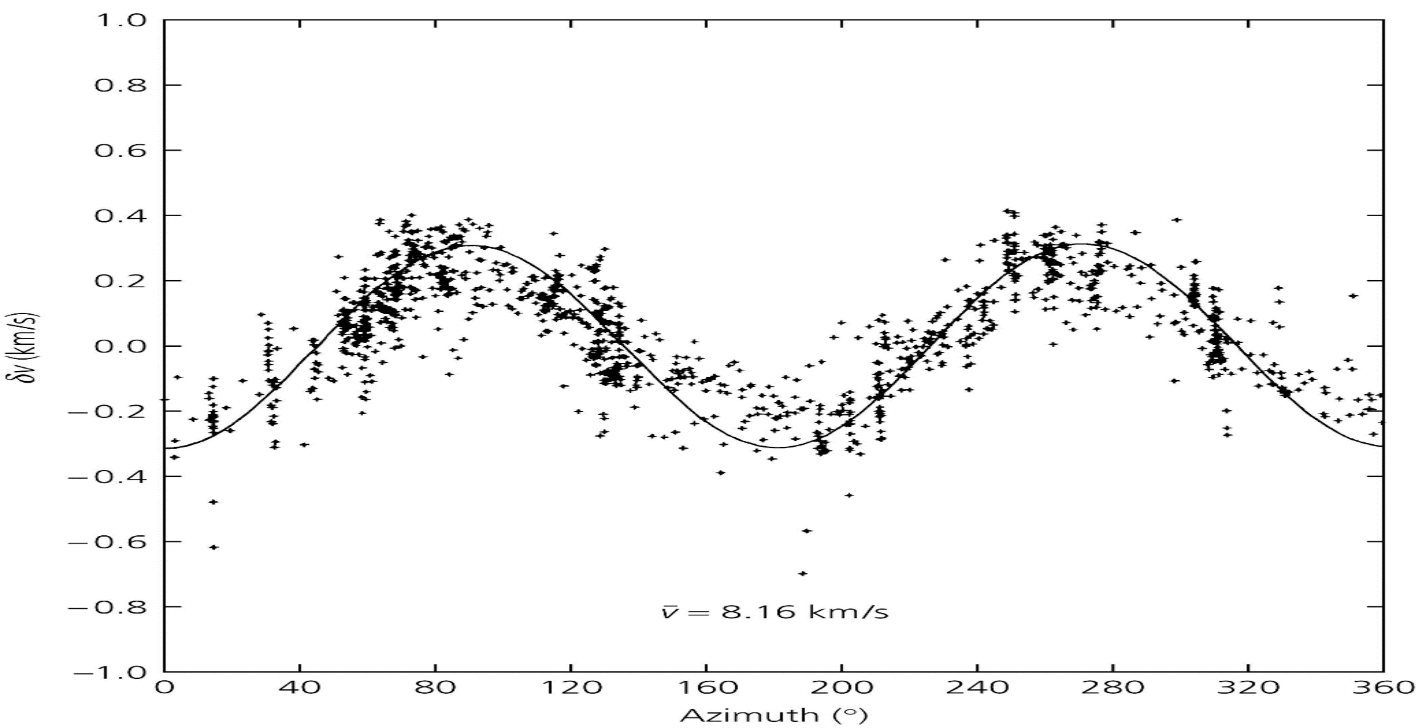
Parallel to Ridge: Slow
Perpendicular to Ridge: Fast



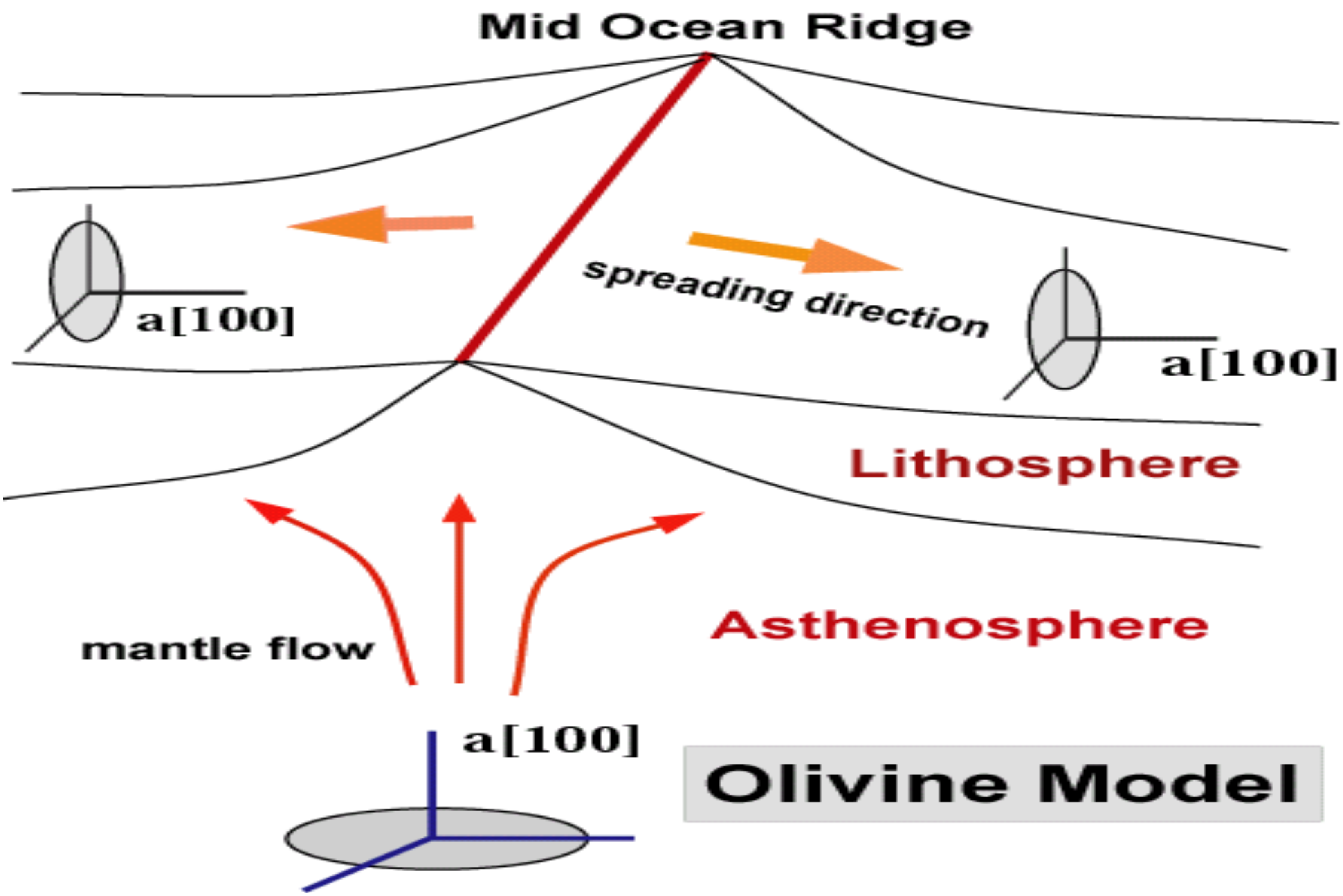
Why? More melt (hot, slow) along the ridge than perpendicular to ridge.

