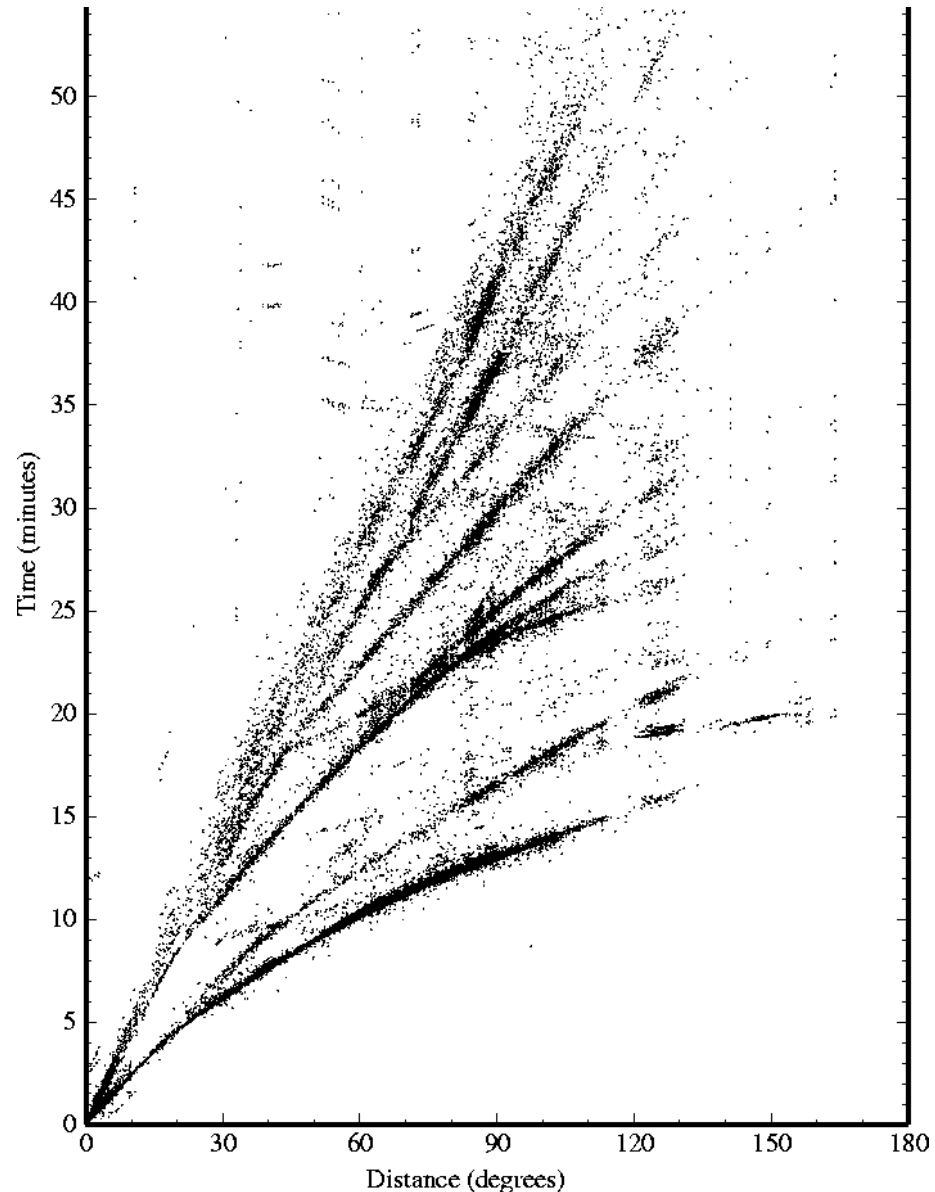


Seismic Phases and 3D Seismic Waves

Main Seismic Phases:

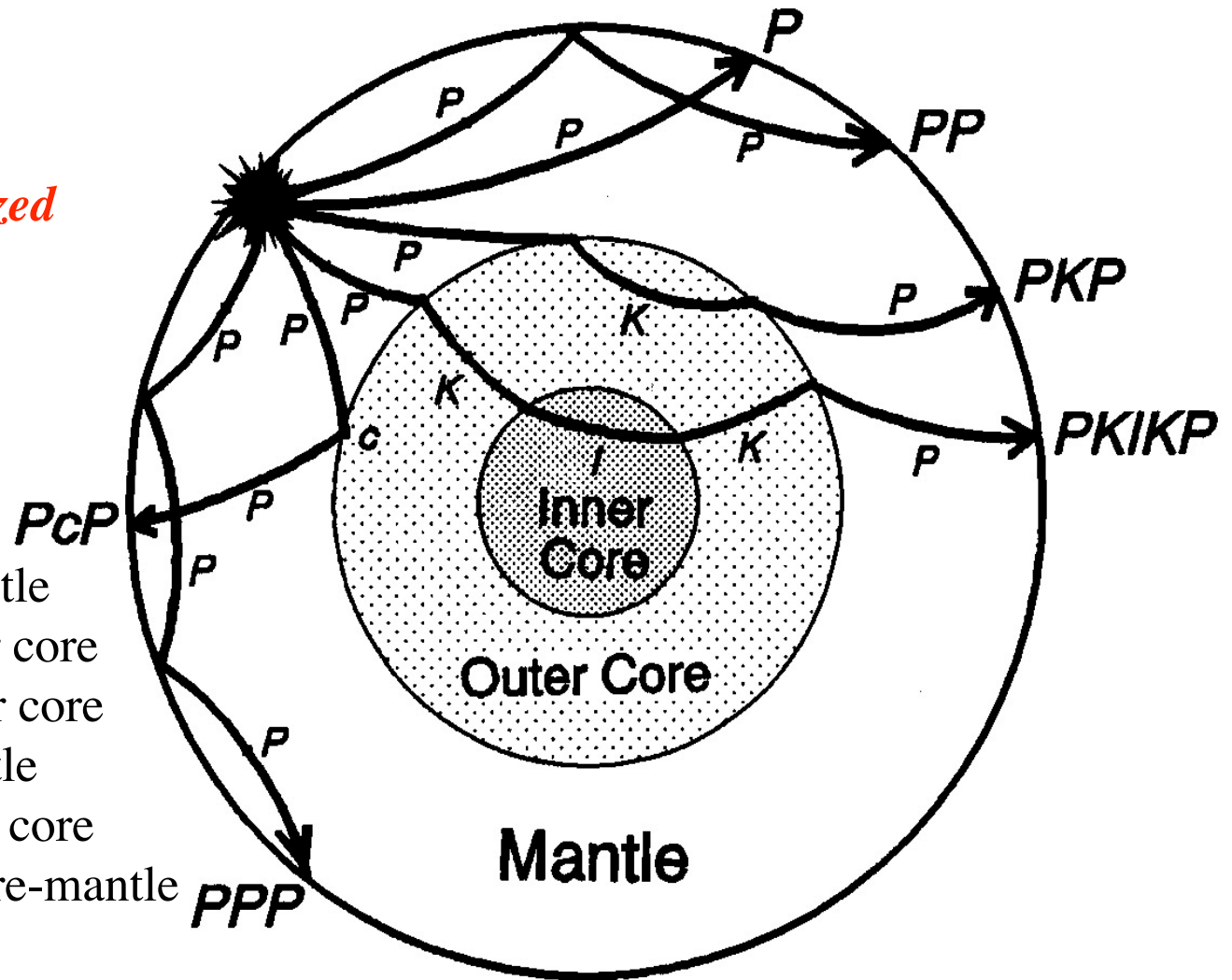
These are stacked data records from many event and station pairs. The results resemble those of predicted travel time curves from PREM (Preliminary Reference Earth Model, by Dziewonski and Anderson)



Naming conventions

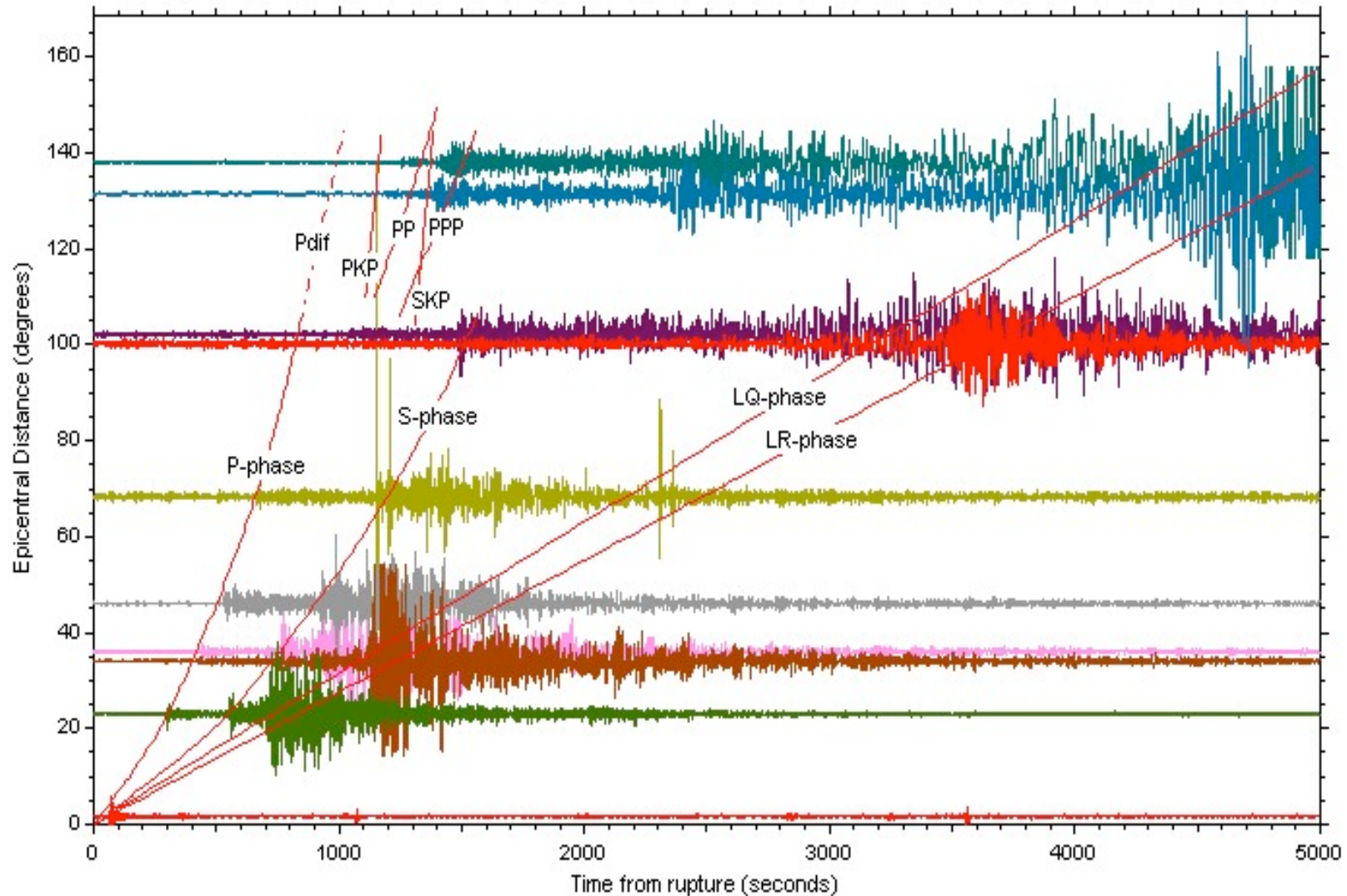
(Note: small and capitalized letters do matter!)

- P: P- wave in the mantle
- K: P-wave in the outer core
- I: P-wave in the inner core
- S: S-wave in the mantle
- J: S-wave in the inner core
- c: reflection off the core-mantle boundary (CMB)
- i: reflection off the inner-core boundary (ICB)
- PmP: Reflection off of Moho
- Pn: refracted wave on Moho
- Pg: direct wave in the crust.



**Different waves have different paths,
hence are sensitive to different parts of the
Earth**

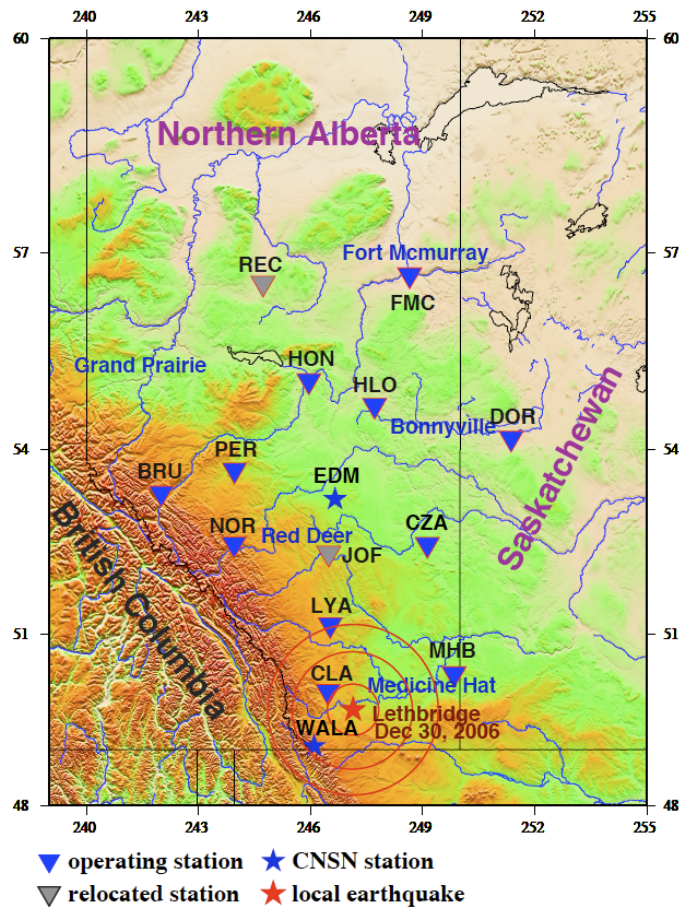
Arrival Time of Seismic Phases



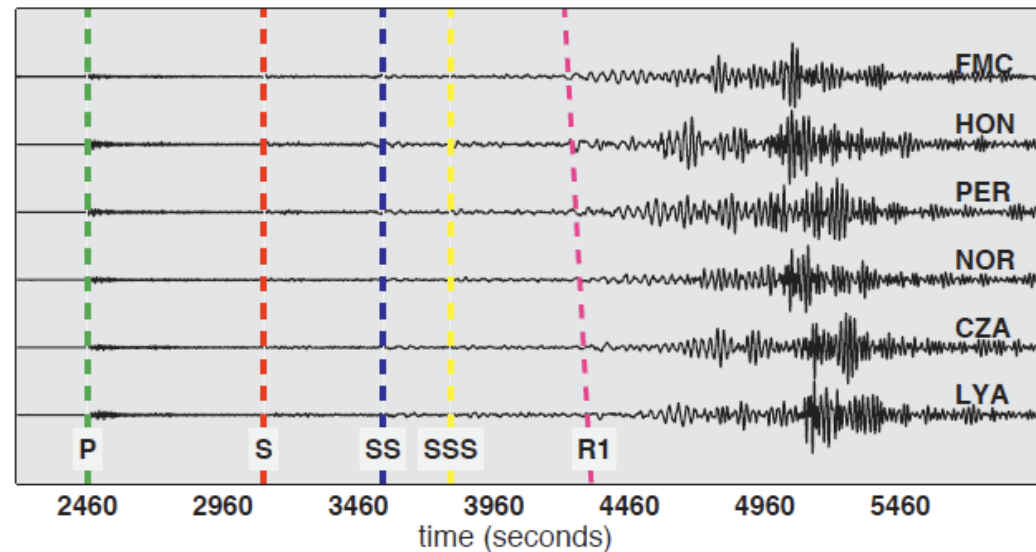
Terminology: *Onset time* (time for the beginning, not max, of a seismic phase. It is where waves begins to take off.) LQ, LR are surface-related wave types.

