As x increases, CD becomes longer → arrival is delayed

- steeper slope on a distance-time plot
Reciprocity: wave equation (that governs propagations of deformation) obeys a reciprocal relationship with respect to source and receiver locations.

For both horizontal and dipping layers, IF position of a source and a receiver are interchanged, the travel time (and path) remains the same. This is the “Condition of Reciprocity”, heavily used in refraction surveys in the case of dipping/undulating layers. It is also used in reflection seismics to (e.g., data reconstruction)
reciprocity in horizontal layers
reciprocity in horizontal layers

First arrival times

refracted

direct

offset
No dip

Dip of 10 degrees
• two observations: slope and y-intercept
• three unknowns:  \( v_2, z \) and \( \Phi \) \( \rightarrow \) **NON-UNIQUE**

No dip (3200 m/s)  
5° dip (3700 m/s)  
10° dip (4450 m/s)
Solution: reverse the refraction experiment

- fire a shot at the downdip end of the seismic line and record the arrivals updip

- as offset \((x)\) becomes bigger, BA will become smaller
  - shallower slope (faster \(V\)) of the arrival on time-distance plot
T-x in dipping layers
Effective Acquisition Designs

Reverse Profile Shots
(fire shots at the ends)

Split spread
(receivers on both directions of shots)
The **downdip travel time** is (see book for extensive derivations):

$$t_d = \frac{x \sin(\theta_c + \phi)}{v_1} + \frac{2z \cos \theta_c}{v_1}$$

On distance-time plot:

**slope** = \( \frac{\sin(\theta_c + \phi)}{v_1} = \frac{1}{v_{2d}} \)

**y-intercept** = \( \frac{2z \cos \theta_c}{v_1} \)

**Inverse of slope** \( (v_{2d}) \):

\( v_{2d} = \frac{v_1}{\sin(\theta_c + \phi)} \leq v_2 \)

**“Apparent velocity”**

→ steeper slope, therefore \( v_{2d} \) is less than real velocity
Travel time:

\[ t_u = \frac{x \sin(\theta_c - \phi)}{v_1} + \frac{2z' \cos \theta_c}{v_1} \]

Slope → updip velocity

\[ V_{2u} = \frac{V_1}{\sin(\theta_c - \phi)} \geq V_2 \]

“Apparent velocity”

→ shallower slope, therefore \( v_{2u} \) is greater than real velocity
Horizontal vs. dipping interface

No dip (3200 m/s)

10° dip (4450 m/s)

• forward profiles are the same, reverse profiles are different
• recognize a dipping layer by a change in slope of the arrival on forward and reverse profiles
• arrival times at ends of profile are equal ($t_{AD} = t_{DA}$) $\rightarrow$ RECIPROCITY
Determining the dip and velocity

- four observations: slope and y-intercept (two directions)
- three unknowns: $v_2$, $z$ and $\Phi$
- can now obtain a unique solution

$$V_{2d} = \frac{V_1}{\sin(\theta_c + \phi)}$$

$$V_{2u} = \frac{V_1}{\sin(\theta_c - \phi)}$$

$$\theta_c = \frac{1}{2} \left[ \sin^{-1} \left( \frac{V_1}{V_{2d}} \right) + \sin^{-1} \left( \frac{V_1}{V_{2u}} \right) \right]$$

$$\phi = \frac{1}{2} \left[ \sin^{-1} \left( \frac{V_1}{V_{2d}} \right) - \sin^{-1} \left( \frac{V_1}{V_{2u}} \right) \right]$$
Data analysis

1. Direct arrival: slope = $1/v_1$
2. Refracted arrivals:
   - downdip: slope = $1/v_{2d}$
   - updip: slope = $1/v_{2u}$
3. Calculate dip and $\theta_c$
   \[
   \theta_c = \frac{1}{2} \left[ \sin^{-1} \left( \frac{v_1}{v_{2d}} \right) + \sin^{-1} \left( \frac{v_1}{v_{2u}} \right) \right]
   \]
   \[
   \phi = \frac{1}{2} \left[ \sin^{-1} \left( \frac{v_1}{v_{2d}} \right) - \sin^{-1} \left( \frac{v_1}{v_{2u}} \right) \right]
   \]
4. Calculate $v_2$ : $v_2 = v_1 / \sin \theta_c$
Example – find $v_1$, $v_2$, and dip

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<th>$t_{ref}$ (s)</th>
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Non-planar interfaces

Equations for horizontal and dipping planar layers are nice
... but not applicable to many geological situations

- if the refraction arrival times are **not a straight line** on a time-distance plot → irregular (non-planar) interface

*(Kearey et al., 2002)*
Definition of delay time

TRAVEL TIME:

\[ t = t_{AB} + t_{BD} + t_{DE} \]

ANOTHER WAY TO THINK ABOUT IT:

\[ t = \text{time taken to travel a distance} \ x' \ \text{at velocity} \ v_2 \]

PLUS extra time to travel downward/upward through layer 1 at velocity \( v_1 \) over some distance unit

“DELAY TIME”
Single horizontal interface:

TRAVEL TIME:

\[ t = t_{AB} + t_{BC} + t_{CD} \]

\[ t = \frac{x}{v_2} + \frac{2z_1 \cos \theta_c}{v_1} \]

Or:

\[ t = \frac{x}{v_2} + \frac{z_1 \cos \theta_c}{v_1} + \frac{z_1 \cos \theta_c}{v_1} \]

Think of this as:

- travelling a distance \( x \) at velocity \( v_2 \)
- plus extra time to go from A-B and C-D at velocity \( v_1 \)

A-B term = shot delay time (\( \delta_{ts} \))

C-D term = detector delay time (\( \delta_{td} \))
DELAY TIMES:

At shot: $\delta_{ts}$ – extra time to travel AB compared to CB  
(longer path + slower velocity)

At detector: $\delta_{td}$ – extra time to travel DE compared to DF

TRAVEL TIME: $t = x'/v_2 + \delta_{ts} + \delta_{td}$

For layers with relatively low relief: $x' \approx x$

$\Rightarrow$ applicable to any interface
What is the delay time?