Figure 3.6-4: Example of anisotropy in the oceanic lithosphere.

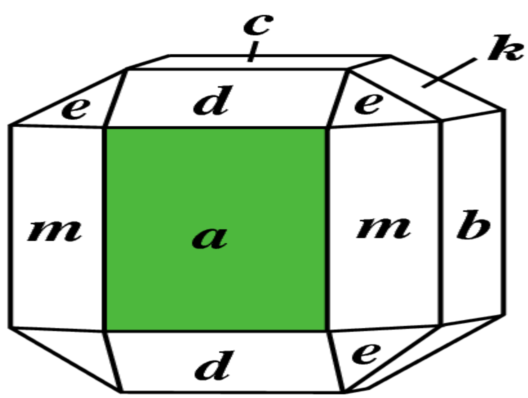




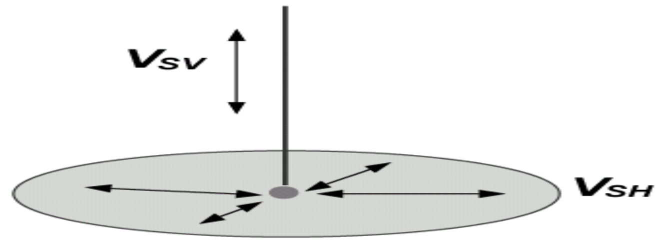
Pn analysis for P (head waves) traveling along Moho


Mid Ocean Ridge
a[100]
spreading direction
a[100]
Lithosphere
mantle flow
Asthenosphere
a[100]
Olivine Model

Olivine



transverse isotropy with a vertical symmetry axis



$$A = \rho V_{PH}^2$$
$$C = \rho V_{PV}^2$$
$$N = \rho V_{SH}^2$$
$$L = \rho V_{SV}^2$$
$$F = \eta(A - 2L)$$

Conventional wisdom:

Flow direction is parallel to direction of fast shear or compression velocity

Radial anisotropy:

Simplest anisotropy with 5 independent elastic parameters

S waves:

L corresponds to μ for Z direction (think of the first two indices as normal in X or Y pointing to Z, the last two indices represents response to strain along X or Y with directional derivative in Z)

→ $$V_{SH}^2 = L/\rho$$

N corresponds to Vsv

→ $$V_{SV}^2 = N/\rho$$

This means waves will travel in different speeds depending on the polarization, or particle motion.

F is more complex and depends on velocities at intermediate incidence angles, usually quantified by $\boldsymbol{\eta = N/(A-2L)}$, where η varies between 0 and 1.

Simple questions:

Why choose transverse isotropy with a vertical symmetry axis?

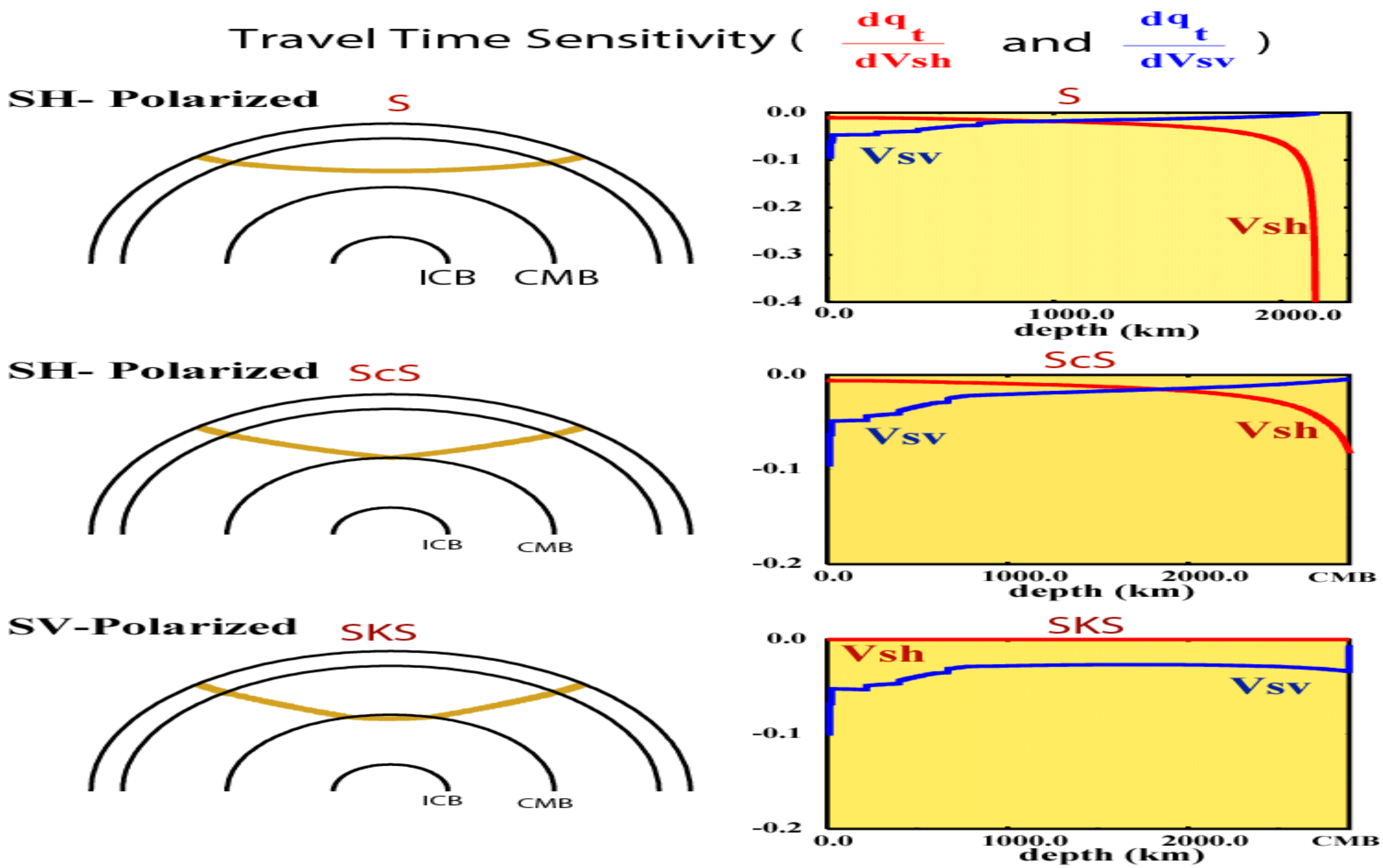
Answer: things settle down in layers (or so called the *principle of horizontality*). It is all based on common knowledge.

