Temporal Methods for Eye Movement Analysis

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Abstract

Eye movements unfold over time, yet many classic measures collapse that time course into static counts and durations. This chapter introduces methods that keep the temporal structure of looking behaviour intact. It has four parts. First, we review traditional characterisations of eye movements, namely measures of fixations and saccades and area-of-interest analyses, and explain what each can and cannot reveal. Second, we present methods for analysing the temporal patterns of fixations and saccades and, in particular, the changing relationship between the eyes and the head, in both the short term (the sub-second range around a saccade) and the long term (seconds to minutes), showing across several converging methods that during passive scene viewing the eyes typically lead the head. Third, we introduce recurrence quantification analysis (RQA) and show step by step how recurrence relates to fixation patterns, how its measures (recurrence, determinism, laminarity and centre of recurrence mass) are computed and interpreted, and how they discriminate viewing conditions. Finally, we extend the approach to cross-recurrence quantification analysis (CRQA), which characterises the temporal relationship between two event sequences, such as the eyes and the head or two observers viewing the same scene. Throughout, we aim to give beginning researchers both the conceptual grounding and the concrete, worked steps needed to apply these methods to their own data.

Abbreviations

The following abbreviations are used in this chapter:

AOI	Area of interest
CLUST	Clustering
CORM	Center of recurrence mass
CRQA	Cross-recurrence quantification analysis
DCRP	Diagonal cross-recurrence profile
DET	Determinism
hLAM	Horizontal laminarity
hTT	Horizontal trapping time
IDT	Identification based on dispersion threshold
LAM	Laminarity
MANOVA	Multivariate analysis of variance
REC	Recurrence
RQA	Recurrence quantification analysis
vLAM	Vertical laminarity
VOR	Vestibulo-ocular reflex
vTT	Vertical trapping time

1 Introduction and Learning Objectives

This chapter introduces methods for the temporal analysis of eye movements, with an emphasis on preserving the temporal structure of looking behaviour. We begin by reviewing traditional eye movement measures before introducing methods for analysing temporal patterns of eye and head movements, including recurrence quantification analysis (RQA) and cross-recurrence quantification analysis (CRQA). Throughout, we focus on the concepts underlying these approaches as well as the practical steps required to apply them to eye-tracking data.

With the information provided in this chapter, readers will be able to understand

- The different methods for characterizing eye movements in the temporal domain
- The different methods for characterizing the temporal relation between eye and head movements
- The fundamentals of recurrence quantification analysis for characterizing eye movement dynamics
- The fundamentals of cross-recurrence quantification analysis for characterizing the relationship between different event sequences.

2 Historical Annotations

We first present fundamental measures of eye movements, in particular the detection of fixations and saccades. Then we show how fixations can be related to the content of scenes using an area-of-interest analysis and discuss simple fixation transitions using Markov models.

2.1 Fundamental Measures of Eye Movements

From a psychological perspective, the most important eye movement events are fixations and saccades (see also Alexander & Martinez-Conde, Chapter 3 in this volume). A saccade is a rapid, ballistic motion of the eyes from one point to another, while a fixation is a brief (around 200 millisecond) pause between saccades during which most visual information is gleaned.

In the analysis of eye behavior, we focus on the detection of fixations and saccades, but there are other ocular events, such as smooth pursuit, micro-saccades, and blinks that we do not cover here (see for example, Andersson et al., 2017; Holmqvist et al., 2011; Hooge et al., 2018, 2022). There are two fundamentally different approaches to eye movement analysis. The first approach starts with the detection of fixations, and saccades are defined as differences between successive fixations. The second approach starts with the detection of saccades, and fixations are defined as stable points between saccades.

In the first approach, a popular method for the identification of fixations is the Dispersion-Threshold (IDT) algorithm (Komogortsev et al., 2010; Salvucci & Goldberg, 2000), which assumes that the dispersion of gaze points during a fixation is relatively small (e.g., 2.5 - 3°) and that the duration of fixations exceeds a minimum duration (e.g., 80 *ms*). Specifically, the algorithm proceeds as follows:

1. Initialize a window of gaze points to cover duration threshold. Drop the data points if the dispersion of the gaze points exceeds the dispersion threshold.
2. Add further gaze points to the window as long as the dispersion of the gaze points does not exceed the dispersion threshold.
3. Define the fixation position as the centroid of gaze points.

4. Remove the gaze points of the fixation and start again from step 1.

The second approach begins with the detection of saccades, and fixations are defined as stable points between saccades. The detection of saccades is based on the assumption that motion above a velocity threshold is assumed to be part of a saccade. Specifically, the algorithm proceeds as follows:

1. Calculate the gaze velocities between all successive gaze points.
2. Detect peak velocities (which are assumed to define the middle of a saccade).
3. Add velocities immediately before the peaks and immediately after the peaks as long as they exceed a velocity threshold. Velocities below that threshold are assumed to be part of a fixation.
4. Peak velocities must be below a certain limit to exclude artefacts, such as blinks.
5. Finally, fixations are defined as the relatively stable positions between saccades.

In contrast to eye movements, head movements are usually very smooth and do not permit identifying stable points from the raw data. This may be attributable largely to the difference in mass between the eyes and the head. For this reason, we define a head position as the average head direction during a fixation. Head shifts are then defined as the directional differences between successive head positions. The reader is encouraged to consult further sources of information on event detection in eye movement analysis, including, for example, those by Andersson et al. (2017), Holmqvist et al. (2011), Hooge et al. (2018, 2022) and Nyström & Holmqvist (2010).

In the following sections, head positions are frequently referred to, as well as eye positions (fixations), with the latter defined relative to the head position (eyes-in-head) or relative to the world (eyes-in-world or gaze). It is crucial that the reader keeps these terms in mind when reading this chapter.

2.2 Area of Interest Analysis of Eye Movements

A major approach for relating fixations to the content of visual scenes is based on defining areas of interest (AOIs). In a visual search display, one might define AOIs for each target and each distractor. In images of more complex scenes, for example, in work with social scene stimuli (e.g., Birmingham et al., 2008a, 2008b), the AOIs can include the bodies, faces and eye regions of the persons in the scenes (see Fig. 1).

In a static AOI analysis, we can use the following measures: absolute number of fixations in each AOI, the proportion of fixations in each AOI relative to the AOI area, or the average fixation durations (dwell time) in each AOI. In a dynamic AOI analysis, one can, for example, analyze the transitions between AOIs.