Shot delay time

\[
\delta_{ts} = \frac{AB}{v_1} - \frac{CB}{v_2} = \frac{z}{v_1 \cos \theta_c} - \frac{z \tan \theta_c}{v_2}
\]

If you know \(v_1\), \(v_2\), and \(\delta_{ts}\), you can calculate the depth of the interface below the shot \(\rightarrow\) **perpendicular distance**

\(\delta_{ts} = \frac{z \cos \theta_c}{v_1}\)

Similarly, the **detector delay time** is:

\[
\delta_{td} = \frac{z' \cos \theta_c}{v_1}
\]
For a horizontal interface:

z is the same for shot and detector, so the delay times are equal

\[
\delta_{td} = \delta_{ts} = \frac{z \cos \theta_c}{V_1}
\]

Travel time:

\[
t = \frac{x}{V_2} + \delta_{ts} + \delta_{td}
\]

\[
t = \frac{x}{V_2} + \frac{2z \cos \theta_c}{V_1}
\]

...as we saw before
Can use the delay times to determine the depth of the interface below the study area

- **one shot** → measure total travel time to each detector
  - can not separate $\delta_{ts}$ and $\delta_{td}$
- **two shots (forward and reverse)** → can find $\delta_{ts}$ and $\delta_{td}$
  - will give you $v_2$ and $z(x)$
Delay Time Method (plus-minus method)

Reciprocal time

depth (m)

S1

v₂

v₁

distance (m)

t_{S1-D}

t_{S1-S2}

t_{S2-D}

L

x
At detector D, there are three observations:

\( t_{S1D}, t_{S2D}, t_{S1S2} \)

Equations:

1. Forward shot:
   \[
   t_{S1D} = \frac{X}{V_2} + \delta_{S1} + \delta_d
   \]

2. Reverse shot:
   \[
   t_{S2D} = \frac{(L - X)}{V_2} + \delta_{S2} + \delta_d
   \]

3. Reciprocal time:
   \[
   t_{S1S2} = \frac{L}{V_2} + \delta_{S1} + \delta_{S2}
   \]
MINUS TERM (to get $v_2$)

(1) Forward shot: $t_{S1D} = \frac{x}{v_2} + \delta_{S1} + \delta_d$

(2) Reverse shot: $t_{S2D} = \frac{(L - x)}{v_2} + \delta_{S2} + \delta_d$

(1) – (2) will get rid of $\delta_d$

$$t_{S1D} - t_{S2D} = \frac{x}{v_2} + \delta_{S1} - \frac{L}{v_2} + \frac{x}{v_2} - \delta_{S2}$$

$$t_{S1D} - t_{S2D} = \frac{2x}{v_2} - \frac{L}{v_2} + \delta_{S1} - \delta_{S2}$$

$$t_{S1D} - t_{S2D} = \frac{2x}{v_2} + K$$

(equation of a line)

Plot of $t_{S1D}-t_{S2D}$ vs. $2x$ → straight line, slope $1/v_2$
PLUS TERM (to get $\delta_d$)

(1) Forward shot: \[ t_{S1D} = \frac{x}{v_2} + \delta_{S1} + \delta_d \]

(2) Reverse shot: \[ t_{S2D} = \frac{(L - x)}{v_2} + \delta_{S2} + \delta_d \]

(1) + (2): \[ t_{S1D} + t_{S2D} = \frac{L}{v_2} + \delta_{S1} + \delta_{S2} + 2\delta_d \]

Reciprocal time (eq’ n 3):

\[ t_{S1S2} = \frac{L}{v_2} + \delta_{S1} + \delta_{S2} \]

\[ t_{S1D} + t_{S2D} = t_{S1S2} + 2\delta_d \]

Re-arranging… \[ \delta_d = \frac{(t_{S1D} + t_{S2D} - t_{S1S2})}{2} \]
Delay Time Method (summary)

- Collect data in forward and reverse profiles
- For each detector, observe $t_{S1D}$ and $t_{S2D}$
- Also observe $t_{S1S2}$
- DIRECT WAVE $\rightarrow v_1$
- MINUS TERM ($t_{S1D} - t_{S2D}$)
- Plot $(t_{S1D} - t_{S2D})$ vs. $2x \rightarrow v_2$
- PLUS TERM ($t_{S1D} + t_{S2D}$)
- Detector delay time ($x$): $\delta_d = \frac{(t_{S1D} + t_{S2D} - t_{S1S2})}{2}$
- Interface depth: $\delta_d = \frac{z \cos \theta_c}{v_1}$ $\rightarrow$ Perpendicular depth to interface $\approx$ true depth for low relief (dip $< 10^\circ$)
Comments on delay time method

- **only use refracted arrivals** → straight line on MINUS TERM plot (lateral variations in velocity can also affect line)

- depth is the **perpendicular depth** to the interface ($\approx$ true depth for low relief)

- produces a **smoothing** of the interface

- **vertical scale** for depth plot → try to reduce vertical exaggeration so that the actual relief is properly represented