The UofA efforts: Canadian Rockies and Alberta Network (CRANE)

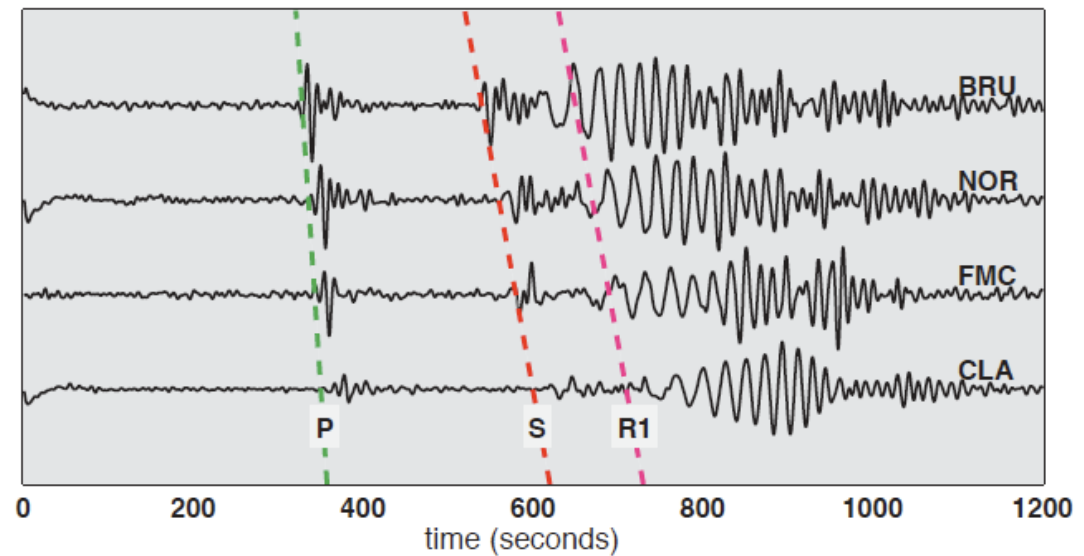
CRANE Station Distribution



Sichuan, China (Mw=7.9, May 12, 2008)

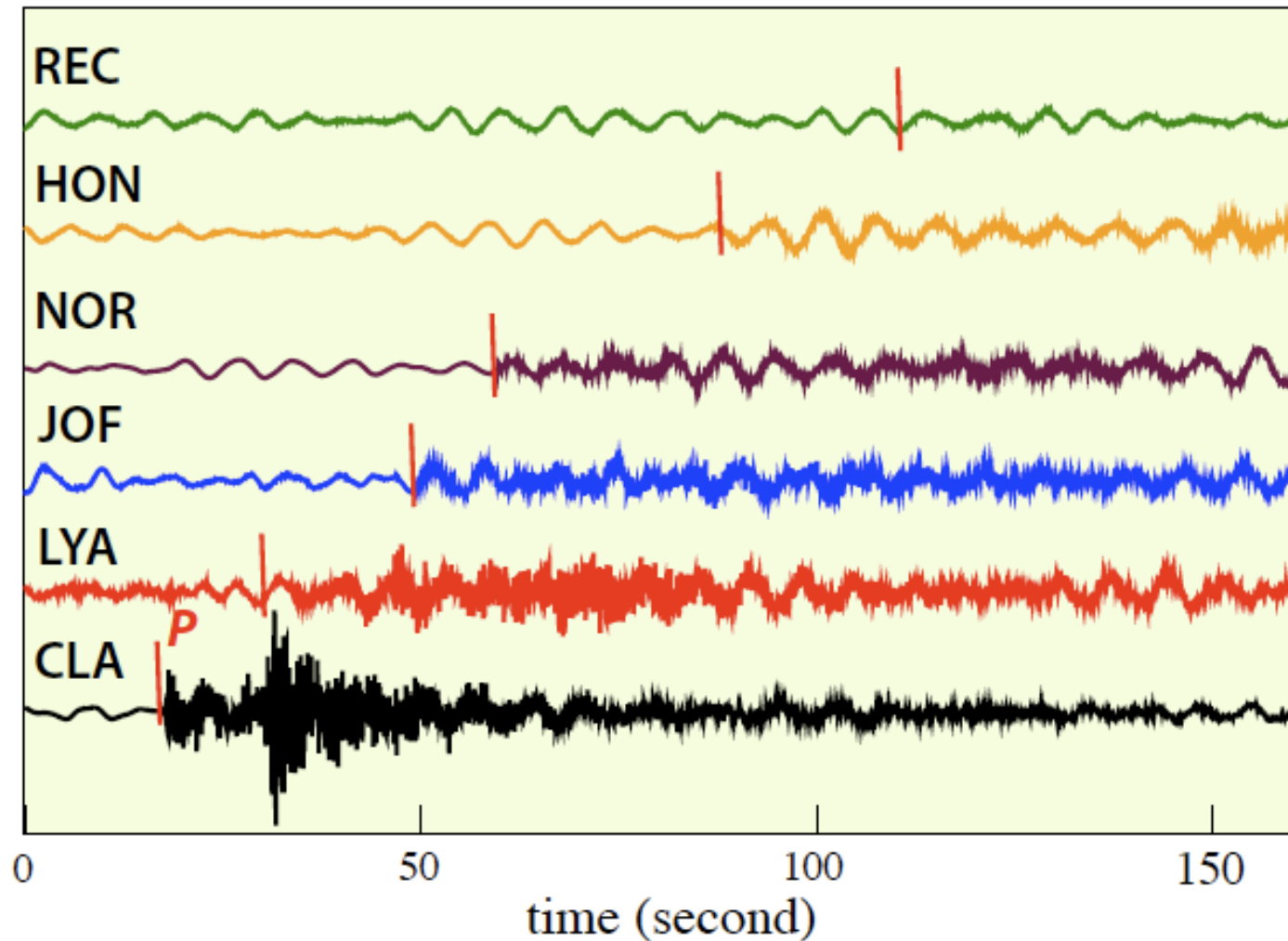


Kodiak Island, Alaska (Mw = 5.0, April 30, 2009)



Not all phases are easily distinguishable: Below is the result from a 3.5 event in Lethbridge. Only P and surface waves are seen near Claresholm region. Strong decay due to sedimentary cover and attenuation.

Regional Earthquake (Dec 30, 2006, Lethbridge, Mag=3.5)



Travel times of a given seismic arrival can be predicted based on the *travel time table* computed from a spherically symmetric Earth model (e.g., PREM, IASPEI).

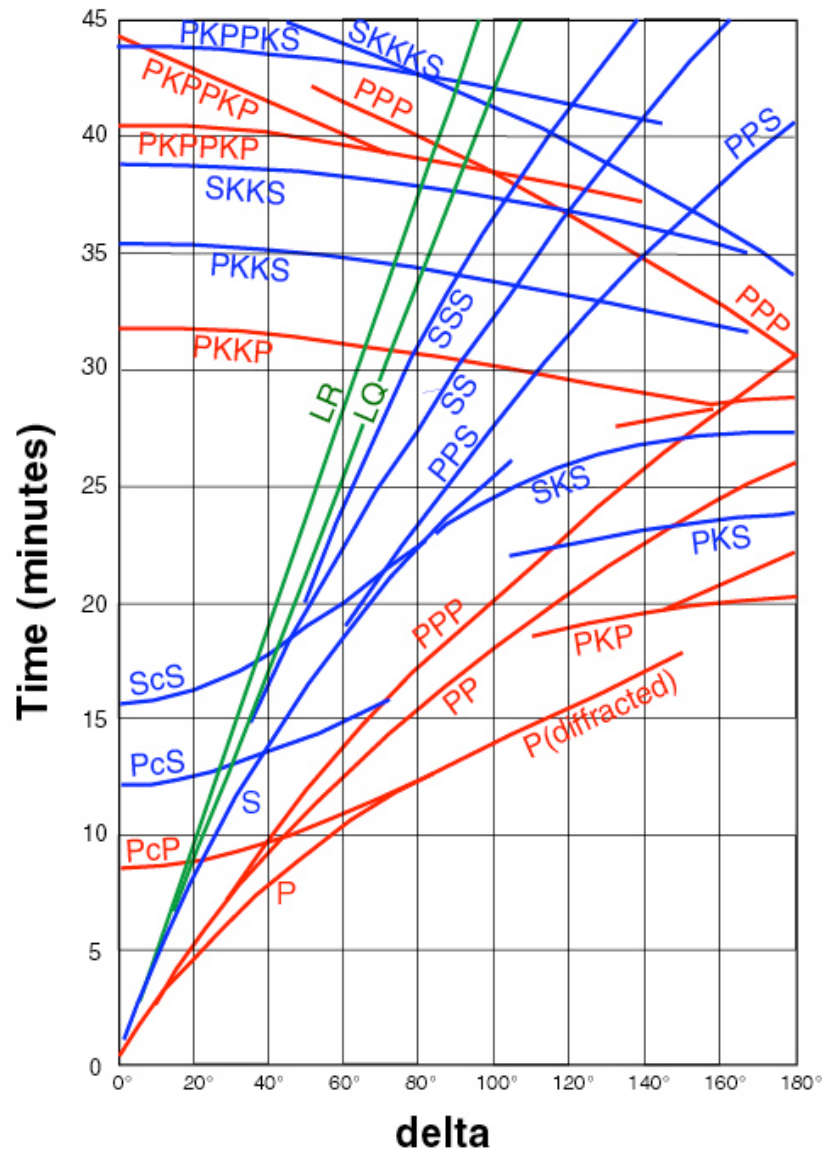
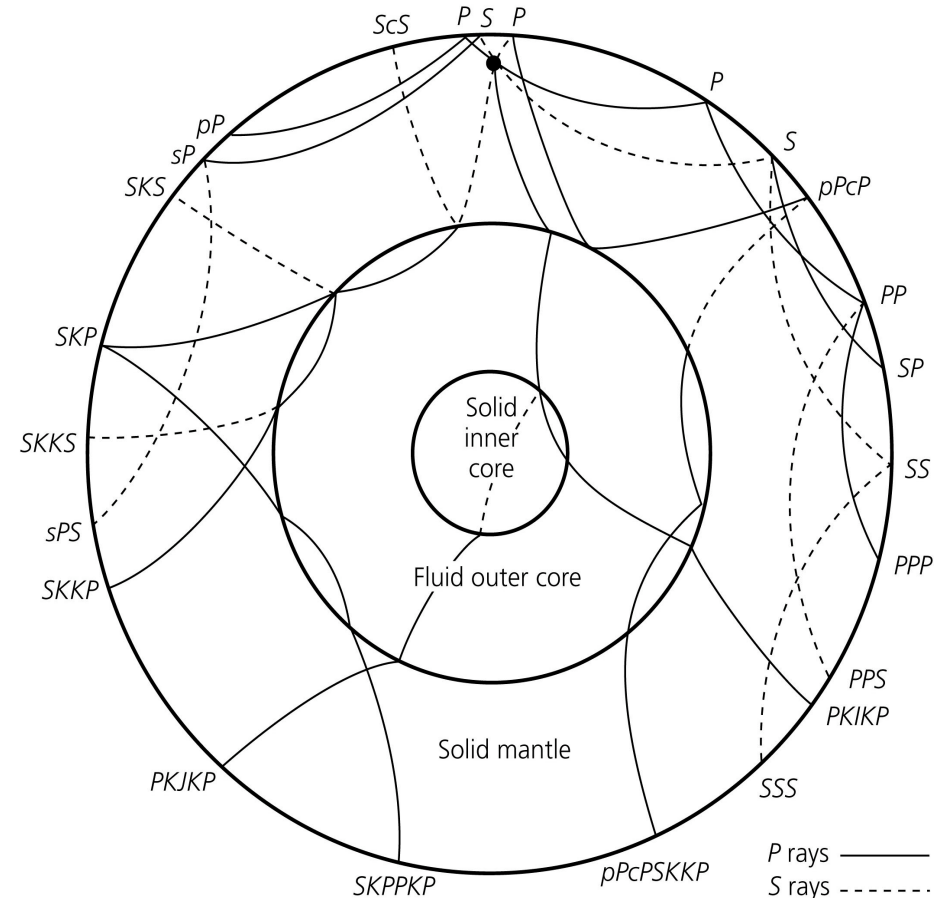


Figure 3.5-5: Illustration of various body wave phases.



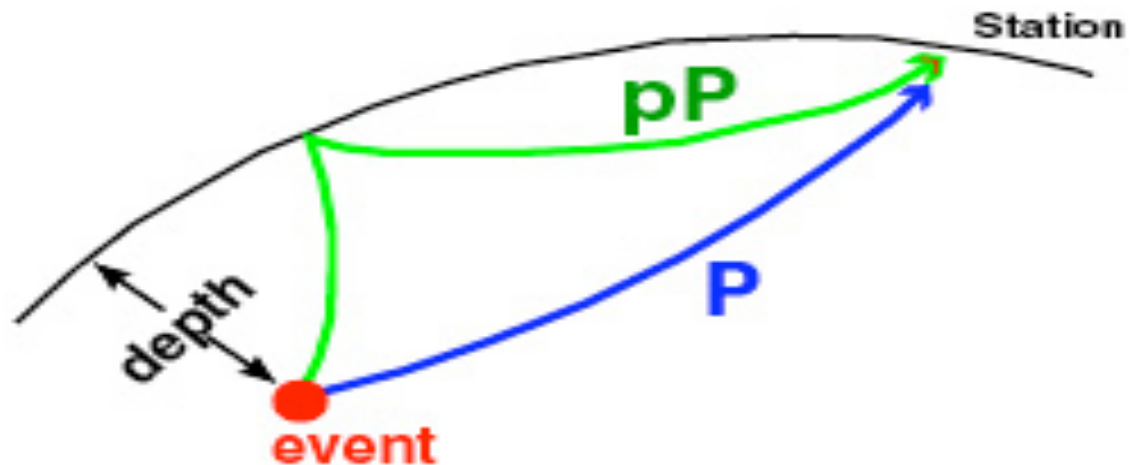
Differential time between two phases (arrivals) are particularly useful for many reasons, some listed here:

- (1) Their time difference is essentially INSENSITIVE to source effect (since they came from the same source, regardless of the complexity of source. This means less error!
- (2) Their difference usually can be used to approximate the depth of major jumps (discontinuities) inside the earth.

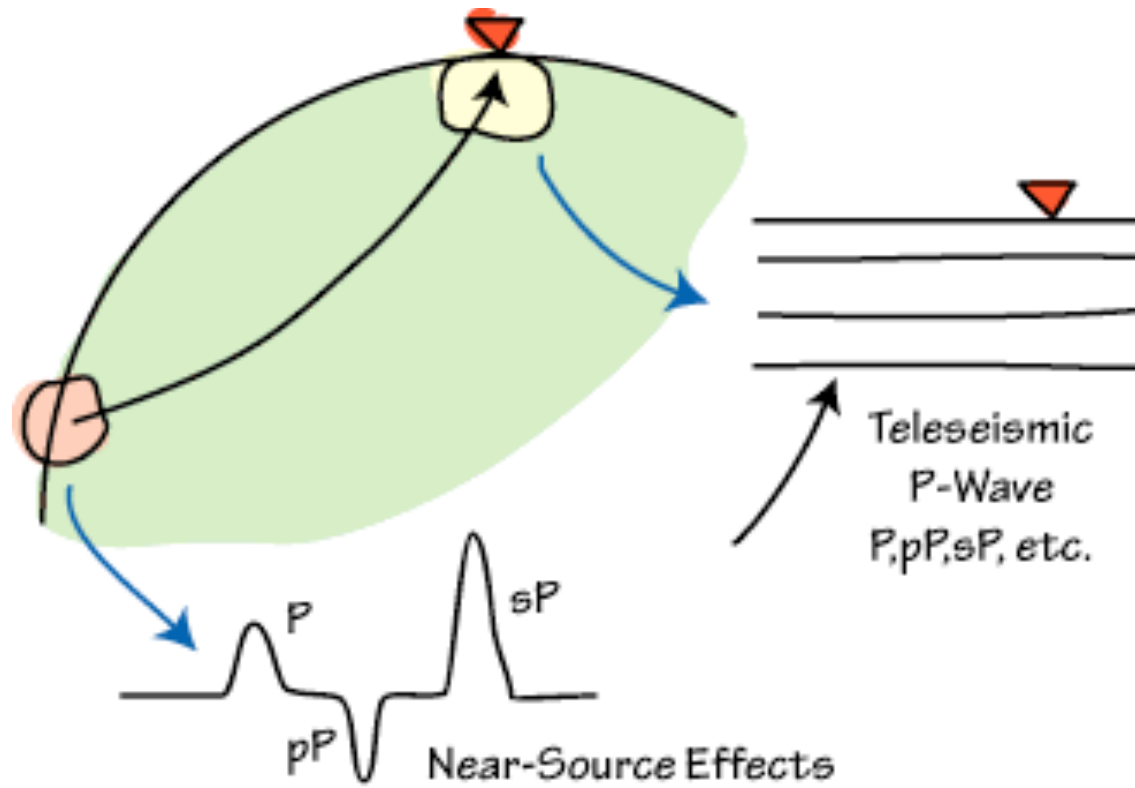
pP-P time indicates depth

Hypocenter: True location of earthquake

Epicenter: Projection of earthquake on the Earth's surface



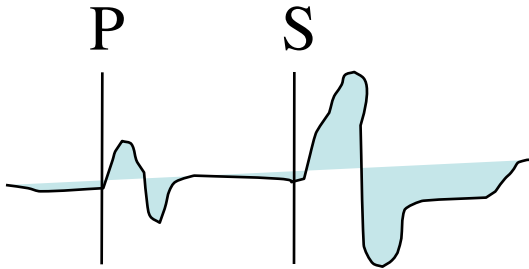
- (3) The magic number “8” for regional seismic arrivals. That is, if P arrival is X sec, S phase is Y sec, then we can approximate the source-station distance by $(X - Y) * 8$ km (**only works for local distances!**).



From eqseis.geosc.psu.edu

Why does 'magic 8' work?

Similar to finding the timing difference between thunder and lightening!



Timing difference:
 $T_s - T_p$

Now, assume typical
crustal speeds:

$V_p = 5.5 \text{ km/s}$, $V_s = 3.2 \text{ km/s}$

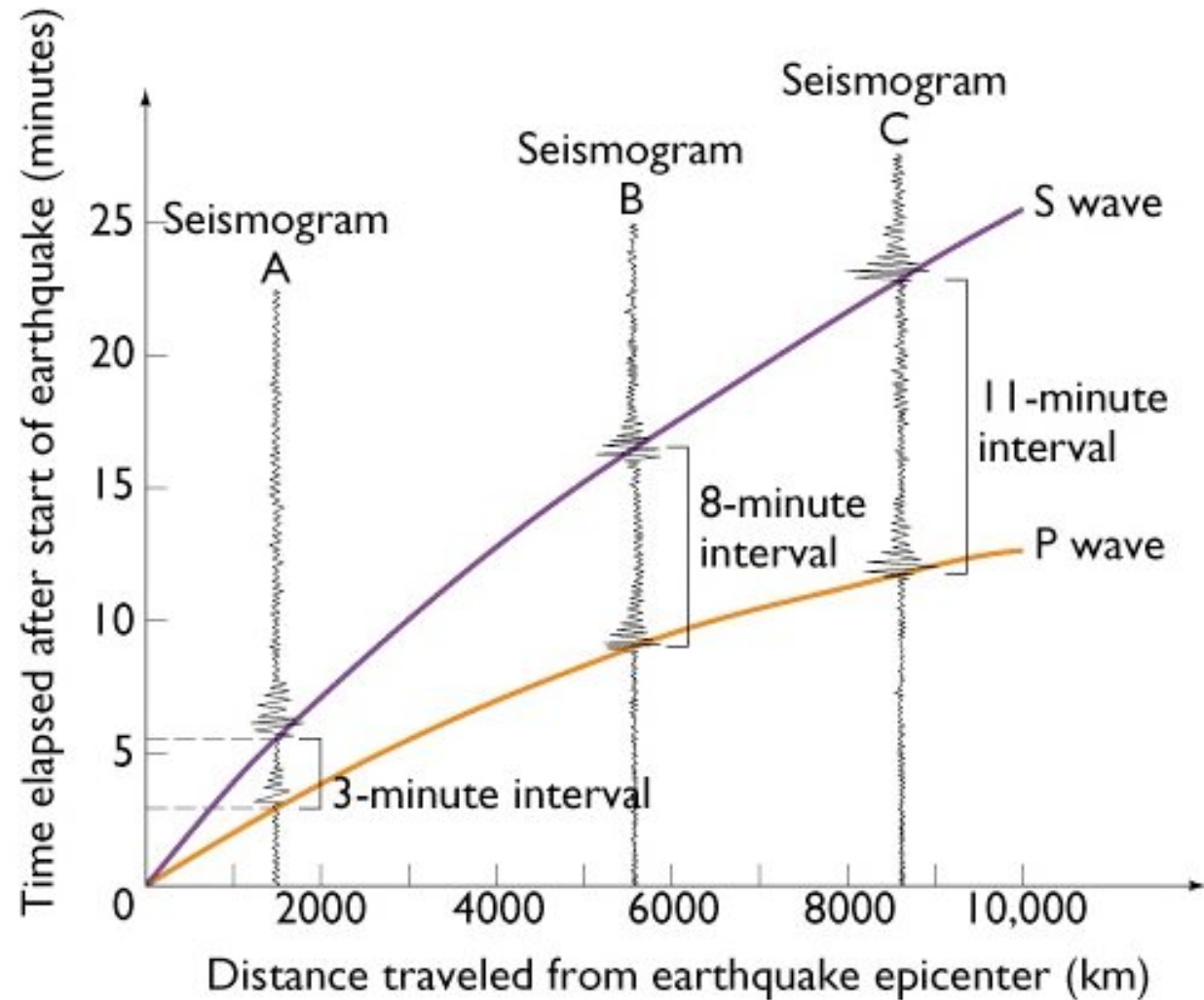
Need to find distance X .