Which velocity is higher, VSV or VSH in general?

Answer: near surface, usually VSH.

Body Waves under Radial Anisotropy

Gu et al. 2005, EPSL



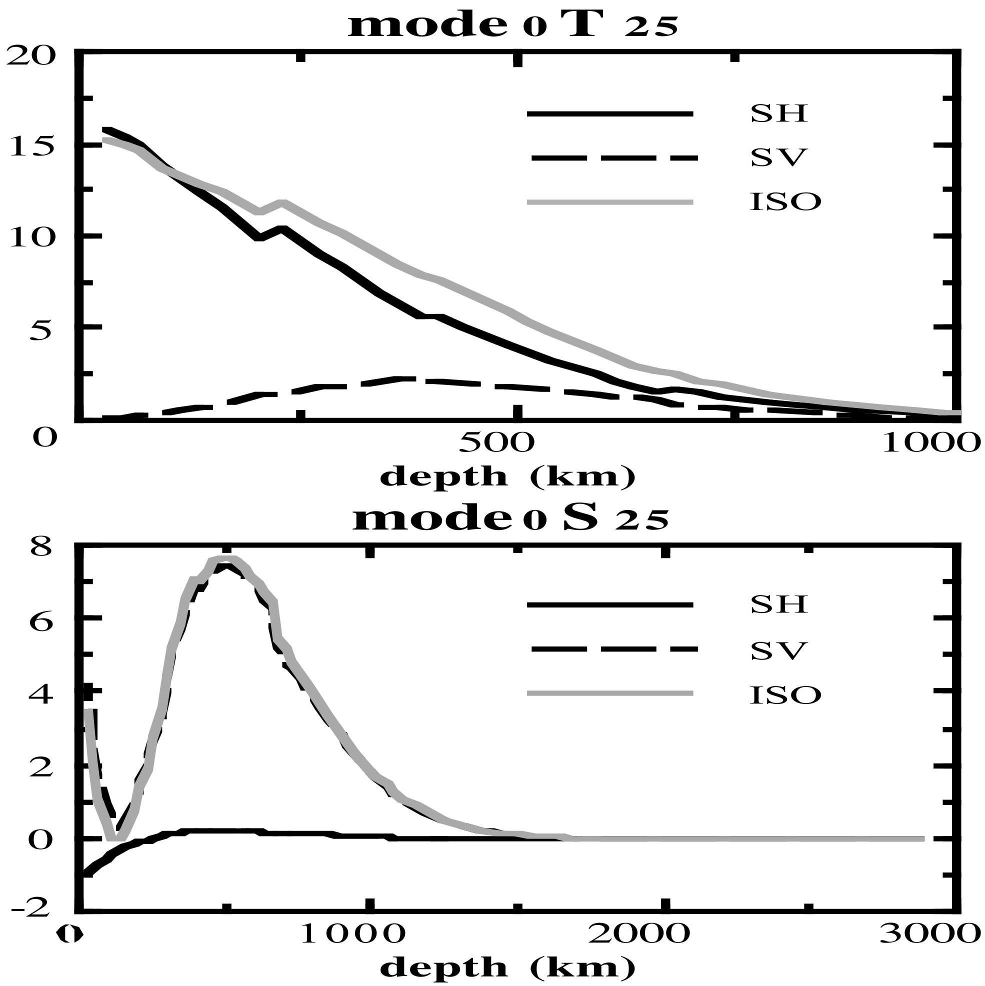
(1) Travel time of a SH-Polarized wave is sensitive to both Vsh and Vsv

(2) Sensitivity to Vsh and Vsv depends both on polarization and direction of the ray path

(3) SV-polarized waves are only sensitive to vertical wave speed (Vsv).

