Gravity anomalies of some simple shapes

Reminder: we are working with values about...
0.01-0.001 mgal ≈ 10^{-8} - 10^{-9} g_N!!!
Applications of Gravity Surveys

- Exploration of fossil fuels (oil, gas, coal)
- Exploration of bulk mineral deposit (mineral, sand, gravel)
- Exploration of underground water supplies
- Engineering/construction site investigation
- Cavity detection
- Glaciology
- Regional and global tectonics
- Geology, volcanology
- Shape of the Earth, isostasy
- Military (e.g., detection of underground bunkers)
Gravity and gravity anomalies

Gravity forward problems:

**Density contrast**
(variation in subsurface mass distribution relative to some “average” Earth)

**Gravity anomaly**
(difference between observed gravity and expected gravity)

We computed the gravity anomaly of a spherical ore body at X (immediately on top of the ore body).

→ How does the gravity anomaly vary laterally?
Calculating gravity anomalies

On Top of Point Mass

- $g$ ("normal" gravity)
- $\Delta g$ ("extra" gravity due to high density point mass)

Away from Point Mass

- $\Delta g$ - depends on distance ($h$) to point mass (inverse square law)
- $z$, $h$, $\theta$
Also, $\Delta g$ is usually small

- $\theta$ is small
- $\Delta g_x$ is small

→ consider only anomalies in vertical gravity

(for exploration studies...for tectonic studies, the horizontal component can be significant, e.g., in mountainous areas)
For a point mass at distance \( r \):

\[
\Delta g = \frac{G \Delta m}{h^2}
\]

\( \Delta m \) is mass difference w.r.t. surrounding material

Vertical component is:

\[
\Delta g_z = \frac{G \Delta m}{h^2} \cos \theta = \frac{G \Delta mz}{h^3}
\]

Note:

\[
\cos \theta = \frac{\Delta g_z}{\Delta g} = \frac{z}{h}
\]
Gravity profile for a point mass
Gravity anomaly of any body

- divide body into “point masses”
- at each position
  - calculate gravity anomaly due to each point mass
  \[ \Delta g_z = \frac{G \Delta m z}{h^3} \]
- add up all anomalies

Mathematically (in 3D):

\[ \Delta g_z = \iiint G \Delta \rho \frac{(z' - z)}{h^3} \, dx' \, dy' \, dz' \]

Where: \( h = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2} \)

& (x, y, z) is the observation point
Gravity anomaly of a buried sphere

Sphere is **more dense** than surrounding material – therefore has a **mass excess** \((M_S)\)

Can show that the vertical component of gravity will vary as:

\[
 g_z(x) = \frac{G M_S z}{(x^2 + z^2)^{\frac{3}{2}}}
\]

**NOTE:**
- \(x\) is the horizontal distance from the **center** of sphere to the point of measurement
- (just like for a point mass)
\[ g_z(x) = \frac{GM_Sz}{\left(x^2 + z^2\right)^{3/2}} \]

**SOME VALUES**

\[ \begin{align*}
z &= 100 \text{ m} \\
r &= 50 \text{ m} \\
\Delta \rho &= 2000 \text{ kg m}^{-3} \\
M_S &\sim 10^9 \text{ kg} \\
\end{align*} \]

*Where:*

\[ M_S = \left(\frac{4}{3} \pi r^3\right) \times (\Delta \rho) \]

\[ g_z(x) = \text{___________________} \]
Gravity anomaly of a buried sphere

• maximum $g_z$ at point above the centre of the sphere ($x=0$ m)
• $g_z$ decreases rapidly away from the sphere
• maximum value of $g_z$ is:

$$g_z^{\text{max}} = \frac{GM_S}{Z^2}$$
Gravity anomaly of a buried sphere

- maximum $g_z$ at point above the centre of the sphere ($x=0$ m)
- $g_z$ decreases rapidly away from the sphere
- maximum value of $g_z$ is:
  \[ g_z^{\text{max}} = \frac{GM_S}{z^2} \]

- **half-width** of the curve is where $g_z = g_z^{\text{max}} / 2$
  \[ x_{1/2} = 0.766 z \]
  - can show that $x_{1/2} = 0.766 z$
  - equivalently: $z = 1.305 x_{1/2}$
Gravity anomaly of a buried sphere

Forward problem:
Given $z$, $r$, and $\Delta \rho$

◊ can calculate $g_z (x)$

Inverse problem:
Observe $g_z (x) \rightarrow$ find $g_z^{\text{max}}$, $x_{1/2}$

• can now determine depth:
  
  $z = 1.305 \times x_{1/2}$

• and $M_S$: 
  
  $M_S = \frac{g_z^{\text{max}}}{G} \times z^2$

\[ g_z^{\text{max}} \]

\[ x_{1/2} \]

\[ X \]

\[ \rho \]

\[ \rho_0 \]

\[ z \]
Changing the depth of the sphere...

\[ g_z^{\text{max}} = \frac{GM_S}{z^2} \]

\[ x_{1/2} = 0.766 \, z \]

Deeper sphere
\( \rightarrow \) broader, smaller amplitude gravity anomaly

(Lowrie, 2007)
The width (wavelength) of the anomaly is related to the depth of the body that causes it.