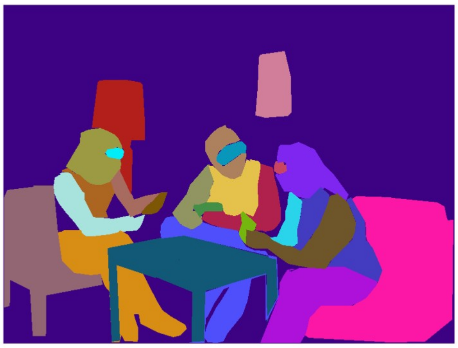


Fig. 1. Areas of interest of the social scene. The color of each area has been chosen randomly.

2.2.1 Advantages of AOI analyses

AOI analyses have proven useful for assessing eye movement patterns with simple and complex images. Fixation frequencies or dwell times on different AOIs can be compared between different experimental groups or conditions and even between different stimuli provided they have the same image structure, i.e. they contain the same types of regions (for example, people, faces or eyes in social scenes). Assuming a uniform distribution of fixations across images, the number of fixations or dwell time in an AOI is proportional to the AOI area. If fixation counts or durations need to be compared between different AOIs, then it is advisable to normalize the number or duration of fixations by the AOI area and obtain the number of fixations or dwell time per unit area in each AOI.

The definition of AOIs is relatively straight-forward with static images and many eye tracking systems provide software for defining static AOIs. With dynamic stimuli, the definition of AOIs becomes more difficult. Example of such cases include movies and real-world experiments, where participants are wearing a mobile eye tracker and fixations are projected onto the recorded scenes by the eye tracking software. This is illustrated in Fig. 2, where the rider changes position and size in every movie frame, leading to an enormous effort for defining the AOI rider for long frame sequences. One possible solution to this problem relies on so-called keyframing. In this technique, the relevant AOIs are defined only in certain frames (key frames), and the computer then interpolates the AOI boundaries for all the frames in between (Igarashi et al., 2005). Alternatively, in the special case where an AOI analysis is limited to persons, one can rely on more recent methods for the detection of faces or persons (e.g., Deng et al., 2020; Lin et al., 2021) for the definition of AOIs. Even if these methods are not error-free they can be used in a semi-automatic approach and substantially reduce the coding effort for dynamic AOI analyses.

In mobile eye tracking it is also possible to utilize markers (unique patterns printed on a piece of paper) to define regions of interest in the real environment. These markers can then be used to translate world-view fixation coordinates to fixation coordinates based on the area around a marker instead (e.g., Krassner et al., 2014; Haraped et al., 2025).

Fig. 2. Three frames from the opening scene of the movie “The Good, the Bad and the Ugly”. The size and location of the rider change in each frame, increasing the effort of defining areas of interest.

2.2.2 Disadvantages of AOI analyses

The AOI analysis is useful for the comparison of fixation patterns for identical or structurally similar stimuli with a common set of elements. It can, however, be difficult to use AOIs to derive meaningful results if highly diverse stimuli are being used, e.g., mixtures of social scenes, landscapes and artificial stimuli. In this case, there is no obvious way of comparing AOIs across the set of stimuli. Furthermore, when hand coding is involved, they are vulnerable to systematic biases (Dawson et al., 2021).

2.3 Markov Analysis of Eye Movements

Temporal eye movement analysis is concerned with the temporal sequence of fixations and saccades as they unfold over time (i.e., the scanpath). In the context of AOI analysis, it is concerned with the transitions between different AOIs and typically focuses on AOI transitions matrices. For example, with a set of stimuli derived from an image of two people sitting at a table (see Fig. 1), the regions head1, torso1, head2, torso2, table, and chairs, could yield a transition matrix between the AOI regions as shown in Table 1.

	head1	torso1	head2	torso2	chairs	table
head1	0.1	0.4	0.3	0.1	0.1	0.0
torso1	0.2	0.2	0.1	0.2	0.2	0.1
head2	0.3	0.1	0.3	0.3	0.0	0.0
torso2	0.1	0.1	0.4	0.2	0.1	0.1
chairs	0.1	0.3	0.1	0.3	0.1	0.1
table	0.3	0.1	0.3	0.1	0.2	0.0

Table 1: Example of a transition matrix for six AOI regions

Each row in Table 1 indicates the probability of saccades from the AOI on the left to each of the AOIs, with each row of probabilities adding up to 1. Note that these are conditional probabilities: each entry is the probability that the next fixation falls in a given AOI, given the current AOI, computed over this set of AOIs. The values therefore depend on the set of AOIs over which the conditional probabilities are computed; consequently, adding or removing an AOI changes the probabilities. For example, if a fixation at time *t* is in the region head2, then the next fixation at time *t+1* is in the region head1 with a probability of 0.3, in the region torso1 with a probability of 0.1, and so on. The AOI transitions in Table 1 characterize the dynamic sequence of eye movements for a given experimental condition, and comparisons to other conditions can be made by assessing the similarity of the transition matrices. It is important to point out, however, that such transition matrices capture only the overall characteristics of the transitions, and not, for example, their change over time.

In specific applications, the transition matrices have been modeled using Markov and hidden Markov models (Stark and Ellis, 1981; Holmqvist et al., 2011; Boccignone & Ferraro, 2014), for example, in face recognition (Chuk et al., 2014). As pointed out earlier, such AOI-based analyses are feasible only when the class of stimuli is sufficiently restricted, e.g., contain a common set of regions. Otherwise, the transition matrices cannot be compared across different stimuli.

With the Markov model, we have presented a simple model of transitions between AOIs while looking at images or scenes. Extensions of this model are described in two other chapters of this volume. Boccignone et al. (Chapter 12) model gaze shift patterns using Markov Processes, the Wiener process, and related random walks models. Then they present expanded models rooted in statistical physics for analyzing and modelling eye movements in terms of non-Gaussian random walks and modern foraging theory, and finally using statistical machine learning techniques. Foulsham (Chapter 6) first presents spatial analyses of fixation patterns based on saliency and saliency maps, and then he considers analyses of sequential fixations with scanpaths and discusses the different methods for comparing scanpaths across different conditions.

3 Temporal Analysis of Eye Movements

The temporal analysis of eye movements is concerned with the dynamic aspects of eye movements, the temporal characteristics of fixations and saccades, and the temporal sequence of fixations and saccades as they unfold over time. Here we present methods for relating eye movements to head movements and discuss in detail how to relate them in the short term and in the long term.

3.1 Temporal Profile of Fixations and Saccades

Popular temporal measures of eye movements include the duration of fixations (sometimes referred to as inter-saccade interval) and the duration of saccades (sometimes referred to as inter-fixation interval). Fixation and saccade durations for one of our studies (Bischof et al., 2024), based on approximately 65,000 measurements, are shown in Fig. 3. Note that there are no fixation durations below 80 *ms*, which is due to the fact that we used the IDT algorithm (see Section 2.1) with a minimum duration of 80 *ms*.

