University of Alberta
Department of Mathematical & Statistical Sciences

Stochastic Processes
STAT 580-A1, 34806
Fall 2022
©Yaozhong Hu

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Office Hours:	Tuesdays and Thursdays 9:30am - 10:40am or by appointment
Lecture Room:	C E5-36
Lecture Time:	Tuesdays and Thursdays 11:00am - 12:20pm

Course Description:

This is a three credit course on some basic stochastic processes which is a mathematical object usually defined as a collection of random variables. The key mathematical ideas in this course are explained through some basic examples and facts. We will try to balance the two aspects between mathematical intuition and rigorous proofs or mathematical details.
This course plans to cover discrete time Markov chains with finite and countable state spaces; continuous time Markov chains through three examples (Poisson process, finite state space Markov chain and birth-and-death processes). We will also introduce martingales and associated maximal inequalities, Renewal theory and so on. We shall also briefly introduce Brownian motion, stochastic integration and some other topics in stochastic analysis, Ito's formula, Girsanov transformation, and the Feynman-Kac formula and so on.

Course Prerequisites:

The prerequisites to take this course are only a good calculus-based undergraduate course in probability and a course in linear algebra. Probability theory based on Measure theory such as STAT 571 is not necessarily required but will

be very much helpful.

Course Objectives and Expected Learning Outcomes:

Students are expected to understand the basic material of stochastic processes and know what is its goals and what is the mathematical tools it uses. stochastic processes can be a very theoretical mathematical branch. But it also has many immediate applications to various fields. Students are expected to have a solid background and analytic skills. This will also be necessary for their future study of some other fields such as statistics, engineering, mathematical finance, machine learning.

Lecture Schedule & Assigned Readings:

The pace of lectures will be adapted accordingly throughout the semester. The ambitious intention of the course is to cover chapter 1 to Section 9.7 of the book.

Required Learning Resources:

The main book to be used in this course is

Lawler, Gregory F.
Introduction to stochastic processes. Second edition.
Chapman & Hall/CRC, Boca Raton, FL, 2006. xiv+234
ISBN: 978-1-58488-651-8; 1-58488-651-X

Recommended or Optional Learning Resources:

There are many other nice books on general basic stochastic processes. Students are encouraged but not required to read one or some of them.

1. Hoel, Paul G.; Port, Sidney C.; Stone, Charles J. Introduction to stochastic processes. The Houghton Mifflin Series in Statistics. Houghton Mifflin Co., Boston, Mass., 1972. x+203 pp.

2. Borodin, Andrei N. Stochastic processes. Original Russian edition published by LAN Publishing, St. Petersburg, 2013. Probability and its Applications. Birkhäuser/Springer, Cham, 2017. xiv+626 pp. ISBN: 978-3-319-62309-2; 978-3-319-62310-8

3. Durrett, Richard Essentials of stochastic processes. Third edition. Springer Texts in Statistics. Springer, Cham, 2016. ix+275 pp. ISBN: 978-3-319-45613-3; 978-3-319-45614-0.

4. Brémaud, Pierre Fourier analysis and stochastic processes. Universitext. Springer, Cham, 2014. xiv+385 pp. ISBN: 978-3-319-09589-9; 978-3-319-09590-

5. Varadhan, S. R. S. Stochastic processes. Courant Lecture Notes in Mathematics, 16. Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2007. x+126 pp. ISBN: 978-0-8218-4085-6

6. Gikhman, Iosif I.; Skorokhod, Anatoli V. The theory of stochastic processes. I. Translated from the Russian by S. Kotz. Reprint of the 1974 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2004. viii+574 pp. ISBN: 3-540-20284-6
The theory of stochastic processes. II. Translated from the Russian by S. Kotz. Reprint of the 1975 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2004. viii+441 pp. ISBN: 3-540-20285-4
The theory of stochastic processes. III. Translated from the Russian by Samuel Kotz. Reprint of the 1974 edition. Classics in Mathematics. Springer, Berlin, 2007. x+387 pp. ISBN: 978-3-540-49940-4.

7. Ross, Sheldon M. Stochastic processes. Second edition. Wiley Series in Probability and Statistics: Probability and Statistics. John Wiley & Sons, Inc., New York, 1996. xvi+510 pp. ISBN: 0-471-12062-6

8. Rao, M. M. Stochastic processes: general theory. Mathematics and its Applications, 342. Kluwer Academic Publishers, Dordrecht, 1995. xii+623 pp. ISBN: 0-7923-3725-5.