Gu et al, 2004

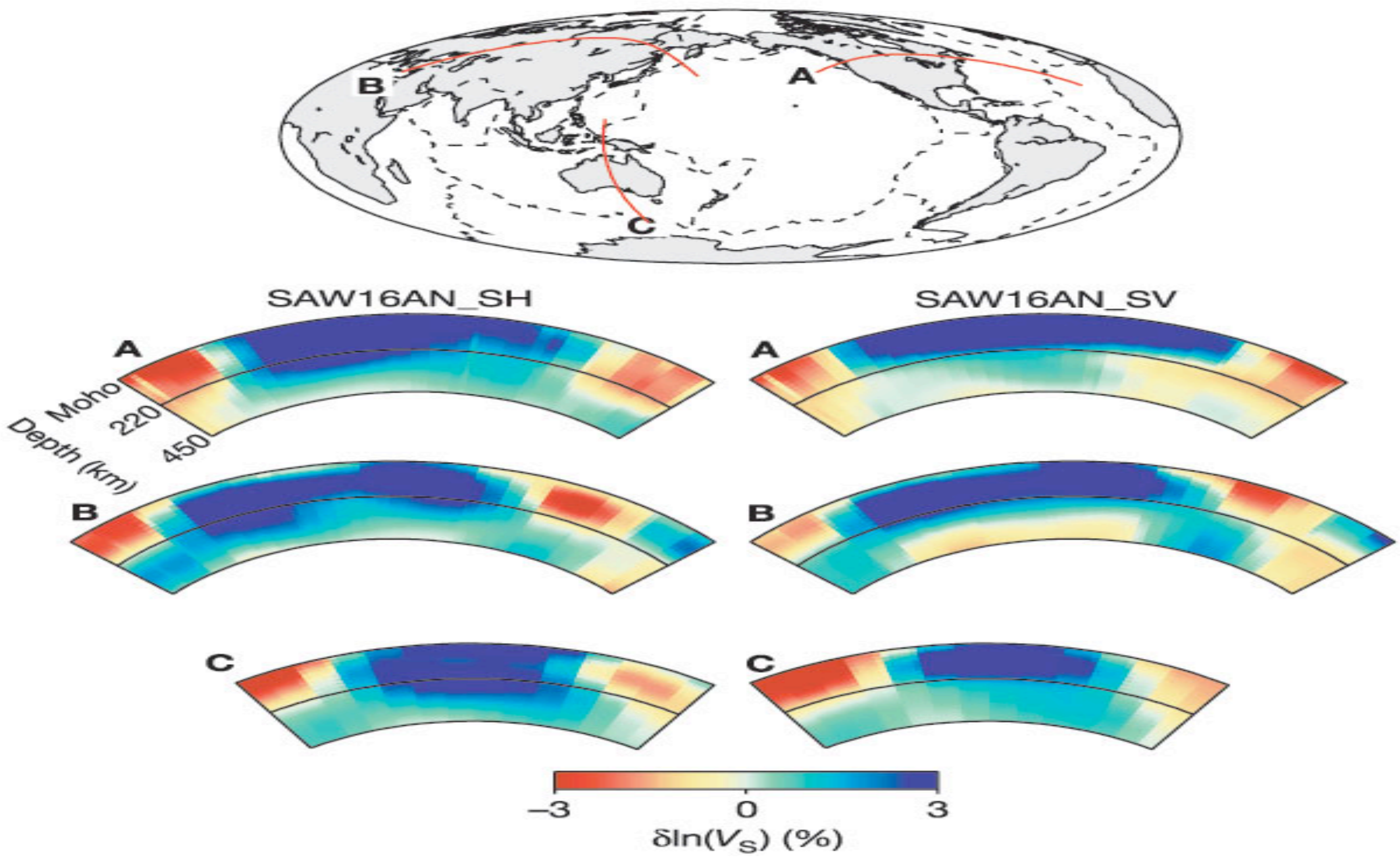
Radial sensitivityof Mode eigenfrequency to SV and SH speed of the earth structure.



Gu et al., 2005 (EPSL)

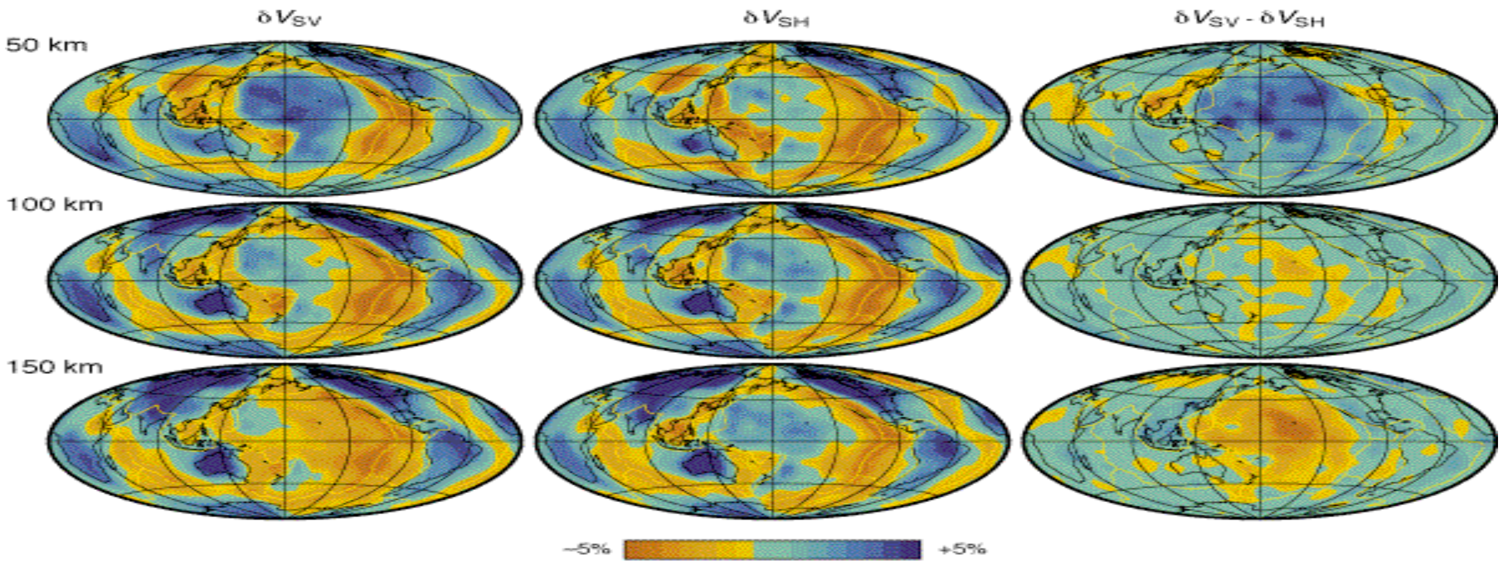
For toridal modes, mode is sensitive to both, but for sphoridal modes, mostly sensitive to SV.