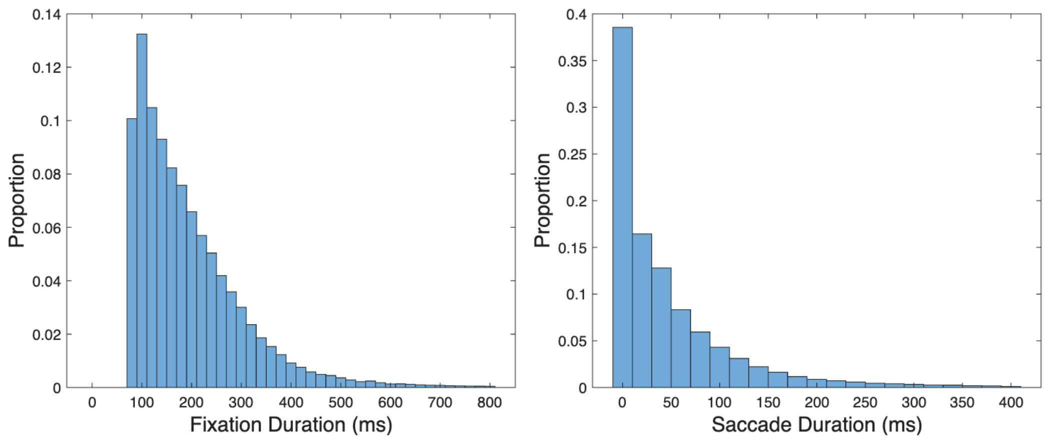


Fig. 3. Duration histograms of fixations and saccades. Note the minimum fixation duration of 80 *ms* due to the IDT algorithm.

Apart from these two measures, one can find many different temporal measures related to fixations and saccades, including, for example, the time to the first fixation after onset of a stimulus, the duration of the first fixation, the average duration of all fixations, the fixation rate (the number of fixations per second), the saccade rate (the number of saccades per second), and more (see Rigas et al., 2018, for a review).

3.2 Temporal Relation between Eyes and Head

Much of what we know about visual attention and eye movement control is derived from studies that require people to look at images presented on a computer monitor while their head is held steady. There is growing recognition, however, that normal looking behaviour involves the movement of the eyes, head and body (e.g., Bischof et al., 2024; Foulsham et al. 2011; Foulsham & Kingstone, 2017; Hooge et al., 2019; Kingstone et al., 2008; Land & Hayhoe, 2001; Land & Tatler, 2009; Risko et al., 2016). As recently noted by Mehrotra et al. (2025), while these and other studies have addressed foundational questions about the complex dynamics between eyes and head (and body) in fields of view that extend beyond the standard computer monitor (see Anderson et al., 2023, for a recent review) this previous research has largely been concerned either with the spatio-temporal coordination of eyes and head during simple gaze shifts (e.g., Freedman, 2008; Land, 2004; Sidenmark & Gellersen, 2019) or a consideration of the eyes positioned within the head during active navigation in complex real world settings that are hard to control (e.g., Einhäuser et al., 2009; Foulsham et al., 2011; Pelz et al., 2001). These studies allowed for the precise calculation and differentiation of how different movement systems contribute to overall attentional control.

In the following sections, we present methods for analyzing the temporal relation between eyes and head. We distinguish between temporal coordination in the short term, i.e., in a sub-second range, and in the long term, i.e., in time intervals between several seconds to minutes.

3.2.1 Short-term Temporal Relation between Eyes and Head

We first analyze the temporal relation between eyes and head in the short term, i.e., in a sub-second range. In other words, we analyze how the distance between eye position relative to the head, and head position, varies over time, aligned to the start of a saccade. Figure 4 shows that eyes-in-head eccentricity varies systematically in temporal relation to saccade initiation. Eyes-in-head eccentricity is the angular offset between the eye-in-head direction and the straight-ahead head direction; it is zero when the eyes point straight ahead in the head and grows as the eyes rotate away from that axis. Before the start of a saccade, during a fixation, eyes-in-head eccentricity diminishes gradually as the trailing head catches up to the eye, reaching a minimum of about 17.5° at the beginning of a saccade, followed by a rapid increase. The minimum eccentricity varies with participants and with studies, but the shape of the curve remains pretty much the same: An analysis by participants revealed that all participants showed the same effect, with the minimum eccentricity (in a range of approximately 11° to 22.5°) coinciding with saccade starts. We call this systematic dip in eyes-in-head eccentricity at saccade onset, followed by a sharp rise, the temporal eccentricity effect. Shown in Fig. 4, it can be explained as follows: Head movements are smooth and follow the eyes with a lag of about one fixation. The eyes jump in saccades and stay more or less locked in place during fixations. Hence the head has time to catch up somewhat with the eyes during a fixation, until the eyes jump in a saccade to the next position.