9. Resnick, Sidney Adventures in stochastic processes. Birkhäuser Boston, Inc., Boston, MA, 1992. xii+626 pp. ISBN: 0-8176-3591-2.

10. Doob, J. L. Stochastic processes. Reprint of the 1953 original. Wiley Classics Library. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1990. viii+654 pp. ISBN: 0-471-52369-0.

11. Wong, Eugene; Hajek, Bruce Stochastic processes in engineering systems. Springer Texts in Electrical Engineering. Springer-Verlag, New York, 1985. xi+361 pp. ISBN: 0-387-96061-9.

12. Karlin, Samuel; Taylor, Howard M. A second course in stochastic processes. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York-London, 1981. xviii+542 pp. ISBN: 0-12-398650-8.

Grade Evaluation:

All midterm and final exams will be in the classroom where the lectures are held. Final exam time is decided by the university and will be announced when it decided.
The course mark will be calculated based on the following breakdown:

Course Component	**Weight**	**Date**
Assignments	30%	Due every Tuesday unless otherwise announced
Midterm 1	25%	Thursday October 6, 2022
Midterm 2	25%	Thursday November 3, 2022
Final Exam	20%	To be determined

*Note: The date of the final examination is set by the Registrar and takes precedence over the final examination date reported in this document. Students must verify this date on BearTracks when the Final Exam Schedule is posted.
The final letter grade will be determined from the course mark as follows:
Grades are unofficial until approved by the Department and/or Faculty offering the course.

Assignments:

(i) Homework will be assigned after each lecture.

(ii) Homework is submitted electronically through "Assign2". You can access to the eclass through https://eclass.srv.ualberta.ca using your ccid.

(iii) The electronic submission of homework is due on Monday at 23:00pm for the two assignments of the previous week unless otherwise stated.

Exam Format:
The format (not the content) of all midterm exams and final will look like

TOTAL: _____

Student Name : _____

Exam for Stat 580, Stochastic Processes
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Problem 1 A bowl contains twenty cherries, exactly fifteen of which have had their stones removed. A greedy pig eats five whole cherries, picked at random, without remarking on the presence or absence of stones. Subsequently, a cherry is picked randomly from the remaining fifteen.

(a) What is the probability that this cherry contains a stone?

(b) Given that this cherry contains a stone, what is the probability that the pig consumed at least one stone?

Problem 2 A purse contains 12 quarters and 2 pennies. All the coins are to be drawn, one at a time, without replacement. You keep all the quarters that are drawn between the two pennies, which is denoted by X (The total number of quarters between the two pennies). Find $\mathbb{E}(X)$ and $\text{var}(X)$.

Exam Aids:

All exams are close book exam. This means it is not allowed to use textbook or any other books. However, Students are allowed to use all kinds of calculators (but no laptop or cellphone can be used). They are also allowed to use one normal size (8×11) page of note which can be written on both sides.

Excused Absence Where the Cause is Religious Belief:

For an excused absence where the cause is religious belief, a student must contact the instructor(s) within two weeks of the start of Fall or Winter classes to request accommodation for the term (including the final exam, where relevant).

Instructors may request adequate documentation to substantiate the student request.

Missed Term Work

A student who cannot write a midterm To apply for an excused absence, a student must inform the instructor within two working days following the scheduled date of the term work or term exam missed, or as soon as the student is able, having regard to the circumstances underlying the absence. In all cases, instructors may request adequate documentation to substantiate the reason for the absence at their discretion.
An excused absence is a privilege and not a right; there is no guarantee that an absence will be excused. Misrepresentation of Facts to gain an excused absence is a serious breach of the *Code of Student Behaviour*.

Missed Final Examination:

A student who cannot write the final examination due to incapacitating illness, severe domestic affliction or other compelling reasons can apply for a deferred final examination. Students who failed at the start of term to request exam accommodations for religious beliefs are expected to follow the normal deferred final examination process. Such an application must be made to the student's Faculty office within two working days of the missed examination and must be supported by a Statutory Declaration (in lieu of a medical statement form) or other appropriate documentation (Calendar section 23.5.6). Deferred examinations are a privilege and not a right; there is no guarantee that a deferred examination will be granted. Misrepresentation of Facts to gain a deferred examination is a serious breach of the *Code of Student Behaviour*.