Global shear anisotropic inversions

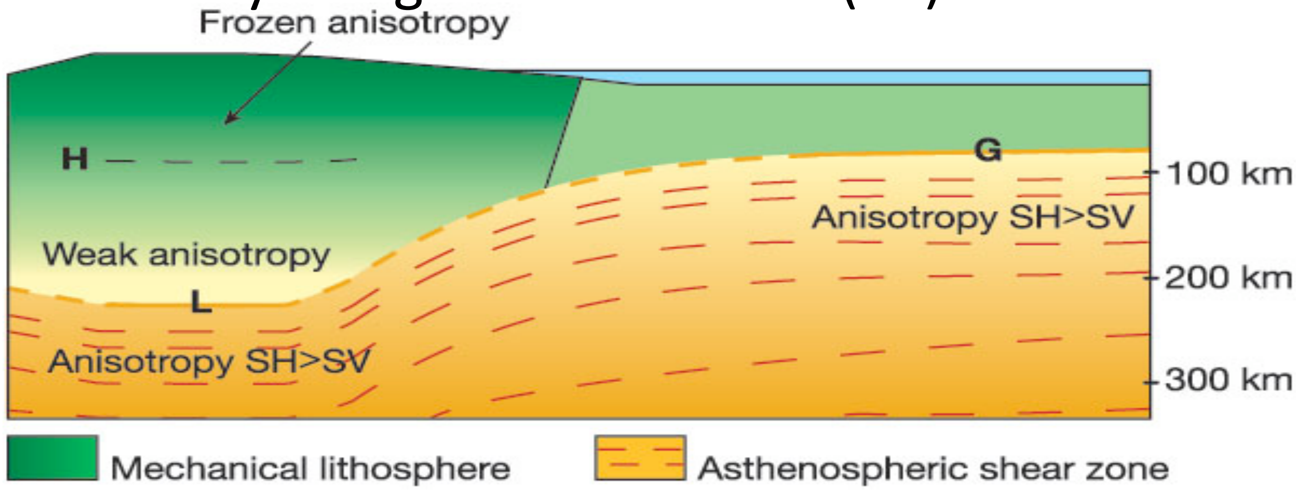


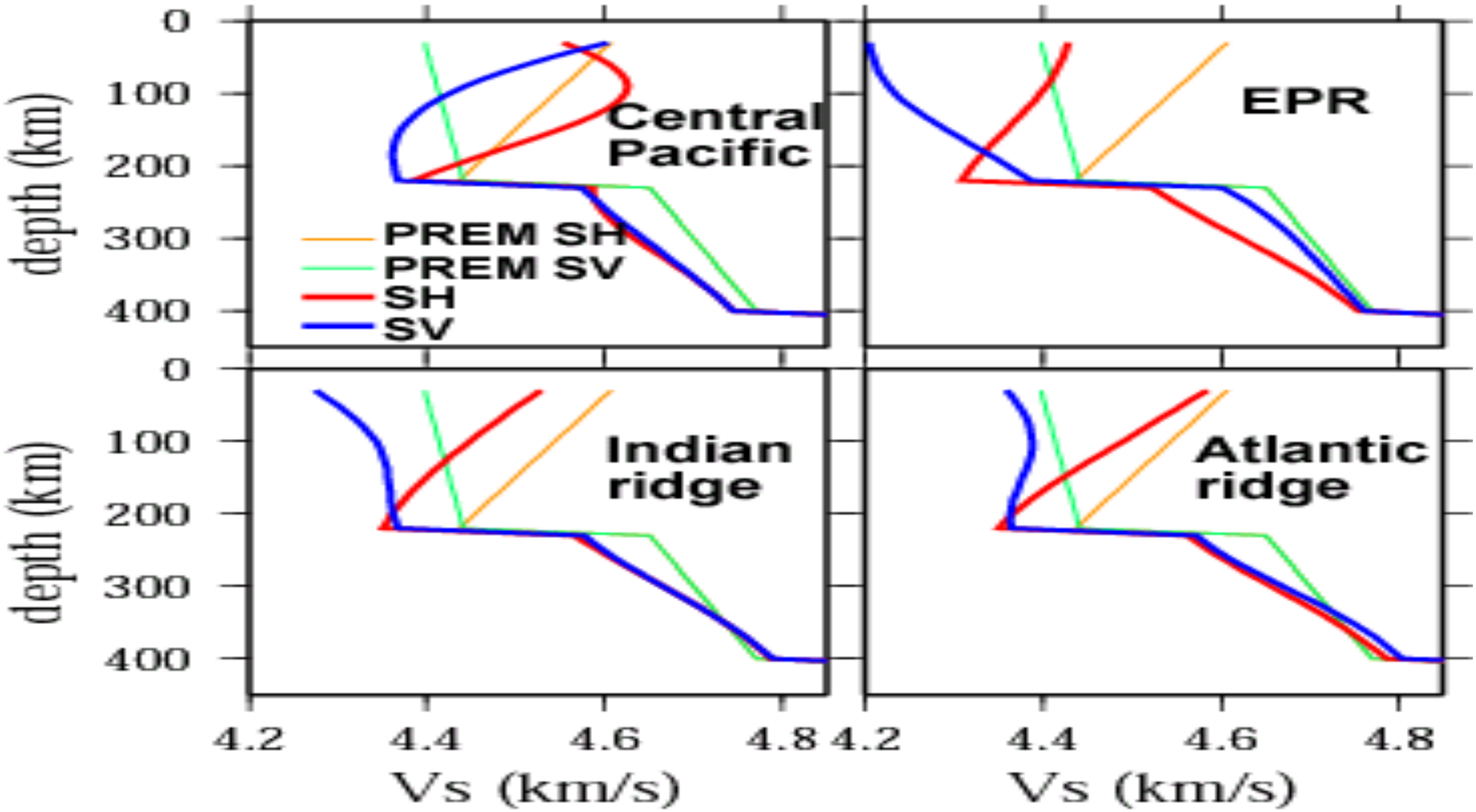
Global shear anisotropic inversions

Ekstrom & Dziewonski, 1998



Interpretation by Gung & Romanowicz (04)