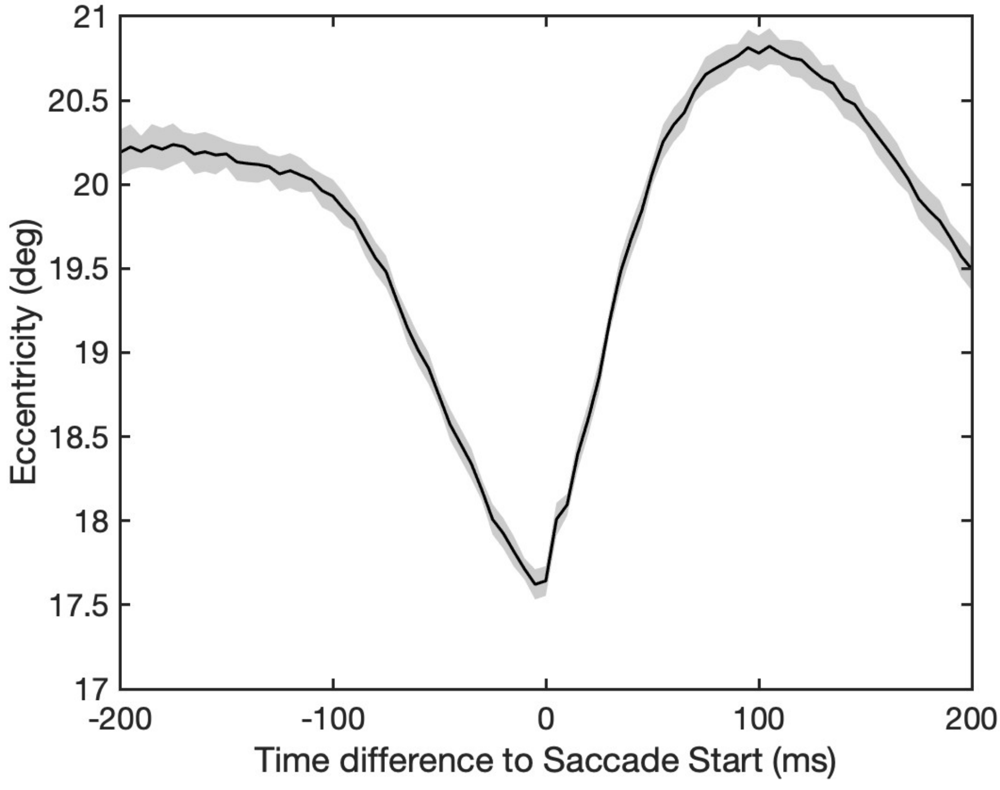


Fig. 4 Average eyes-in-head eccentricity in relation to the saccade starts, with the grey band indicating its 95% confidence interval. Eccentricity diminishes during

fixations, reaching a minimum just when a new saccade is initiated (at time difference 0), followed by a rapid increase.

Note that the finding above should not be confused with the vestibulo-ocular reflex (VOR, Barnes, 1979; Laurutis & Robinson 1986). This reflexive mechanism moves the eyes in the opposite direction of the head movement, in order to stabilize the perceptual input. Figure 5 shows the angle between head direction and eye direction for different ranges of head velocities. For head velocities < 200 °/s, most head directions and eye directions are aligned, but for head velocities > 300 °/s, eye directions are opposite to the head direction, with a gradual transition in the range from about 200 °/s to 300 °/s. The latter effect is due to the VOR. The results show that the VOR occurs mostly for relatively large head velocities.

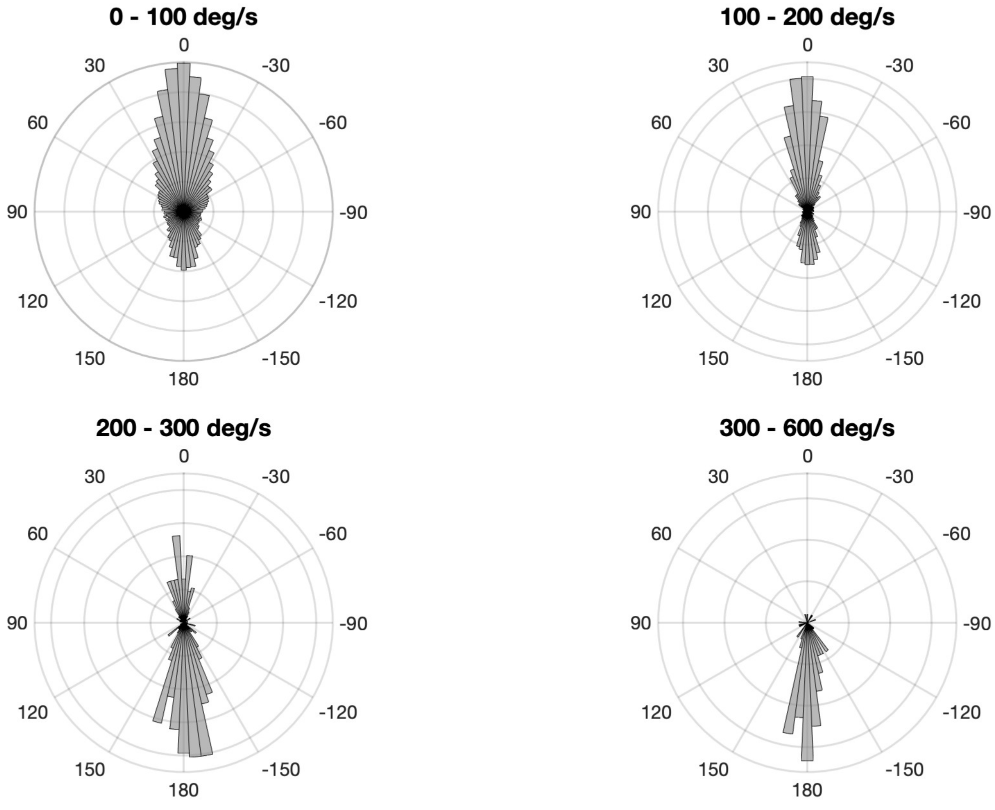


Fig. 5. Angles between head direction and eye direction for different ranges of head velocities. For velocities < 200 ° / sec, most head directions and eye directions are aligned, but for head velocities > 200 ° / sec, most eye directions are opposite to the head direction, showing the vestibulo-ocular reflex.

3.2.2 Long-term Temporal Relation between Eyes and Head

One critical type of information is the long-term spatio-temporal relationship between the eyes and head, as the extent to which eye movements lead or follow head movements may have cognitive consequences. That is, it has long been thought that the relative timing of eye and head movements indicates whether attentional selection is reflexive or volitional – see Solman and Kingstone (2014); Solman et al. (2017); Zangemeister & Stark (1982); Doshi & Trivedi (2012); Freedman (2008). In the following sections, we present several methods for determining the spatio-temporal relation between eyes and head during the passive viewing of scenes: the minimum distance method (Bischof et al., 2020), the direction method (Bischof et al., 2024), and the tracking method (Bischof et al., 2024). In addition, we report the average distance method proposed by Kangas et al. (2022). Each of these methods measures the temporal relation between eyes and head, i.e., they determine whether the eyes are leading the head or vice-versa.

3.2.2.1 Direction Method

This method determines the relation between eyes and head based on directional information, as illustrated in Fig 6. Figure 6a shows a series of fixations in black and a series of head positions in white. The black dots $g_1, g_2, \ldots$ show the position of successive fixations and the white dots $h_1, h_2, \ldots$ show the head averages during the fixations. The grey angles indicate the angle between the lines $g_i - h_i$ and the lines connecting successive head positions $h_i - h_{i+1}$, and the dashed line indicates the smooth movement of the head positions. If the head lags behind the eyes then the eye positions (fixations) must be more or less ahead of the head positions. This is illustrated in the upper panel of Fig. 6A. In contrast, if the head leads the eyes then the eye positions (fixations) must be more or less behind the head positions. This is illustrated in the lower panel of Fig. 6A.