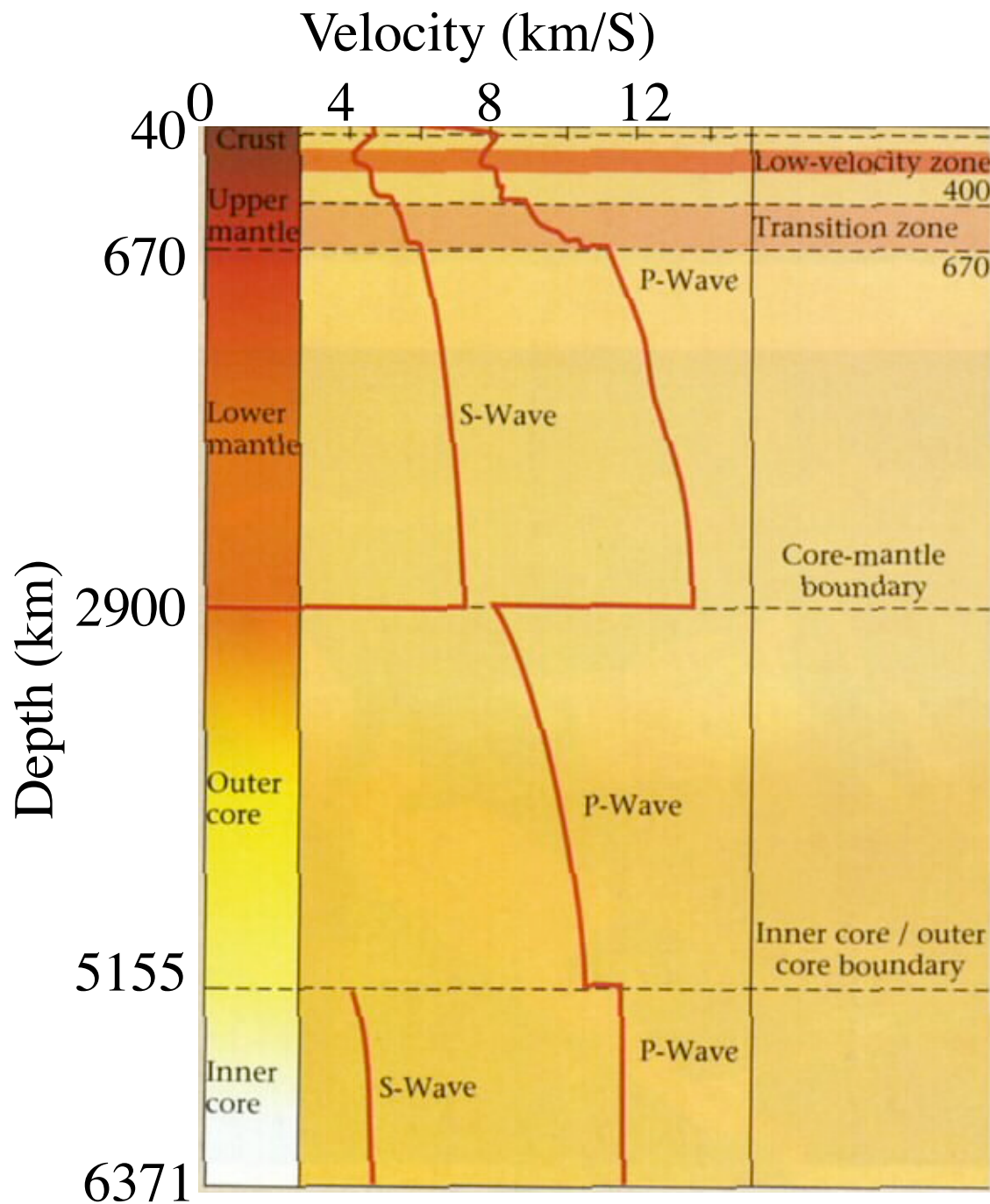
$$T_s - T_p = X/3.2 - X/5.5$$

$$= X(1/3.2 - 1/5.5) \sim X(0.3 - 0.18) \sim \mathbf{X/8}$$



Finding distance from P-S travel time difference



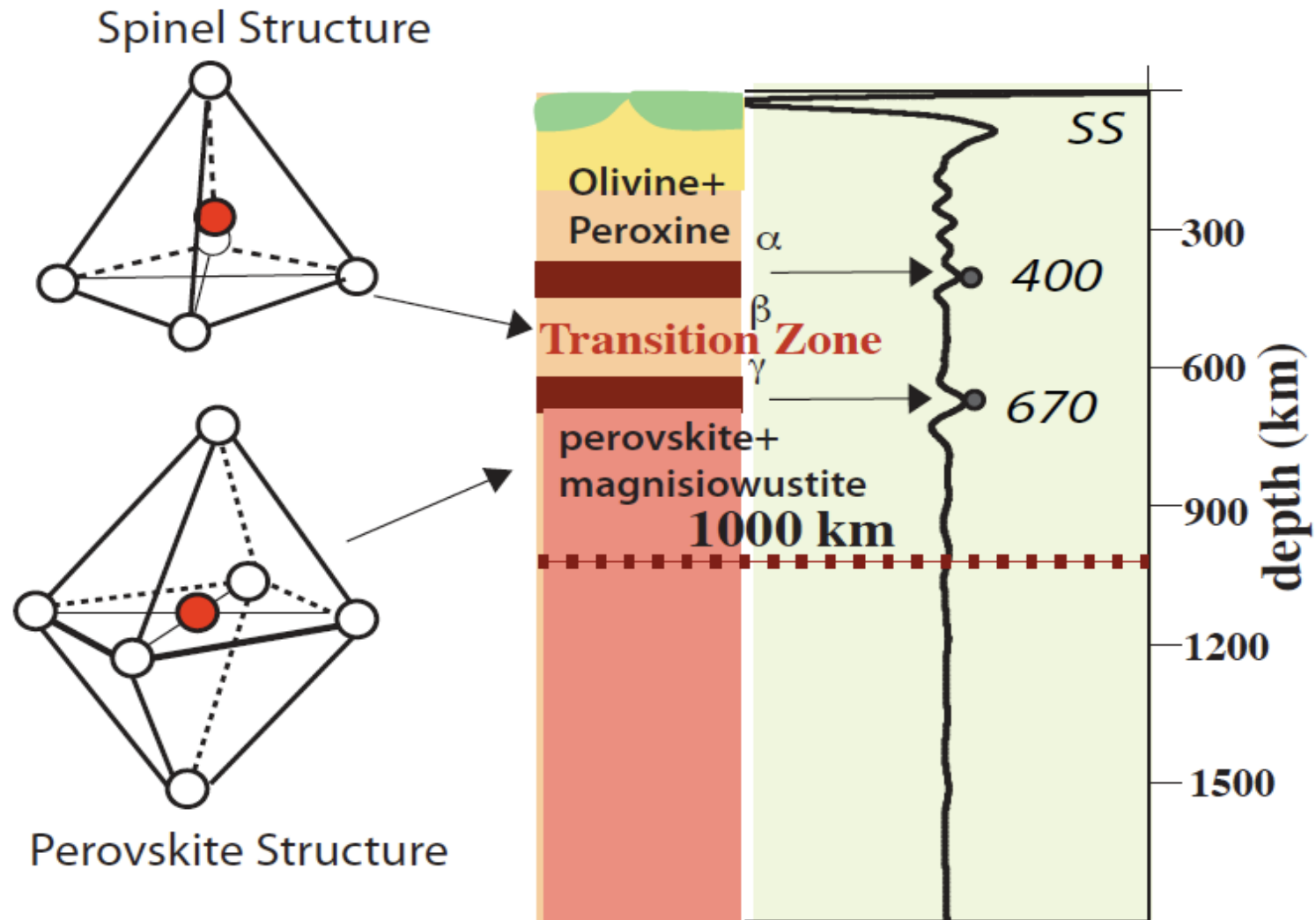


Velocity Structure of the Earth

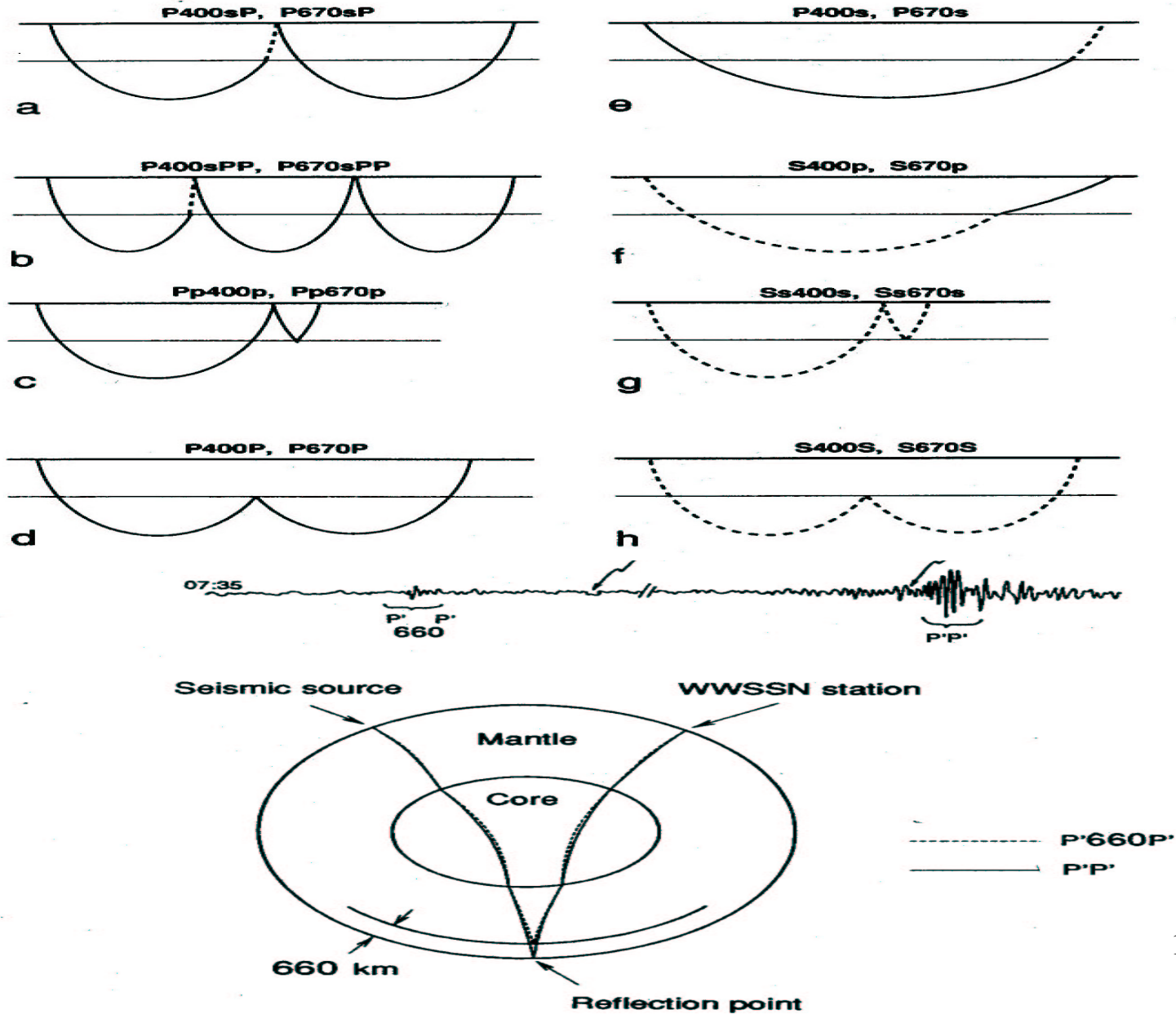
- Upper mantle
 - P waves 8-10 km/s;
 - S-waves 4-6 km/s
- Lower mantle
 - P-waves 12-14 km/s
 - S-waves 6-7 km/s
- Outer Core
 - P-waves 8-10 km/s
 - S-waves - Do not propagate
- Inner Core
 - P-waves 11 km/s
 - S-waves 5 km/s

Taken from

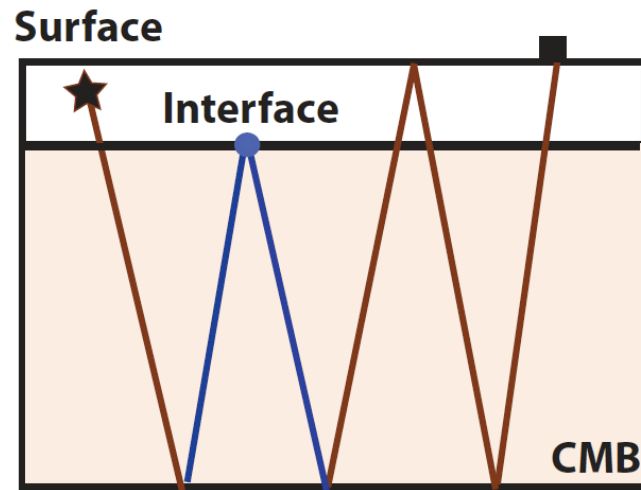
Seismology & Mineralogy of the “Transition Zone”



Secondary arrivals: by the same token, many small phases are useful
useful to constrain internal boundaries, but they are usually *small* in
amp