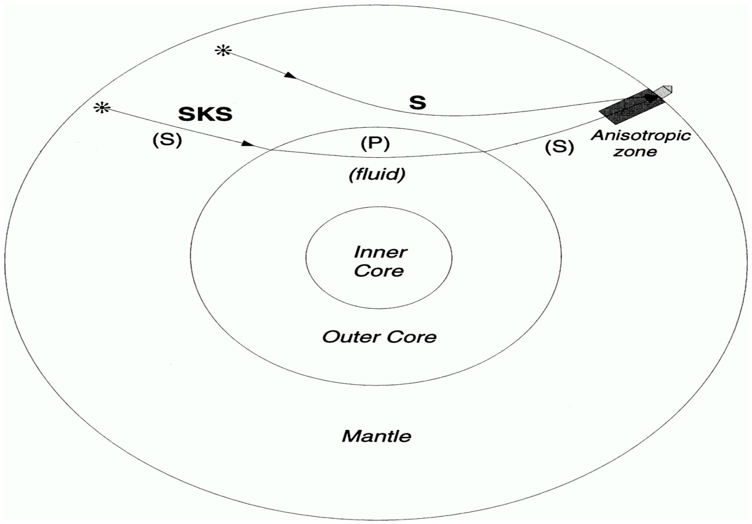


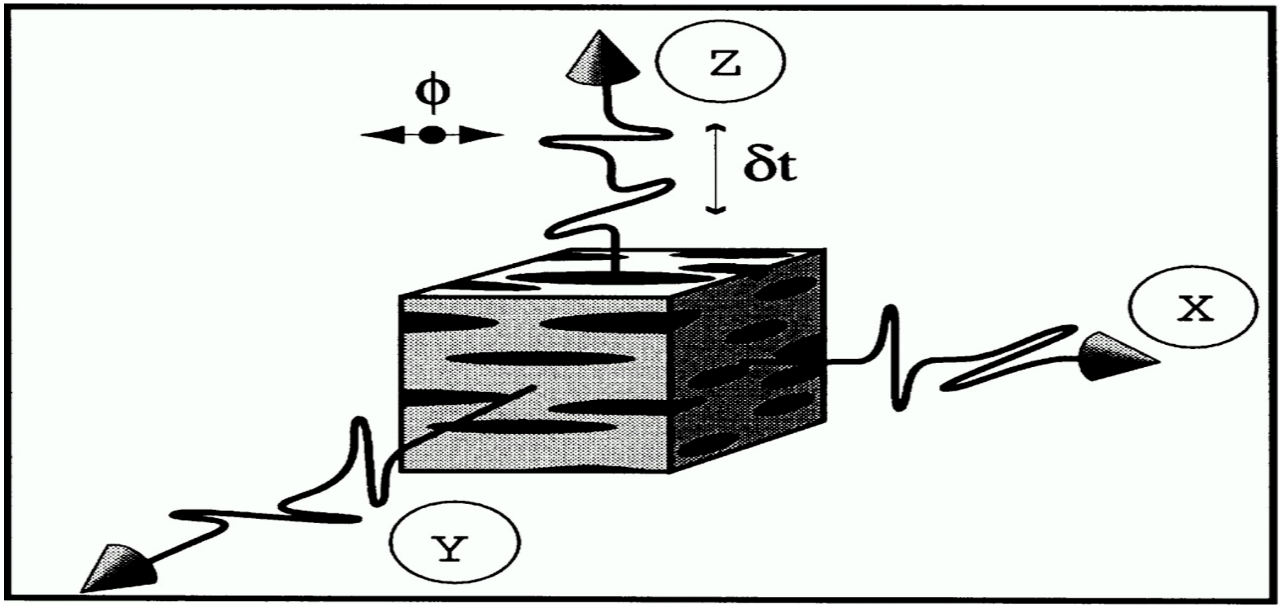


1D wave velocities (PREM) comparing with tomographic result at a given region (VSH >VSV, reflecting horizontality globally).

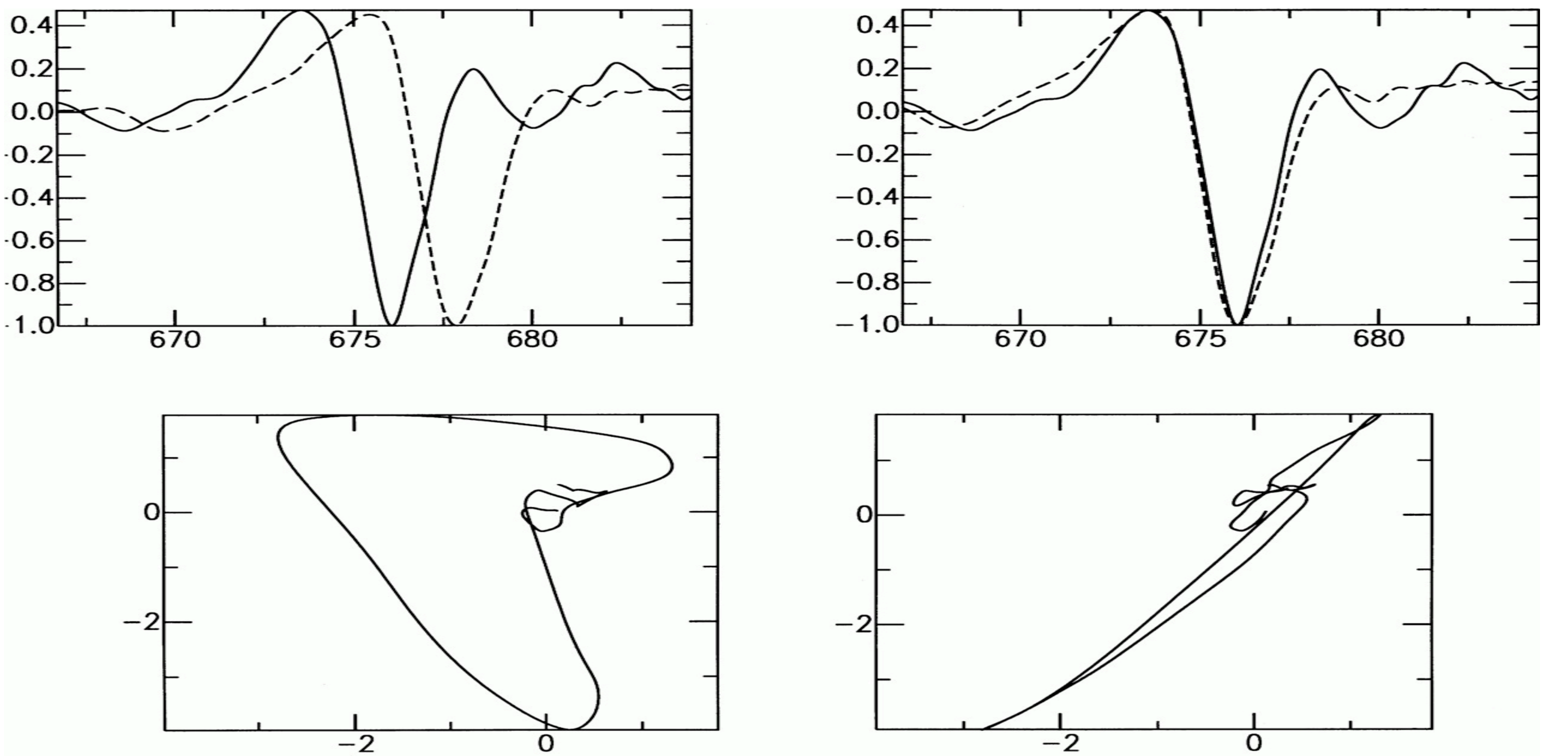
Azimuthal Anisotropy: changes in shear Velocity inside horizontal layers.