The upper panel of Fig. 6A shows examples of the eye leading head, with the distribution of angles centered around 0°, as shown in the upper panel of Fig. 6B. In contrast, the lower panel of Fig. 6A shows examples of the eye trailing head, with the distribution of angles centered around ±180°, as shown in the lower panel of Fig. 6B. In summary, the angles between lines $g_i - h_i$ and $h_i - h_{i+1}$ are centered around 0° if the eyes are leading head and centered around ±180° if the eyes are lagging behind head. The empirical histogram of the angles is shown in Fig. 6C, with a majority of cases centered around 0°, indicating that the eyes are leading head. It should be added that the direction method cannot produce a temporal estimate of the eyes-head delay and only indicates the extent to which the eyes are leading or lagging the head. In summary, the Direction Method indicates that during passive viewing of scenes, the eyes typically lead the head.

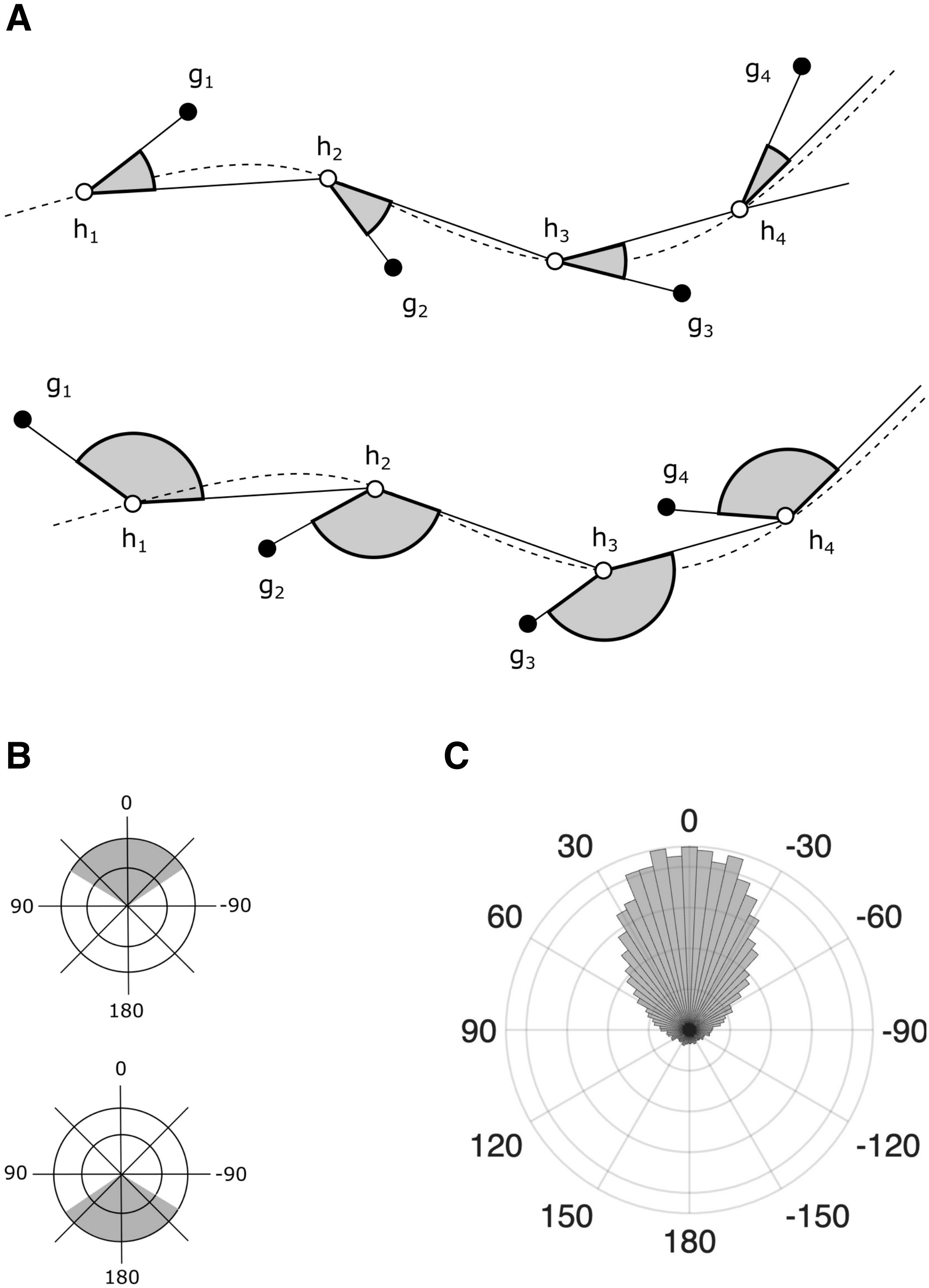


Fig. 6. **A** The black dots g_1, g_2, … show the position of successive fixations and the white dots h_1, h_2, … show the head positions during each fixation. The grey angles indicate the angle between the lines g_i - h_i and the lines connecting successive head positions $h_i - h_{i+1}$, and the dashed line indicates the smooth movement of the head

positions. The small angles in the upper sketch indicate that eye is leading head, whereas the large angles in the lower sketch indicate that eyes are trailing head. **B** If eyes are leading head, then the gray angles are on average small, as indicated by the upper sketch of a polar histogram. If eyes are lagging behind head, then the grey angles are on average large, as indicated by the lower sketch of a polar histogram. **C** Empirical polar histogram.

3.2.2.2 Minimum-Distance Method

This method determines whether the eyes or the head move first during scene viewing by comparing the location of each fixation with the average head positions surrounding it. Specifically, for each fixation g_i, the method examines the average head positions h_j recorded both before and after that fixation and identifies the head position h_{min} that lies closest to g_i. If h_{min} occurs after the fixation (a positive lag), the eyes reached that location first and the head followed; if h_{min} occurs before the fixation (a negative lag), the head moved first and the eyes followed.

One can see the details of how this analysis is done in Fig. 7A. The black and grey points indicate fixations, the dashed line indicates the smooth movement of head position, the white points indicate average head positions during each fixation, and the black lines show the distance between fixations and average head positions. As noted, the minimum distance method determines, for each fixation g_i, the closest average head position h. In this example, h_{i+1} is closest to g_i, indicating that the eyes are leading head by a lag of about one fixation. The histogram of eyes-head lags (Fig. 7B) shows that most lags are positive, that is, eyes are leading head in most cases. An analysis of negative lags showed that they occur exclusively when the head is moving very slowly. The results show that the lag peak is around one fixation, suggesting that the eye leads head by one fixation or approximately 200 *ms*. In summary, the Minimum Distance method indicates that during passive viewing of scenes, the eyes lead the head.