“Low Res” approaches



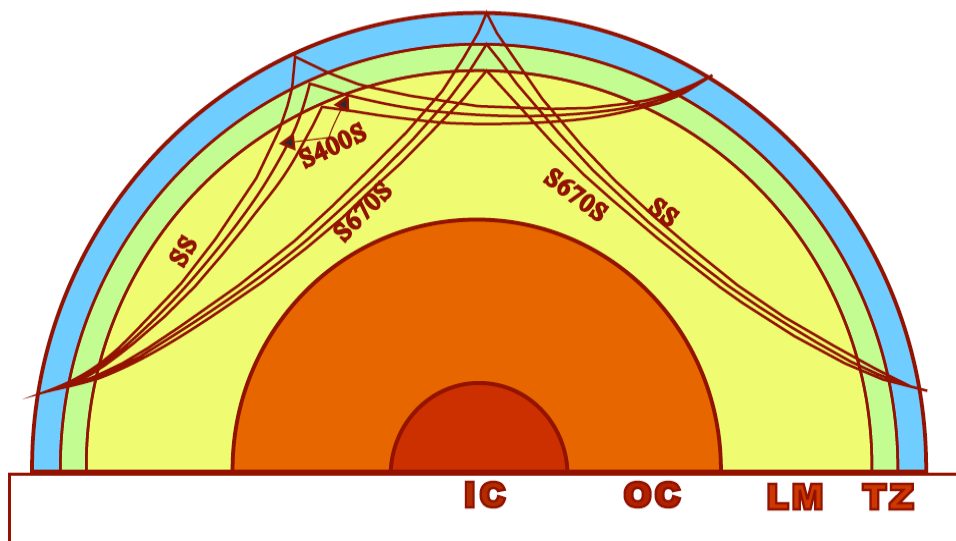
ScS reverberations

Period: 15 sec +

Distance: varies

Plus: versatility, path sensitivity to mantle

Minus: Low resolution



SS/PP precursors

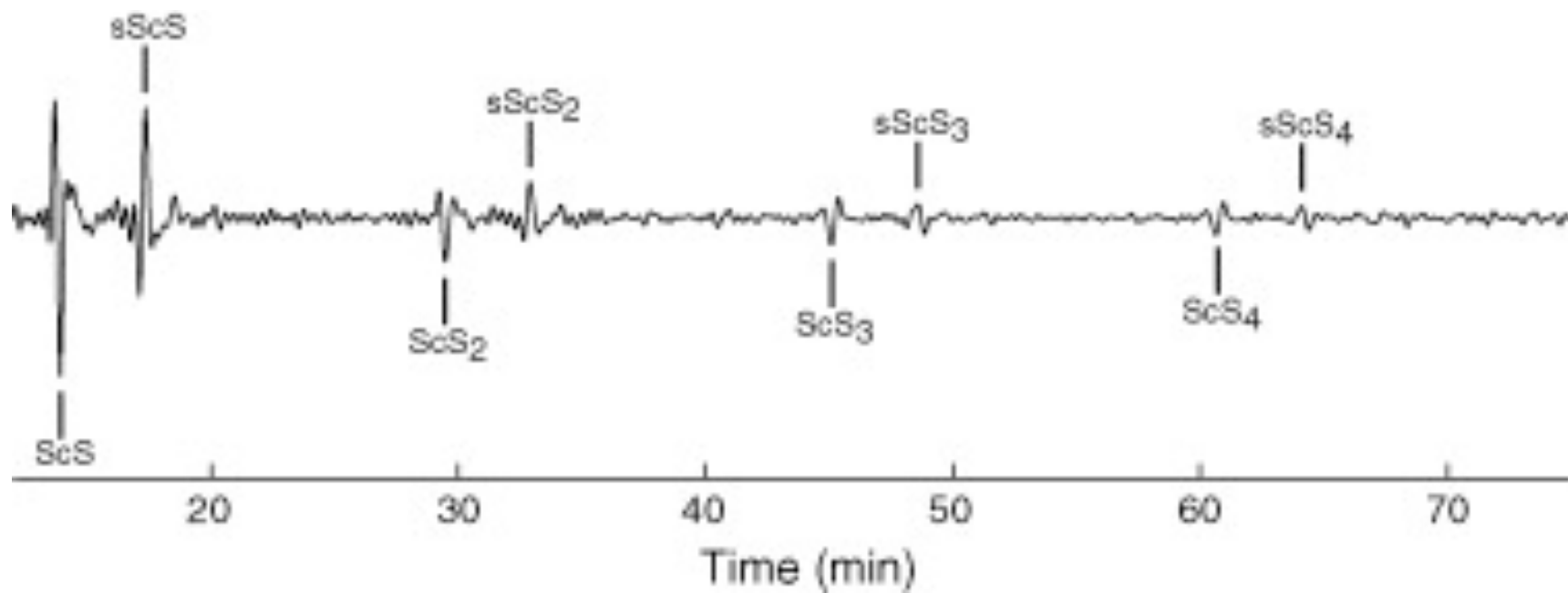
Period: 12 sec +

Distance: 90-170 deg

Plus: True global coverage

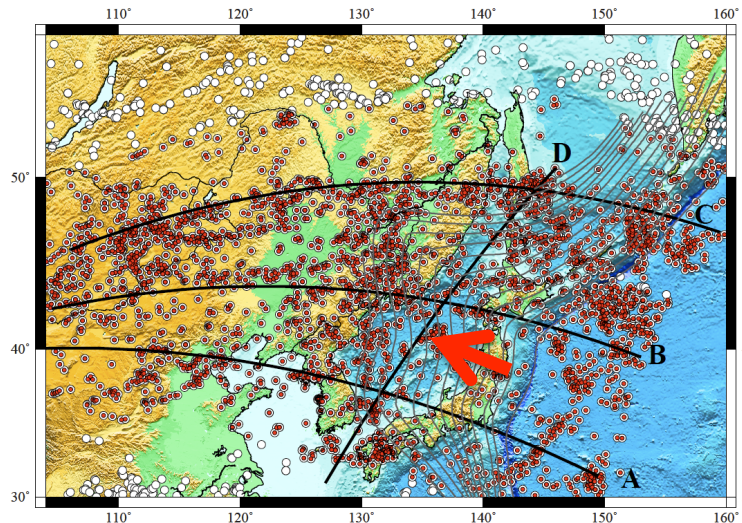
Minus: Low resolution

Complex Fresnel zone

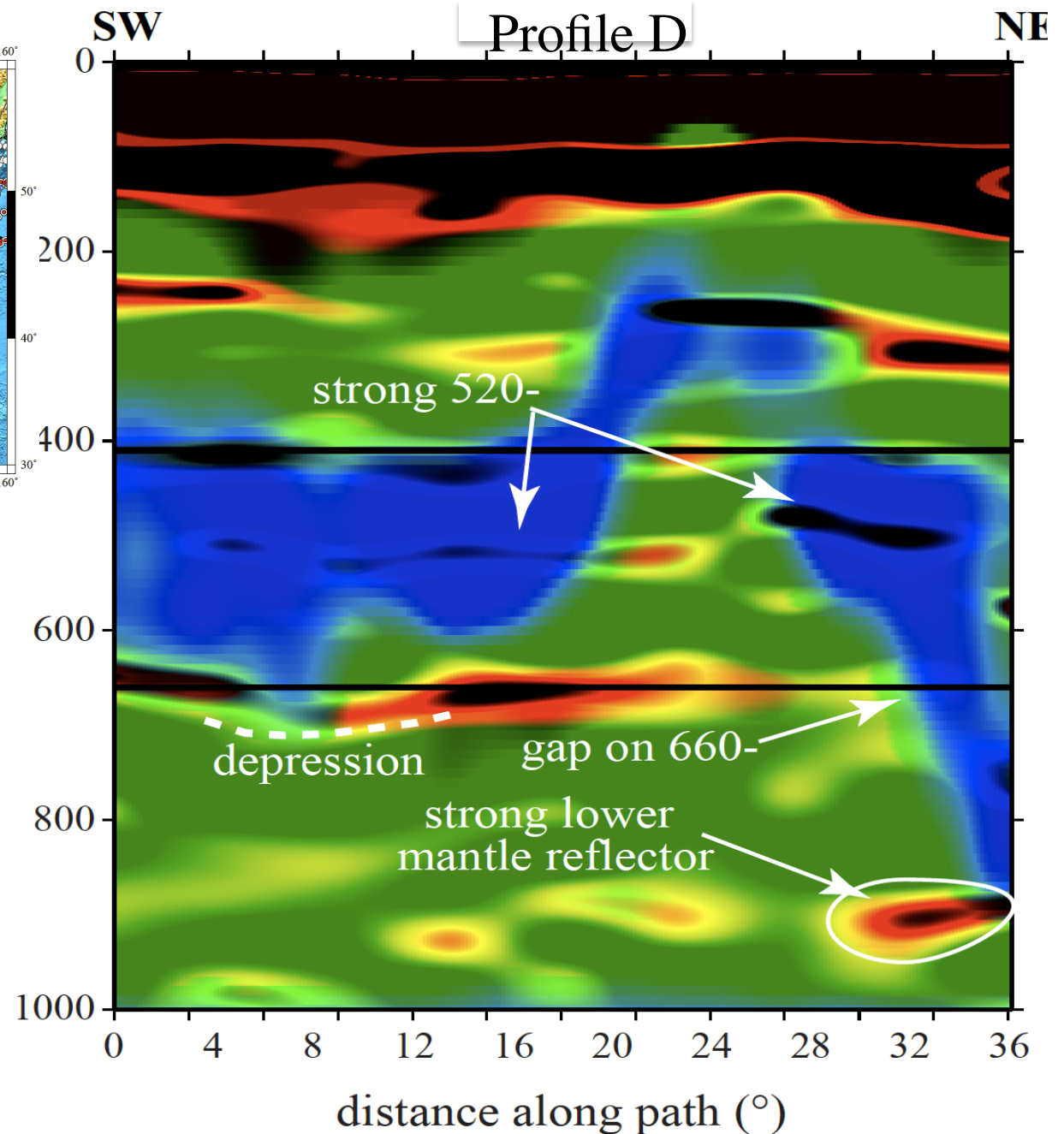


From Bagley et al., 2009

Data & Cross-section

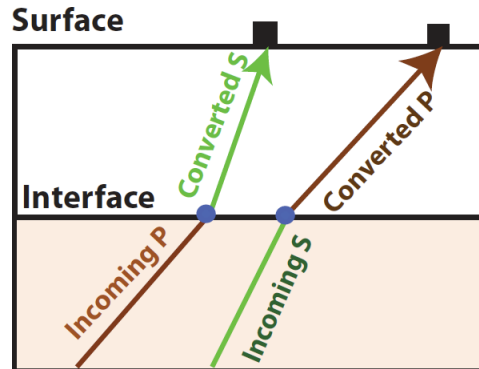


This shows remarkable connections between seismic velocity (background colors) and the major reflectors (yellow horizontal lines). While slab ‘ponding’ is suggested in central Honshu (south), the Kuril subduction zone (north) shows clear signs of a slab that penetrates into the lower mantle.



Gu et al., 2012, EPSL

‘High Res’ Examples



Receiver Functions

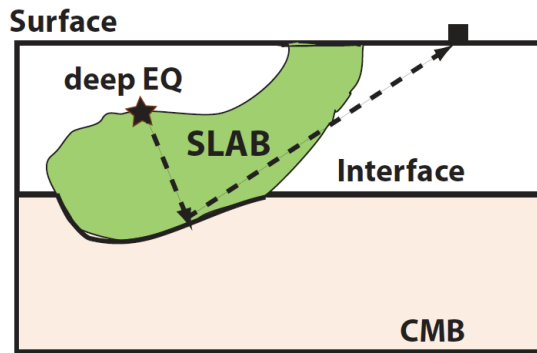
Period: 1-5 sec (high-freq study)

10+ sec (global survey)

Distance: 30-90

Plus: high-resolution

Minus: poor coverage in oceans



Scattered Phases

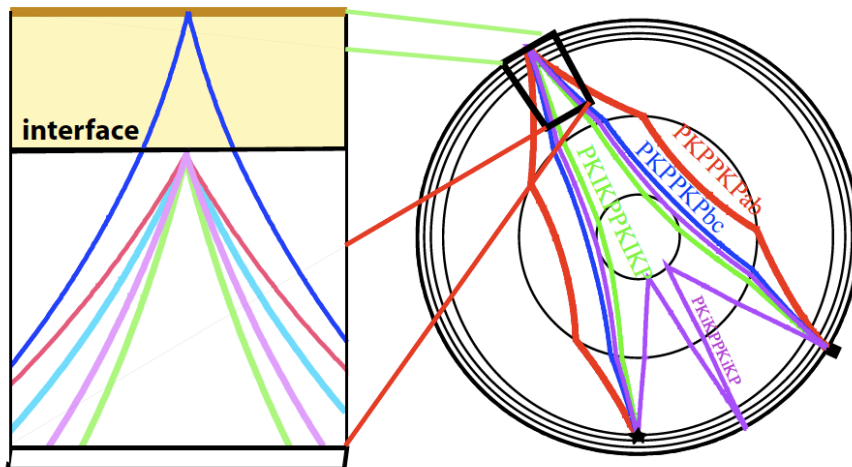
Period: 1 sec +

Distance: varies

Plus: high resolution

Minus: limited spatial/depth coverage
hard to identify

surface



P'P' (aka PKIKPPKIKP) Precursors

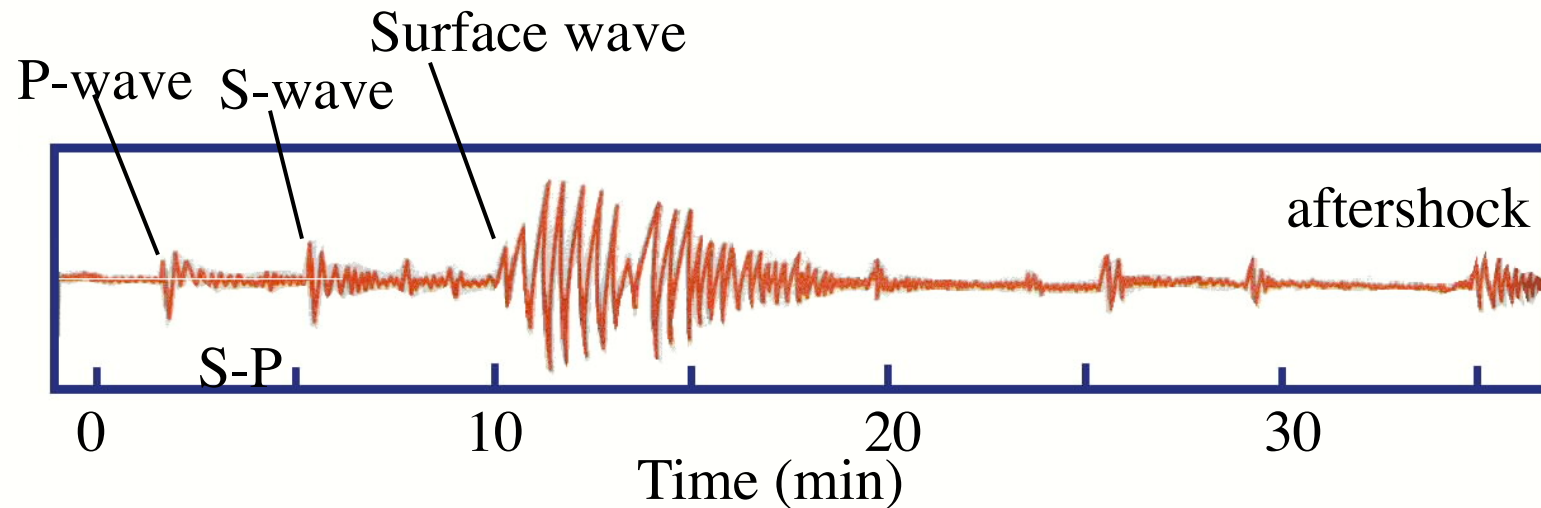
Period: 1 sec

Distance: 60-75 deg

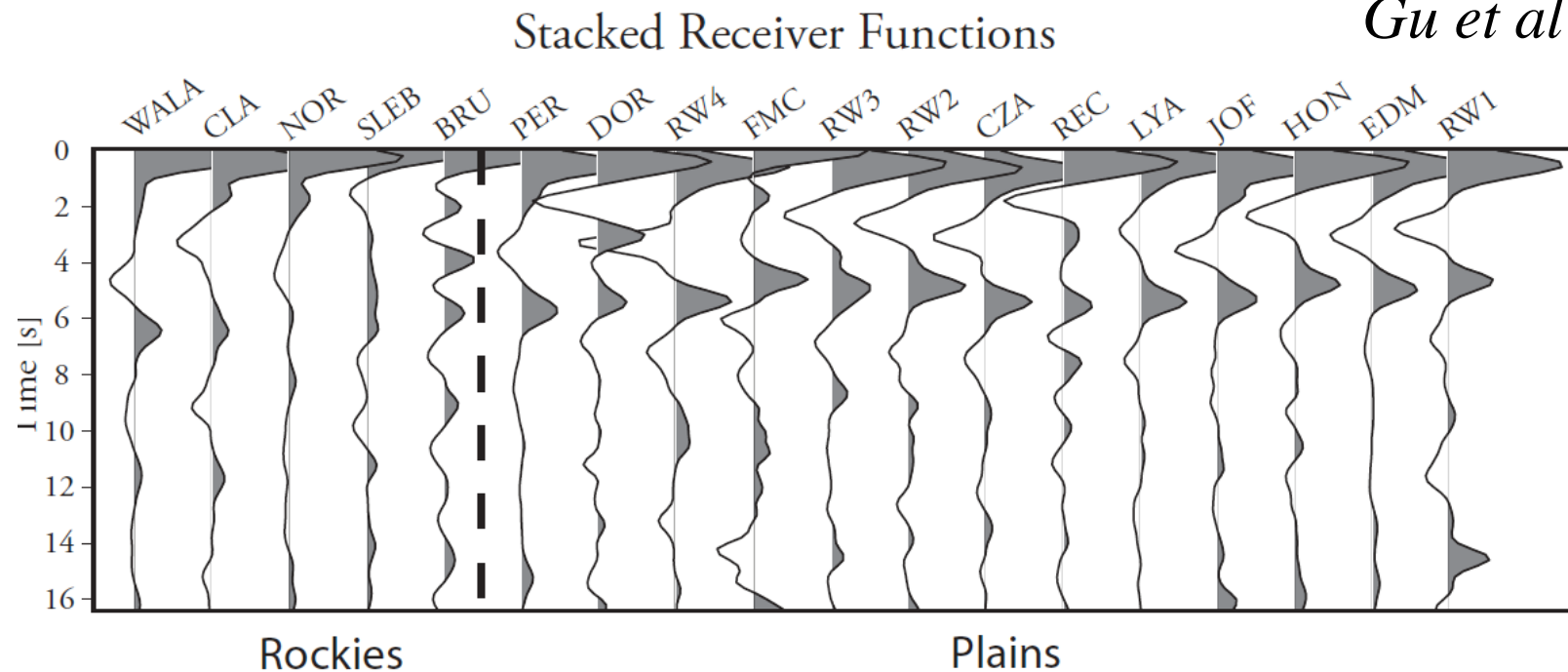
Plus: high resolution

Minus: limited spatial/depth coverage
hard to identify identification

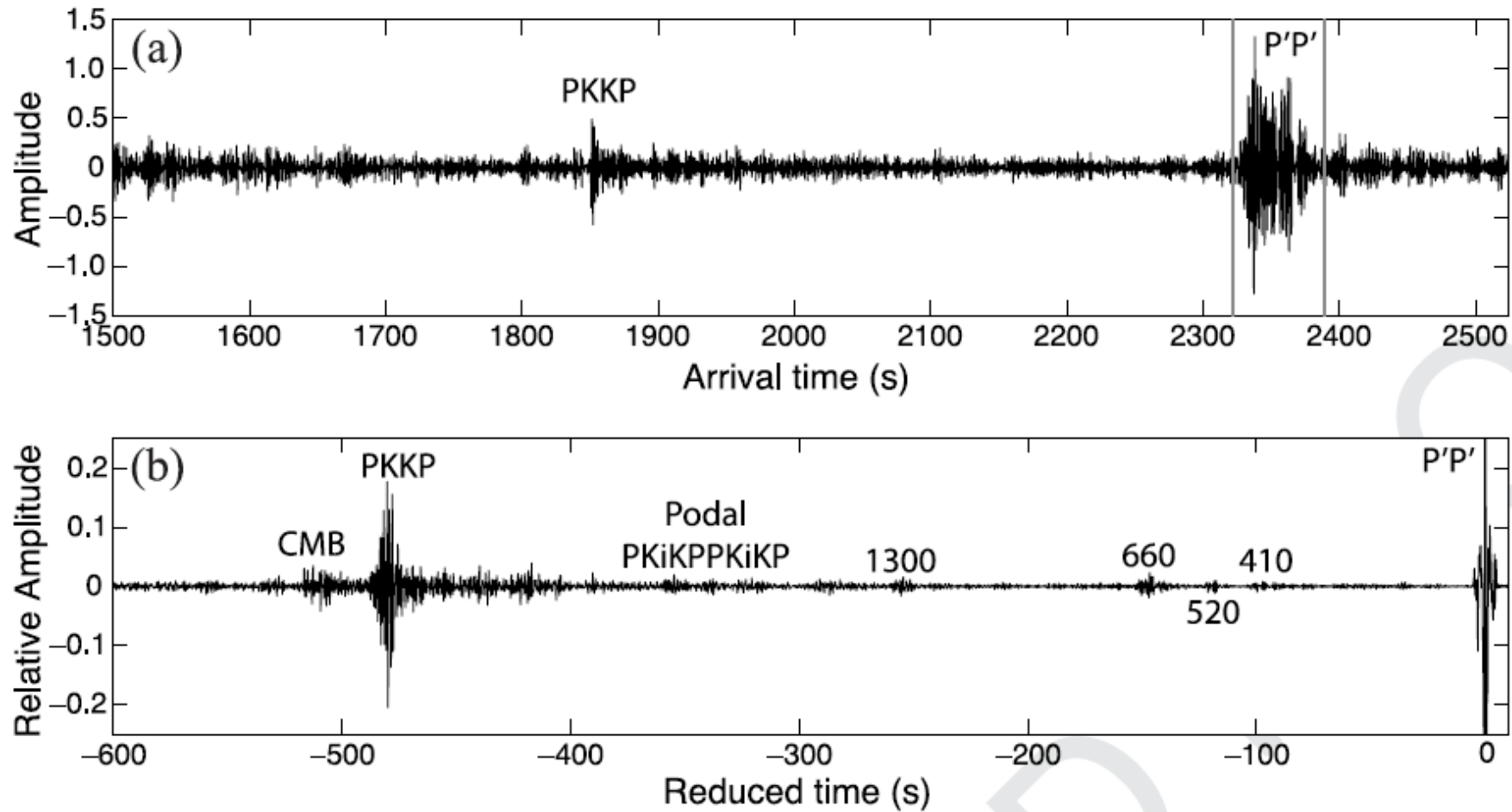
RECEIVER FUNCTIONS (P-to-S or S-to-P converted Waves)



Gu et al., 2013



Sample high-frequency P'P' Precursor Observations



Schultz and Gu, 2013

Basic Idea: Earth is a lot more complex than people give it credit for. There are a lot more reflective features that are either results of chemical or thermal variations. The basic stratifications (220, 400, 660, crust) are just a start!

