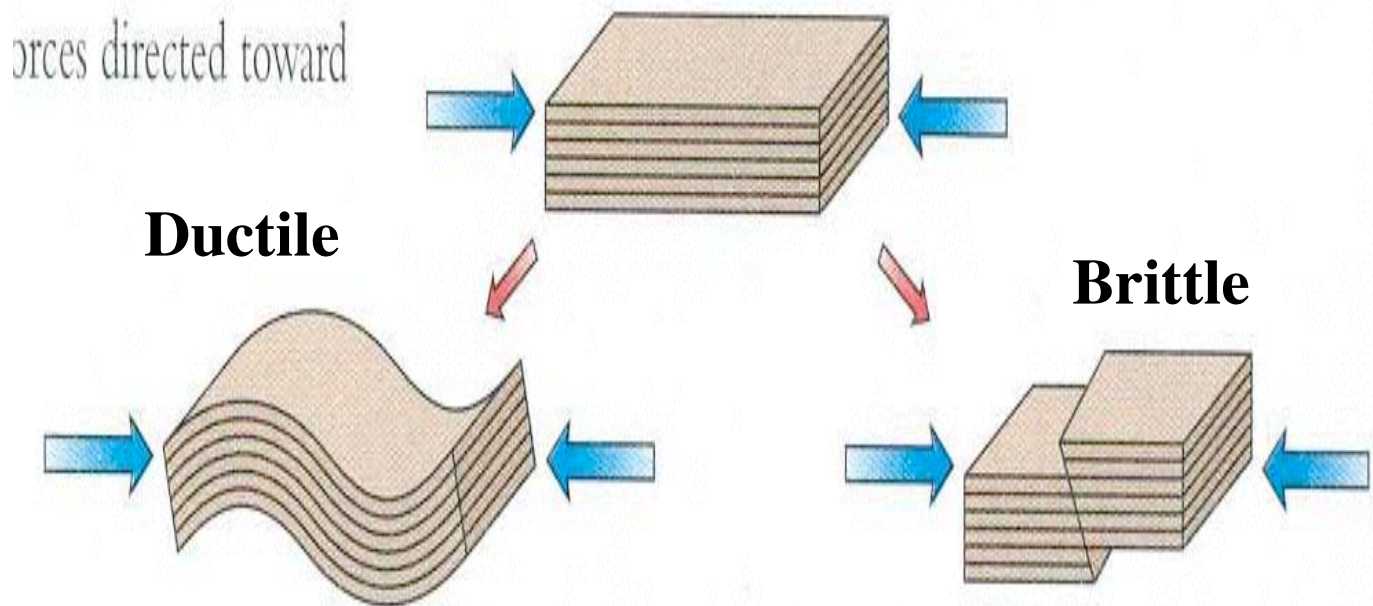


Earthquakes

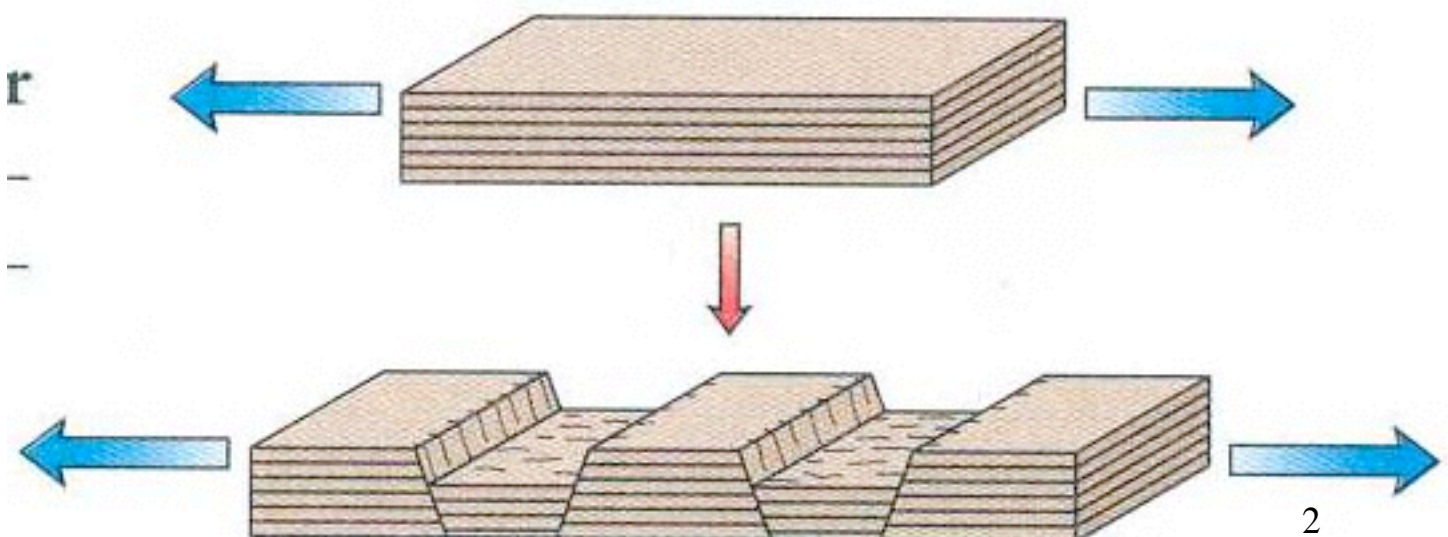


Nope, earthquakes are not caused by the work of Thor and other gods/demons.

Compressional stress induces shortening and/or *reverse faulting*



Extensional stress induces normal faulting



Real Causes: Stress accumulation and sudden release associated with plate motion.

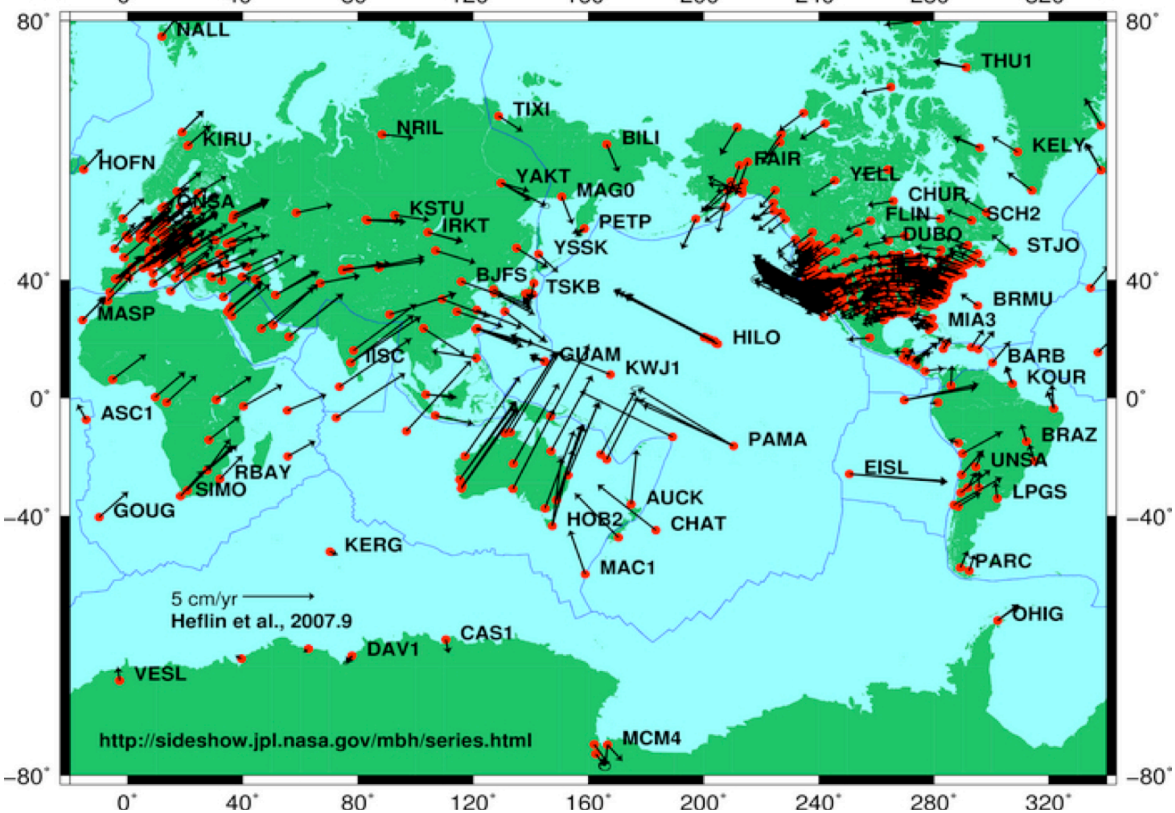
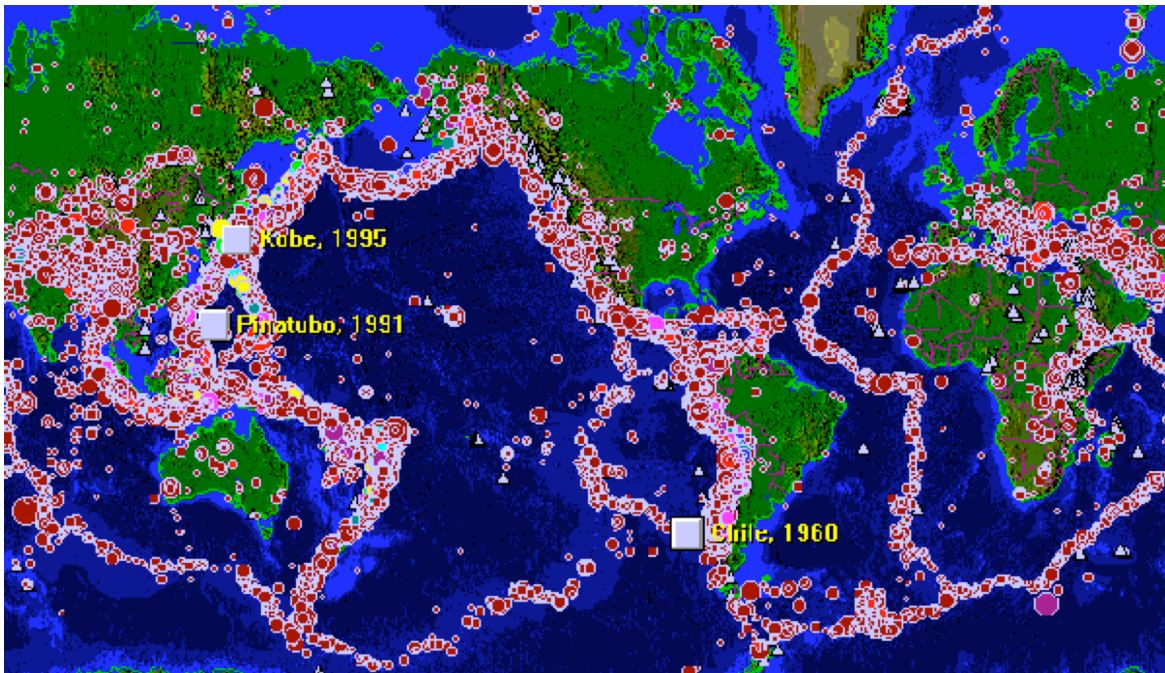
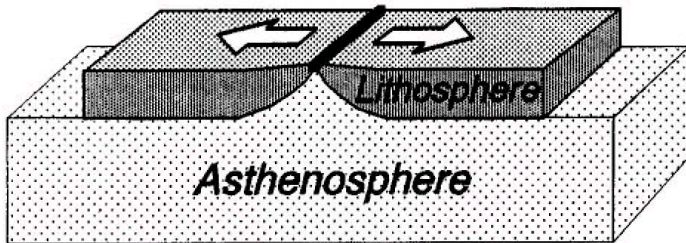
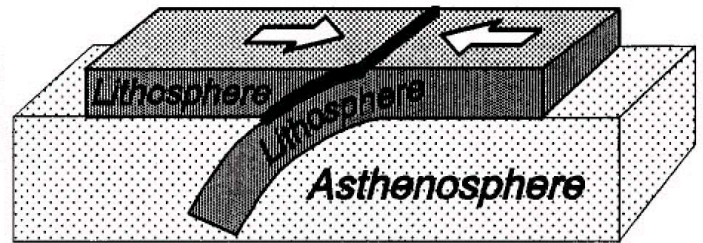


Plate Boundary Types and Associated Earthquakes

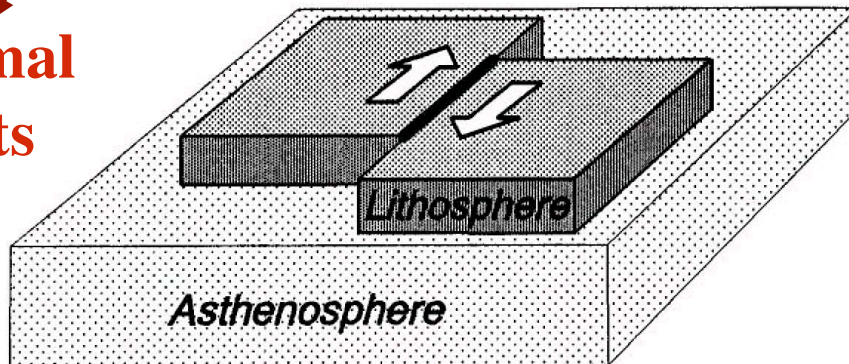
a) Divergent Plate Boundary



b) Convergent Plate Boundary



c) Transform Plate Boundary



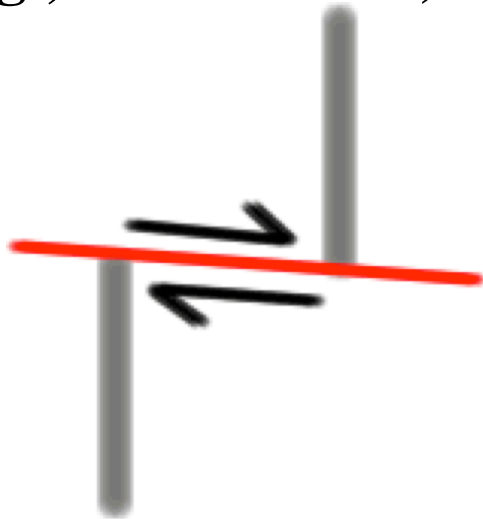
**Normal
faults**

**Reverse
faults**

Strike-slip fault

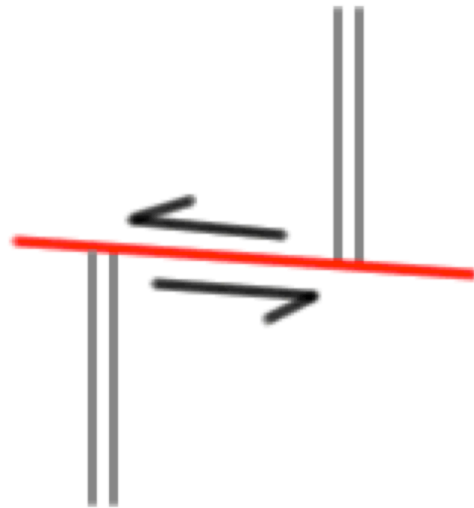
Thrust fault = shallow dipping reverse fault

Regular Strike-Slip Fault (e.g., San Andreas)

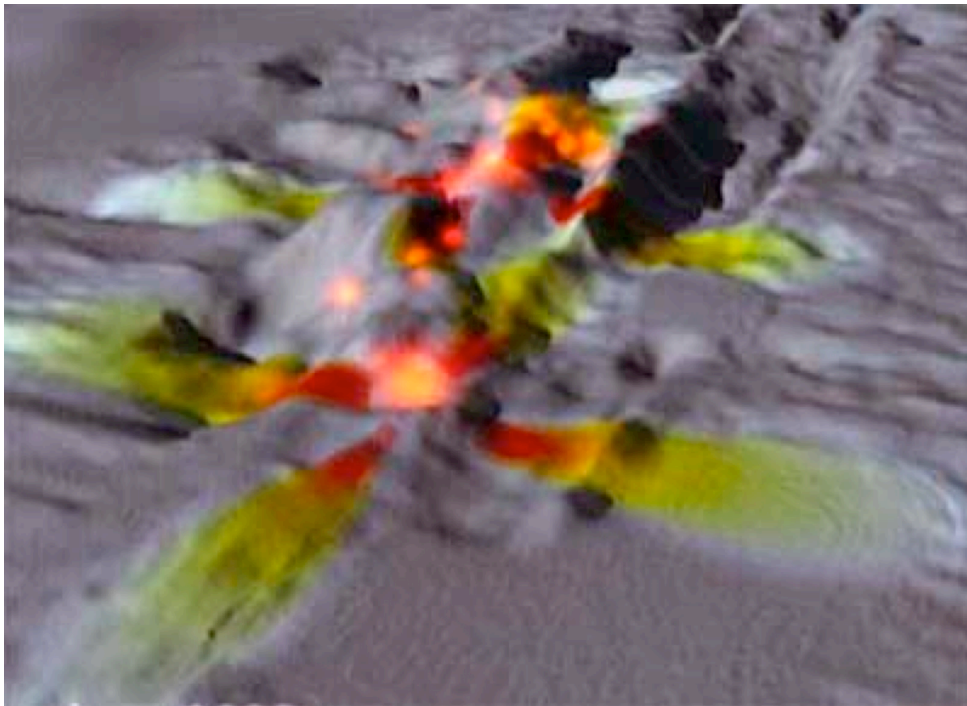


Strike-Slip Fault

Ocean Transform



Transform Fault

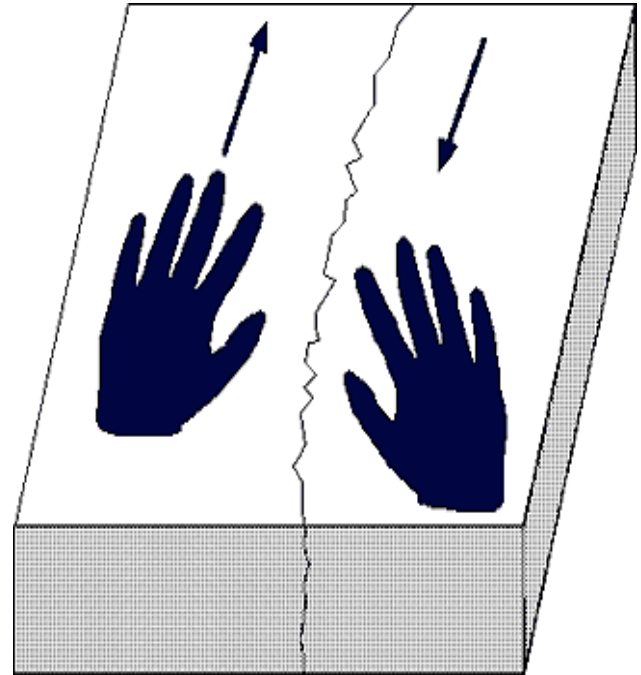


Geophysicists from the Woods Hole Oceanographic Institute (WHOI) have found new evidence that the fragmented structure of seafloor faults, along with previously unrecognized volcanic activity, may be dampening the effects of these quakes.