A

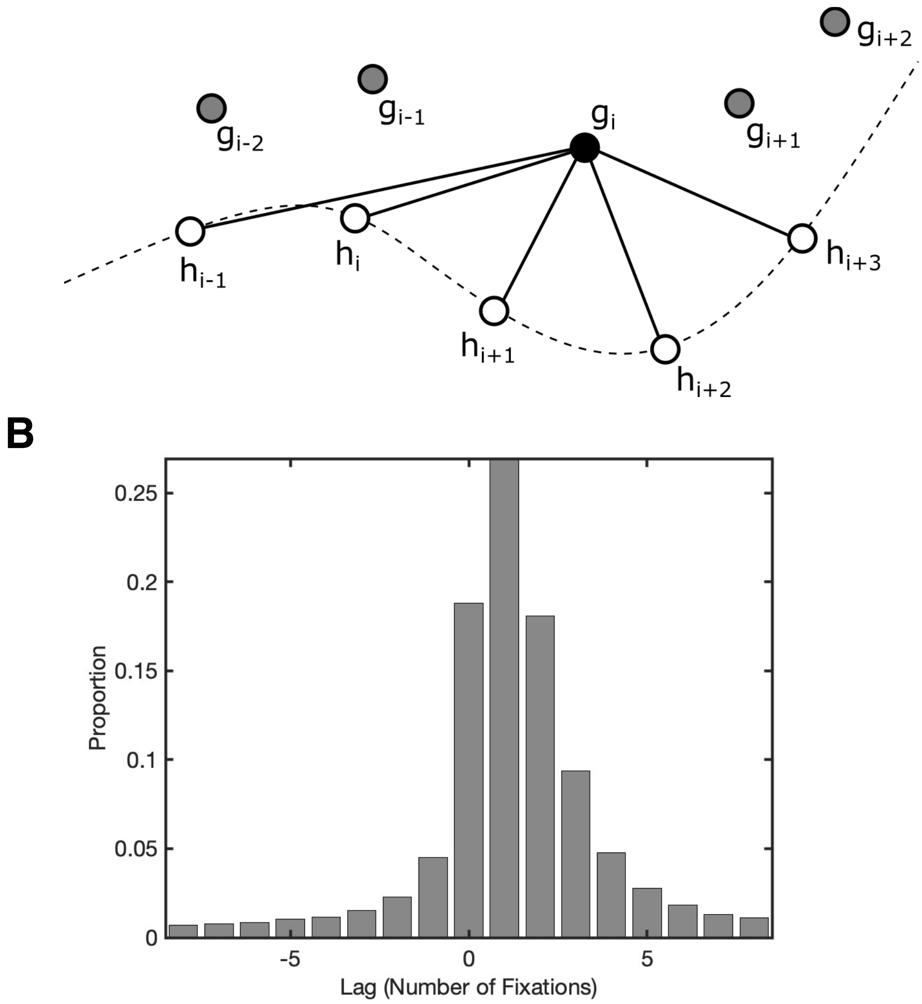


Fig 7. **A** The black dot shows the position of a fixation g_i, the grey dots g_{i-2}, g_{i-1}, g_{i+1}, g_{i+2} show the position of fixations preceding and following g_i, and the white dots h_{i-1}, h_i, … show the head averages during the fixations. The dashed line indicates the smooth movement of the head positions. The black lines show the distances between the fixations g_i and the head positions. In this case, the distance $g_i - h_{i+1}$ is minimal, indicating the gaze is leading head at this time point. **B** Histogram of gaze-head lags with the lag expressed in number of fixations. If gaze has a positive lag, then gaze is leading head, and if it has a negative lag, then gaze is trailing head. The results show that the eyes are leading head in most cases.

3.2.2.3 Average-distance Method

A method related to our minimum-distance method was proposed by Kangas et al. (2022). It estimates the eyes-head delay based on the minimum distance between fixations and head, averaged over participants and images (see Kangas et al., 2022;

Figure 5 and Equation 1). The results of this method with the data from one of our studies (Bischof et al., 2024) are shown in Fig. 8 and indicate an average delay of approximately 170 *ms* between eyes and head. The results confirm the positive lag between eyes and head. In summary, the Average Distance method indicates that during passive viewing of scenes, the eyes lead the head.

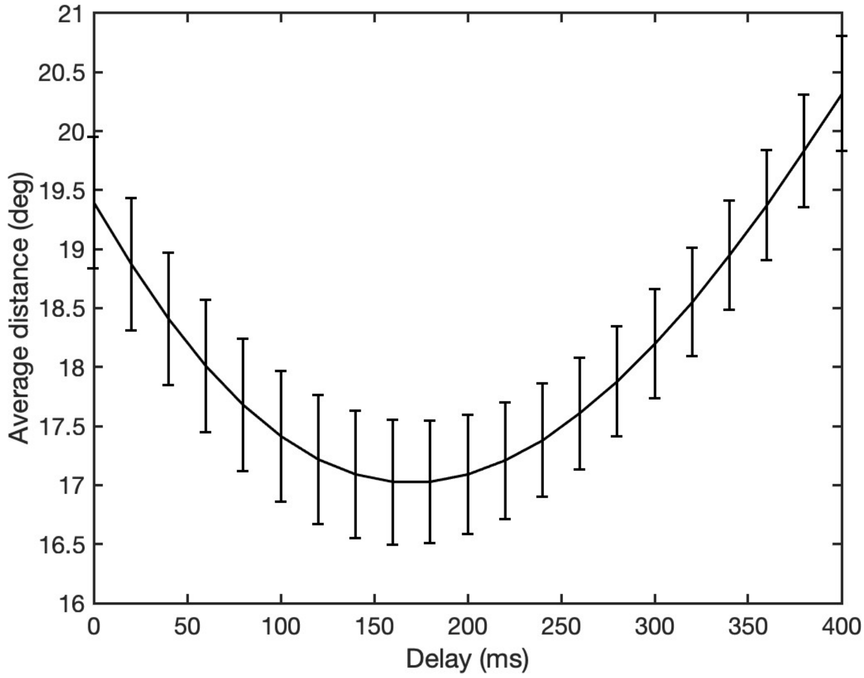


Fig. 8. The curve shows the Kangas et al. (2022) distance curve for Bischof et al.'s (2024) study, averaged over participants and scenes, with the bars indicating ± 1 standard error. The minimum is reached at a delay of about 170 *ms*, indicating that the eyes lead the head.