Any deferred final examinations are scheduled as follows:
All approved Deferred Final Exams for the upcoming Fall Term 2022 will be held on Saturday 7 January 2023.

Date: Saturday 7 January 2023
Time: 9:00am
Location: TBA

Re-examination:

A student who writes the final examination and fails the course may apply for a re-examination. Re-examinations are rarely granted in the Faculty of Science. These exams are governed by University (Calendar section 23.5.5) and Faculty of Science Regulations (Calendar section 192.5.3). Misrepresentation of Facts to gain a re-examination is a serious breach of the *Code of Student Behaviour*.

STUDENT RESPONSIBILITIES

Academic Integrity:

The University of Alberta is committed to the highest standards of academic integrity and honesty. Students are expected to be familiar with these standards regarding academic honesty and to uphold the policies of the University in this respect. Students are particularly urged to familiarize themselves with the provisions of the *Code of Student Behaviour* (online at www.governance.ualberta.ca) and avoid any behaviour which could potentially result in suspicions of cheating, plagiarism, misrepresentation of facts and/or participation in an offence. Academic dishonesty is a serious offence and can result in suspension or expulsion from the University.
All forms of dishonesty are unacceptable at the University. Any offense will be reported to the Senior Associate Dean of Science who will determine the disciplinary action to be taken. Cheating, plagiarism and misrepresentation of facts are serious offenses. Anyone who engages in these practices will receive at minimum a grade of zero for the exam or paper in question and no opportunity will be given to replace the grade or redistribute the weights. As well, in the Faculty of Science the sanction for **cheating** on any examination will include **a disciplinary failing grade** (NO EXCEPTIONS) and senior students should expect a period of suspension or expulsion from the University of Alberta.

Collaboration on Assignments:

Students should work alone for each of their assignments. But they can collaborate to work on the homework assignments. However, the midterm exams are not collaborative and close book.

Exams:

Students will not be allowed to begin an examination after it has been in progress for 30 minutes. Students must remain in the exam room until at least 30 minutes has elapsed. Electronic equipment cannot be brought into examination rooms.

Cell Phones:

Cell phones are to be turned off during lectures, labs and seminars. Cell phones are not to be brought to exams.

Audio or Video Recording:

Audio or video recording, digital or otherwise, of lectures, labs, seminars or any other teaching environment by students is allowed only with the prior written consent of the instructor or as a part of an approved accommodation plan. Student or instructor content, digital or otherwise, created and/or used within the context of the course is to be used solely for personal study, and is not to be

used or distributed for any other purpose without prior written consent from the content author(s).

Students Eligible for Accessibility-Related Accommodations (students registered with Student Accessibility Services – SAS):

Eligible students have both rights and responsibilities with regard to accessibility-related accommodations. Consequently, scheduling exam accommodations in accordance with SAS deadlines and procedures is essential. Please note adherence to procedures and deadlines is required for U of A to provide accommodations. Contact SAS (www.ssds.ualberta.ca) for further information.

Student Success Centre:

Students who require additional help in developing strategies for better time management, study skills, or examination skills should contact the Student Success Centre (2-300 Students' Union Building).

Decima Robinson Support Centre for Mathematical & Statistical Sciences:

Students who require additional help with assignments or have questions about the course material in general are encouraged to visit the Decima Robinson Support Centre (528 Central Academic Building). Graduate students will be available to provide one-on-one help. In order to get maximum help during each visit, students are asked to be specific about the problem with which they are seeking help. The Centre is open Monday to Friday, 9:00–15:00.

Policy about course outlines can be found in section 23.4(2) of the University Calendar.

Important dates of Fall 2022

September 1	Thursday	First day of class	
September 5	Monday	Labor day	University buildings closed
September 30	Friday	in honor of Truth and Reconciliation	University buildings closed
October 10	Monday	Thanksgiving Day	University buildings closed
November 7-10	Whole week	Reading week	No class
November 11	Friday	Remembrance Day	University buildings closed
December 8	Thursday	Last day of the class	
December 12 - 21		Final exams	

Disclaimer:

Any typographical errors in this Course Outline are subject to change and will be announced in class.

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