Shear Wave Splitting



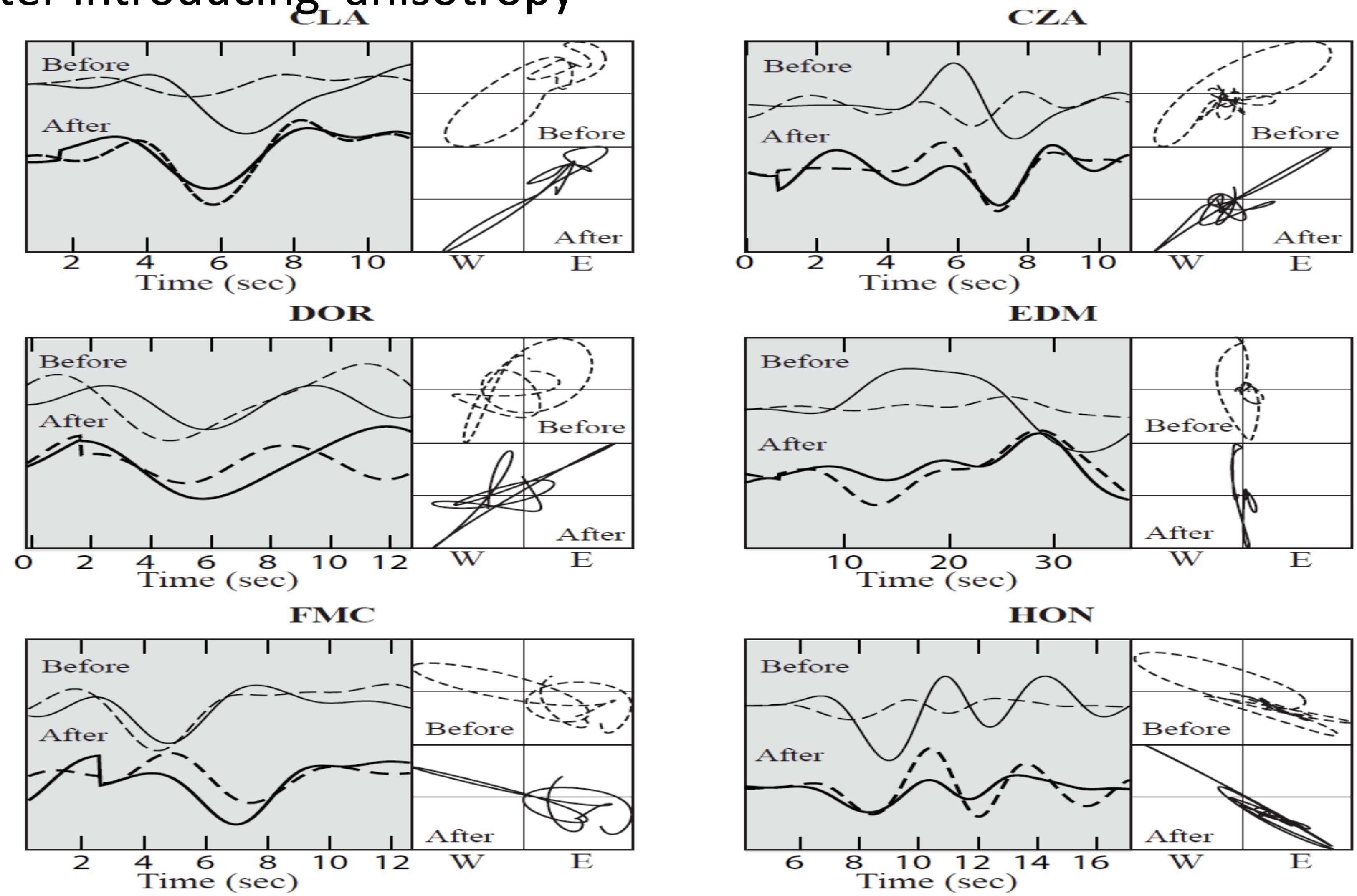


Particle motions before and after splitting measurements

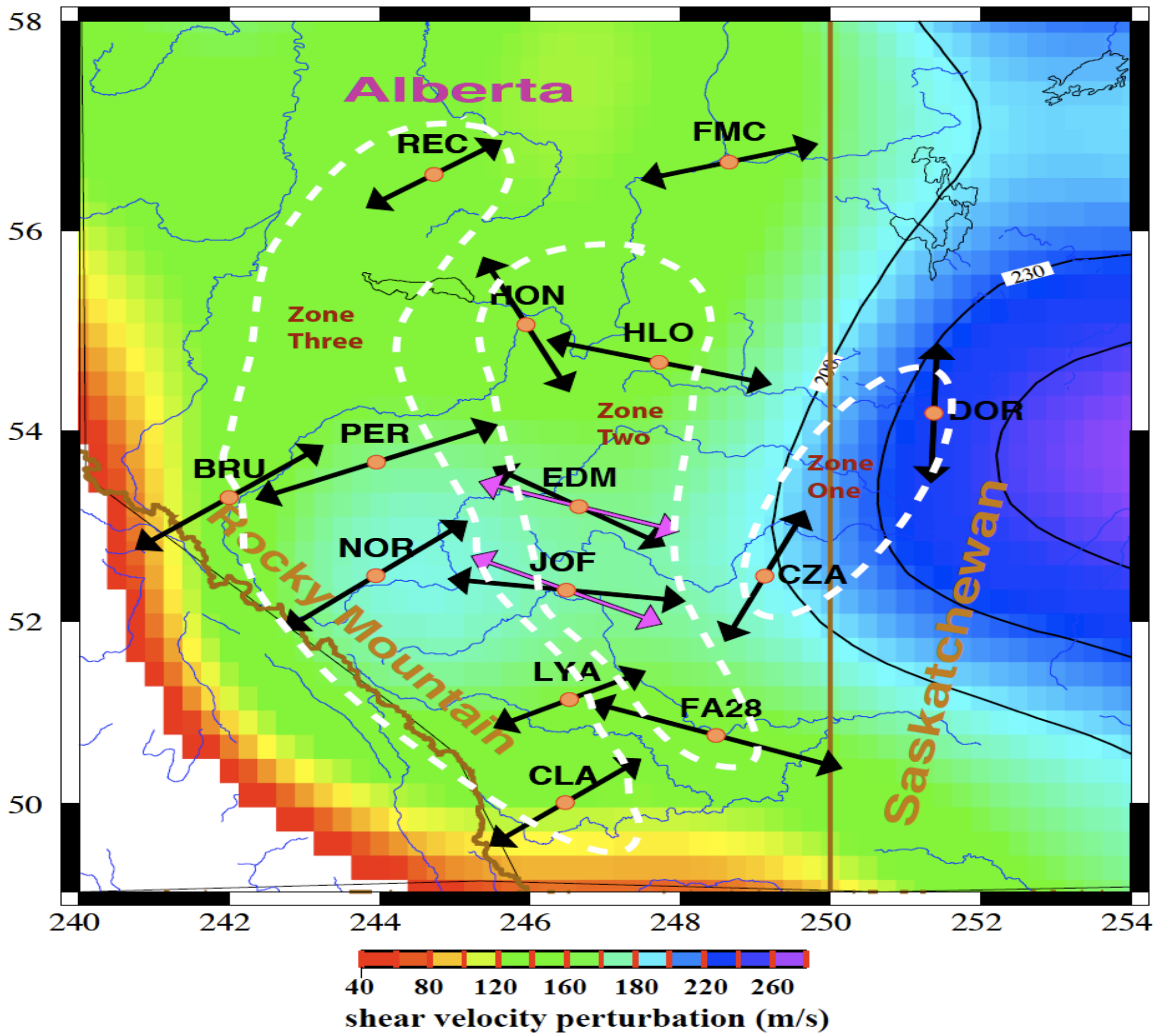


Plotted above are comparisons of radial and transverse components. The right panels show no anisotropy, which result in a 45 deg straight line. The left shows strong anisotropy prior to time shift (anisotropy) and rotation of the records to null axis.

Data fit: Shaded indicate “cross-convolution functions” & non-shaded plots are particle motions of Radial vs. Transverse before & after introducing anisotropy