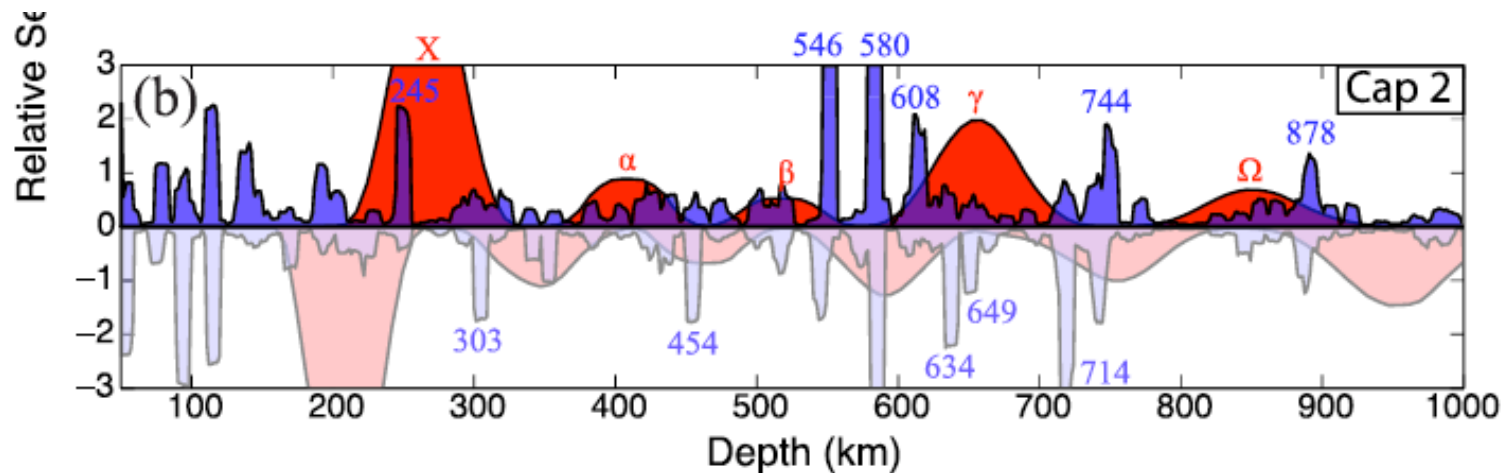


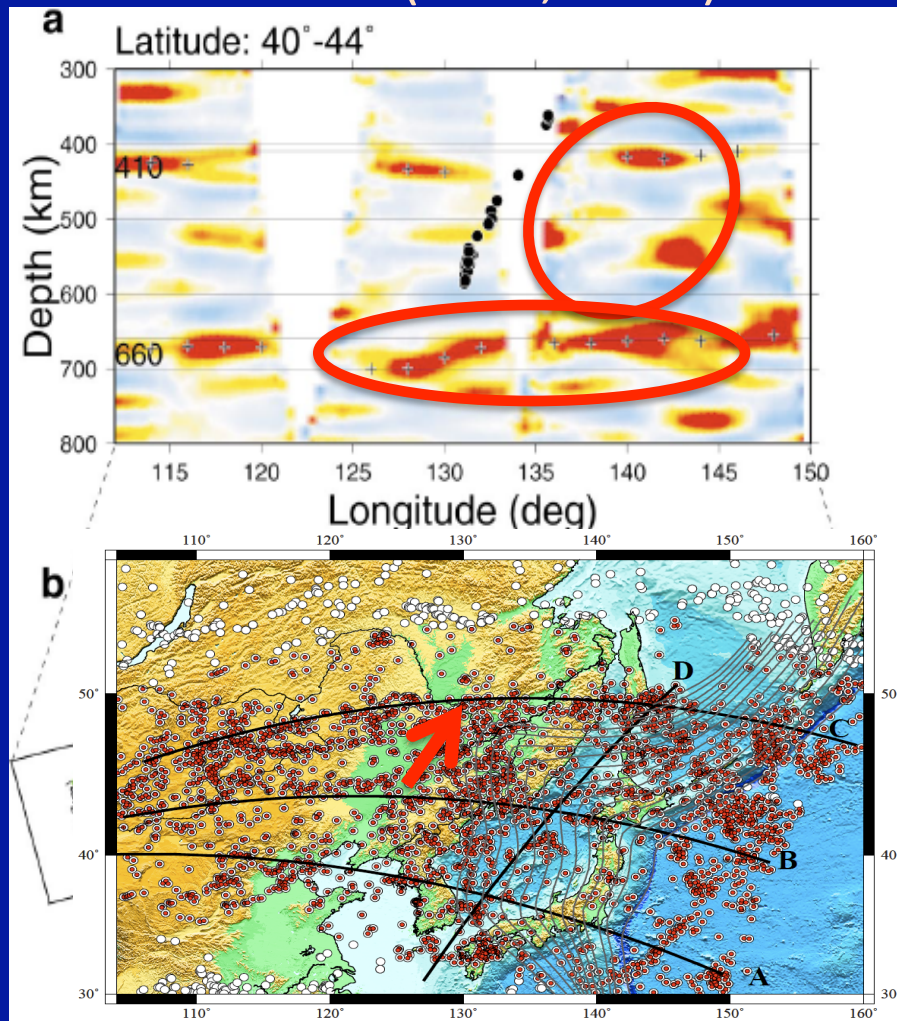
Figure 1. Migration profiles for Cap 1 and 2. Migration for SS (red-filled traces) and P'P' (blue-filled traces) is repeated for both their positive (upper half) and negative (faded lower half) amplitude information. Positive amplitude information is used to infer mantle structure, dynamics and composition local to the western Pacific Ocean (a) and subduction zone (b) caps. Letters and numbers (colour-coded) provide labels to phases referred to in the text.

Red: low-res SS precursor reflectivity

Blue: high-res P'P' precursor reflectivity

Schultz and Gu, 2013

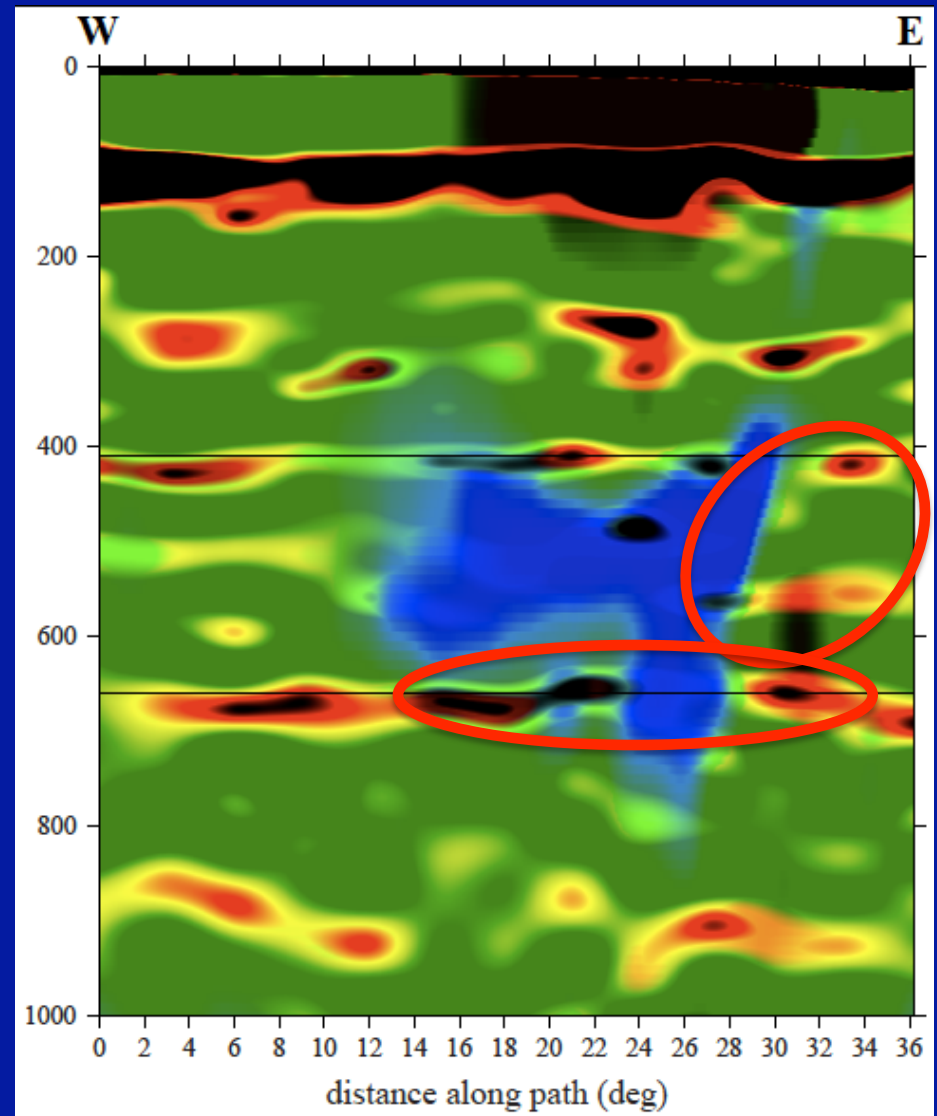
Receiver Function Image Li et al (EPSL, 2003)



Consistency between the above methods
really provides solid evidence for mantle
geodynamics

SS Precursor Migration Image

Blue/red background, velocity
Front, reflectors, Gu et al. (2011)



Requirements for these small phases to work

1. Stacking (phase equalization) is almost always required due to small reflection/conversion amplitudes
2. A reference 'major phase' with similar paths is generally required:
e.g.: SS for SS precursors, PP for PP precursors,
P for P-to-S conversions, S for S-to-P conversions,
P'P' (PKIKPPKIKP) for its precursors

Exception: slab reflected phases

3. A reference P or S Earth model (SS, PP & P'P' precursors)
or both (receiver functions)
4. Approximations could be made if accuracy is not vital:
e.g.: SS/PP differ from precursors by a 2-way travel time above
reflection interface
pP (depth phase) differs with P by 2 way time above hypocenter

5. Proper corrections

Complications:

(1) Surface Topography timing correction

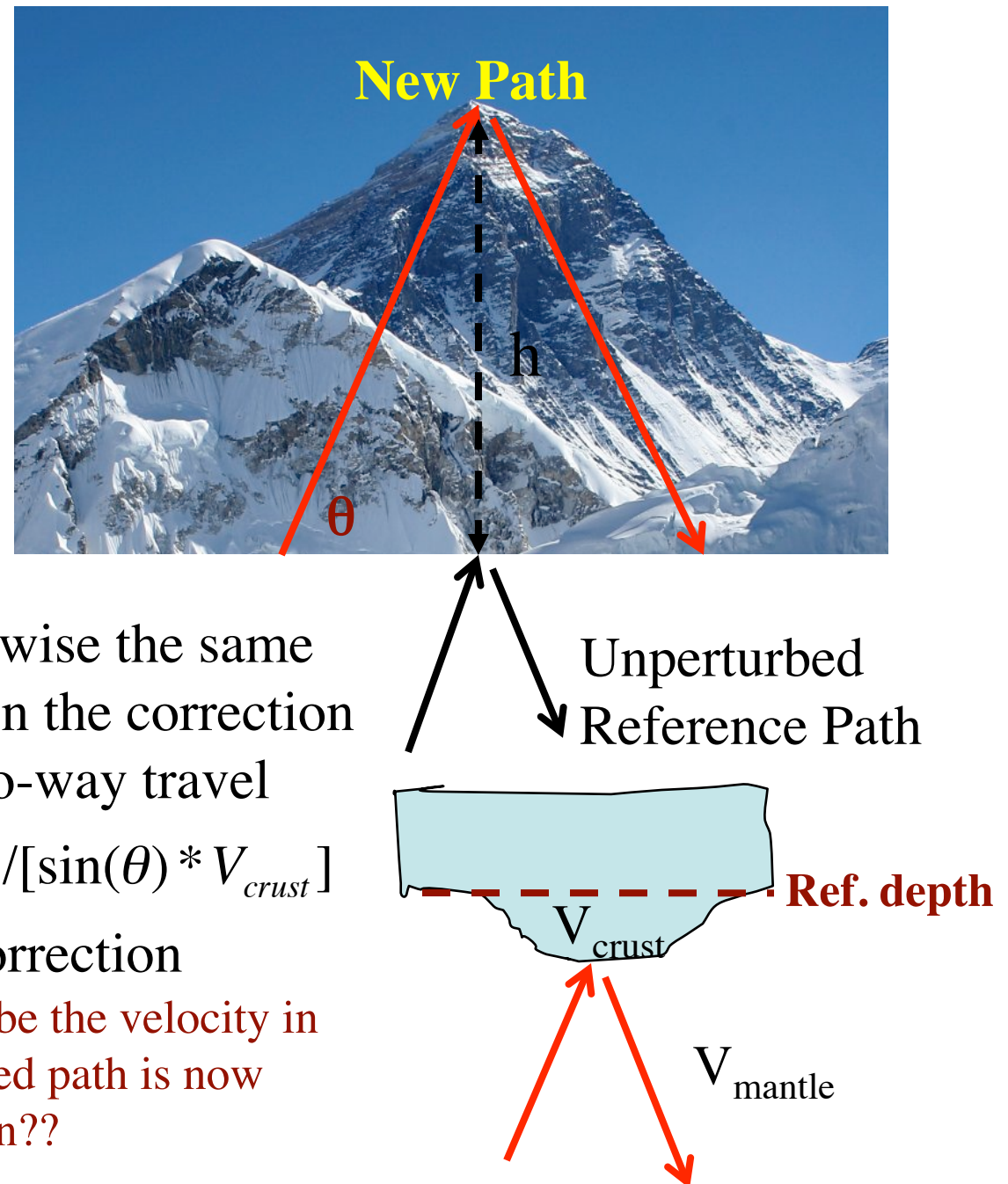
Essentially twice the additional P/S (depending on the wave type) travel time in the mountains. To provide more precise answers:

1. Assume path is otherwise the same
2. If theta is known, then the correction due to additional two-way travel

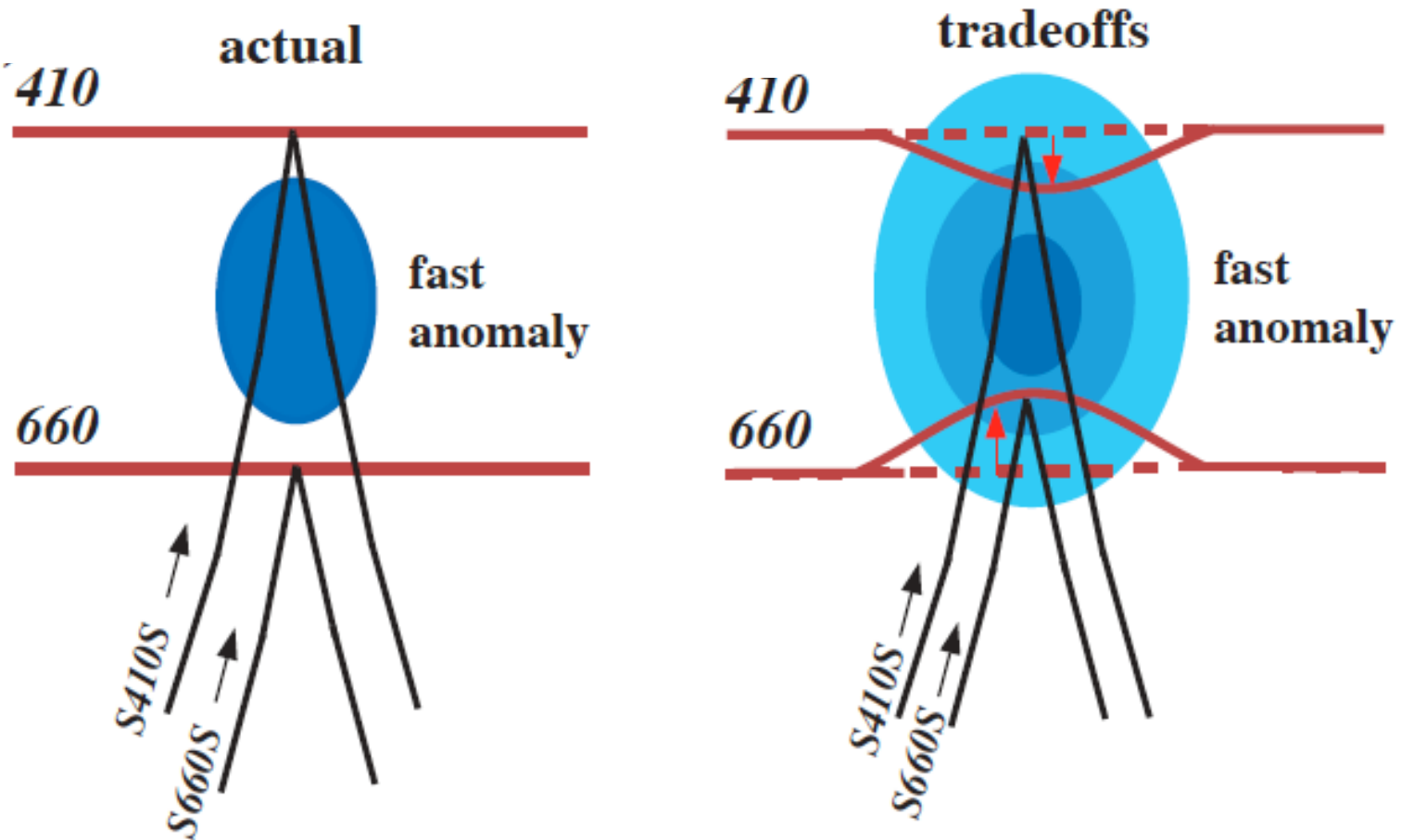
$$\text{time } T_{topocorr} = 2 \times h / [\sin(\theta) * V_{crust}]$$

(2) Crustal depth timing correction

Has to be careful, what should be the velocity in the negative time correction (red path is now faster than flat path) calculation??