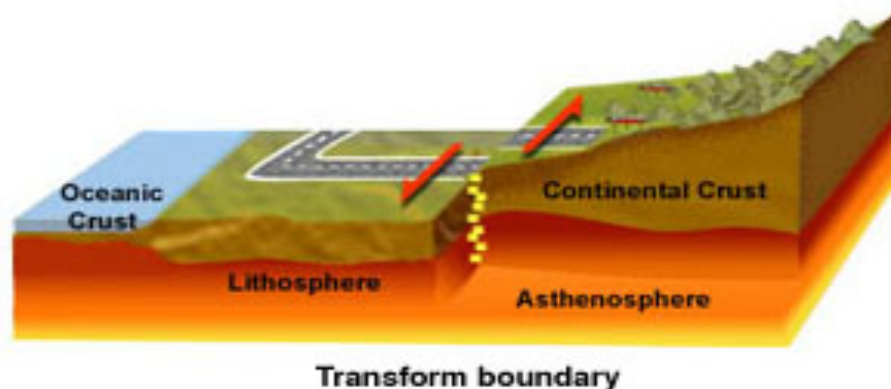
The Real Reasons behind transform faulting: sliding plates or blocks, vertical dip angle

Simple illustration

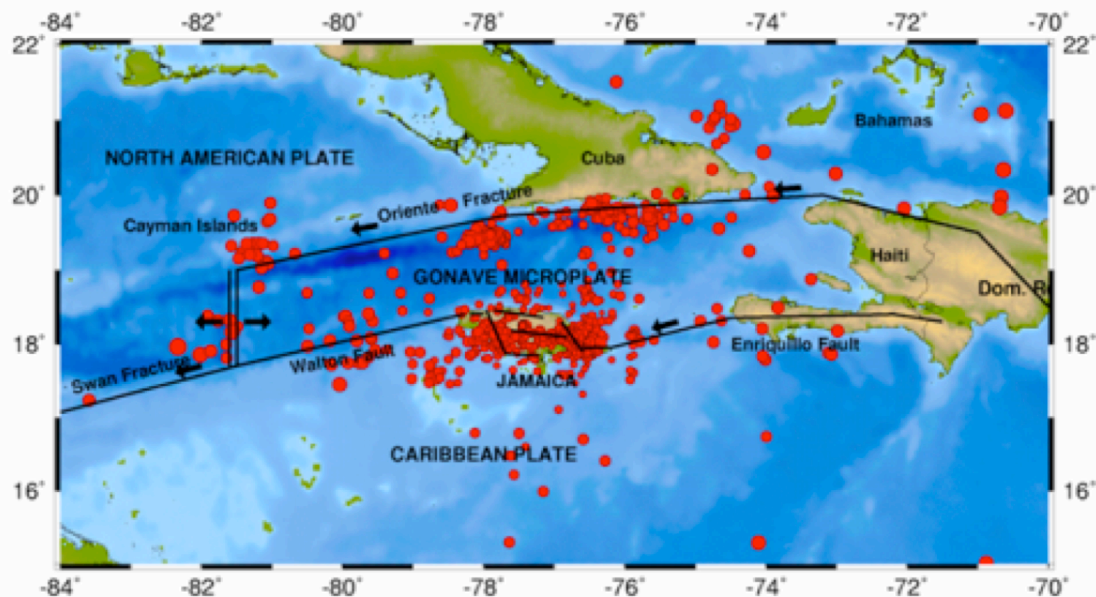
Part of San Andreas Fault



Note: lots of faults are actually “buried” (no fault line on the Earth’s surface).

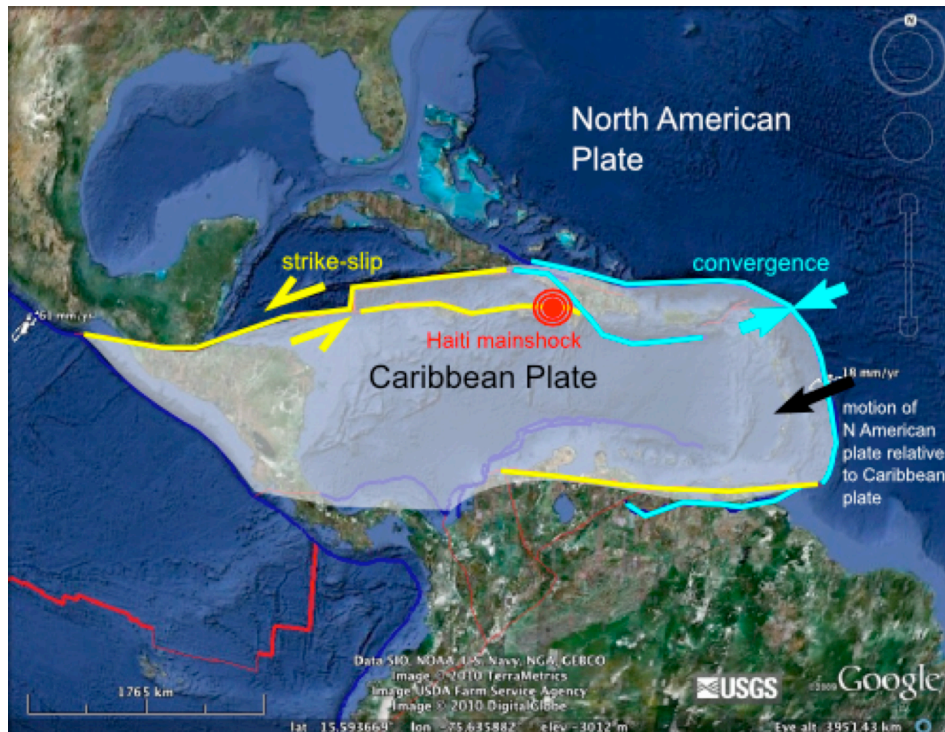


Sample Transform (strike-slip) Earthquake (Haiti Earthquake, 2010, mag = 7.0)



The Caribbean is contained on its own separate little plate; a rather diminutive part of the tectonic jigsaw that is the Earth's crust. It is surrounded on three sides by the much larger North and South American plates, both of which are moving approximately westwards with respect to the Caribbean plate at around 2-3 centimetres a year.

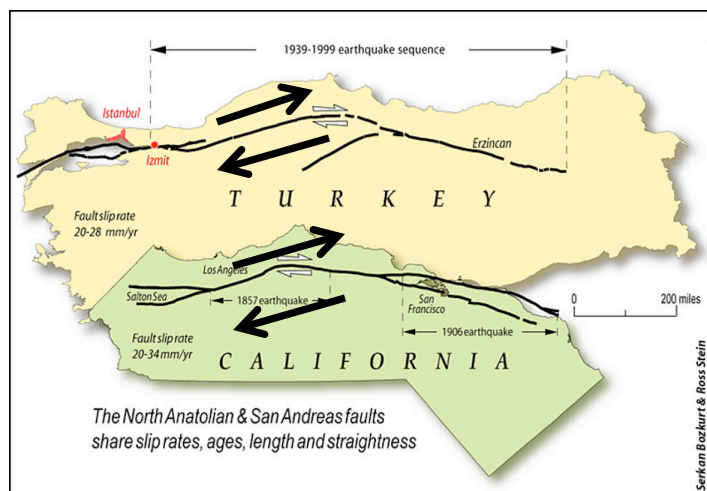
(Haiti Earthquake, 2010, mag = 7.0)



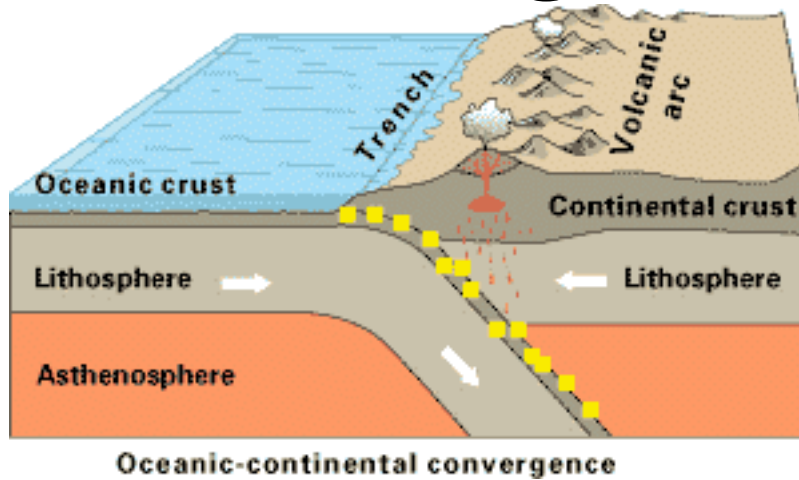
On the Enriquillo fault, strike-slip motion (with eastward relative movement of the North American plate) causes stress accumulation. Some of that stress build-up is released through continuous plate movement, but remaining stress can accumulate over time and cause large events.

Similar case:

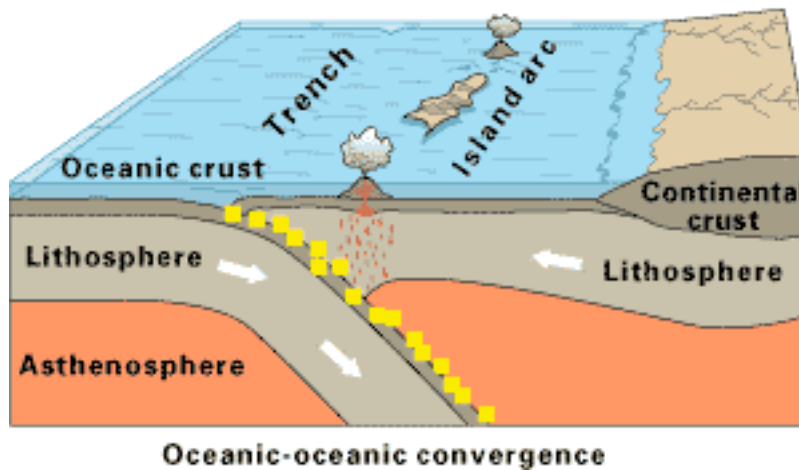
Anatolian Fault (Turkey) which resembles the San Andrea's



Convergence

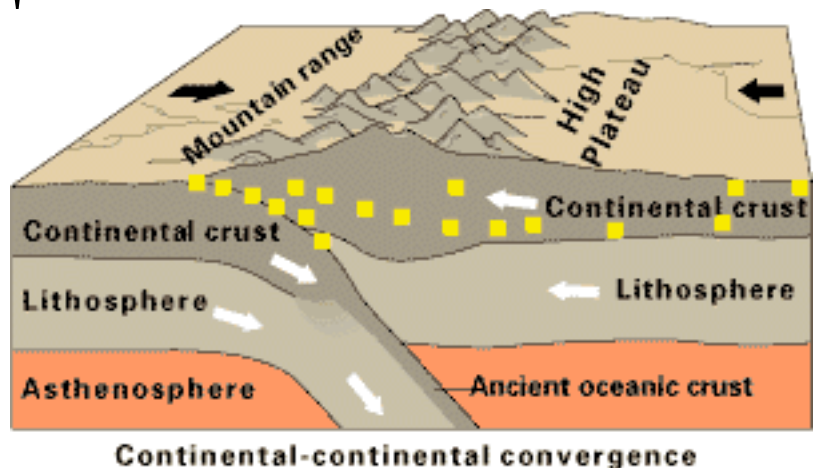


Ocean-continent collision: stronger oceanic plate cuts into continental plate, arc melting happens (e.g., Japan)!



Ocean-ocean collision: the older, colder oceanic plate of the two gets down under, melting occurs (e.g., the Philippines) '

Continent-continent collision: pileup of continental material, less melting (e.g., the Himalayas) !



Complexity: Double Seismic Zone

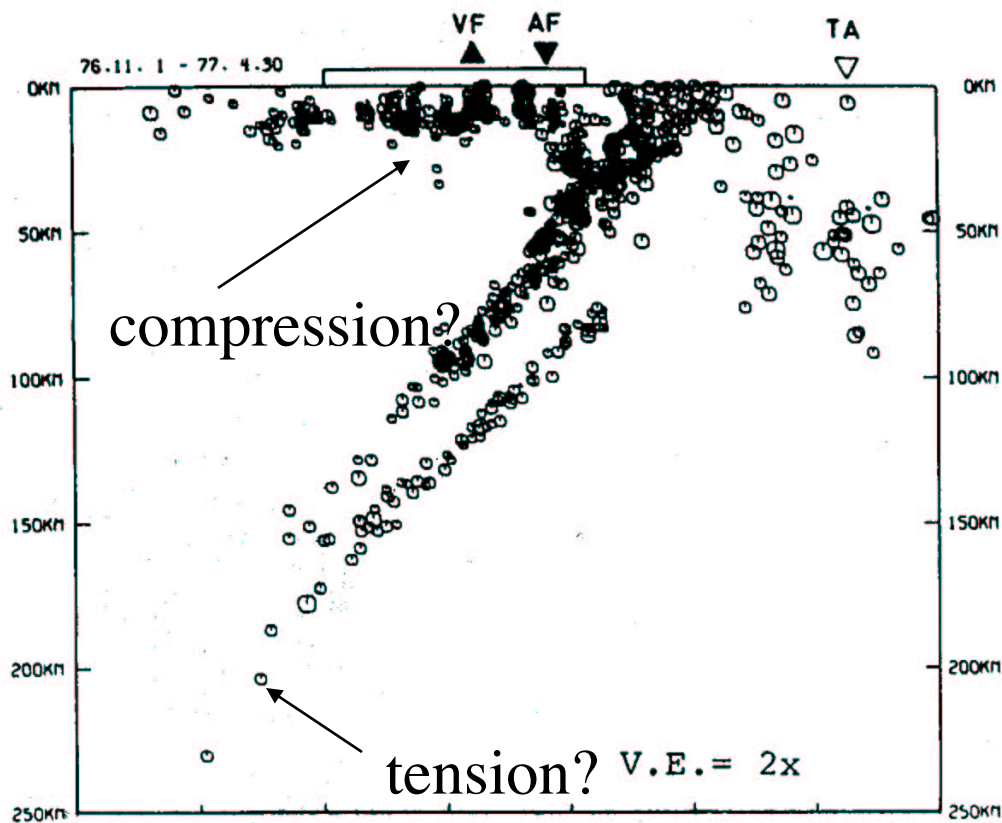
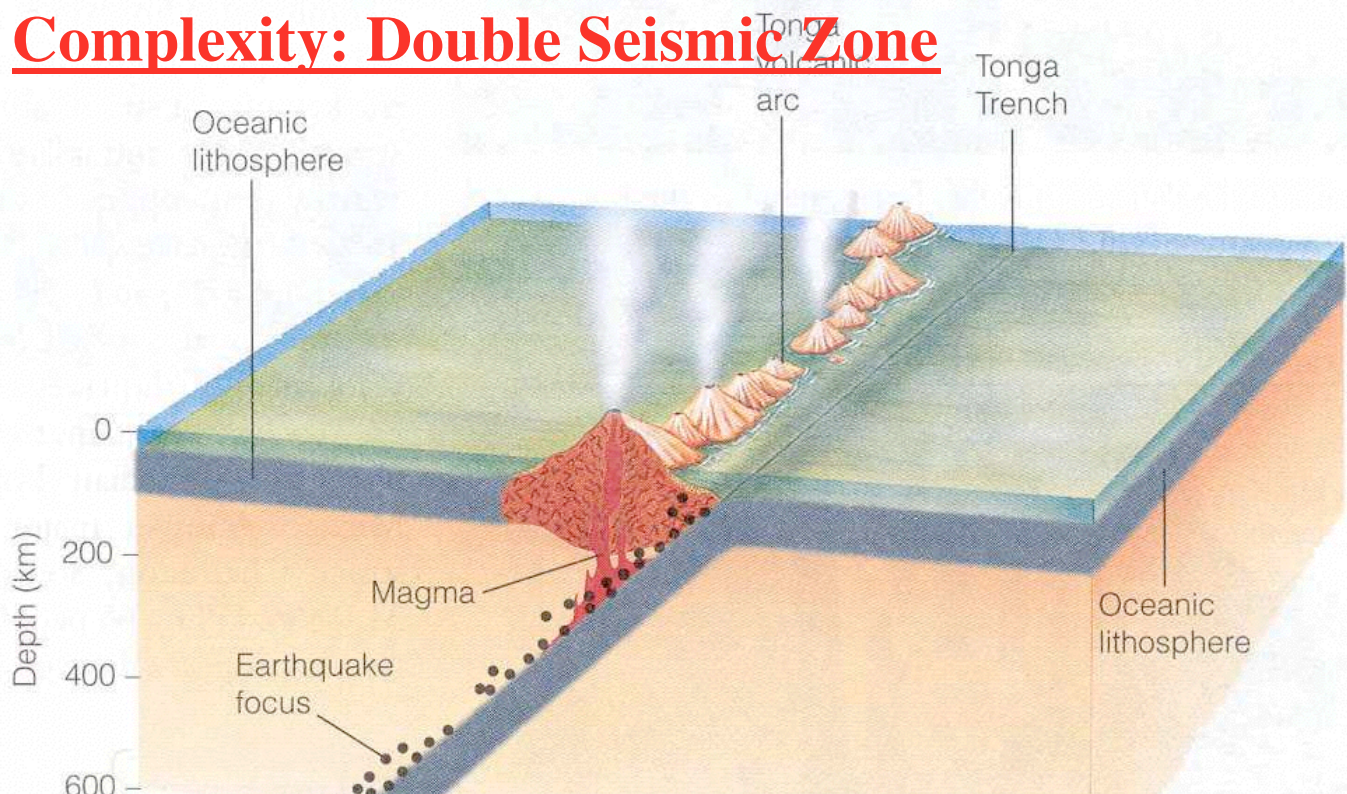
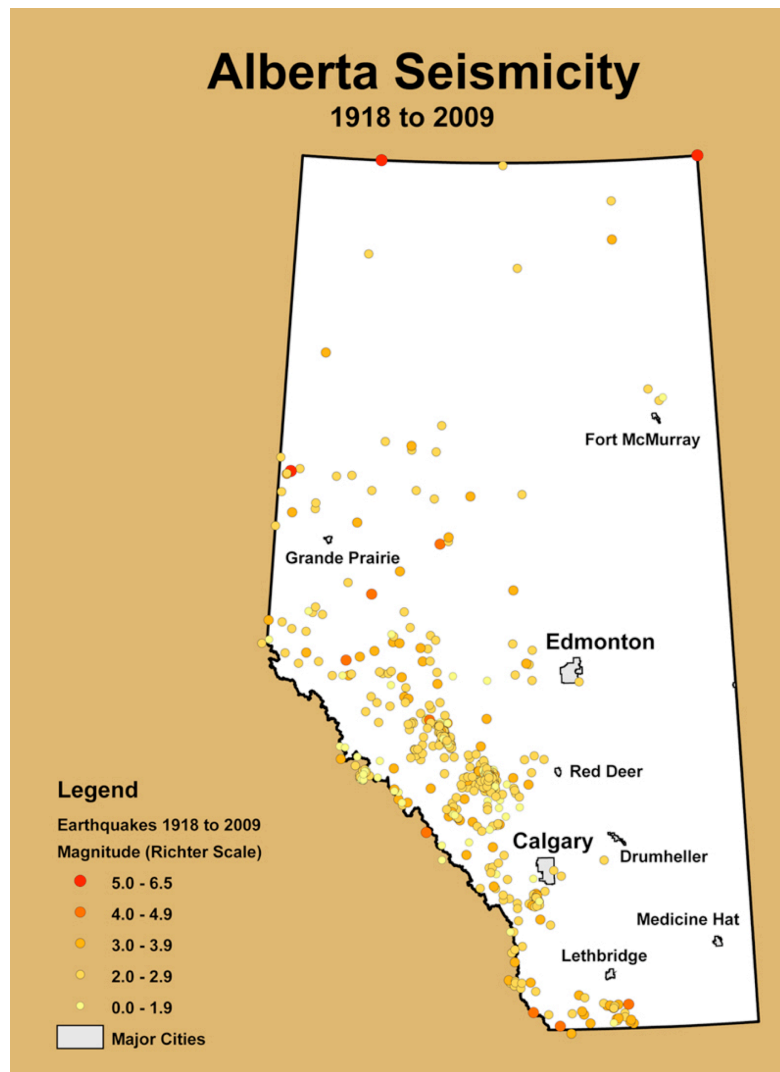


Figure 4.16. The Wadati-Benioff zone under northern Honshu, Japan, showing two parallel planes of earthquake foci. VF indicates the volcanic front, at the centre of the land area. Reproduced by permission from Hasegawa (1989). For a colour plot with greater detail see Hasegawa et al. (1991).

Second type: Intraplate Earthquakes (earthquakes that do not occur near plate boundaries)

These earthquakes are fairly rare, and their causes are not very well understood. They could occasionally reach damaging magnitudes.

Example:



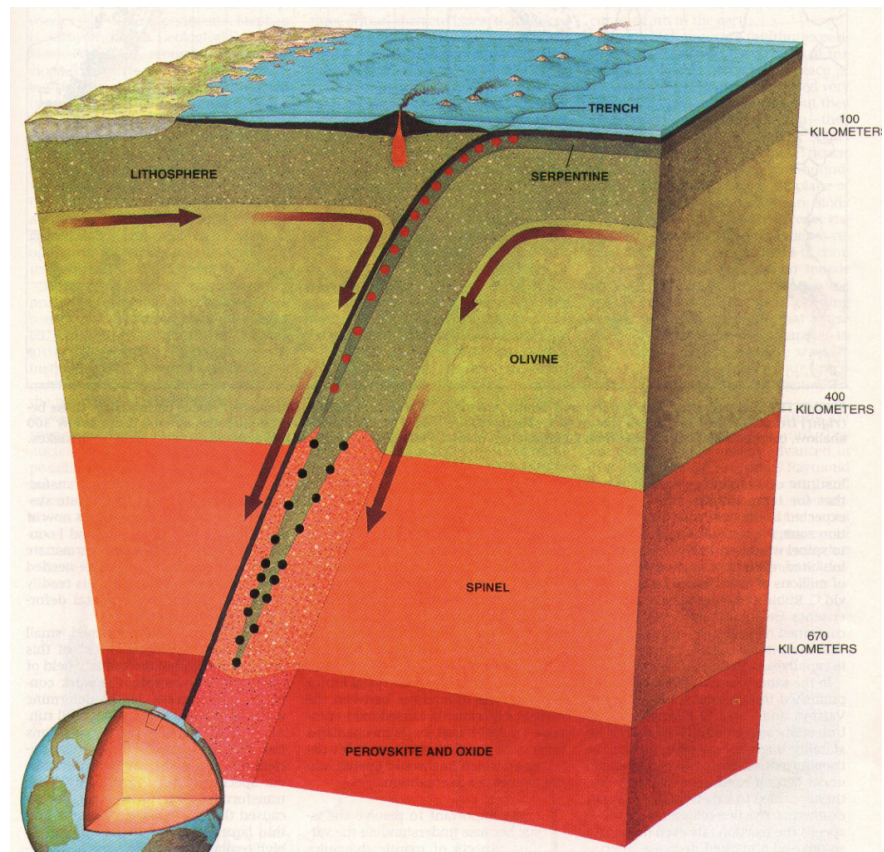
Induced earthquakes due to oil/gas industry? Lubrication of faults?