3.2.2.4 Tracking Method

The fourth method is based on estimating how long it takes the head to move to the head position closest to a fixation. In Fig. 9A, the black dots $g_1, g_2, \ldots$ show the position of successive fixations and the white dots $h_1, h_2, \ldots$ show the head averages during the fixations. The dashed line indicates the smooth movement of the head positions. The black lines are obtained by projecting the lines g_i - h_i onto the continuous lines of head positions, and one can measure how long (in *ms*) it takes the head to reach the end position of the projected line. In this example, g_1, g_2, and g_4 are leading head while g_3 is trailing head. Fig. 9B shows the histogram of eyes-head lags (in *ms*). Most of the values are positive, indicating that eyes are leading head, and only a small proportion of cases indicate that the eyes are trailing head. In summary, the Tracking method indicates that during passive viewing of scenes, the eyes typically lead the head.

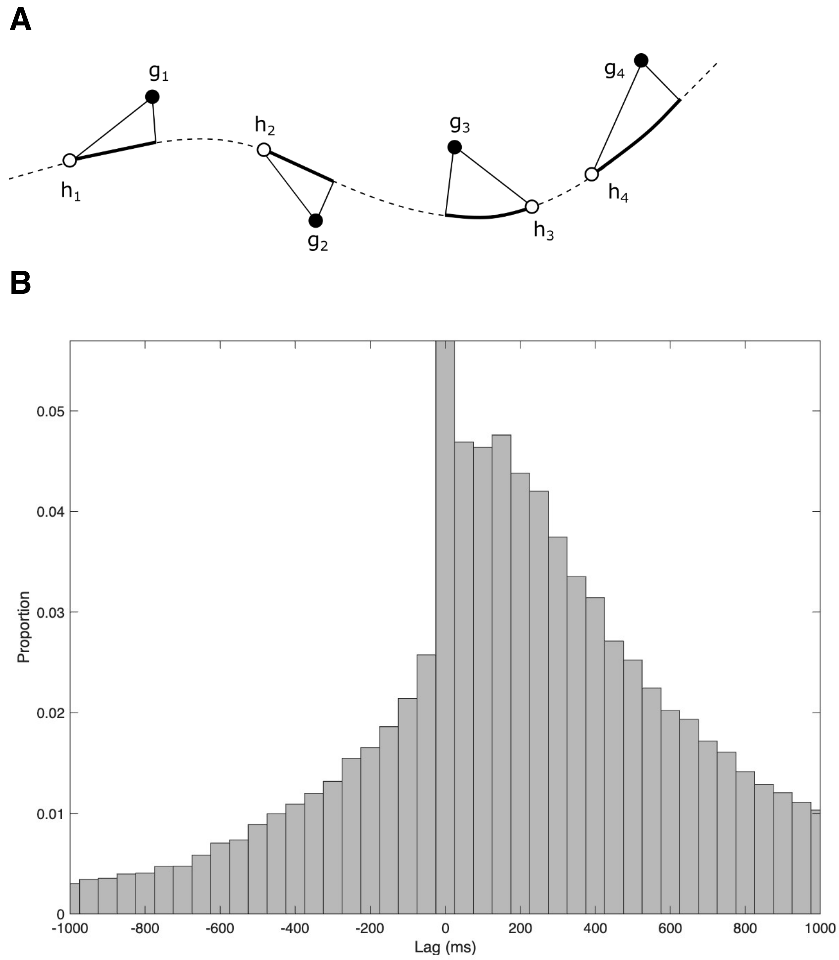


Fig. 9. A The black dots g_1, g_2, … show the position of successive fixations and the white dots h_1, h_2, … show the head averages during each fixation. The dashed line indicates the smooth movement of the head positions. The black lines are obtained by projecting the lines g_i - h_i onto the continuous lines of head positions. In this example, g_1, g_2, and g_4 are leading head while g_3 is trailing head. **B** Histogram of eye-head lags (in *ms*). Most of the values are positive, indicating that the eyes are leading head.

Note that all methods indicate that eyes were leading head in a majority of cases. In other words, they indicated that during passive viewing of scenes, the eyes lead the head in most cases.

Previous work on the coordination between eye and head movements suggests that, for small eye movements (< 45°), the eye leads the head, and for larger shifts (> 60°) the initiation of the two tends to be more synchronous (e.g., Barnes, 1979). In both cases, however, the eyes terminate in advance of the slower head movements owing to longer contraction times for the neck muscles and the greater inertial forces acting on the head compared to the eye (Bizzi et al., 1971; Freedman, 2008; Gilchrist et al., 1998). Interestingly, the conditions that result in the head leading the eyes are relatively few, including, for example, preparation for a specific task-oriented event, such as shoulder checking in a car before changing lanes (Doshi & Trivedi, 2012) or choosing to move the eyes into space that is outside visible range, such as when looking at the world through binoculars (Sidenmark & Gellersen, 2019; Solman et al., 2017). It remains an open question whether the paucity of observed head-led behavior is due to the types of research we conduct in the lab or whether it is generally applicable.

4 Recurrence Quantification Analysis (RQA)

Recurrence analysis has been used successfully as a tool for describing complex dynamic systems, for example, climatological data (Marwan & Kurths, 2002), electrocardiograms (Webber & Zbilut, 2005), or postural fluctuations (Riley & Clark, 2003, Pellecchia & Shockley, 2005), which are inadequately characterized by standard methods in time series analysis (e.g., Box et al., 2008). It has also been used for describing the interplay between dynamic systems in cross-recurrence analysis, e.g., for analyzing the postural synchronization of two persons (Shockley et al., 2003; Shockley et al., 2007; Shockley & Turvey, 2005). The fundamental idea of recurrence analysis is to analyze the temporal pattern of repeated (recurrent) events, for example, the same tidal height in tide analysis, the same waves in the ventricular cycle in electrocardiogram analysis, the same postural relation in the analysis of postural synchronization. In the sections below, we describe a simplified version of recurrence analysis based on categorical data applied to fixations. In this context, RQA can be used to describe fixation patterns over time, e.g., the temporal pattern of re-fixations and the relative durations of fixations and re-fixations. In reading research, for example, RQA can be used to study regressions back to material already read. In image understanding, RQA can be used to study the importance of certain image locations for image understanding. At the end of this section, all the steps of a complete recurrence quantification analysis are presented in a textbox. The reader is advised to consult this textbox as each new concept is introduced in this section to better understand its role in the complete analysis.

4.1 Categorical Recurrence Quantification

Richardson, Dale and colleagues have generalized recurrence analysis to the analysis of categorical data and have used it for analyzing the coordination of gaze patterns between individuals (e.g., Cherubini et al., 2010; Dale, Kirkham, & Richardson, 2011; Dale, Warlaumont, & Richardson, 2011; Richardson & Dale, 2005; Richardson et al., 2009; Shockley et al., 2009). For example, Dale, Warlaumont and Richardson (2011) quantified the coordination between a speaker and a listener's eye movements as they viewed actors on a screen (see Fig. 10).