(3) Heterogeneity correction for tradeoffs between velocity and interface depth



Left panel: A faster than expected arrival of SS causes small timing difference with its precursor

Right panel: The same timing difference can be made up by a weaker high-velocity zone with an uplifted 660 boundary

Basic Ray Tracing and Body Waves

Ray theory: Solutions of wave equations wave vectors that describes how short-wavelength seismic energy propagates, where by “short” we mean short relative to any scale lengths in the structure (not counting abrupt jumps in property). P = ray parameter (s/km, sec/deg or sec/radians):

Most important concept: **Snell's Law**

$p = \sin(\theta(z)) / v(z)$ is conserved for a given path

Special cases:

(1) vertical incidence

$$p=0$$

(2) At surface:

$$\theta(0) = \sin^{-1}[p \cdot v(0)]$$

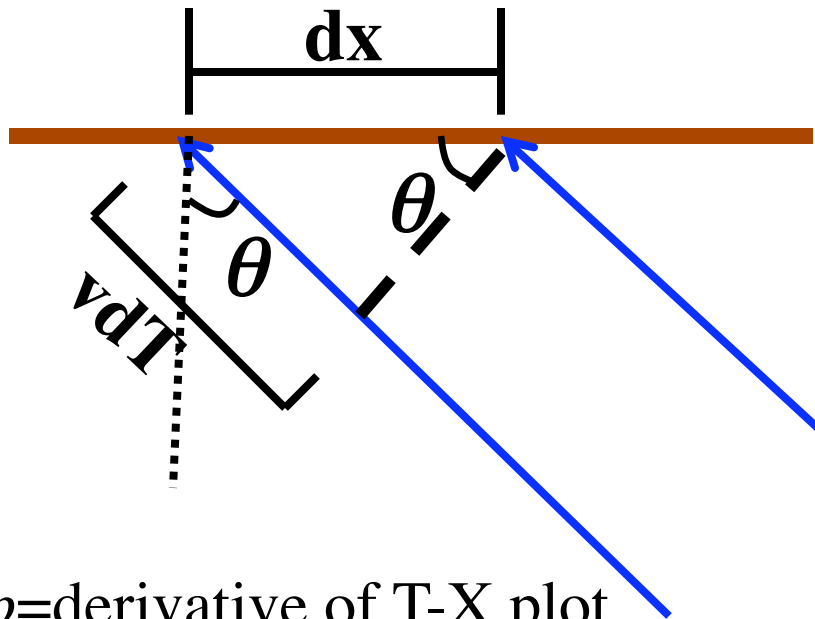
(3) Turning depth

$$v(z_{tp}) = 1/p$$

$$\sin \theta = \frac{v dT}{dx}$$

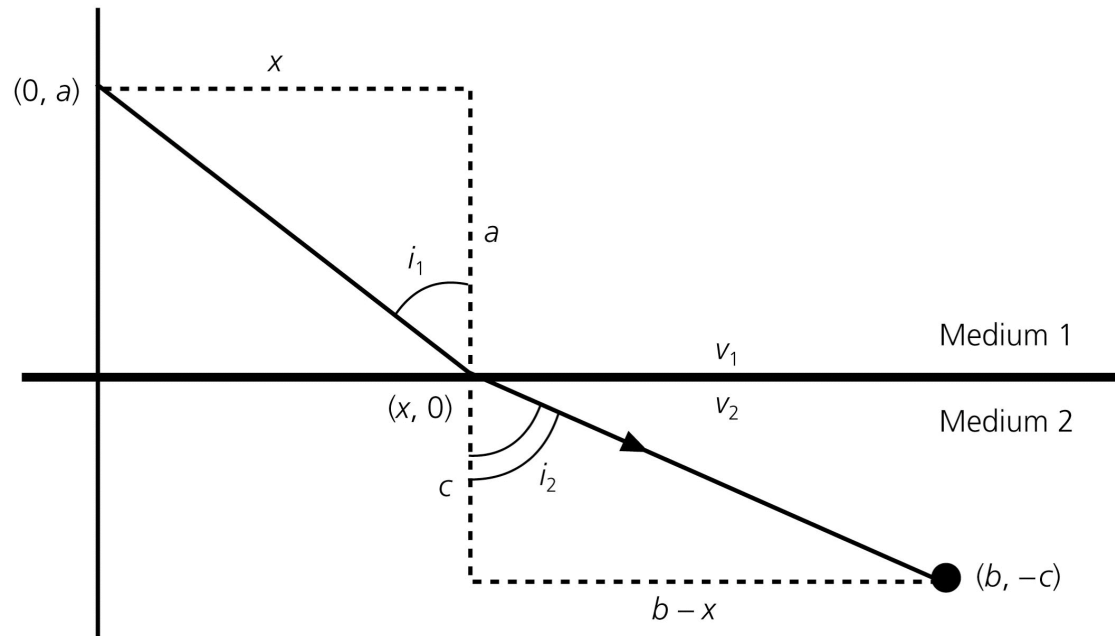
$p = \text{derivative of T-X plot}$

$$p = \frac{\sin \theta}{v} = \frac{dT}{dx} = \frac{1}{c_x}$$



Fermat's Principle: Ray paths between two points are those for which travel time is an extremum (min or max) with nearby possible paths.

Figure 2.5-14: Derivation of Snell's law using Fermat's principle.

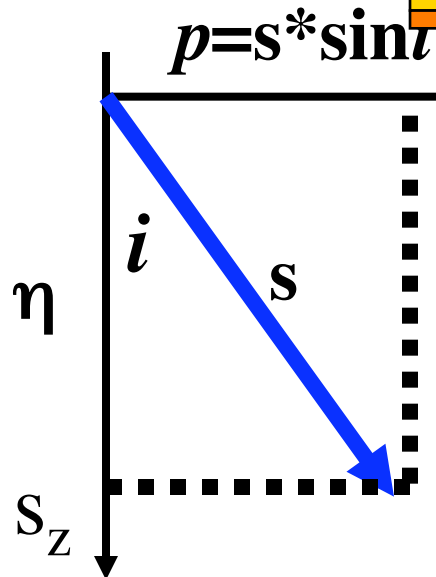
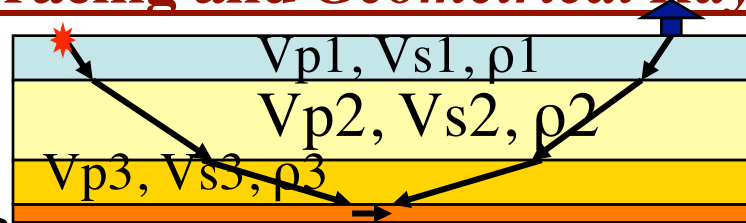


$$T(x) = \frac{(a^2 + x^2)^{1/2}}{v_1} + \frac{[(b-x)^2 + c^2]^{1/2}}{v_2}$$

$$\frac{dT(x)}{dx} = \frac{v_1}{v_1(a^2 + x^2)^{1/2}} - \frac{v_2}{v_2[(b-x)^2 + c^2]^{1/2}} = \frac{\sin i_1}{v_1} - \frac{\sin i_2}{v_2} = 0$$

$$\Rightarrow \frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2} \quad (\text{Snell's Law})$$

Ray Tracing and Geometrical Ray Theory

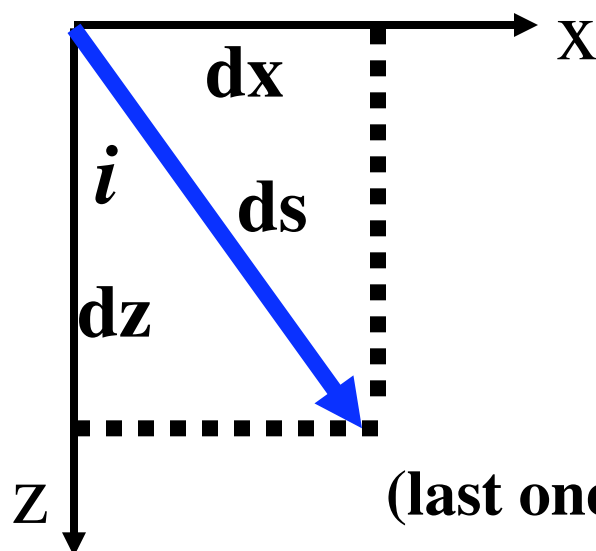


S_x Assume s to be slowness $1/v$

$$\eta = s \cos i = \sqrt{s^2 - p^2}$$

(vertical ray parameter)

Now ds to be a small segment of distance



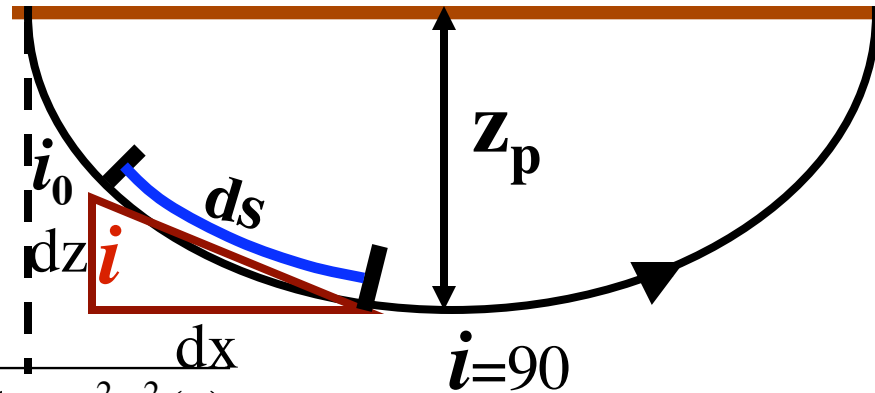
$$\frac{dx}{ds} = \sin i$$

$$\frac{dz}{ds} = \cos i = \sqrt{1 - \sin^2 i}$$

$$\frac{dx}{dz} = \tan i = \frac{dx/ds}{dz/ds}$$

(last one is integratable over depth z)

continuous
distribution of
velocity



$$\sin i = pv(z)$$

$$\cos i = \sqrt{1 - \sin^2 i} = \sqrt{1 - p^2 v^2(z)}$$

Distance: $x(p) = 2 \int_{z=0}^{z_p} \frac{dx}{dz} dz = 2 \int_{z=0}^{z_p} \sin i / \cos i dz$ **Ray Tracing Equations**

$$= 2p \int_{z=0}^{z_p} \left(1/v^2(z) - p^2\right)^{-1/2} dz = 2p \int_{z=0}^{z_p} \left(s^2 - p^2\right)^{-1/2} dz$$

Time:

$$T(p) = 2 \int_{z=0}^{z_p} \left(v(z) \cos i\right)^{-1} dz = 2 \int_{z=0}^{z_p} s^2 dz / \sqrt{s^2 - p^2}$$

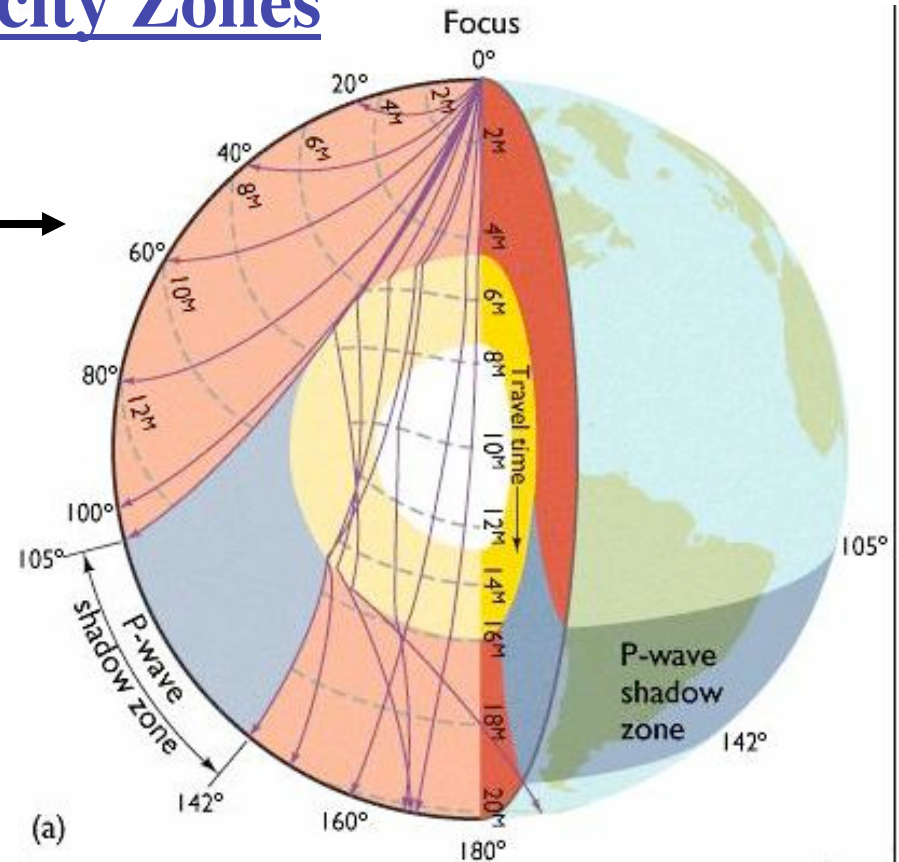
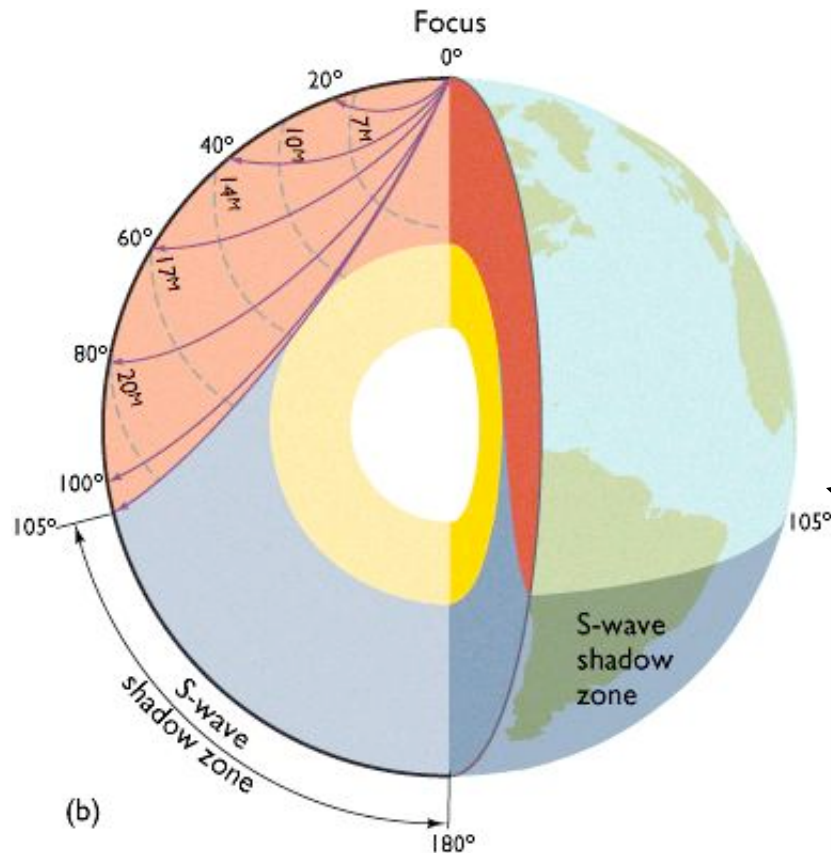
Well-known problem: at turning point, $p=1/v$, so $s^2-p^2=0$, Singularity!