Ultimately, there is a stress build-up and a catastrophic stress release!

Deep Earthquakes

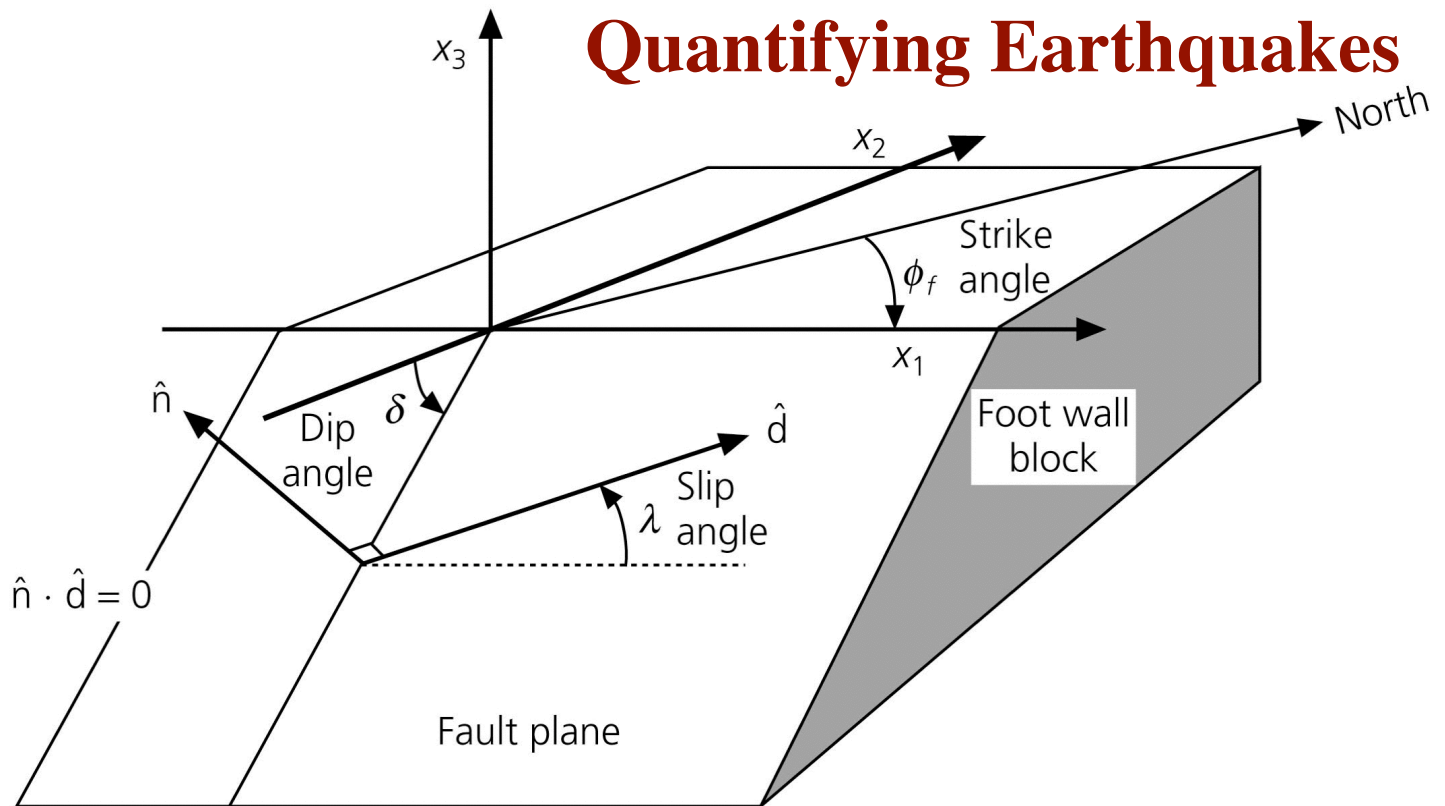
Deepest known earthquakes: Near Tonga region, down to ~680 km, at the base of upper mantle.

Why not below 680: change of mineralogy and overwhelming pressure prevents the existence of faults



Phase change hypothesis: Sudden phase change of olivine can cause a major drop of volume. Therefore “collapse” of space could induce earthquakes.

Figure 4.2-2: Fault geometry used in earthquake studies.



Fault geometry in earthquake studies

Foot wall: What is shown here

Hanging wall: the block that is sliding against this block.

Three angles that uniquely describes a fault:

Strike (ϕ): angle of fault on earth's surface with respect to North (0-360)

dip (δ): fault dip angle measured with -x2 direction (condition = angle must be less than 90).

Rake or Slip (λ): the angle that hanging wall moves against this block, measured with x1 direction (0--360).

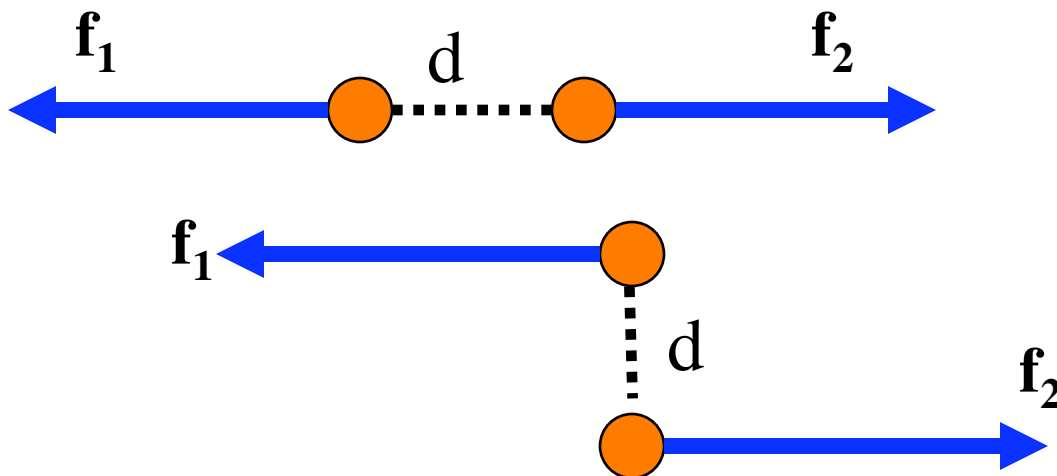
A Better Representation of Source: *equivalent body forces*

Idea of a *point source*: size of source $\ll$ wavelength

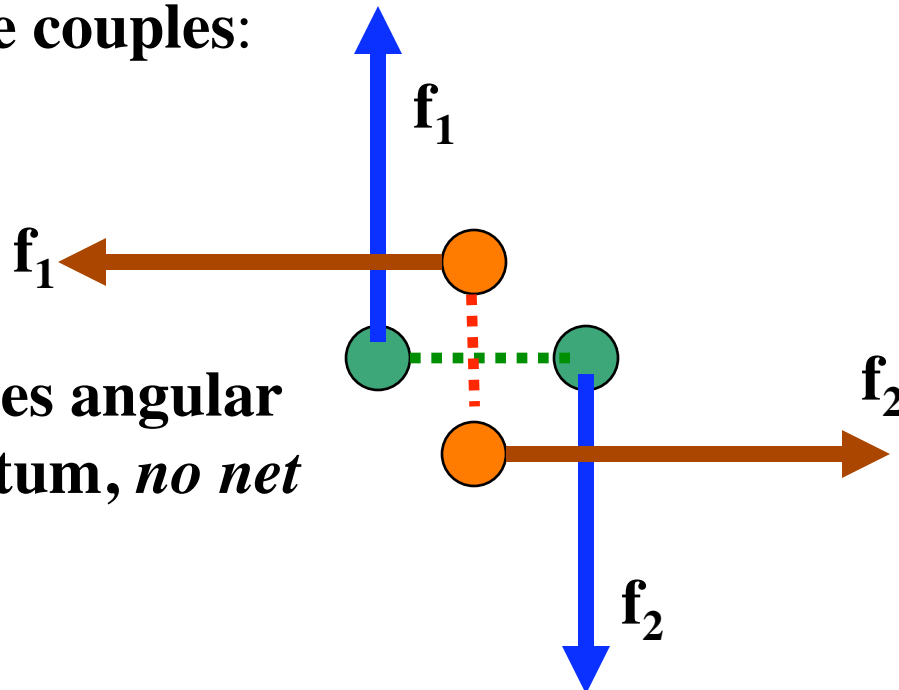
External force ---> generate a single force.

Internal forces: must act in a opposite direction to conserve momentum.

Force couples:



Double couples:



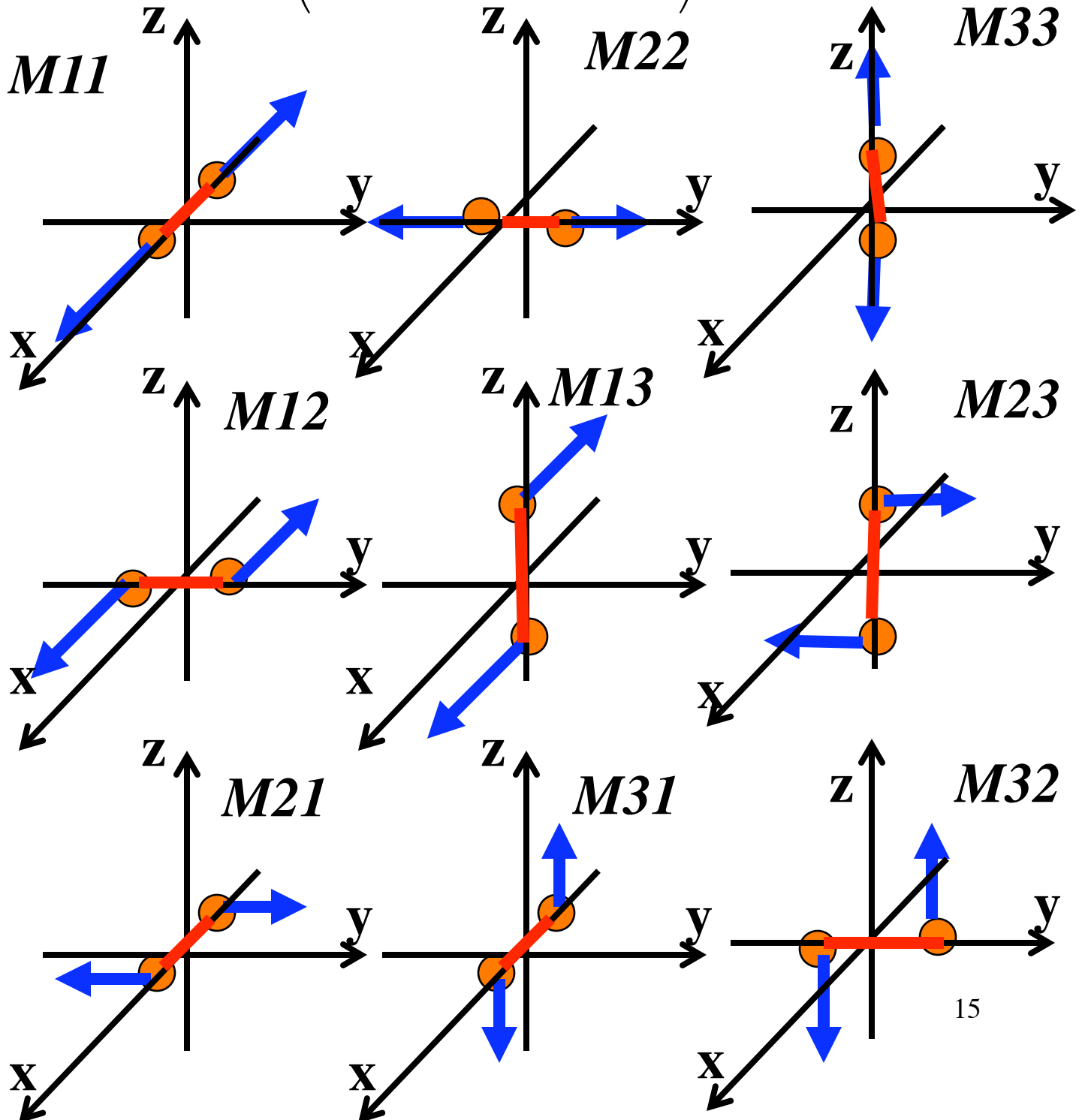
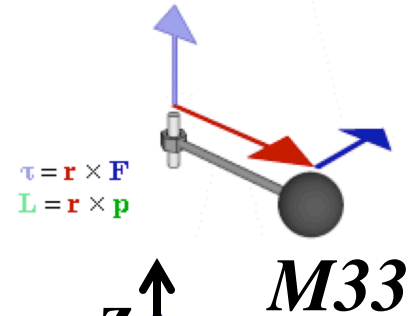
Conserves angular momentum, *no net torque*

Torque (or **Moment**) of the system: $M_{ij} = f_i * d_j$

We can define moment tensor **M**

where,

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$$



Symmetry of the *Moment Tensor*: symmetric matrix due to the fact there is no net torque, so

$$M_{ij} = M_{ji}$$

6 independent element!

Now assume the earthquake is a pure double couple, that means there is no net volume change, which means trace must be 0.

$$M_{11} + M_{22} + M_{33} = 0$$

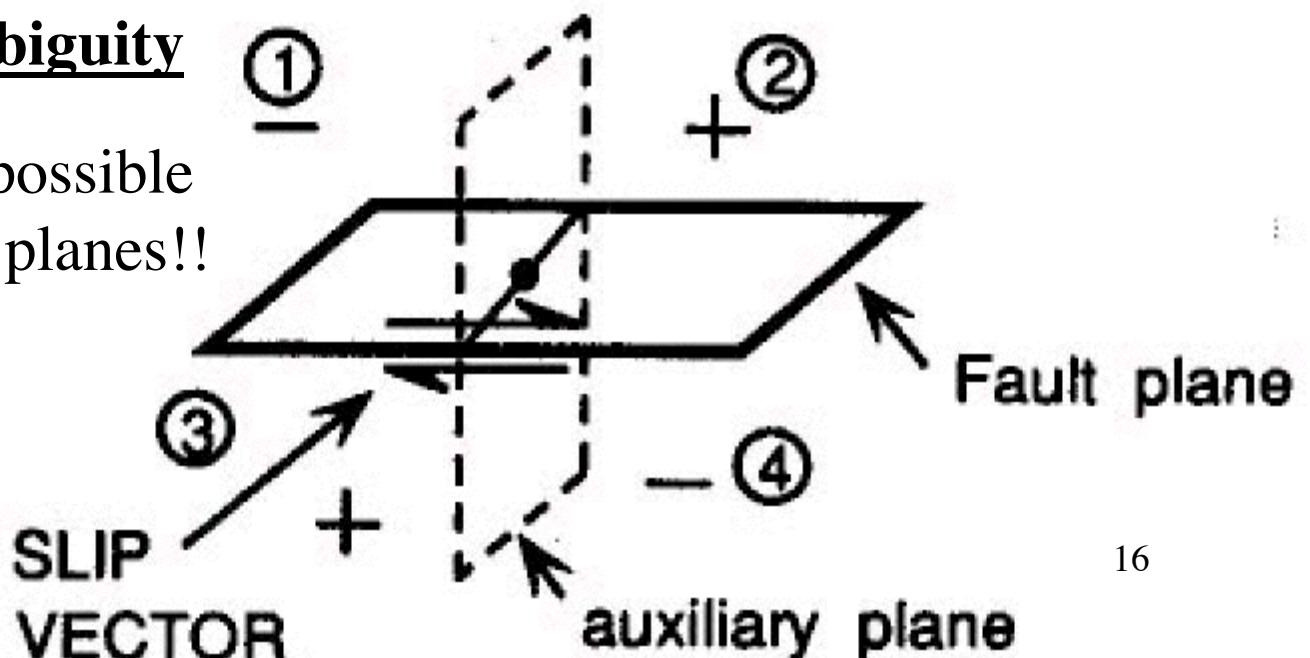
where this relation fails: Explosive source

Scalar moment: Total moment = $M_0 * \underline{\underline{\mathbf{M}}}$

M_0 is ***scalar moment***, which tells about the total moment release of earthquake, in Nm, or dyne cm

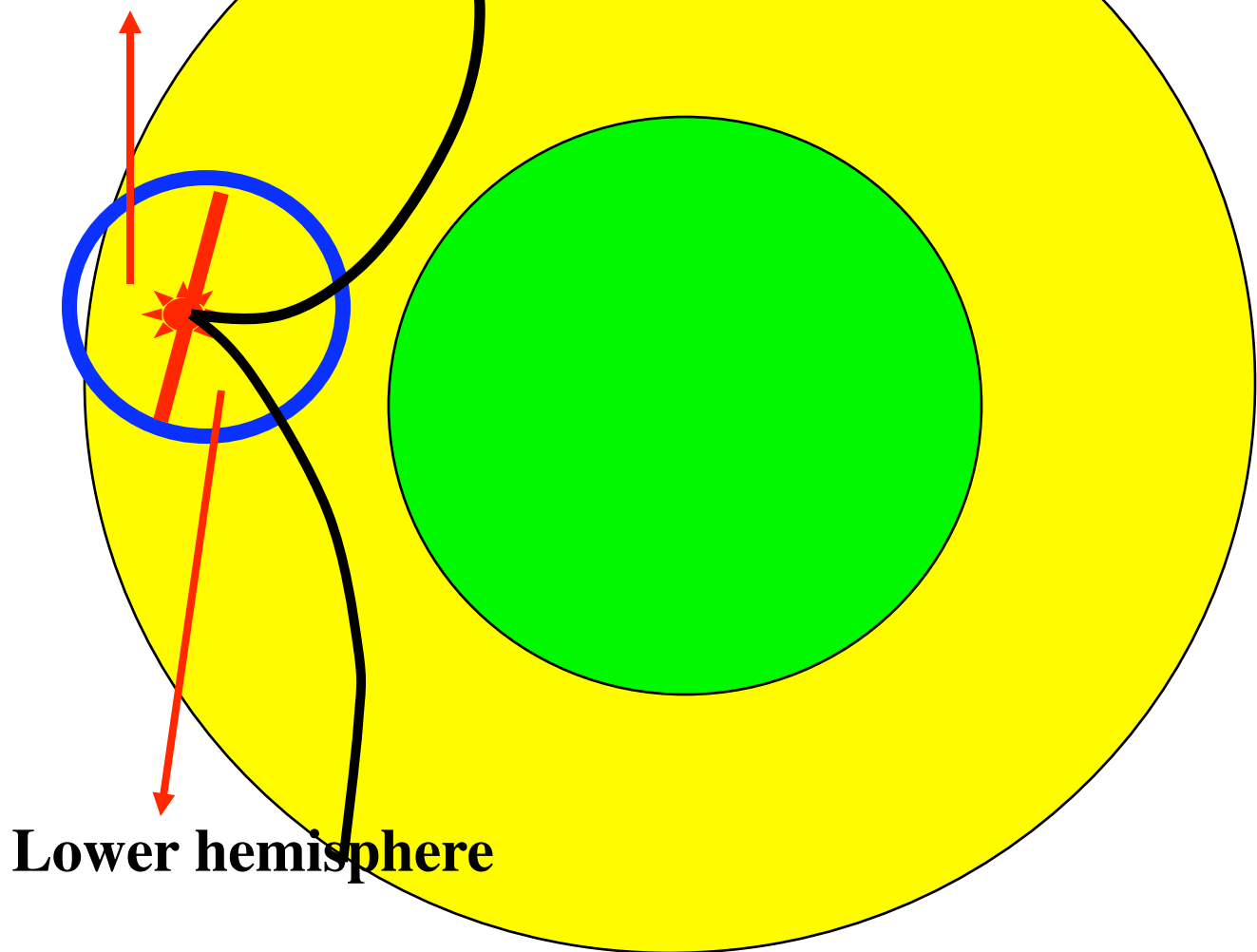
Ambiguity

Two possible fault planes!!