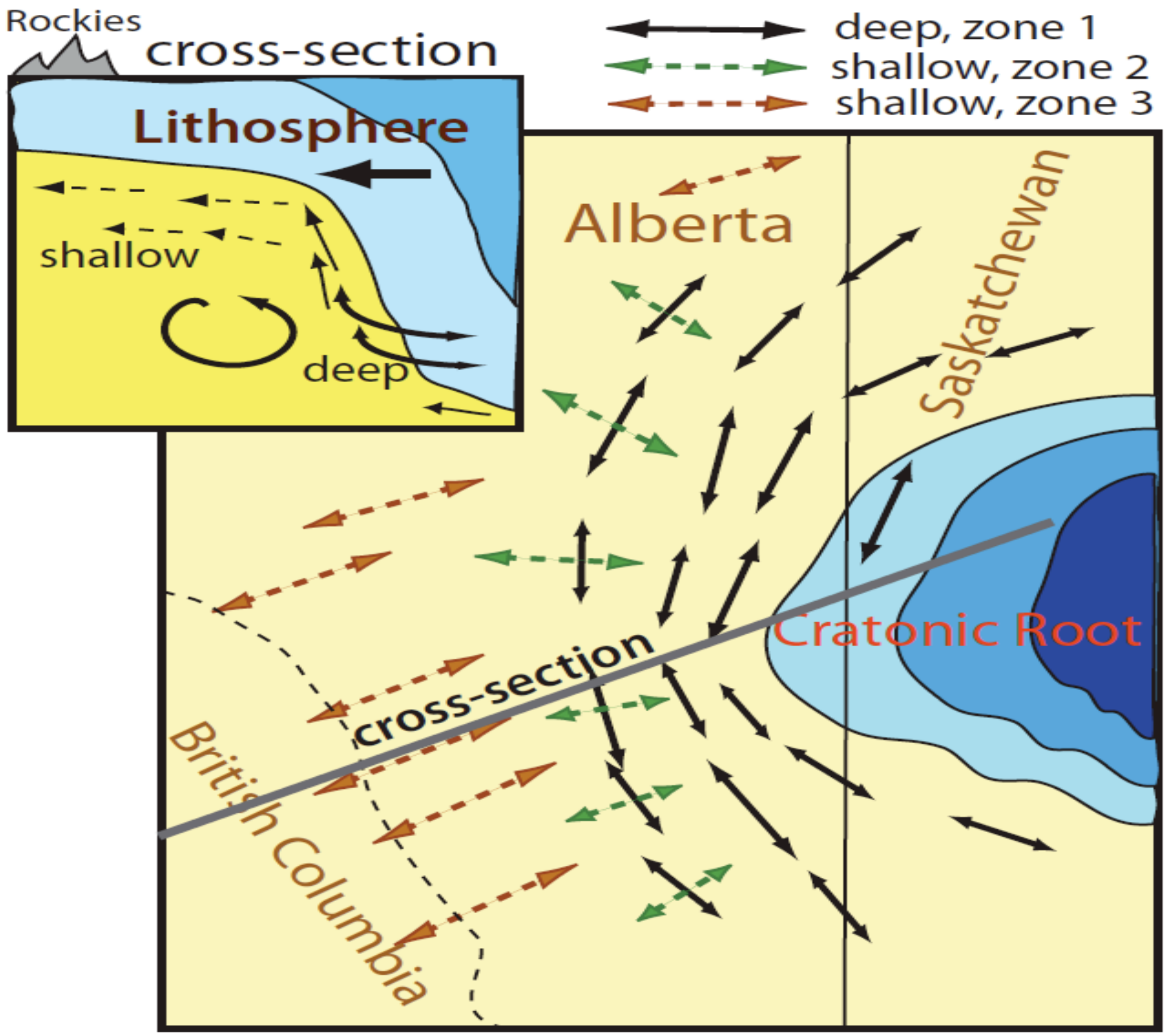
Splitting time & fast direction + velocity



Gu et al., 2011

Splitting angles (fast direction indicated by arrows) show northeast-southwest directions near the foothills of the Rockies, consistent with plate motion of north America. The splitting times of 1+ sec indicate significant horizontal anisotropy. However, the story is not too clear in the Alberta Basin where angles are anomalous!

Interpreted Flow in and around Alberta



Maybe the result of moving continental root (below 100 km) that channels the flow like boat does to water when moving??