Sample solutions: (1) Richardson' deferred approach (extrapolation)
(2) Gauss integration

(1) Complexity with Low Velocity Zones

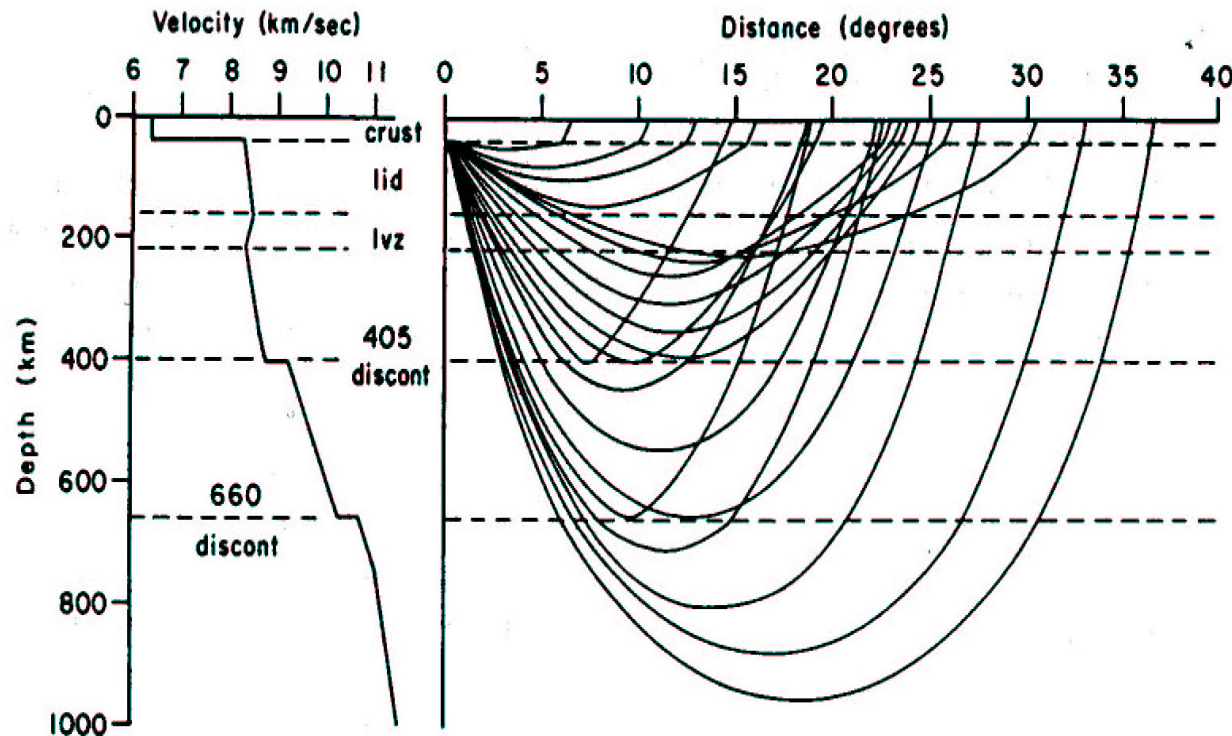
First-order Low velocity Zones

P wave shadow zone is caused by
a decrease of P speed in the outer
core.



S wave shadow zone is
caused by the fact no shear
wave propagates through the
outer core, which is liquid.

Second-order Low velocity zones



Low velocity zones can also cause rays to bunch up and:

- (1) Strong amplitude at distance of multiple arrivals due to superposition
- (2) Triplications can be caused by sudden jumps in material properties, such as an impedance increase (aka a seismic discontinuity). Crustal phases such as Pg, Pn and PmP can triplicate.

(2) Complexity associated with Multiple Layers

One-layer vs. Multiple-layer

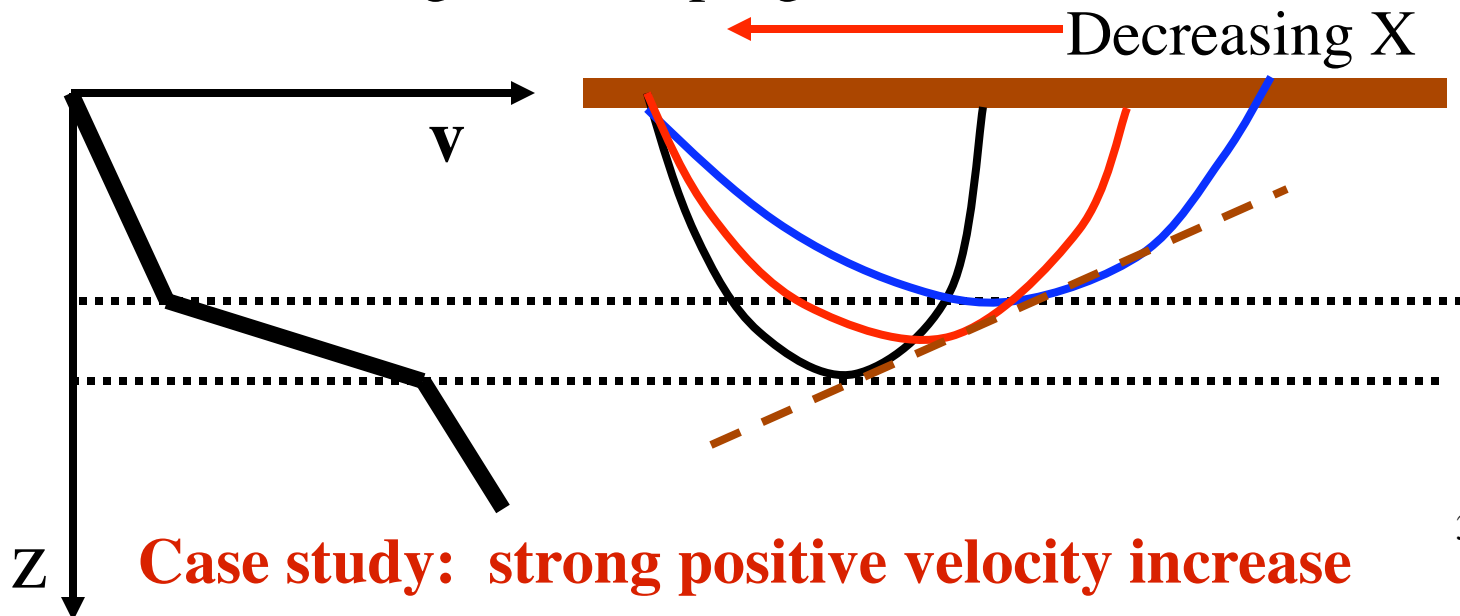
(1) Ray parameter P change for 1

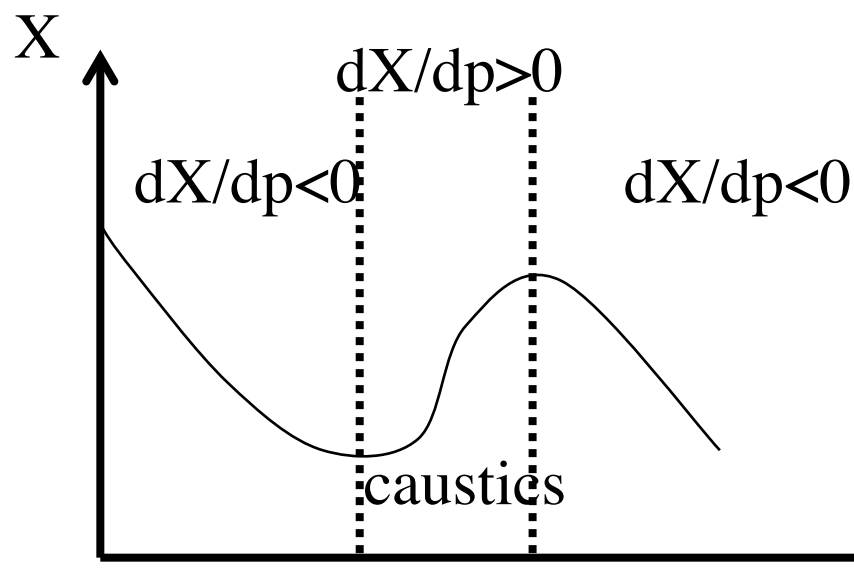
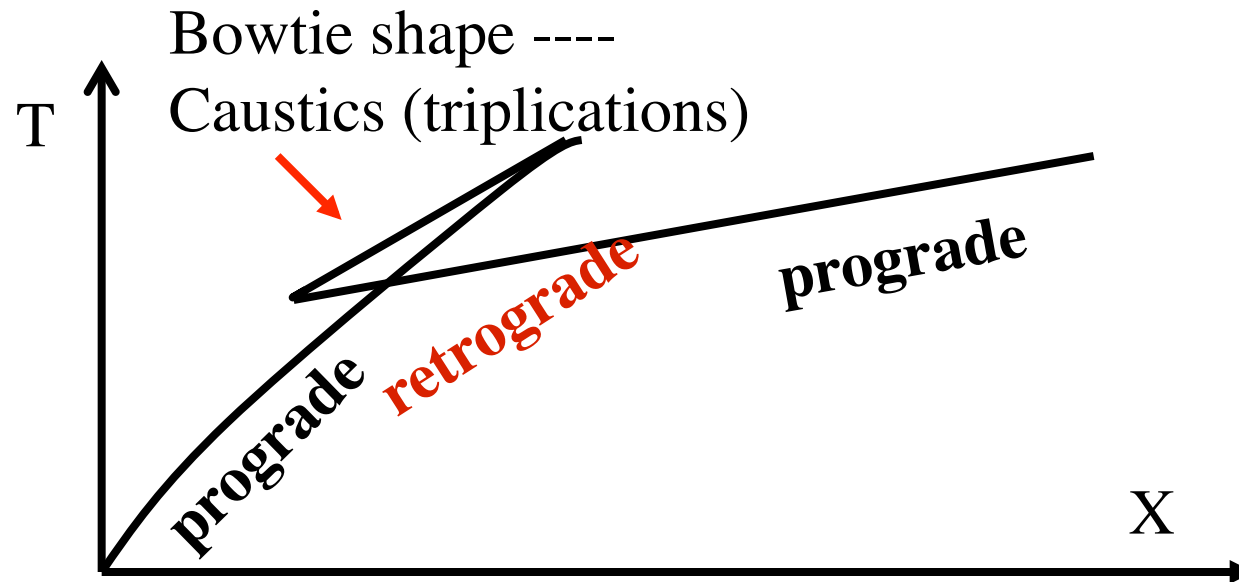
simple layered case $p = \sin i / v$
which means at larger distance,
 $\sin i$ increases, p increases.

Real earth: p decreases as
distance increases.

(2) In **1-Layer case**: always prograde

Real Earth: both retrograde and prograde can exist.





How to avoid dX/dp sign problem?

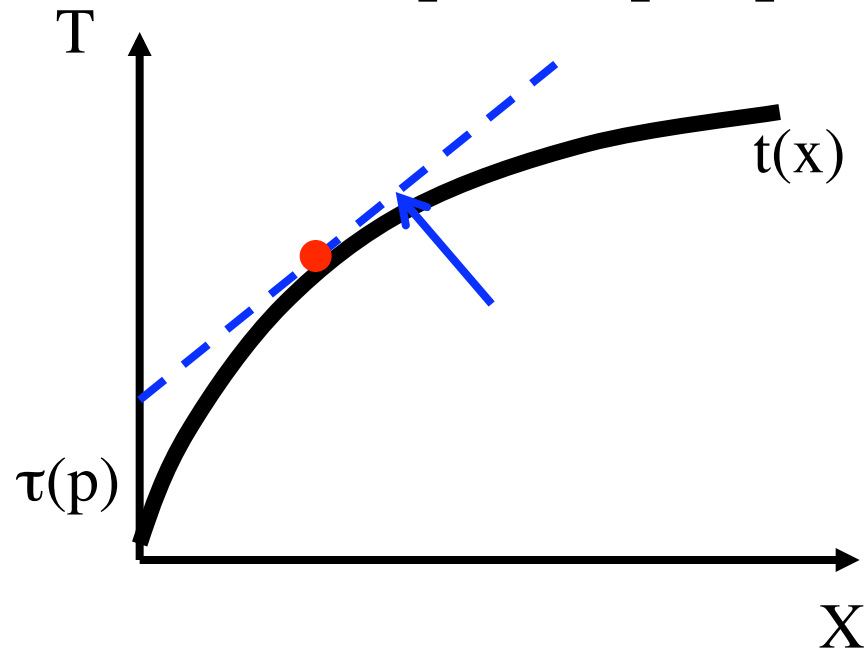
Reduced velocity and $\tau(p)$ function.

Reduction velocity: velocity used to plot seismograms at different different distances on the same plot (**expands time scale**)

$\tau(p)$ is called delay time.

$$\tau(p) = T(p) - p X(p)$$

$$\tau(p) = T(p) - p X(p)$$

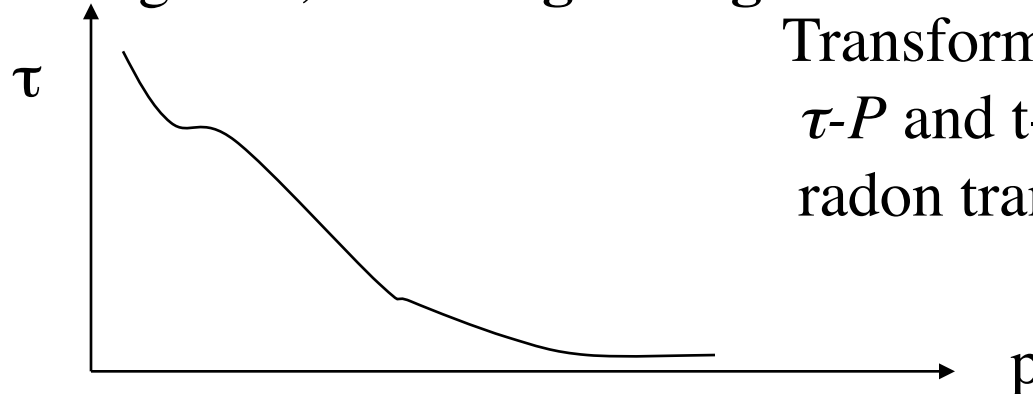


Consider a point on a travel time curve $t(x)$ at distance X and time T . Linear function of $\mathbf{X(p)}$
slope = $-P$ and intercept = $T(p)$

Beauty of this approach:

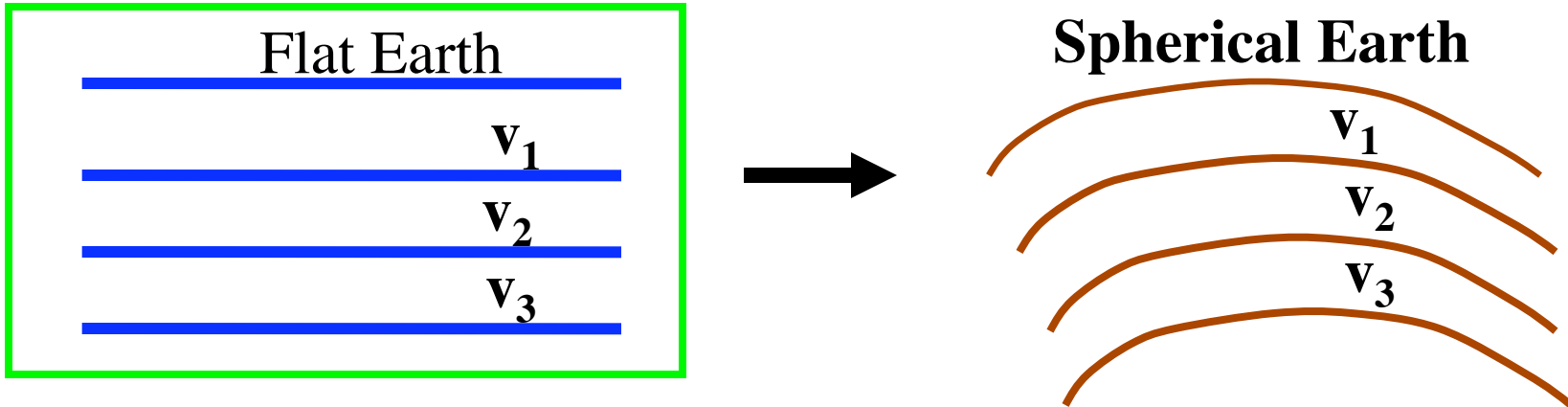
$$d\tau(p)/dp = -X(p)$$

Slope on the plot of τ - P always negative, **no change in sign!**



Transforms between τ - P and t - X is called radon transform

(3) Complexity of Flat Earth vs. Spherical Earth



In flat earth : $p = dT/dX$

In spherical earth : $p = RdT/dX = dT/d\Delta$

where Δ is the angle in radians.