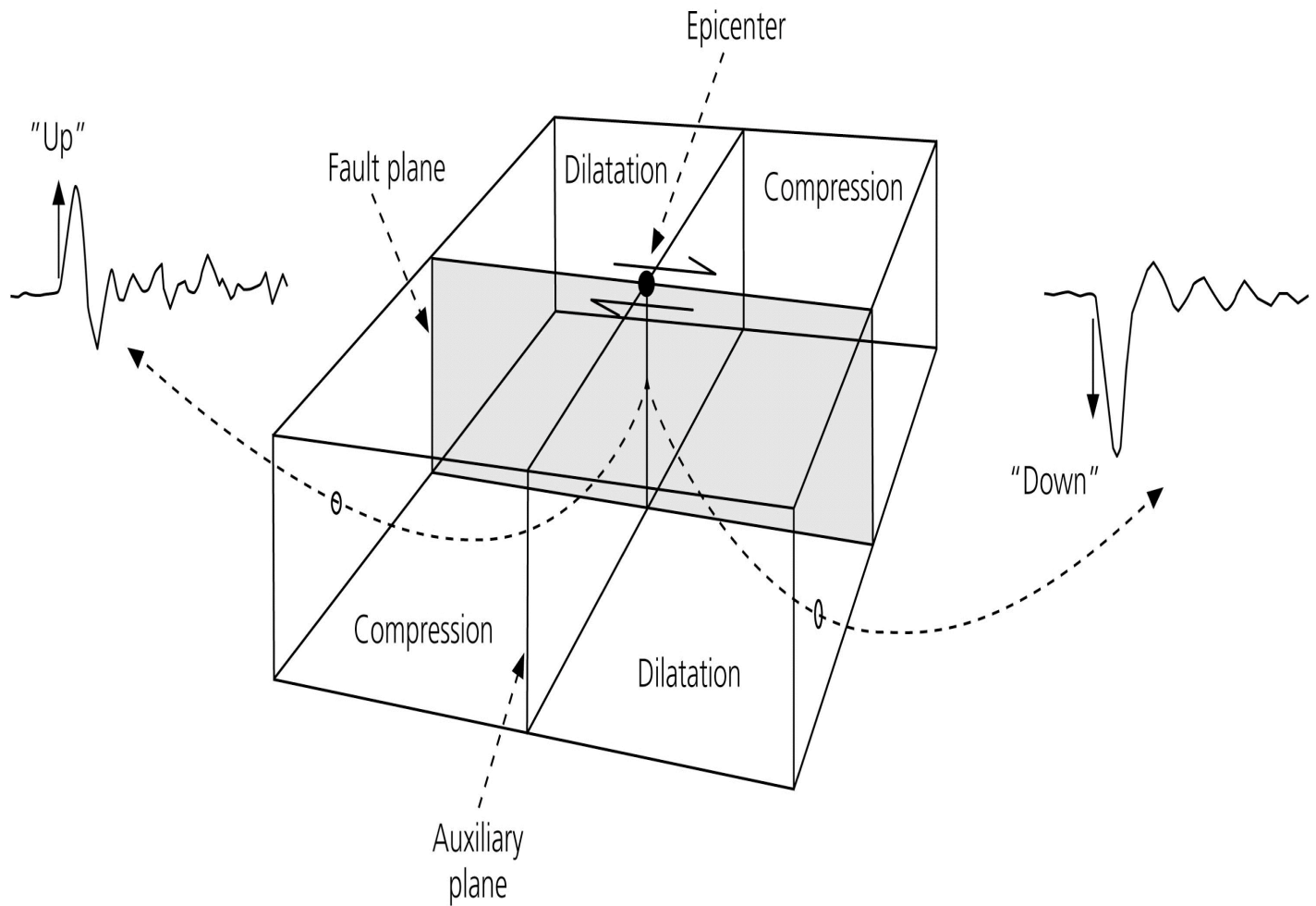
Lower hemisphere convention

Upper hemisphere



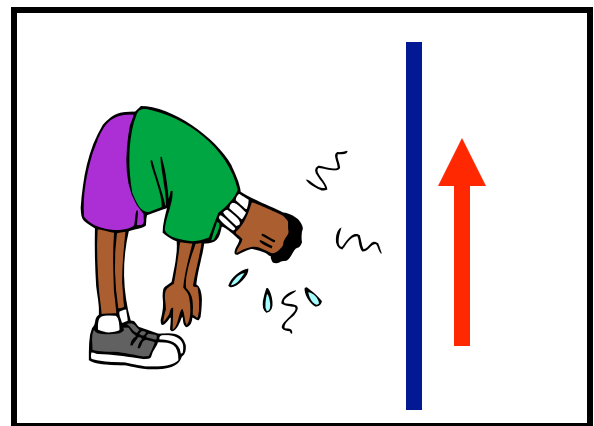
The “beachballs” are lower focal hemisphere projections of the radiation patterns. **Imagine** the dilatational quadrants are transparent, shine flashlight from bottom hemisphere vertically up, the PROJECTION (circle) you¹⁷ see is the moment tensor solution!!

Figure 4.2-4: First motions in relation to fault orientation.

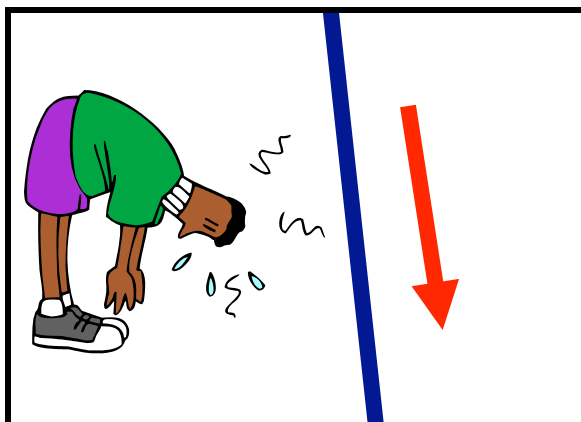


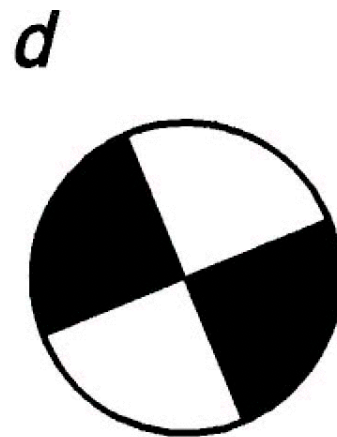
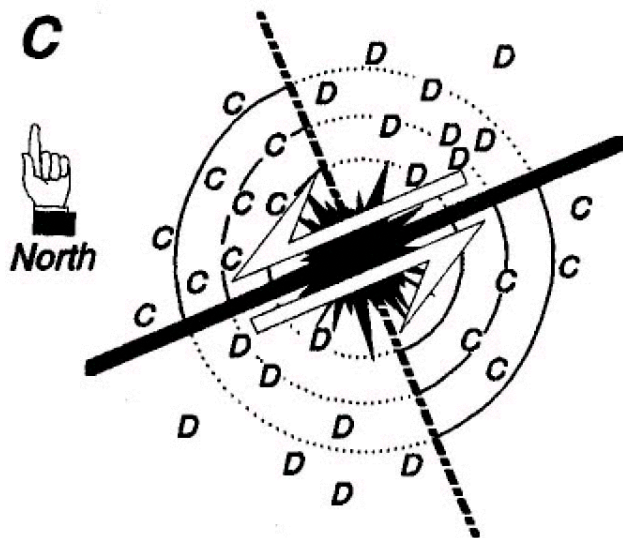
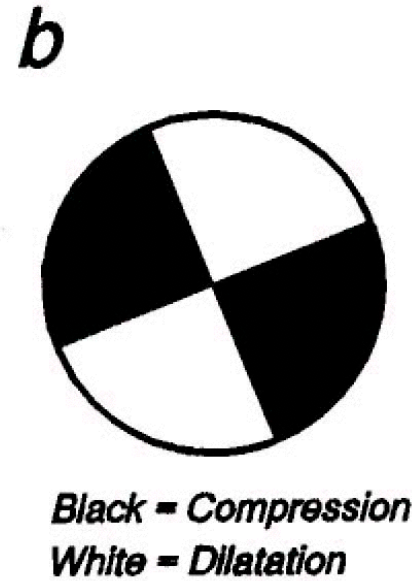
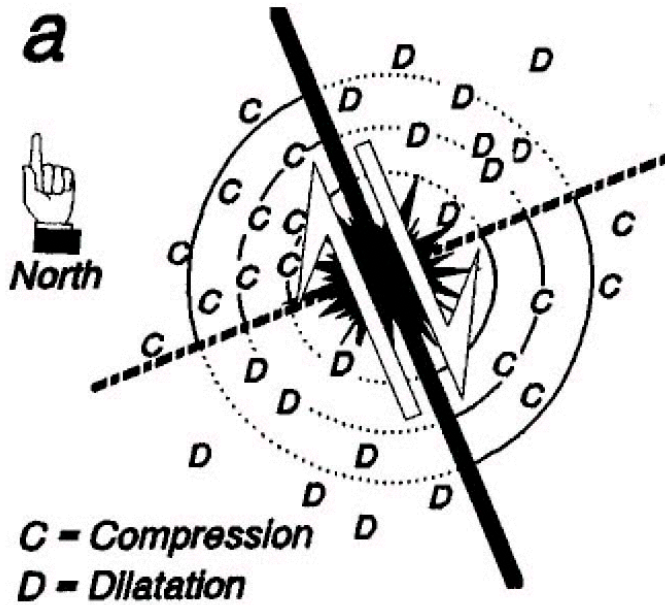
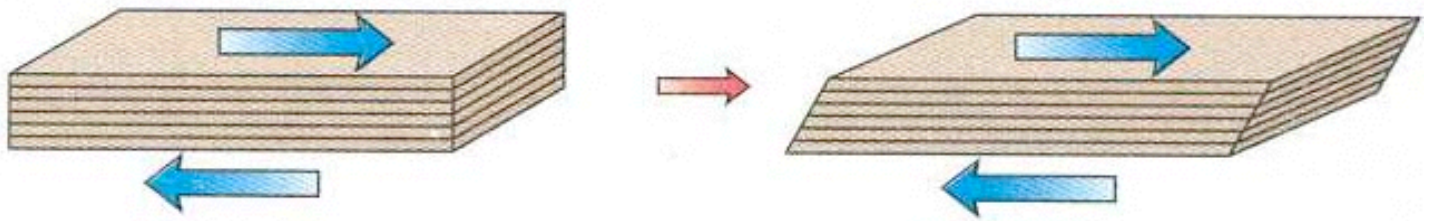
First motions relative to fault orientation

Left lateral faulting:



Right lateral faulting:

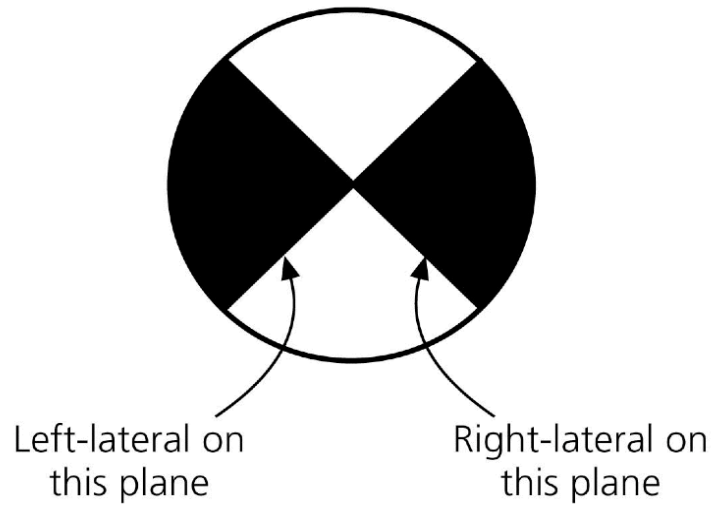




Most Confusing Bit: the Maximum Compressive Axis is at the center of the Dilatation Quadrant. *Why? Imagine a vacuum being squeezed by the atmosphere.*

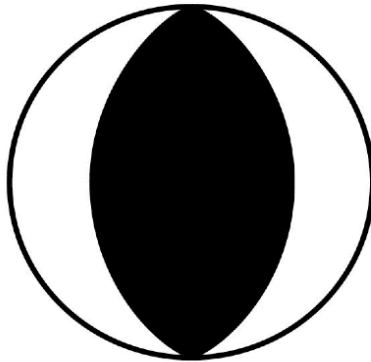
Figure 4.2-14: Focal mechanisms for various fault mechanisms.

Strike-slip fault

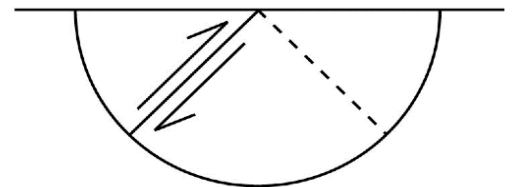


Dip-slip faults

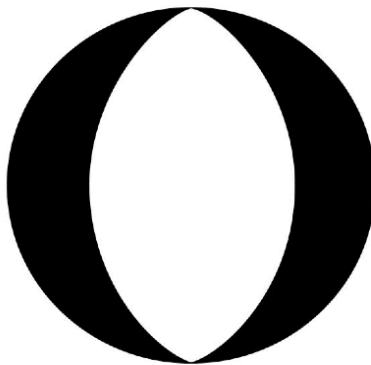
Thrust fault



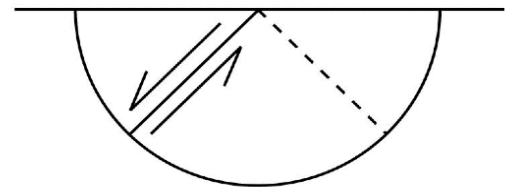
Focal sphere side view



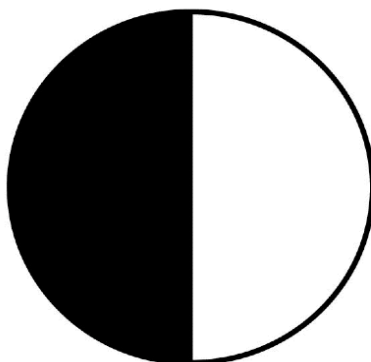
Normal fault



Focal sphere side view



Vertical dip-slip



Focal sphere side view

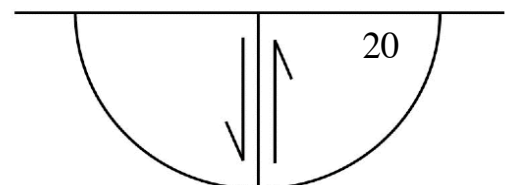
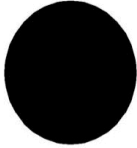
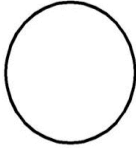
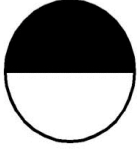
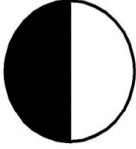
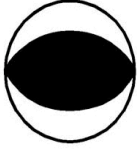
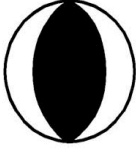
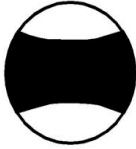
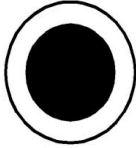
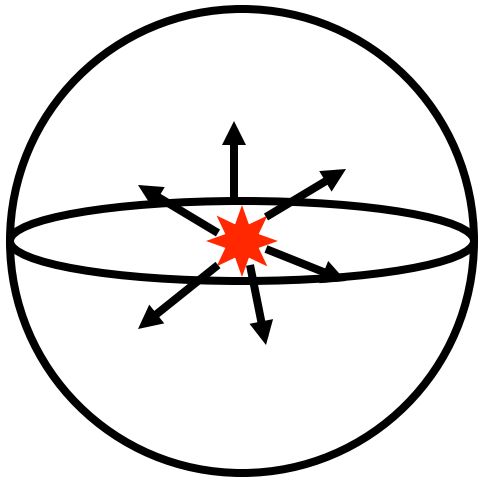


Figure 4.4-6: Selected moment tensors and their associated focal mechanisms.

Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$-\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$-\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$		$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	
$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$		$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$	
$\frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$\frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$\frac{1}{\sqrt{6}} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$\frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$		$-\frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	

The last two rows are usual mechanisms that are related to CLVD components, that is, the sum of two smaller elements are exactly the same as the third. This corresponds to tensile crack, etc.



Idea of a “cavity” generated by a small explosive source

General Momentum Eqn for elastic medium:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \tau_{ij} + f_i$$

Body force term

Consider: a unit force applied at point $\mathbf{x}_0$ at time t_0 , $u(\mathbf{x}, t)$ is measured at the receiver due to the “hand of god”.

$$u_i(\mathbf{x}, t) = G_{ij}(\mathbf{x}, t; \mathbf{x}_0, t_0) f_j(\mathbf{x}_0, t_0)$$

$(\mathbf{x}, t)$ at any point

Starting point

$G_{ij}(\mathbf{x}, t; \mathbf{x}_0, t_0)$ Green function that calculates earth’s response to impulse source

Power of the Eqn: can sum up the point forces linearly and obtain total displacement.

Near vs. Far field terms

$$\Phi(r,t) = \frac{1}{r} f(t \pm r/v) \text{ is a solution to}$$

$$\nabla^2 \Phi(x,t) - \frac{1}{v^2} \frac{\partial^2 \Phi}{\partial t^2} = -4\pi \delta(\mathbf{r}) f(t)$$

Where Φ is the amplitude of the P potential.

$$\text{displacement } u(r,t) = \frac{\partial \Phi}{\partial r} = \frac{1}{r} \frac{\partial [f(t - r/\alpha)]}{\partial r} - \frac{1}{r^2} f(t - r/\alpha)$$

$$= -\frac{1}{r\alpha} \frac{\partial f(t - r/\alpha)}{\partial r} - \frac{1}{r^2} f(t - r/\alpha)$$

where $\tau = t - r/\alpha$ is the *delay time*. The r term is the far-field displacement, and r^2 term, which decays much faster, is near-field displacement

Ultimately (not shown here), the P and S radiation patterns shown on the previous pages are results of :

For a homogeneous layer

P wave

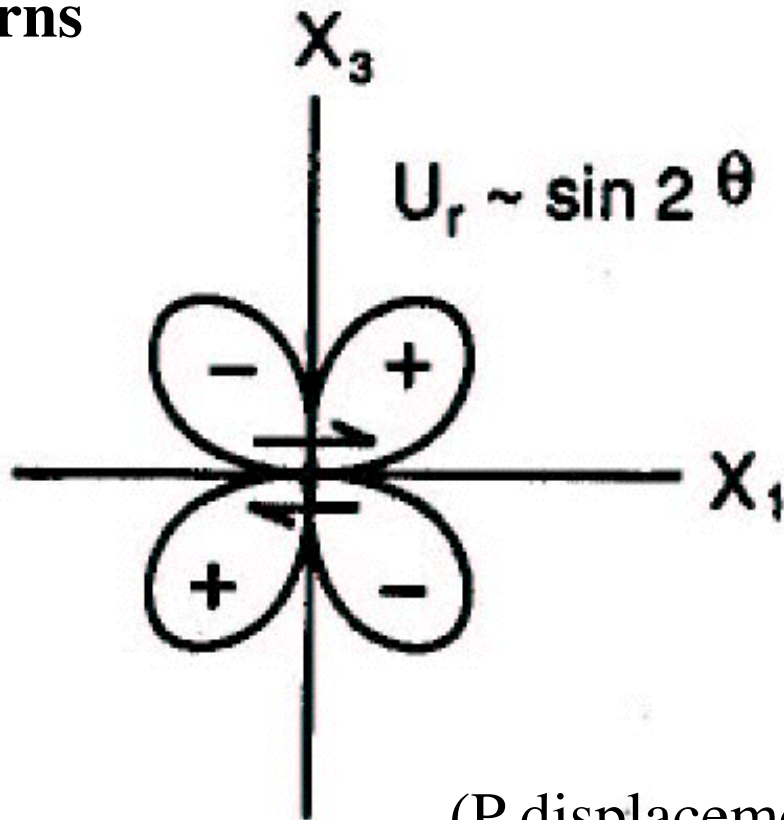
$$u^P(x,t) = \frac{1}{4\pi r \rho \alpha^3} \sin(2\theta) \cos(\phi) \dot{M}(t - r/\alpha) \vec{r}$$