**Deeper body = wider anomaly**

→ measure the half-width ($x_{1/2}$) of an anomaly to get the approximate depth of the body.

2D: cylinder \[ z = x_{1/2} \]

3D: sphere \[ z = 1.305 \times x_{1/2} \]

where \( z \) is the center.

- for other shapes, \( z \) represents the **maximum depth of the top of the body**

(Lowrie, 2007)
Assumption: a 3D anomaly is caused by a point mass (a 2D anomaly is caused by a line mass) at depth $= z$

$x_{1/2}$ gives $z$
Buried horizontal cylinder

Cylinder with radius $a$ is at depth $z$. The density contrast between the cylinder and surrounding material is $\Delta \rho$.

Can show that the vertical component of gravity will vary as:

$$g_z(x) = \frac{2G\pi r^2 z \Delta \rho}{\left(x^2 + z^2\right)}$$
\[ g_z(x) = \frac{2G\pi r^2 z \Delta \rho}{(x^2 + z^2)} \]

**SOME VALUES**

axis at \( x = 0 \) m

\( z = 100 \) m

\( r = 50 \) m

\( \Delta \rho = 2000 \) kg m\(^{-3}\)

(same values that were used for the sphere)
**Important observations**

- maximum $g_z$ is at the axis of the cylinder ($x=0$ m)
- maximum value of $g_z$ is:

$$g_{z \text{ max}} = \frac{2G\pi r^2 \Delta \rho}{Z}$$

- can show that half-width is related to depth of the centre of the cylinder:

$$x_{1/2} = Z$$
Changing the depth

Changing the radius

\[ g_z^{\text{max}} = \frac{2G\pi r^2 \Delta \rho}{z} \]

Half width: \( x_{1/2} = z \)

(see textbook for the parameter values used to generate these plots)
Compare the gravity anomaly profiles:

- why is $g_z^{\text{max}}$ larger for the cylinder?
- if you recorded these profiles in the field, how would you decide if the anomaly was due to a sphere or a cylinder?
Simple code to try on matlab

% a comparison between cylinder and sphere
x = linspace(-250,250,501);
pi=3.1415926;
G=6.67e-11;
z=100;
r=50;
Vs=4*pi*r*r*r/3.0;
drho = 2000;
gz_sph=G*Vs*z*drho./((z*z+x.*x).^1.5*100000;
gz_cyl=2*pi*G*r*r*drho*z.((z*z+x.*x)*100000;
gz_sph_scaled=gz_sph.*2.0953/0.7;
plot(x, gz_sph)
hold on
plot(x, gz_cyl, 'r')
hold on
plot(x, gz_sph_scaled, 'g')
xlabel('x (m)')
ylabel('gravity anomaly gz (mgal)')
legend('sphere', 'cylinder', 'sphere scaled');
Gravity profiles vs. maps

**PROFILE**

**SPHERE**

**MAP**

**CYLINDER**
Example Problem

Buried spherical iron ore body:

• Centre of ore body:
  \[ x = \underline{} \text{ m} \]

• Maximum value of \( g_z \):
  \[ g_z^{\text{max}} = \underline{} \text{ mgal} \]

• Half-width:
  \[ x_{1/2} = \underline{} \text{ m} \]

• Depth of sphere:
  \[ z = \underline{} \text{ m} \]

• Excess mass of sphere:
  \[ M_S = \underline{} \text{ kg} \]

\( G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \)

Is it possible to determine the radius or density contrast of the sphere?
Infinite horizontal layer

Can show that the gravitational attraction of this slab at P is:

\[ g_z = 2\pi G \rho \Delta z \]

→ \( g_z \) does not depend on the x-position of the measurement or on the depth of the layer -- **WHY?**
$g_z$ does not depend on the x-position of the measurement or the depth of the layer \textbf{WHY?}

\[ g_z = 2\pi G \rho \Delta z \]
Infinite Horizontal Layer

Can also think about this in terms of gravity anomaly caused by a density contrast:

\[ \Delta g_z = 2\pi G \Delta \rho \Delta z \]

where \( \Delta \rho = \rho_0 - \rho \)
Compare the gravity response of these two layers:

\[ g_z = 2\pi G \rho \Delta z \]

Gravity value is:

\[ g_z = \]

Both slabs have the same \( g_z \) – can not tell them apart using only gravity data

\( \rightarrow \) non-uniqueness
Horizontal layer cut by a fault on the left
Computer modeling of gravity anomalies

- for more complex geometries, gravity anomalies can be calculated using computer modeling programs
- models can be 2D or 3D
  - for 2D: polygon is assumed to extend infinitely in the strike direction (in and out of page)
Verification of modeling program

- before using any program, must verify that the calculations are accurate
- **HOW?** construct a model where the answer is known to see if the program gives you the right answer
- for example – buried horizontal cylinder

**Why is model $g_z$ less than expected?**

line = model result
dots = expected answer
Verification of modeling program

16-sided “cylinder”

line = model result
dots = expected answer
Example: A sedimentary basin

Density contrast = -400 kg/m$^3$
- minimum $g_z$ is the nearly the same for both basins
- edge effects extend further into Basin 2
FORWARD PROBLEM:

- used to evaluate simple structures and to design surveys

INVERSE PROBLEM:

- most common, remember that structure can be non-unique