Can use a simple flattening transformation to convert to flat earth,

$$v_{flat}(r) = \frac{a}{r} v_0(r)$$

a = radius of the earth, r = radius to the current depth

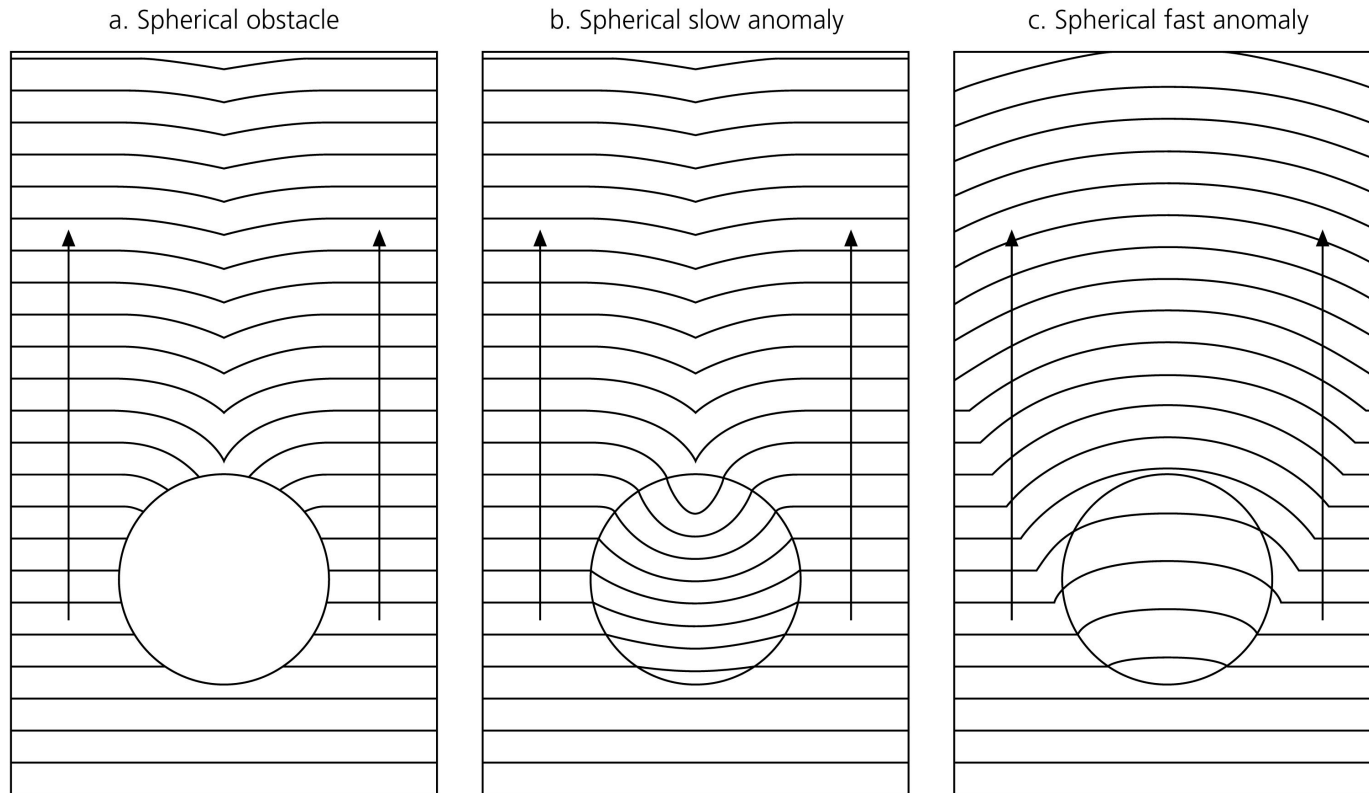
$$\frac{r \sin i}{V} = \text{constant} = p$$

(Snell's Law for spherical earth)

r is radius at a given depth.

(4) Complexity with Wavefront healing

Figure 2.5-19: Waves interacting with a spherical anomaly.



Case 1. Huygen's will allow waves to diffract around a solid obstacle.

Case 2. Small effect of wave distortion and slow-down inside gets "healed" once ray passes the slow anomaly.

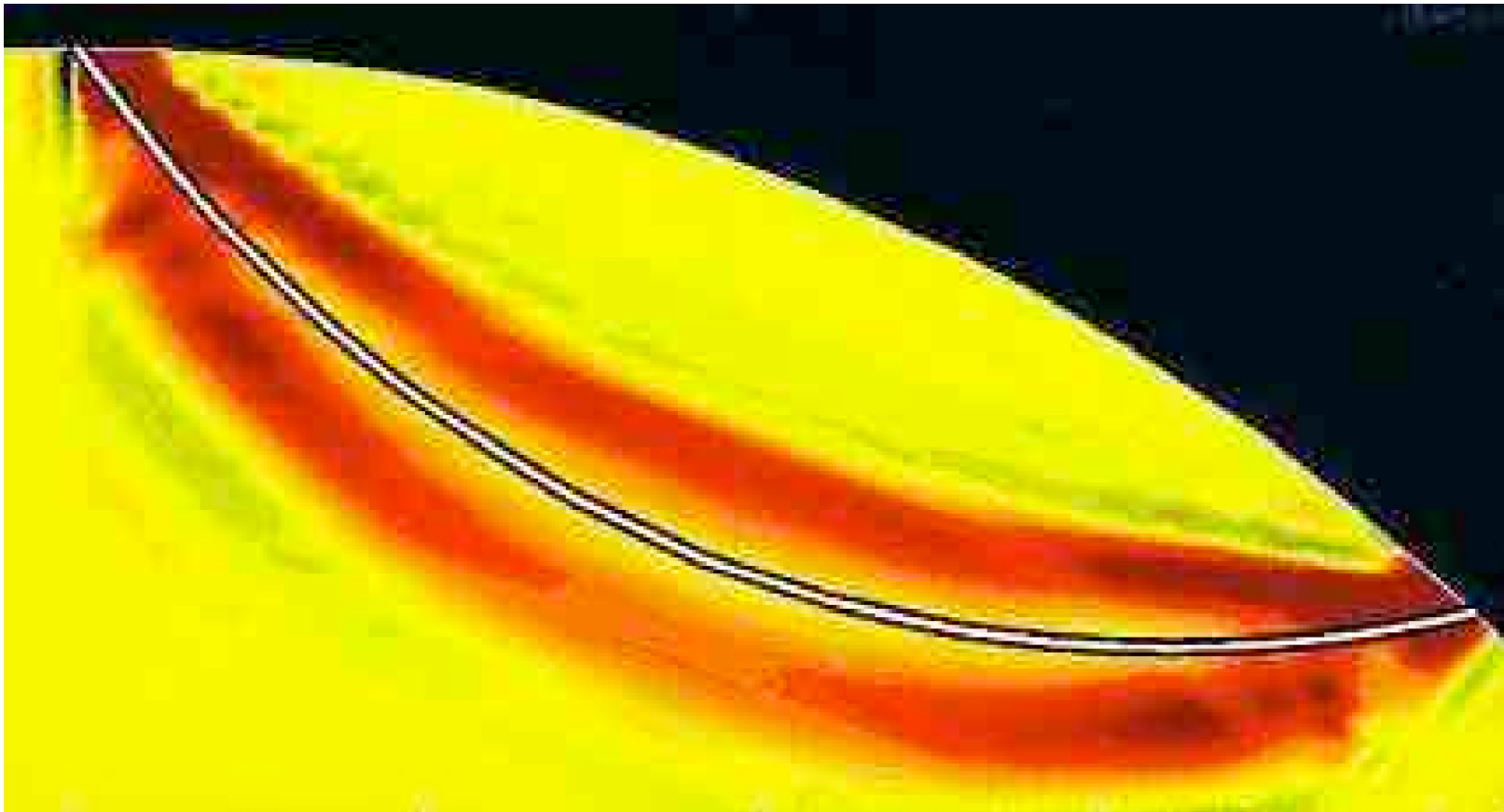
Case 3. Fast anomalies are not easily "healable".

Moral: Subduction zone signatures are easier to observe than ³⁵plumes!

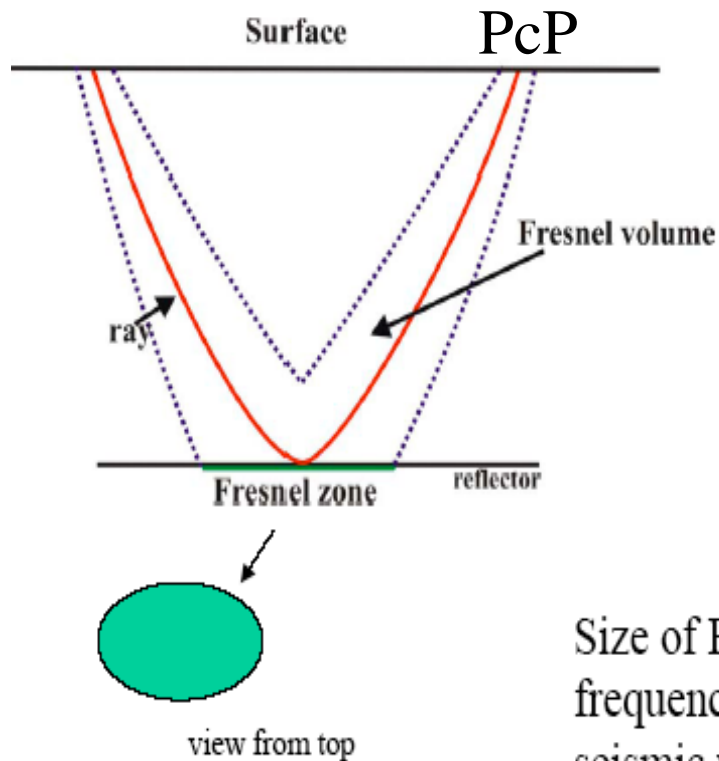
(5) Complexity with Banana and Donut (or Fat Rays)

Drawback of geometrical ray theory:

- (1) no amplitude information
- (2) only good to elastic limits, geometrical ray.
- (3) Doesn't work well in 3D aspherical phenomenon.
- (4) Rays are wide and banana looking

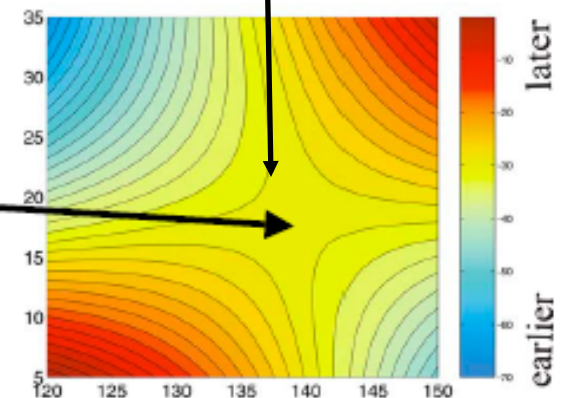
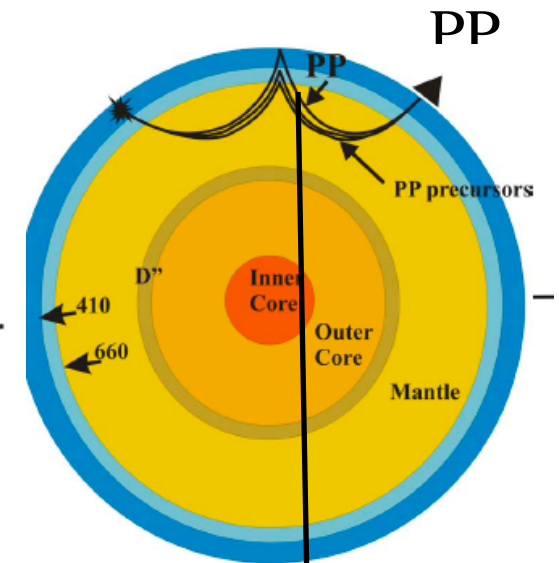


Quantifying “fat rays”



Volume around ray contributes to signal:
Fresnel volume.
 At reflector this volume (in 1D) is the **Fresnel zone.**

Size of Fresnel zone depends on:
 frequency, depth of reflector, and
 seismic velocity ($\Rightarrow$ wavelength).



Fresnel zones can be viewed at different points of the ray, for example, the Fresnel zone of PcP viewed from the top has an oval shape, which is very different from an “underside” reflection, such as PP wave Fresnel zone viewed near the Earth’s surface. The pattern depends on where we make a cut. The whole envelope surrounding the ray at any point is part of the Fresnel volume.

Quantifying “fat rays”

Recall Fermat’s Principle:

- (1) Extremum in time,
Not distance.
- (2) Works for small
perturbations only

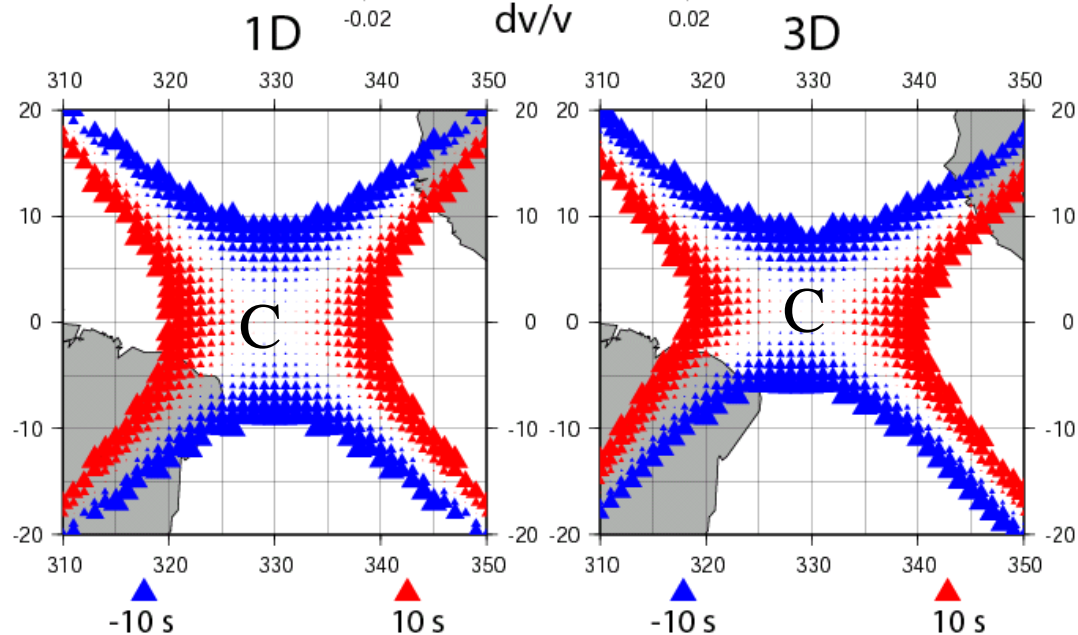
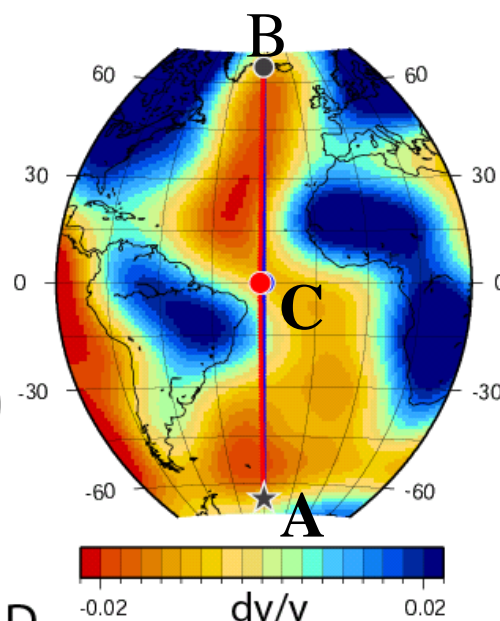
Triangles represents
time difference of
time difference
between the sum of
the segments AC and
BC subtract by
geometric time to
travel from A to B.

First Fresnel zone, or wavefield “footprint”

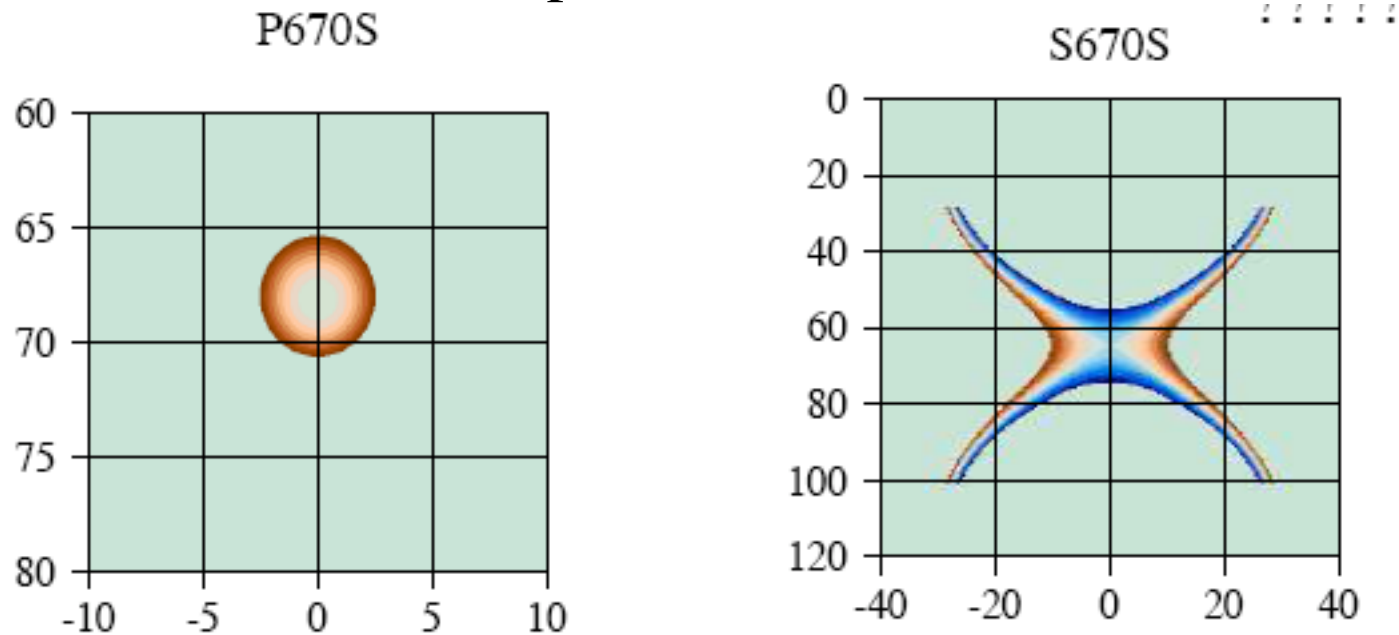
$$(\text{Time_AC} + \text{Time_BC}) - \text{Time_AB} \leq |\text{Wave_Period}/2|$$

Fresnel
Zones

(130 deg, 20s)



Fresnel Zones (footprint, Gu et al. 2008)



The left shows a simple, minimum phase Fresnel zone for a simple, converted phase. It is similar for P waves. Red color means paths that are not “great-circle” are slower than the center point (geometrical prediction). The story is much less obvious in the case of a minimax pattern, a case of a reflected wave where interference is weird. Blue represents a faster reflection path than the geometrical one (0 time).