S wave

$$u^S(x,t) = \frac{1}{4\pi r \rho \beta^3} (\cos 2\theta \cos \phi \vec{\theta} - \cos \theta \sin \phi \vec{\phi}) \dot{M}(t - r/\beta) \vec{r}$$

Radiation patterns

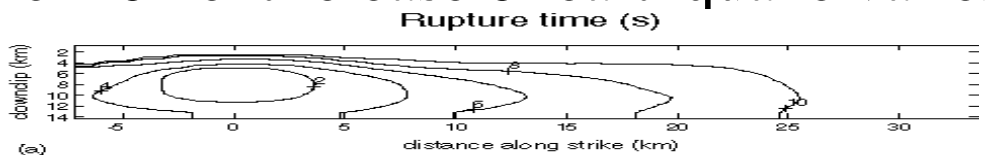
b



$$u_r = \frac{1}{4\pi\rho\alpha^3 r} \dot{M}(t - r/\alpha) \sin 2\theta \cos \phi$$

(P displacement along propagation direction $\mathbf{r}$)

source time function, or moment rate function. It describes how the moment release of earthquake varies with time



Example here shows two subsources

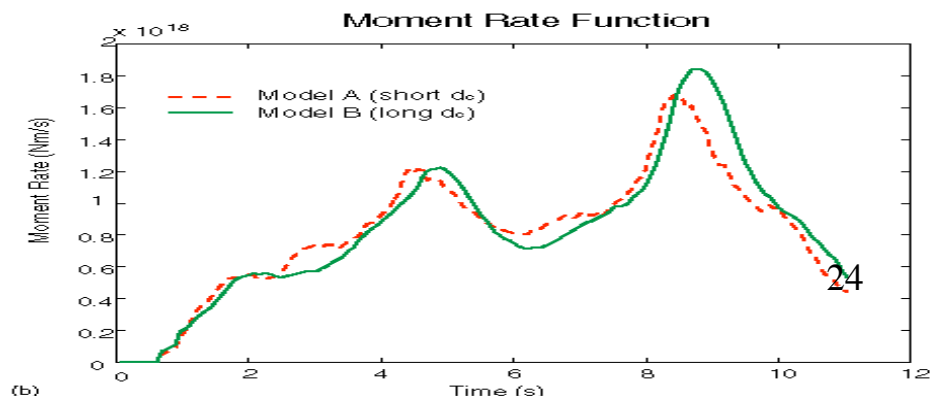
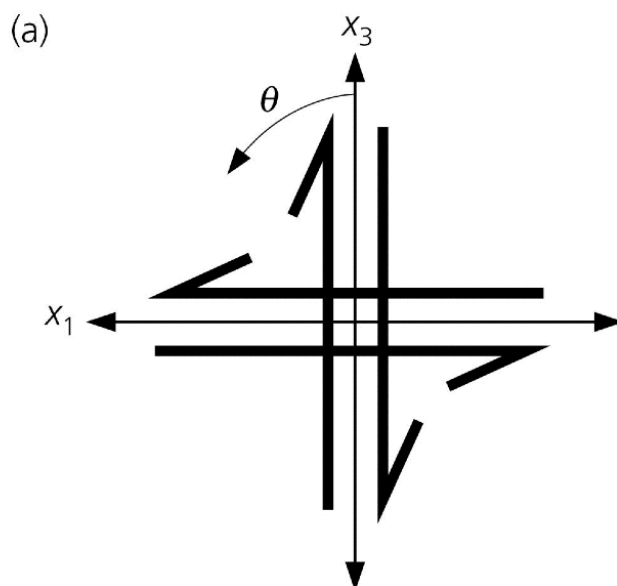
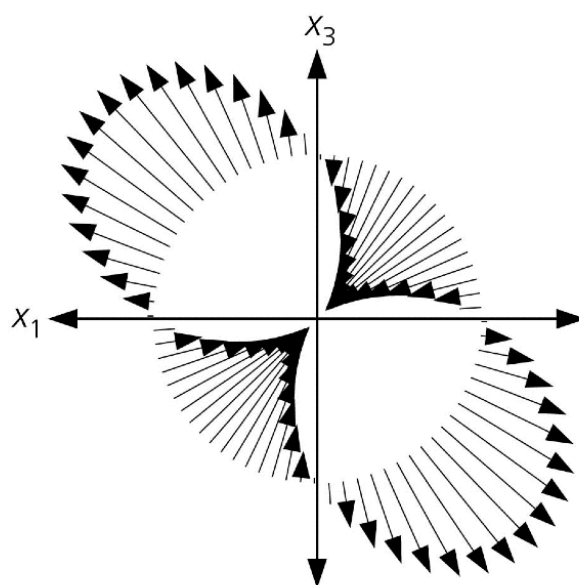
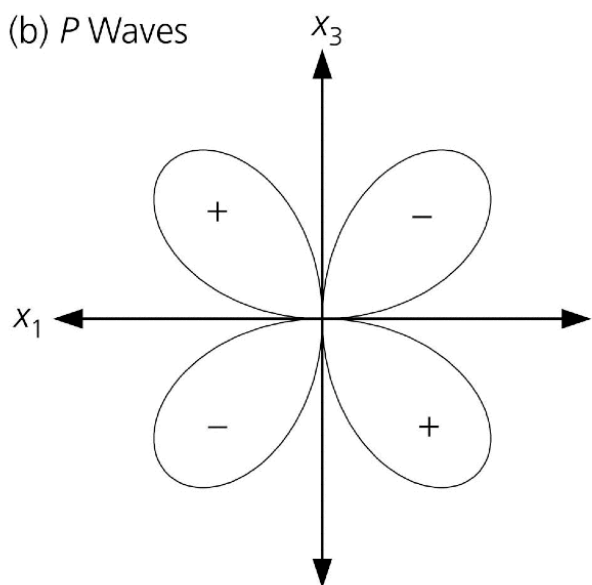


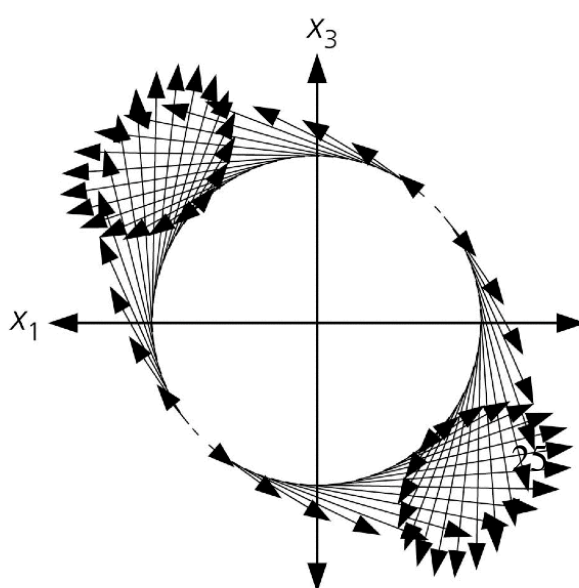
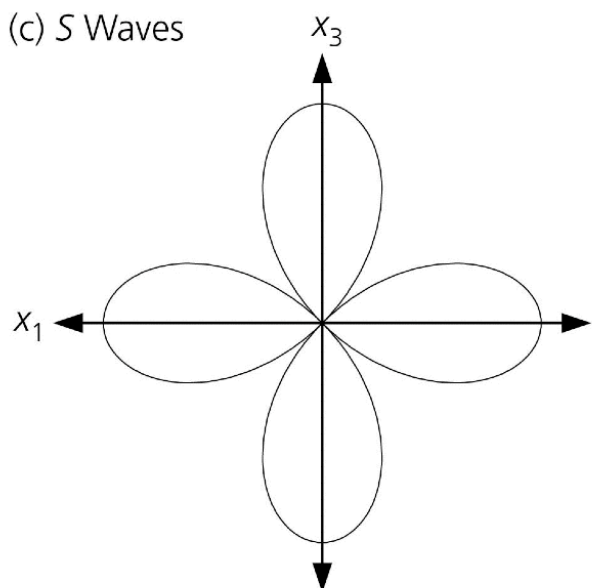
Figure 4.2-7: *P* and *S* radiation amplitude patterns.



(b) *P* Waves



(c) *S* Waves



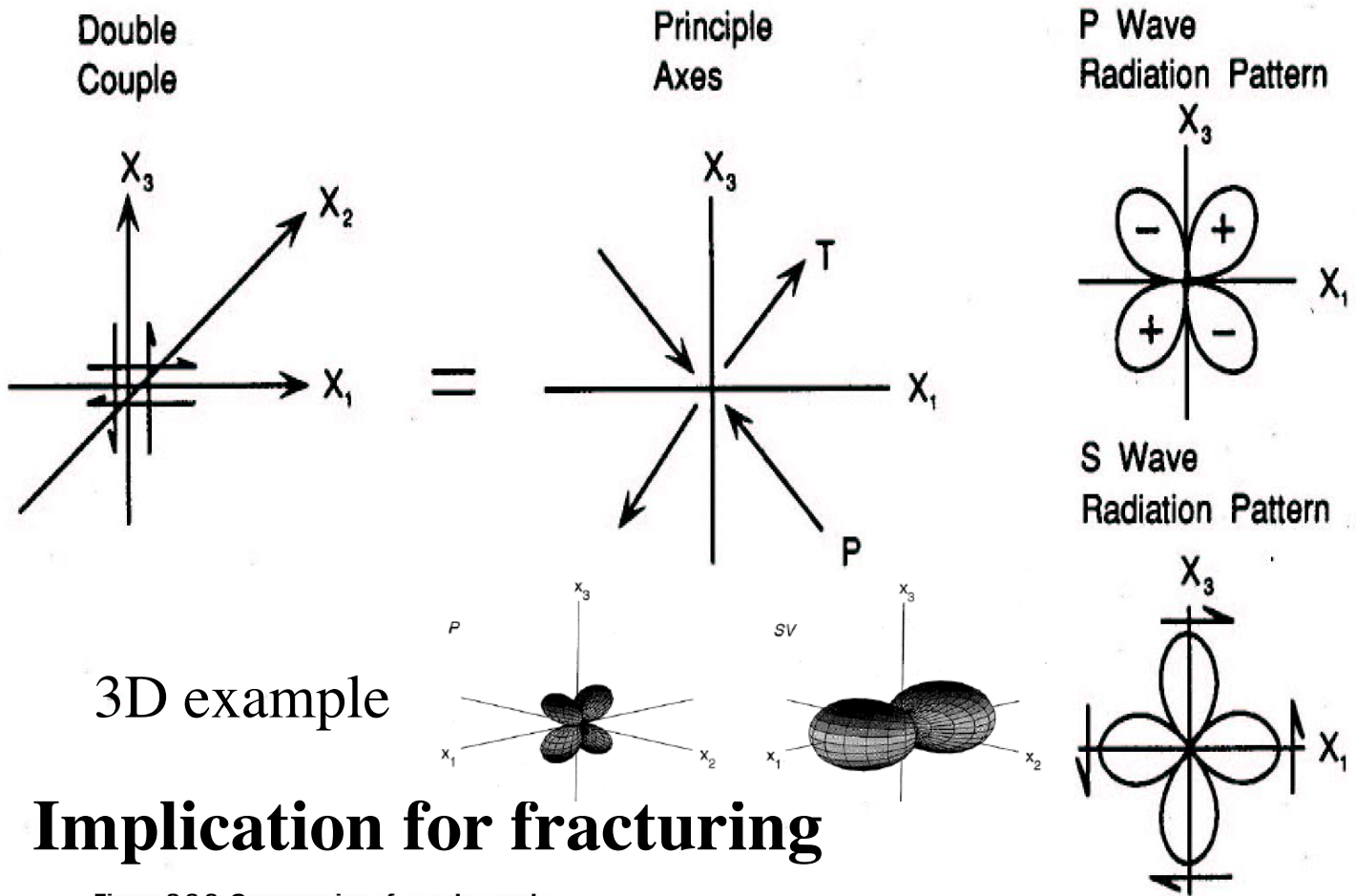
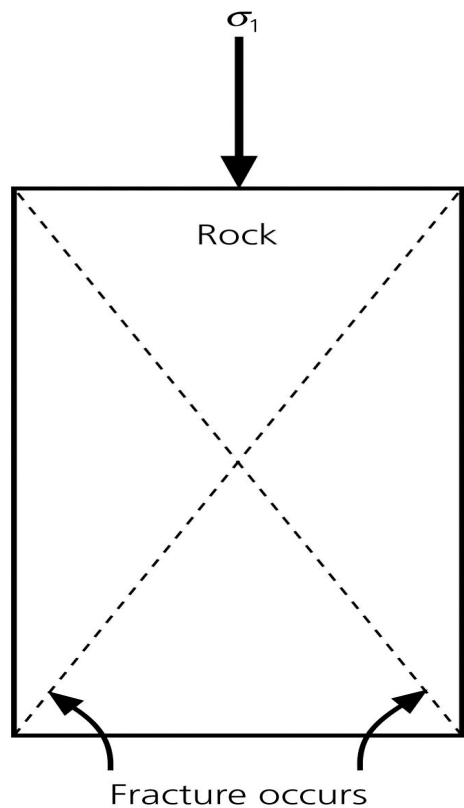


Figure 2.3-8: Compression of a rock sample.

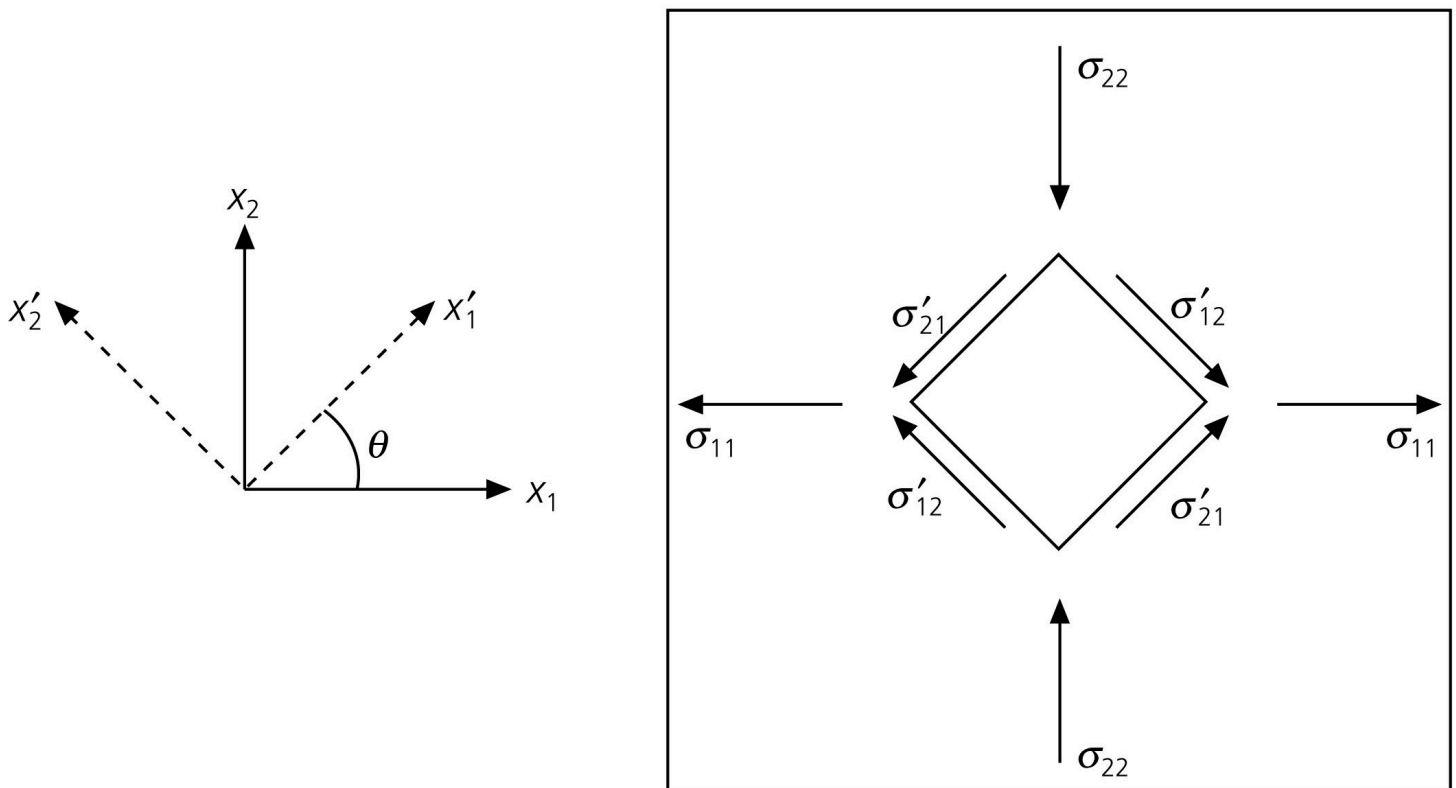


Pressing a rock in the left picture (with max compression in vertical and minimum compression along x-axis) cause 45 deg faults!

The so called *principal axis* (maximum compression & maximum dilatation) are associated with 45 deg angles

Below is a textbook example for principal stress analysis. Imaging maximum compression (pressure in this case) in two vertical vectors and minimum compression in the two horizontal ones. This is the “reverse scenario” where some arbitrary stresses are “partitioned” as shear stresses along the diamond.

Figure 2.3-6: Different stress components in different coordinate systems.



Same concept apply to moment tensor analysis and the concept of *principle moments*.

An important property of real symmetric matrices (e.g., moment tensors, stress tensors, strain tensors):

Diagonalizable (fundamental theorem of real symmetric matrix)

There exist a coordinate transformation matrix U that transforms

$$\mathbf{M}' = \Lambda = \delta_{ij} \lambda_i = \mathbf{U} \mathbf{M} \mathbf{U}^T \quad \delta_{ij}=1 \text{ (if } i=j)$$

Else $\delta_{ij}=0$

$$\mathbf{M}' = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \quad \lambda_s \text{ are the } \textit{principle moments}, \text{ important for Earthquake source.}$$

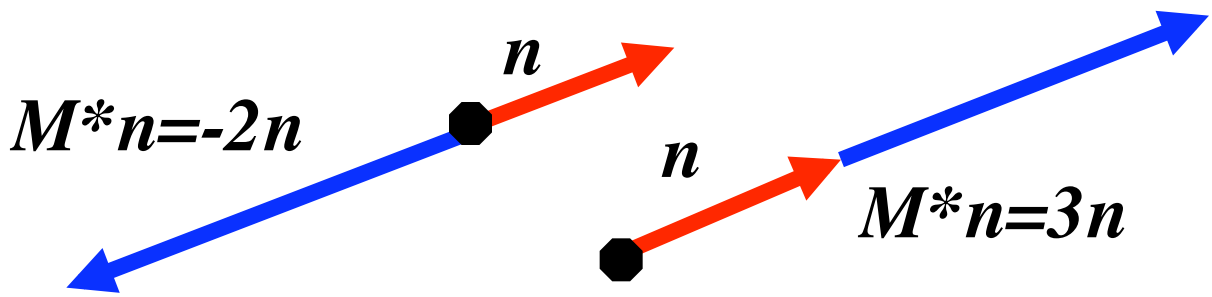
What does it really mean physically?

Force vectors on such special planes are parallel/anti-parallel to the unit normal of these planes.



$$\mathbf{M}_{ij} n_j = \lambda n_i \quad \text{Eigenvalues } \lambda: \text{ Principal forces}$$

Eigenvectors: principal force axes $\mathbf{n}$



n has eigenvalue $\lambda = -2$

n has eigenvalue $\lambda = 3$

$$M_{ij}n_j = \lambda n_j \Rightarrow (M_{ij} - \lambda \delta_{ij})n_j = 0$$

$$\begin{pmatrix} M_{11} - \lambda & M_{12} & M_{13} \\ M_{21} & M_{22} - \lambda & M_{23} \\ M_{31} & M_{32} & M_{33} - \lambda \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{eigenvalue problem!}$$

If there is nontrivial solution for lambda,
determinant has to be 0.

$$\det \begin{vmatrix} M_{11} - \lambda & M_{12} & M_{13} \\ M_{21} & M_{22} - \lambda & M_{23} \\ M_{31} & M_{32} & M_{33} - \lambda \end{vmatrix} = 0$$

$$(M_{11} - \lambda) \begin{vmatrix} M_{22} - \lambda & M_{23} \\ M_{32} & M_{33} - \lambda \end{vmatrix} - M_{12} \begin{vmatrix} M_{21} & M_{23} \\ M_{31} & M_{33} - \lambda \end{vmatrix} + M_{13} \begin{vmatrix} M_{21} & M_{22} - \lambda \\ M_{31} & M_{32} \end{vmatrix} = 0$$

Which gives the characteristic polynomial:

$$\lambda^3 - I_1 \lambda^2 + I_2 \lambda - I_3 = 0$$

Where

$$I_1 = M_{11} + M_{22} + M_{33}$$

$$I_2 = \begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} + \begin{vmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{vmatrix} + \begin{vmatrix} M_{11} & M_{13} \\ M_{31} & M_{33} \end{vmatrix}$$

$$I_3 = |\mathbf{M}| = \text{determinant}$$

The I_1 , I_2 and I_3 are invariants of the moment tensor. Meaning their values **will not change** after coordinate transformations.

$I_1 = \text{trace}$, or overall volumetric force in the system

How to find Source Plane solutions (3 in principle, but there are really two big ones if you make assumptions of double-couple solutions)

Step 1: Express as eigenvalue problem based on the matrix $\mathbf{M}$ ($\mathbf{M}$ is moment tensor)

Step 2. Solve the characteristic polynomial, the roots λ s are the eigenvalues (not always easy).

Step 3. Substitute in λ s into the system of equations, solve for $\mathbf{n}$ that satisfy them (these are *eigenvectors*)

For
$$\mathbf{M}' = \mathbf{U}\mathbf{M}\mathbf{U}^T = \mathbf{\Lambda}$$

$$\mathbf{U} = (\text{eig_v1} \quad \text{eig_v2} \quad \text{eig_v3})$$

i.e., each column vector eig_v is an eigenvector that satisfy the eigen equations. They are orthogonal basis vectors perpendicular in the newly transformed coordinate system.

Typical Global Centroid Moment Tensor (CMT) Solution (the two “fault planes” are results of principle moment eigenvectors (there are 3, but assume two are important for the Double couple assumption”). Below is a typical solution from Global Centroid Moment Tensor (GCMT, Lamont-Harvard) analysis.

010176A KERMADEC ISLANDS REGION

Date: 1976/ 1/ 1 Centroid
Time: 1:29:53.4 GMT
Lat= -29.25 Lon=-176.96
Depth= 47.8 Half duration=
9.4

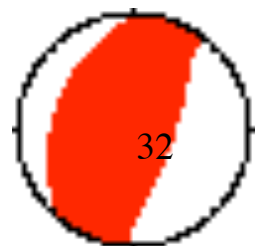
Centroid time minus hypocenter
time: 13.8

Moment Tensor: Expo=26 7.680
0.090 -7.770 1.390 4.520 -3.260

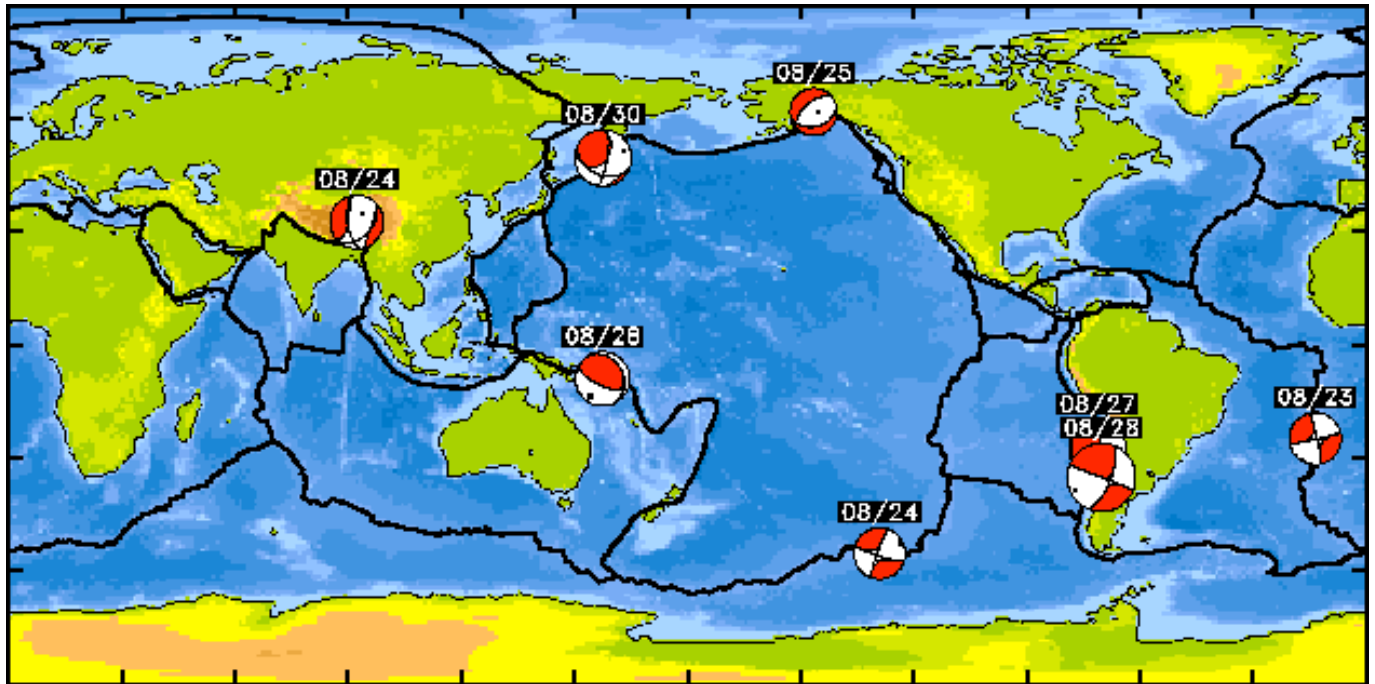
Mw = 7.3 mb = 6.2 Ms =
0.0 Scalar Moment = 9.56e+26

Fault plane: strike=202
dip=30 slip=93

Fault plane: strike=18
dip=60 slip=88



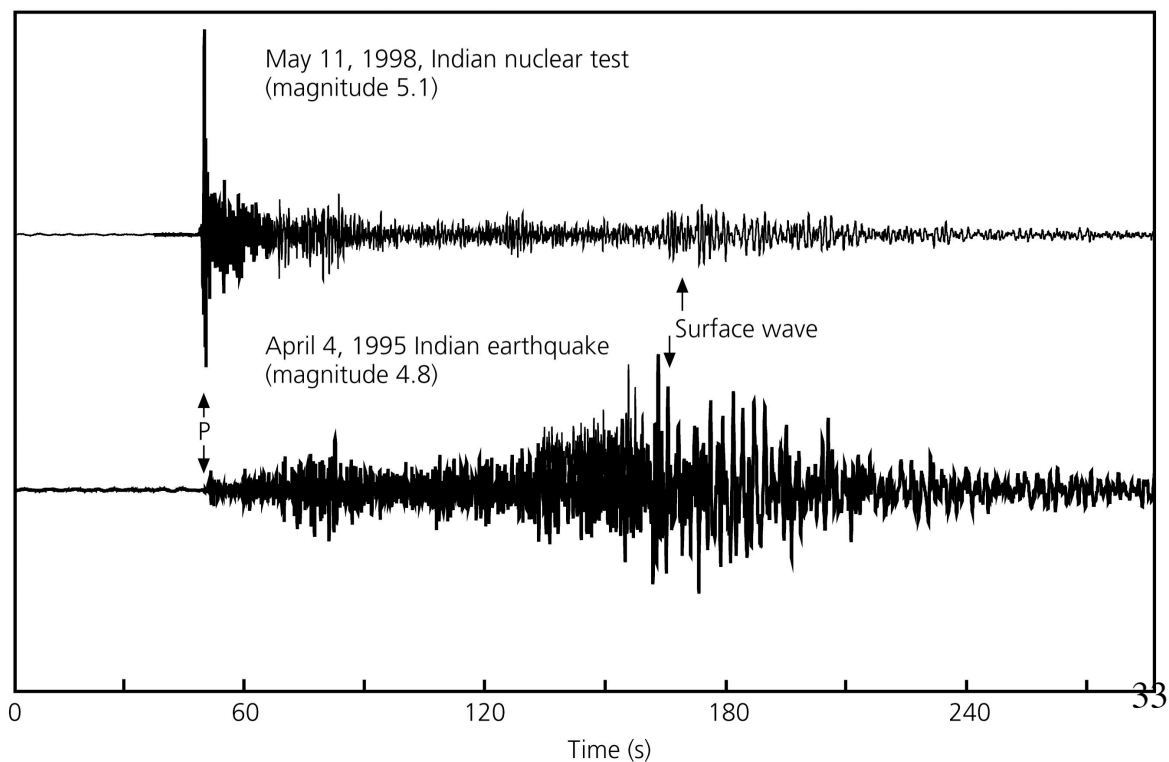
Natural Earthquakes



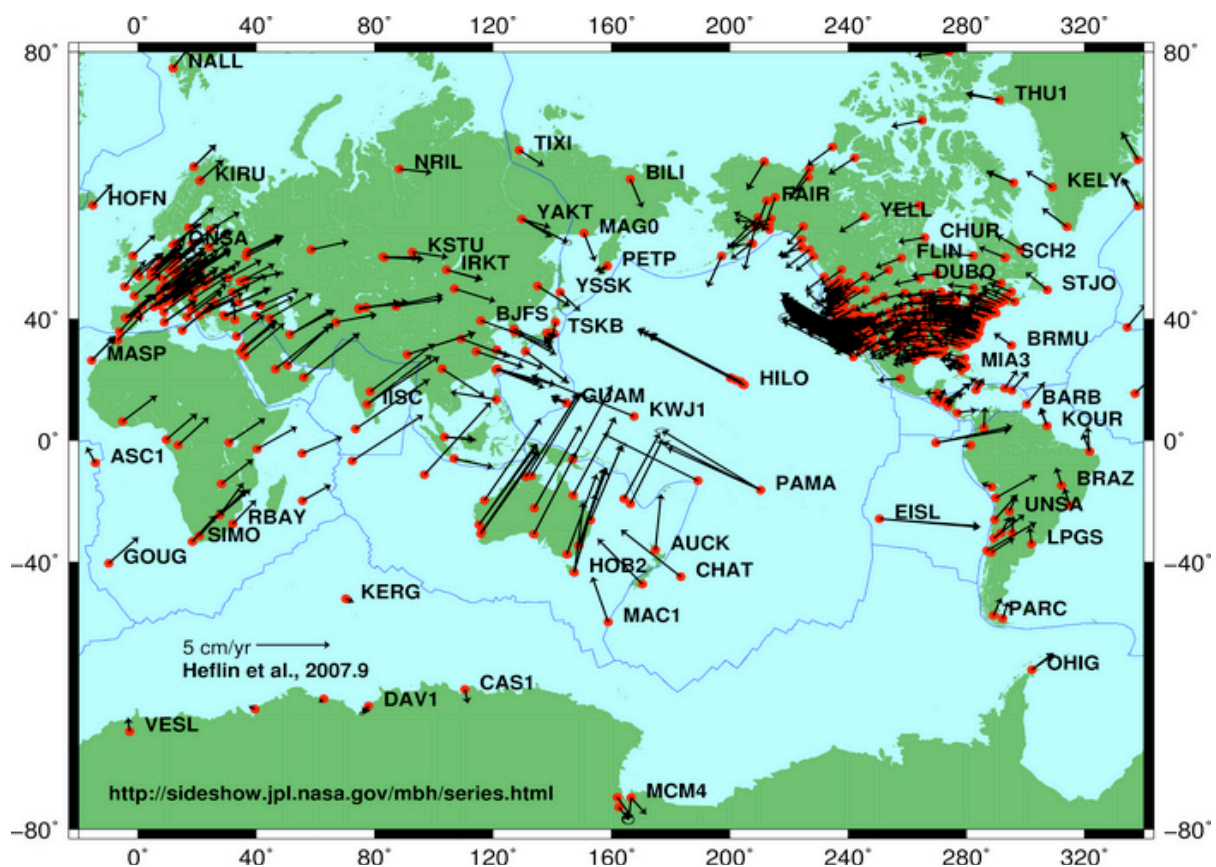
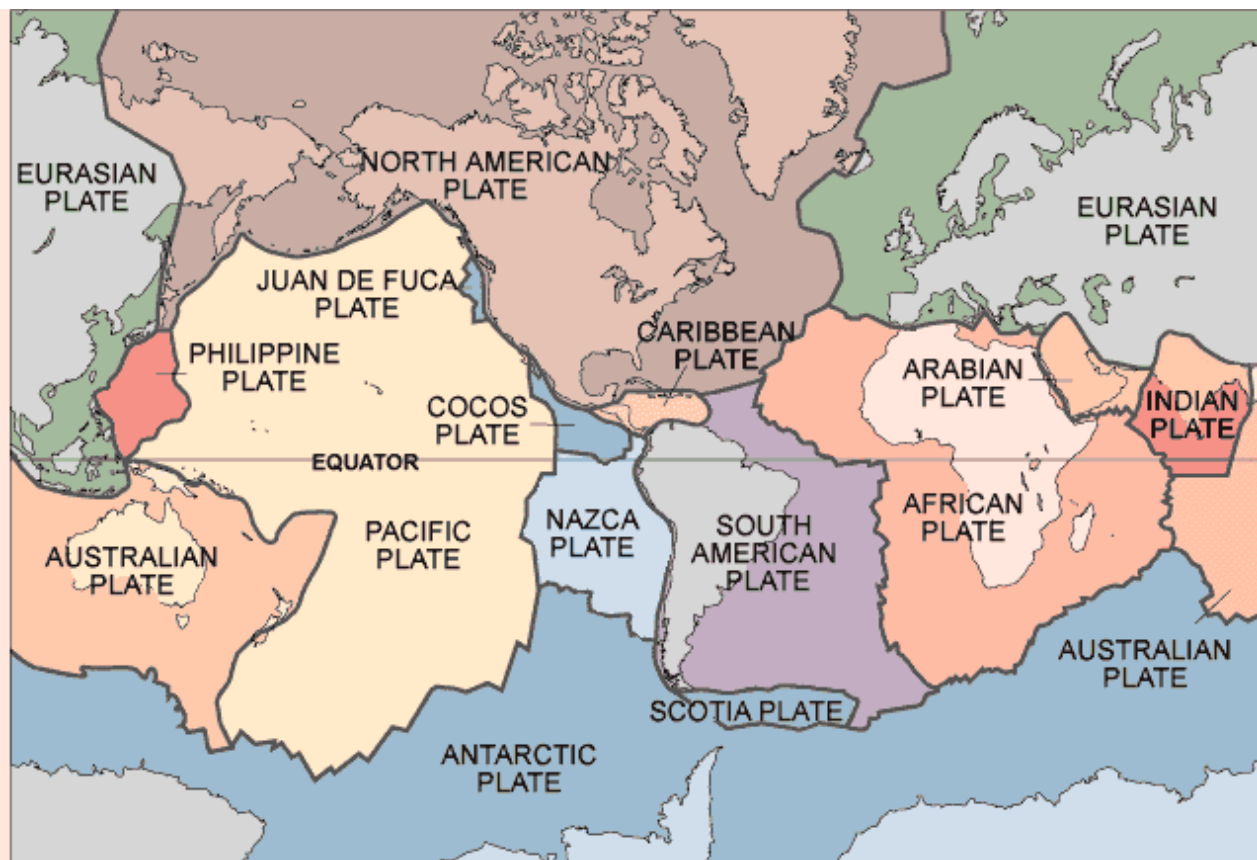
Mechanisms are usually normal or thrust near subduction zones, and strike-slip near ridges.

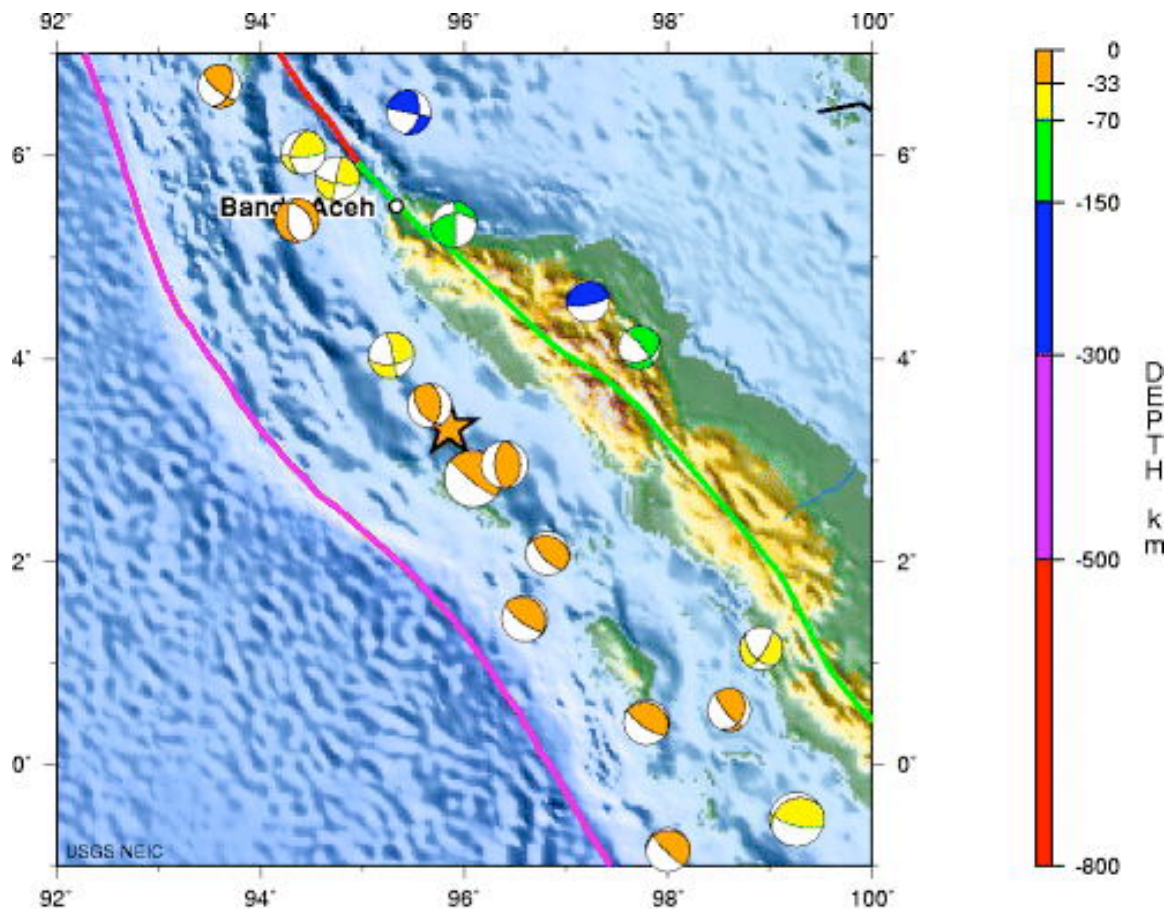
Earthquakes vs. Nuclear Explosions

Figure 1.2-19: Differences in seismic waves from an earthquake and explosion.



Explosions have high P amp, small surface waves.





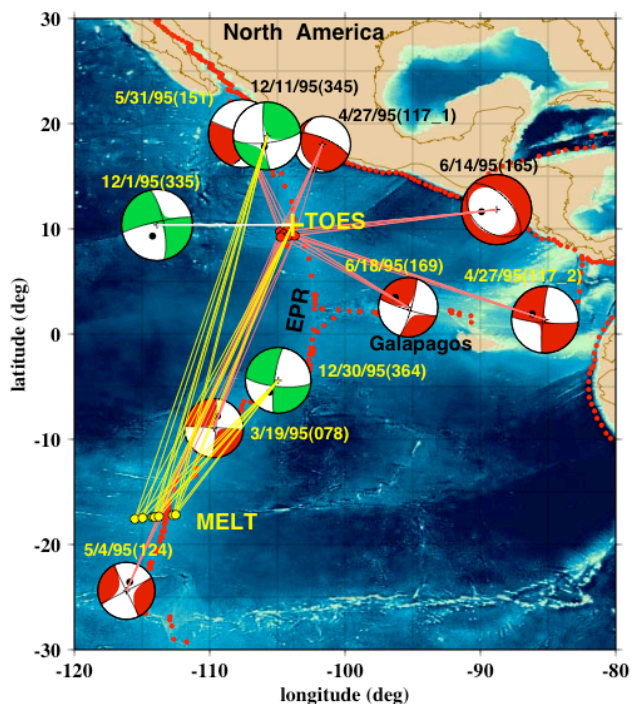
OFF W COAST OF NORTHERN SUMATRA

2004 12 26 00:58:53 UTC 3.32N 95.85E Depth: 30.0 km, Magnitude: 9.0

Historical Moment Tensor Solutions

Major Tectonic Boundaries: Subduction Zones -purple, Ridges -red and Transform Faults -green

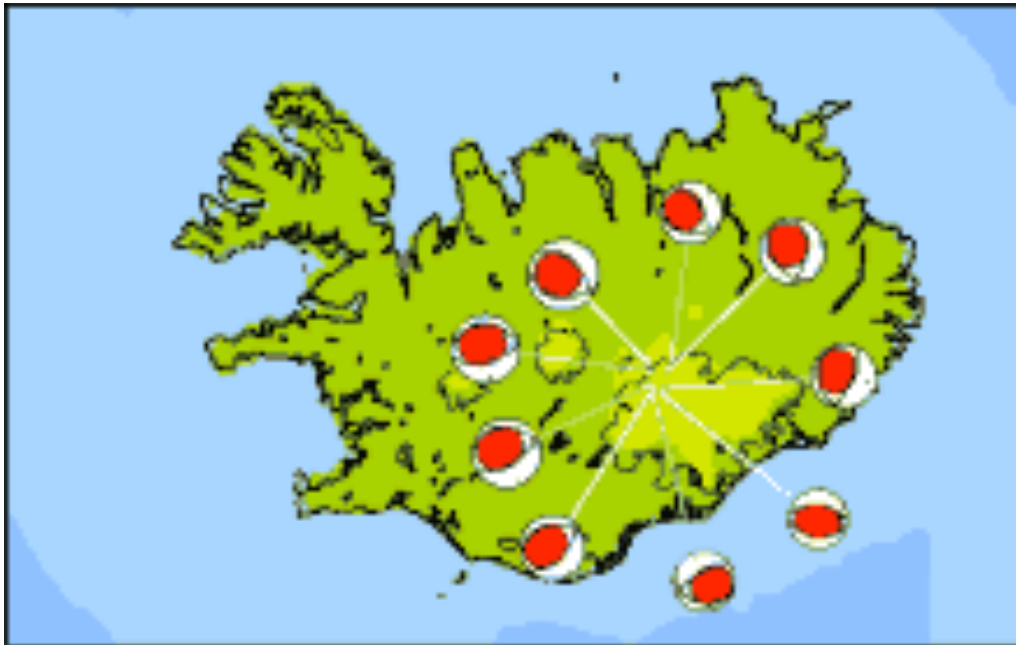
USGS National Earthquake Information Center



Again, here earthquakes are normal or thrust near subduction zones, and strike-slips near ridges. The positions of beachballs represent locations of earthquakes.

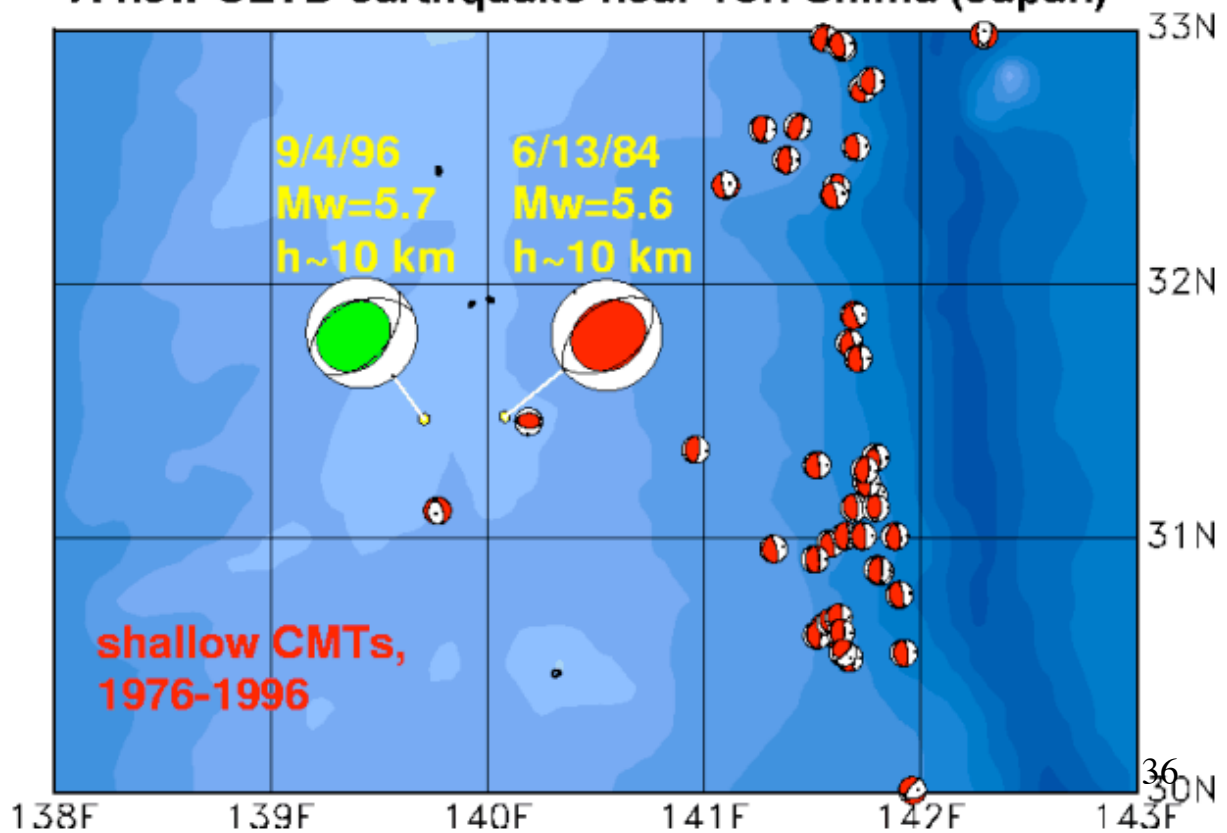
Usual earthquakes

Iceland Bargabunga Volcano



Nettles and Ekstrom, 03

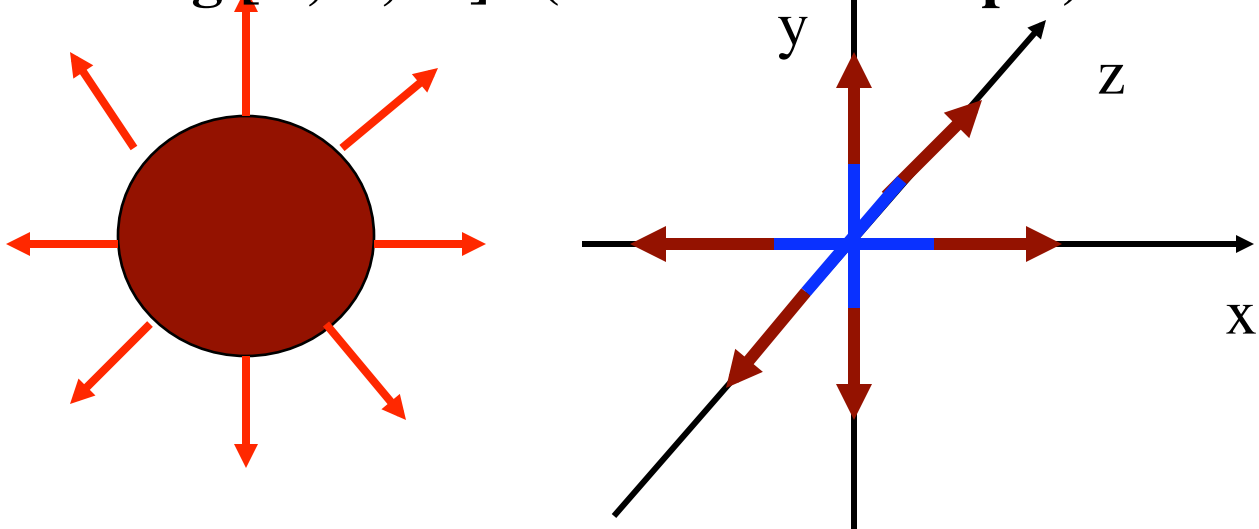
A new CLVD earthquake near Tori Shima (Japan)



Isotropic and CLVD moment tensors

(1) Isotropic tensor (triple vector dipole):

$$\mathbf{M} = \text{diag} [E, E, E] \quad (\text{non double-couple})$$



(2) Another class of non-double couple mechanism.: **Compensated Linear Vector Dipoles (CLVDs)**, I.e., not isotropic, but two sets of dipoles combine to form -2 times the amplitude of the third set.

$$\mathbf{M} = \begin{pmatrix} -2\lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

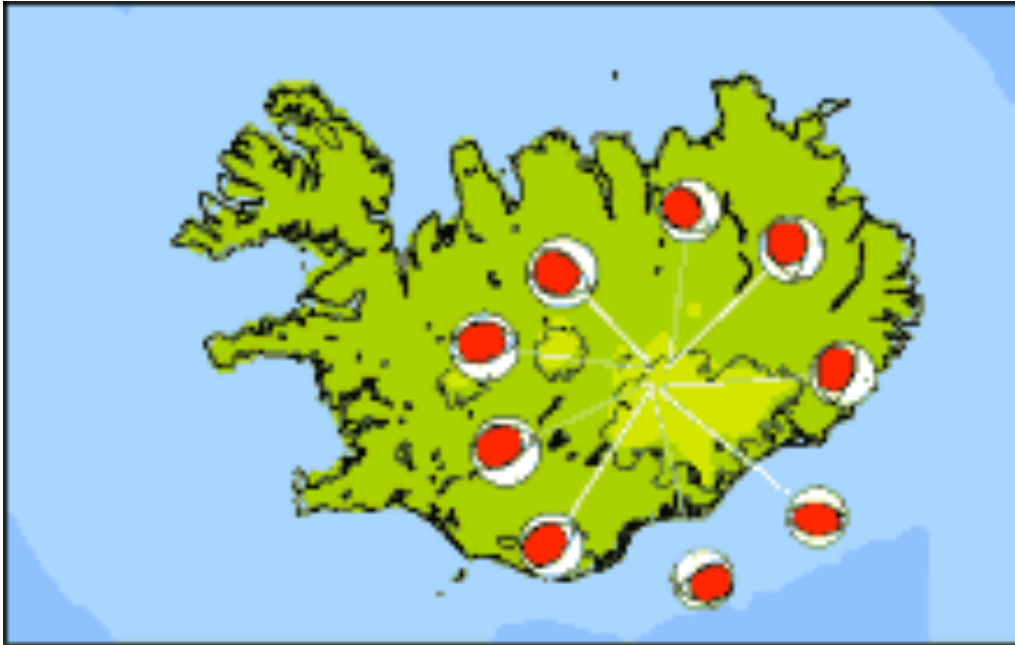
My not-so-original interpretation:
Fried eggs and baseballs!

Conventional wisdom: earthquakes with large CLVD components are rare. But they can be found in volcanic areas where:

- (1) Crack opening under tension
- (2) Sum of two double couple sources.

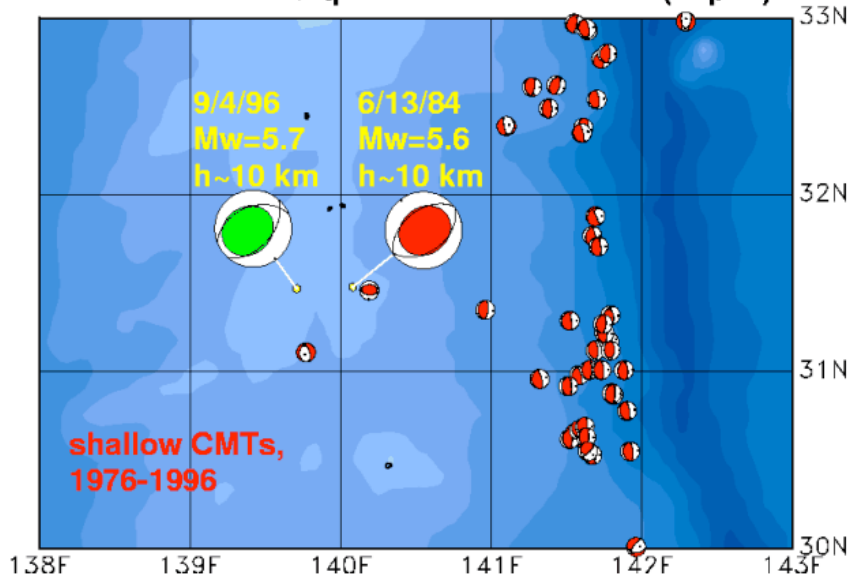
Unusual earthquakes

Iceland Bargabunga Volcano



Nettles and Ekstrom, 03

A new CLVD earthquake near Tori Shima (Japan)

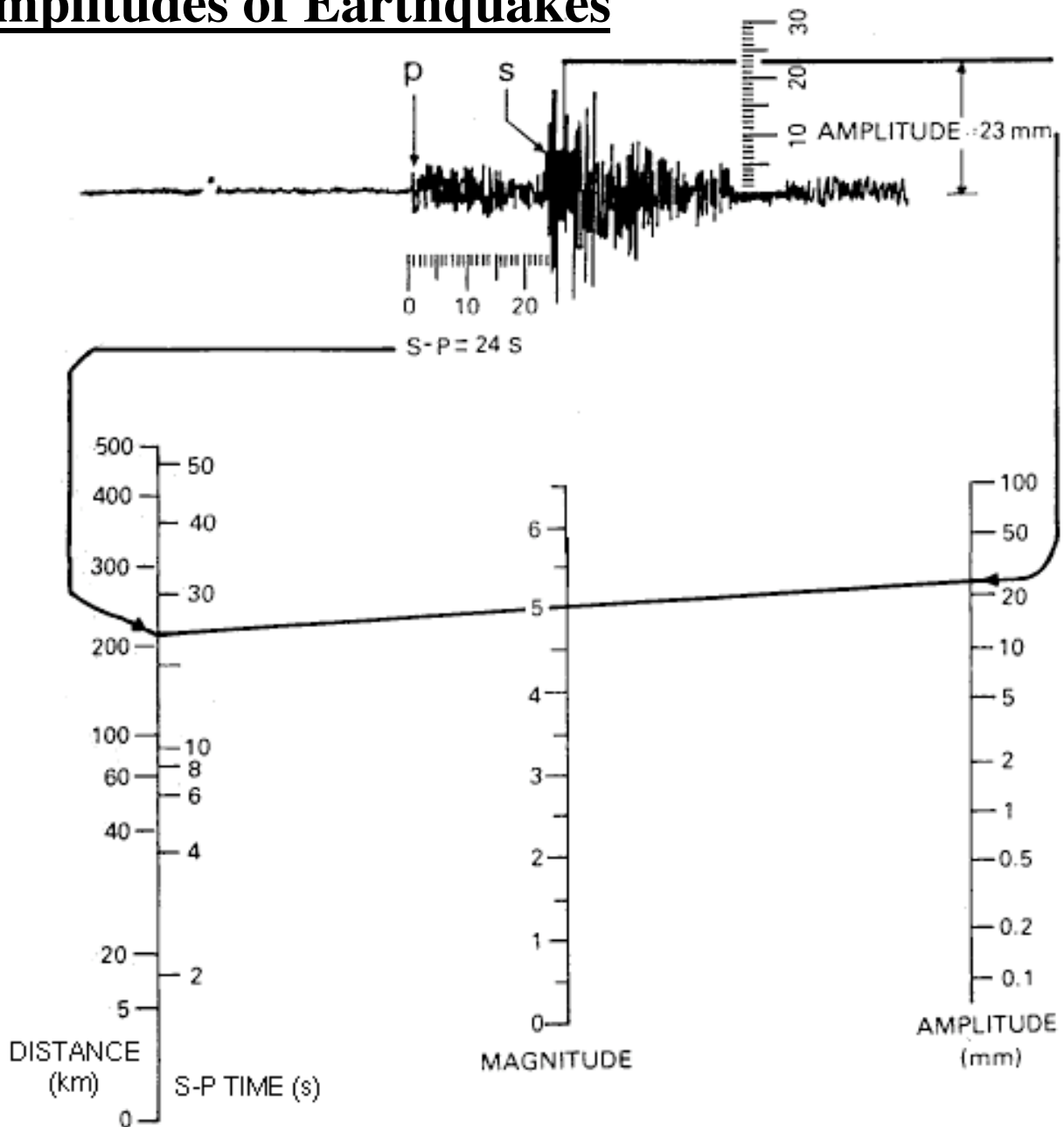


Possible explanation:
deflating magma chamber in cone-shaped setting

Explanation 2

$$\begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & 2M_0 \end{pmatrix} = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & M_0 \end{pmatrix}$$

Amplitudes of Earthquakes



$$M_L = \log_{10} A(\text{mm}) + (\text{Distance correction factor})$$

$$M = \log_{10} A(\text{mm}) + 3 \log_{10} [8 \Delta t(\text{s})] - 2.92 \quad (\text{Richter})$$

M_b and **M_s** are later measures of earthquake magnitudes, **M_b** is body wave magnitude, **M_s** is surface wave magnitude. As name suggests, they³⁹ are measured from seismic records (depends on medium)

Body wave amplitude: $m_b = \log_{10}(A/T) + Q(h, \Delta)$

Amp in
microns

Dominant
Period

Amp vs. range
correction function
(radiation, station
effects, directivity)

Surface wave amplitude:

$$M_s = \log_{10}(A/T) + 1.66 \log_{10} \Delta + 3.3$$

Moment magnitude (most physical and does not rely on station effects).

$$M_w = \frac{2}{3} \left[\log_{10} M_0 \text{ (dyne-cm)} - 16.0 \right]$$

**Another important earthquake relationship:
Magnitude vs. recurrence**

Gutenberg and Richter relationship:

$$\log_{10} N = a - bM_s$$

N = number of events; M_s = surface wave mag

b = **b-values** (typically between 0.8 and 1.2)

if $b=1$, then we expect 10 mag=4 event for each
mag 5 event, and 100 mag=3 events, etc...

$$(Moment) = (Rock\ Rigidity) \times (Fault\ Area) \times (Slip\ Distance)$$

$$M_0 = \mu A d$$

$$(dyne\text{-}cm) = \left[\frac{dyne}{\cancel{cm^2}} \right] \times (\cancel{cm^2}) \times (cm)$$

Now, let's imagine a chunk of rock on a lab bench, the rigidity, or resistance to shearing, of the rock is a **pressure** in the neighborhood of a few hundred billion dynes per square centimeter. (Scientific notation makes this easier to write.)

The **pressure** acts over an **area** to produce a force, and you can see that the cm-squared units cancel. Now if we guess that the distance the two parts grind together before they fly apart is about a centimeter, then we can calculate the moment, in dyne-cm:

$$M_0 = (3 \times 10^{11} \frac{dyne}{\cancel{cm^2}}) (10 \cancel{cm}) (10 \cancel{cm}) (1\ cm)$$

$$M_0 = (3 \times 10^{11}) (10^2) (dyne\text{-}cm)$$

$$M_0 = 3 \times 10^{13} \text{ dyne-cm}$$

$$M_w = \frac{2}{3} \left[\log_{10} M_0 \text{ (dyne-cm)} - 16.0 \right]$$

M_w: Moment magnitude is much more physical since it describes the energy/moment out of the source,⁴¹ and not a measure of the medium away from it.

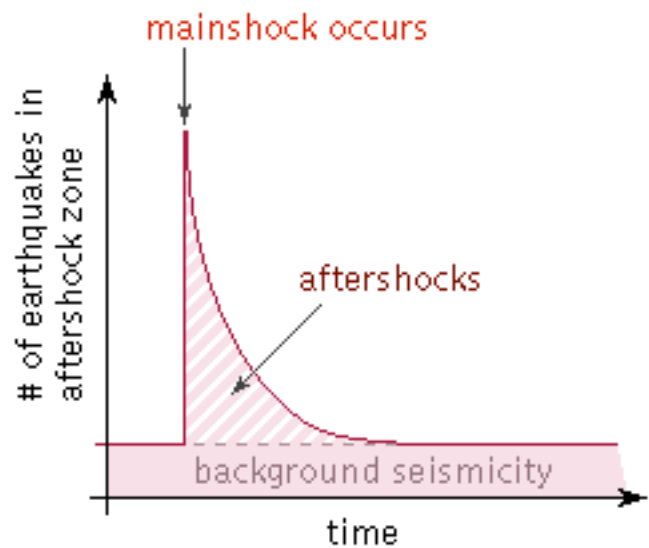
Mainshocks & Aftershocks

Aftershocks: normal earthquakes that follows closely in the wake of a larger earthquake, roughly in the same location. (I.e., no strict way to distinguish a earthquake from its aftershocks).

Aftershock Guideline #1



Aftershock Guideline #2



(1) Aftershocks should occur within some defined zone around the fault area.

(2) Aftershocks should occur before background seismicity return back to normal

An empirical relation governing the decay of

Omori's Law

$$N(t) \propto t^{-p}$$

$$-p \log_{10}(t) = a - bM_s$$

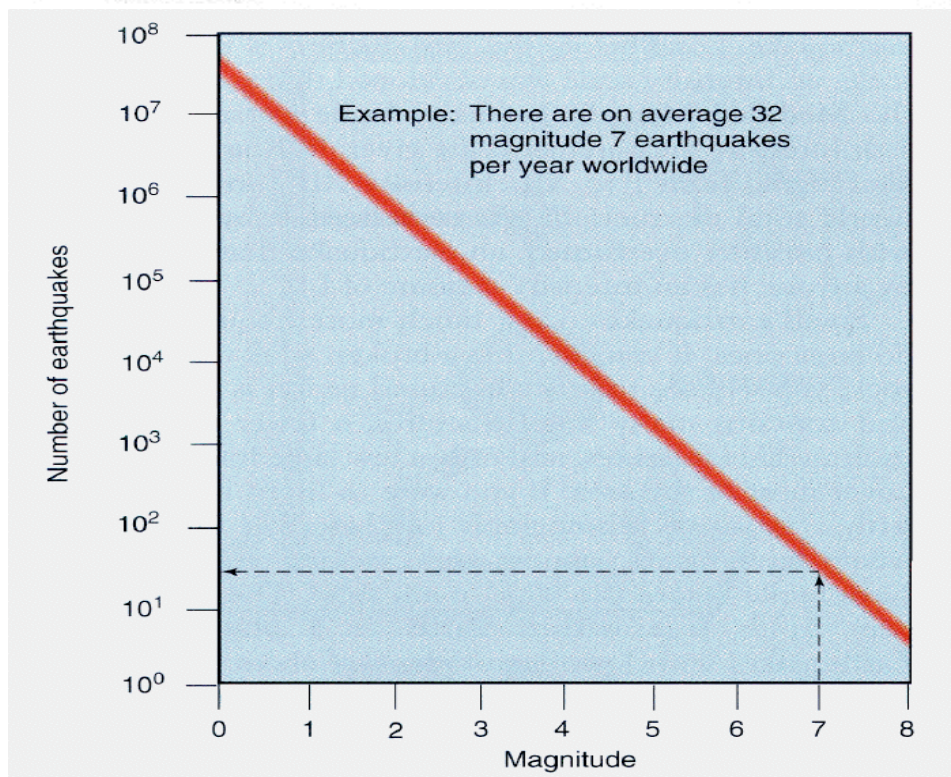
t is time in years, p is a constant called *severity of decay*, relates to b-value!

Earthquake scaling relationships

Table 4.2. *Worldwide mean annual frequency of earthquakes recorded during the period 1918–1945*

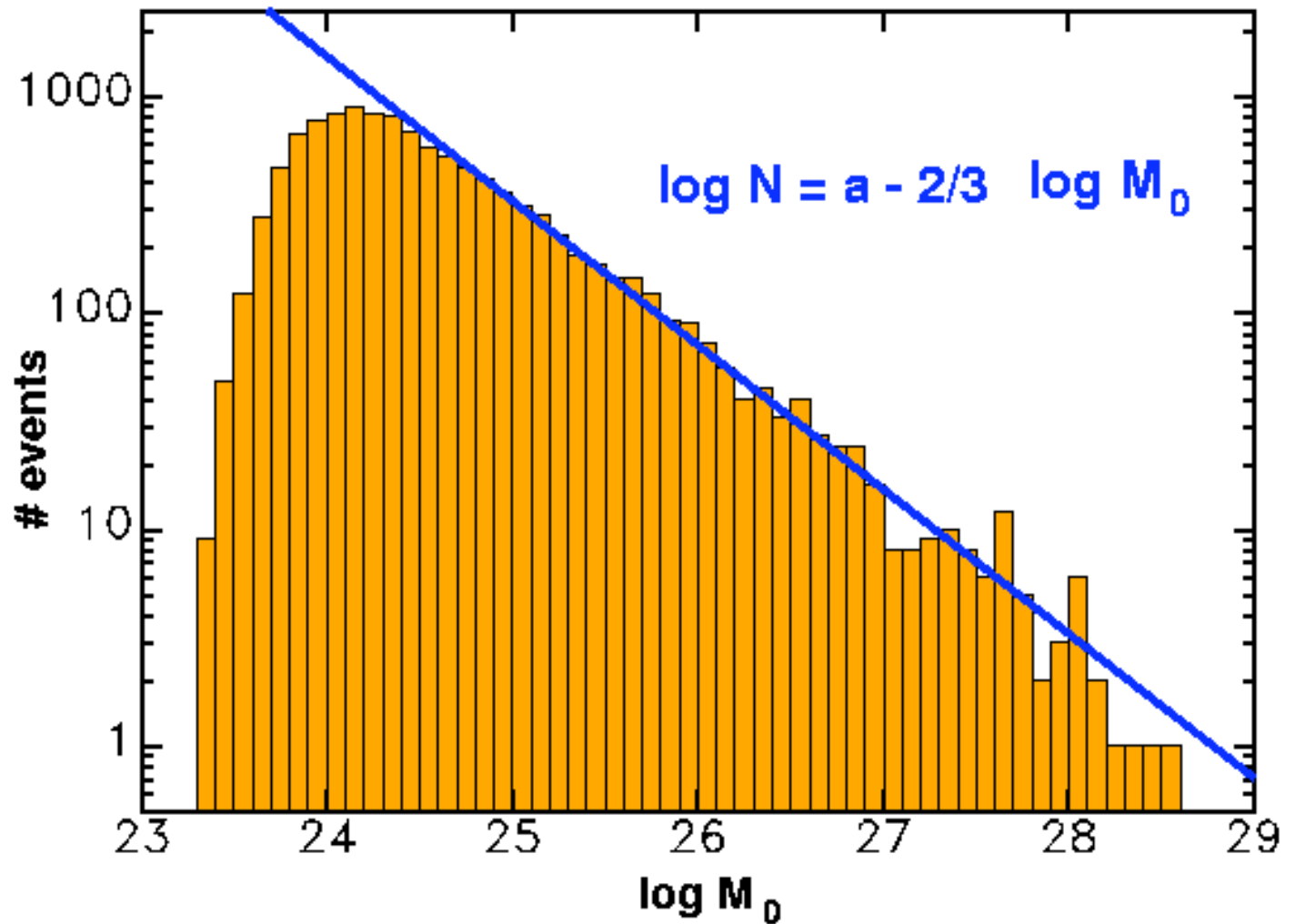
Magnitude	Number per year
> 8.0	1
7–7.9	18
6–6.9	108
5–5.9	800
4–4.9	6,200
3–3.9	49,000
2–2.9	300,000

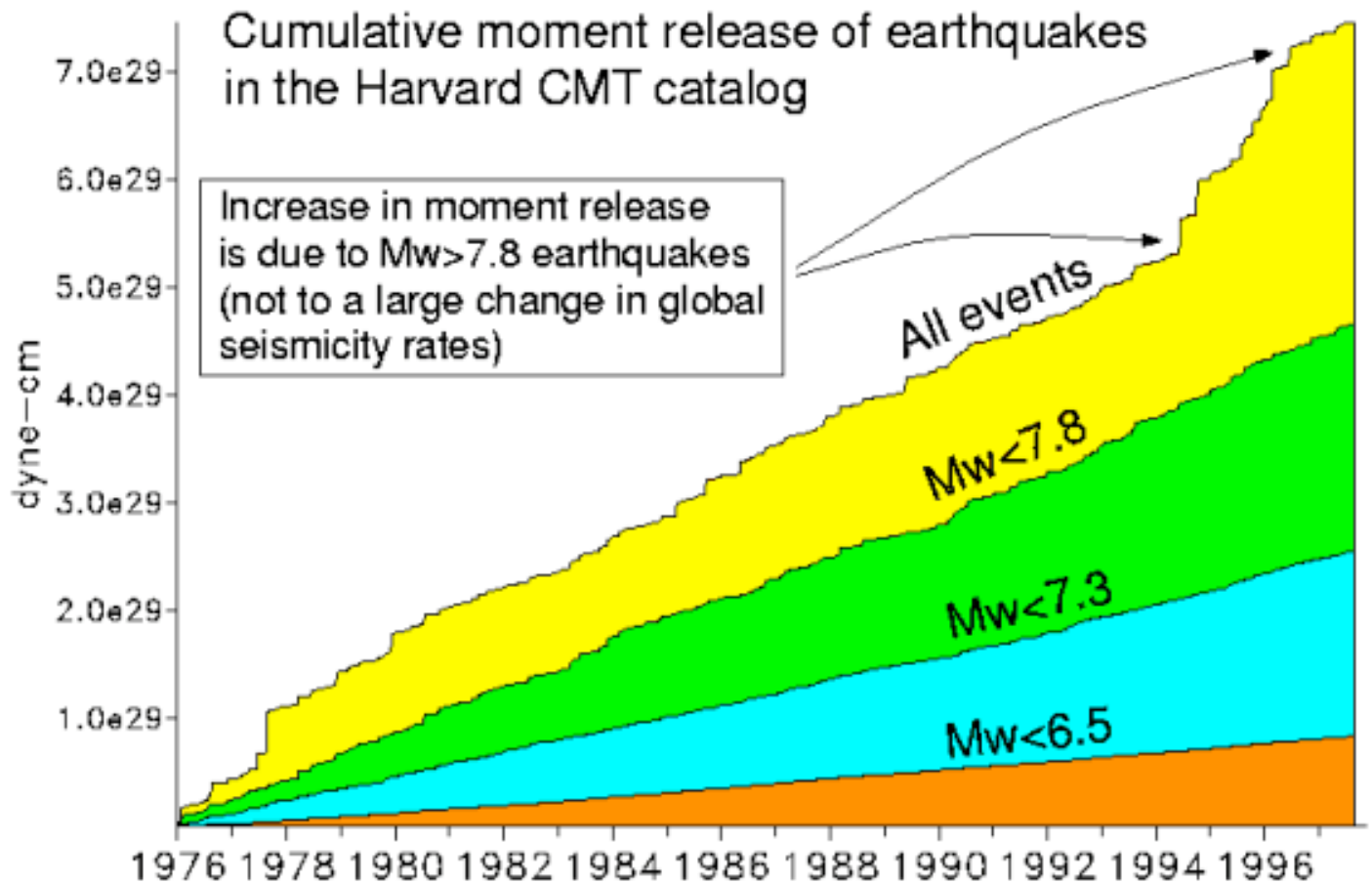
Source: After Gutenberg and Richter (1954).



Magnitude-frequency relationship

Gutenberg-Richter relationship



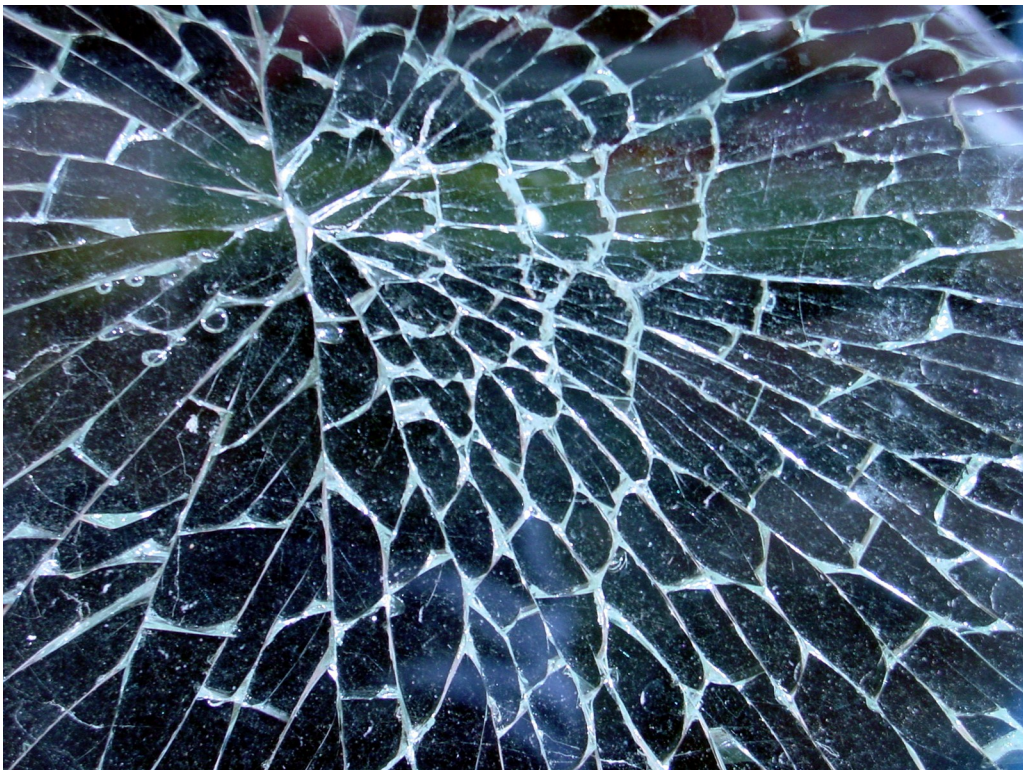


Large earthquakes contribute greatly to the total moment release in the world. This is because the large events have orders of magnitude larger scalar moments than smaller ones.

Haskell Source

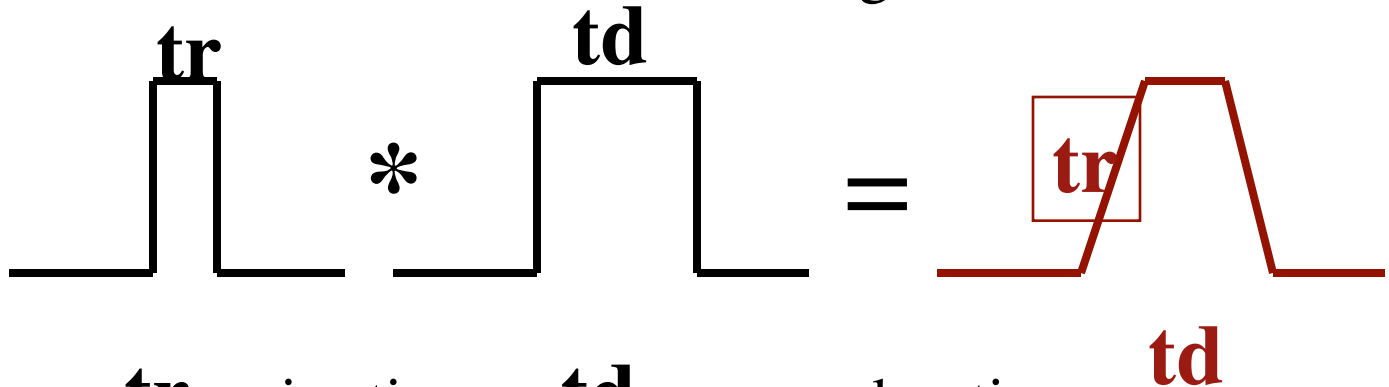


Brune's Model of Faulting



Source Models

Haskell Source: 1 or 2 rectangles



tr = rise time, **td**=source duration

Directivity case 1 : $\text{td}(\text{toward}) = L (1/V_r - 1/\alpha)$

Directivity case 2: $\text{td}(\text{away}) = L (1/V_r + 1/\alpha)$

Crack propagation can affect the signal pulse at a given station. This is the seismology equivalence of the “**Doppler Effect**”.

V_r =speed of medium, α =crack propagation speed
“Doppler shifted” displacement pulse at a station



Other Source Model(S): Brune’s Model

i.e., fault area is not rectangular, but an oval-shaped region. When quake happens, all of it gets cracked!

Source Spectra

Fourier Transform the boxcar (with 1 sec Length) will result in a SINC function (see Shearer, page 181), that is

$$F(B(t)) = \sin(\omega/2)/(\omega/2)$$

This is simply $\text{sinc}(\omega/2)$. Now suppose two boxcars are used to describe the source (Haskell's rectangular model), then we will have a convolution of two boxcars in time. The resulting Fourier transform of the convolution is,

$$F[B(t/t_r)*B(t/t_d)] = t_r t_d \text{sinc}(\omega t_r/2) \text{sinc}(\omega t_d/2)$$

Where t_r and t_d are the rise time and source duration, respectively.

So we can express far-field amplitude spectrum as

$$|A(\omega)| = g M_0 |\text{sinc}(\omega t_r/2)| |\text{sinc}(\omega t_d/2)|$$

g is some scaling factor taking log on both sides, we can separate out the two sinc terms,

$$\log(A(\omega)) = \log(g) + \log(M_0) + \log|\text{sinc}(\omega t_r/2)| + \log|\text{sinc}(\omega t_d/2)|$$

M_0 is scalar moment.

$G = \log(g)$ where g is scaling factor.

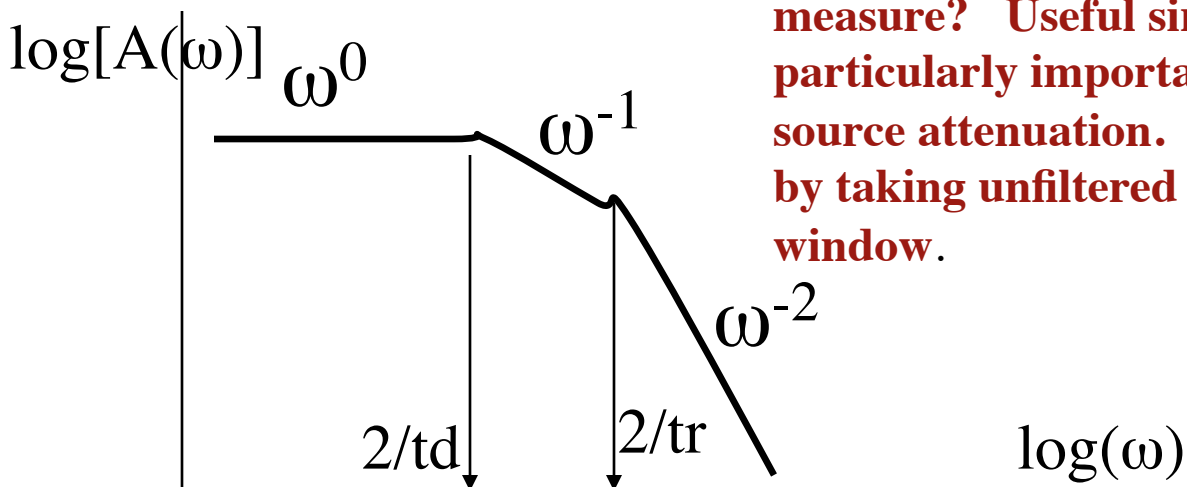
$$\log |A(\omega)| - G \approx \begin{cases} \log M_0 & \omega < 2/t_d \\ \log M_0 - \log \omega & 2/t_d < \omega < 2/t_r \\ \log M_0 - 2\log \omega & 2/t_r < \omega \end{cases}$$

The above is derived from the convenient fact that $|\text{sinc}(x)|$ is close to 1 for $x < 1$ and it is close to $1/x$ as $x > 1$.

Another assumption:

$t_d > t_r$ (source duration > rise time)

Because of the last term of expression above depend on $\log(M_0)$, $-\log(\omega)$, and $-2\log(\omega)$, the last two log expressions are just $\log(\omega^{-1})$ and $\log(\omega^{-2})$, which means if we draw a plot of $\log(A(\omega))$ vs. $\log(\omega)$, three linear segments will appear with slopes of ω^0 (a constant), ω^{-1} and ω^{-2} . See



Why useful and how to measure? Useful since it is particularly important for source attenuation. Measure by taking unfiltered P window.

Appendix 1:

Fourier transform of a box car with unit height and height

$$\begin{aligned} F[B(t)] &= \int_{-1/2}^{1/2} e^{i\omega t} dt = \frac{1}{i\omega} (e^{i\omega/2} - e^{-i\omega/2}) \\ &= \frac{1}{i\omega} [i \sin(\omega/2) - i \sin(-\omega/2) + \cos(\omega/2) - \cos(-\omega/2)] \\ &= \frac{2}{i\omega} [i \sin(\omega/2)] = \frac{\sin(\omega/2)}{\omega/2} = \text{sinc}(\omega/2) \end{aligned}$$

So by Fourier scaling rule, the Fourier signal from a box car with height 1 and length τ

$$F(B(t/\tau)) = \tau \text{sinc}(\omega/2)$$

Appendix 2:

Approximation of sinc(x)

$$x = 30^\circ < 1 \quad \text{sinc}(x) = \frac{\sin(30)}{\pi/6} = \frac{0.5}{0.52} \approx 1$$

$$x = 90^\circ > 1 \quad \text{sinc}(x) = \frac{\sin(90)}{\pi/2} = \frac{1}{\pi/2} \approx \frac{1}{\cancel{50}}$$