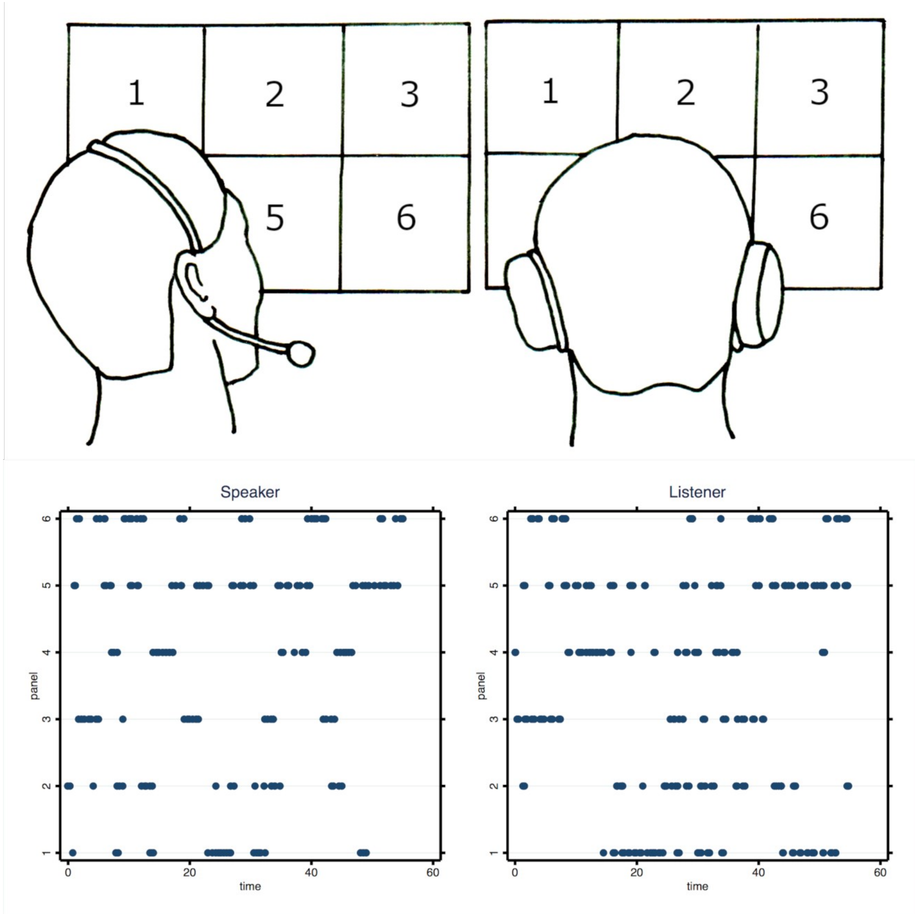


Fig. 10. Dale, Warlaumont and Richardson's (2011) experiment. Upper panel: The left person is speaking about a 2 × 3 grid displaying television characters while the right person listens in. Lower panel: Each interval of the one-minute experiment produces a numeric value representing the panel that was fixated. Using these data, a cross-recurrence analysis compares the panels fixated by speaker and listener over different time lags. (Figure adapted from Dale, Warlaumont & Richardson, 2011)

In this experiment, one person (left) was talking about a particular television character while the other person (right) listened in. Pictures of characters including the one discussed were displayed in a 2x3 grid in front of the speaker and the listener. The eye movements were categorized into a grid corresponding to the six photos of the television characters. The eye movements generated by speaker and listener during a 60 second period are shown in a series of numbers 1-6 corresponding to the six grid locations. Dale, Warlaumont & Richardson (2011) used a cross-recurrence analysis of these fixations and were able to show that the listener tended to follow the same fixation patterns as the speaker, with a delay of approximately 2 seconds. We come back to cross-recurrence applied to the movements of the eyes and head in Section 5, but first we describe how it can be applied to a single scanpath.

4.2 Generalized Recurrence Quantification

Dale et al.'s cross-recurrence analysis can provide an overall measure of similarity across two eye movement sequences (i.e., a form of scanpath comparison that is being discussed by Foulsham (Chapter 6 in this volume). We have introduced a generalized form of Dale et al.'s categorical recurrence analysis to characterize gaze patterns of a single observer (Anderson et al., 2013), and we were able to show that it is a very useful tool for encoding general characteristics of fixation sequences. The essential idea is to consider each fixation as one in a series throughout the image. Recurrence analysis quantifies when one fixation overlaps in space with another, e.g., when a person is looking at the same position twice or more times in the course of looking at a scene. In the following, we first review the fundamentals of RQA for use with categorical eye movement data, with specific consideration of fixation sequences. Then we describe and interpret the main RQA measures.

4.3 Formal Definition of Recurrence

Consider a sequence of N fixations f_i, $i=1, \ldots, N$, with each f_i characterized by its spatial coordinates. Two fixations are considered recurrent, if they are close together. "Closeness" can be defined in several ways, but in general, one can define recurrence r_{ij} as

$$r_{ij} = \begin{cases} 1, & d(f_i, f_j) \leq \rho \\ 0, & otherwise \end{cases}$$

where d is some distance metric (usually Euclidian distance), and ρ is a given radius, i.e., two fixations are considered recurrent if they are within a distance ρ of each other. Guidelines for selecting a value for ρ are introduced further below.

4.4 Recurrence Plot

Recurrence can be represented in a recurrence diagram, which plots recurrences of a fixation sequence over all possible time lags. The essential starting point of a recurrence analysis is a drawing of this plot. While it is not strictly a necessary step, all of the recurrence measures are based on patterns that emerge from this plot. The plot is drawn as follows: If fixations i and j are recurrent (i.e. if $r_{ij} = 1$), then a dot is plotted at position i,j. All fixations are recurrent with themselves (since $d(f_i, f_i)=0$), hence all elements on the major diagonal – the line of incidence - are recurring. Furthermore, since distance metrics are symmetric (i.e., $d(f_i, f_j) = d(f_j, f_i)$) recurrence plots are also symmetric. This is illustrated in Fig. 11. Figure 11A shows a landscape image with a scanpath consisting of 30 fixations, with repeated fixations mainly in the cloud formation. The recurrence plot for the fixation scanpath is shown in Fig. 11B, with each recurrence indicated by a red dot. A recurrence plot can be generated for each sequence of fixations, e.g., for each experimental trial.

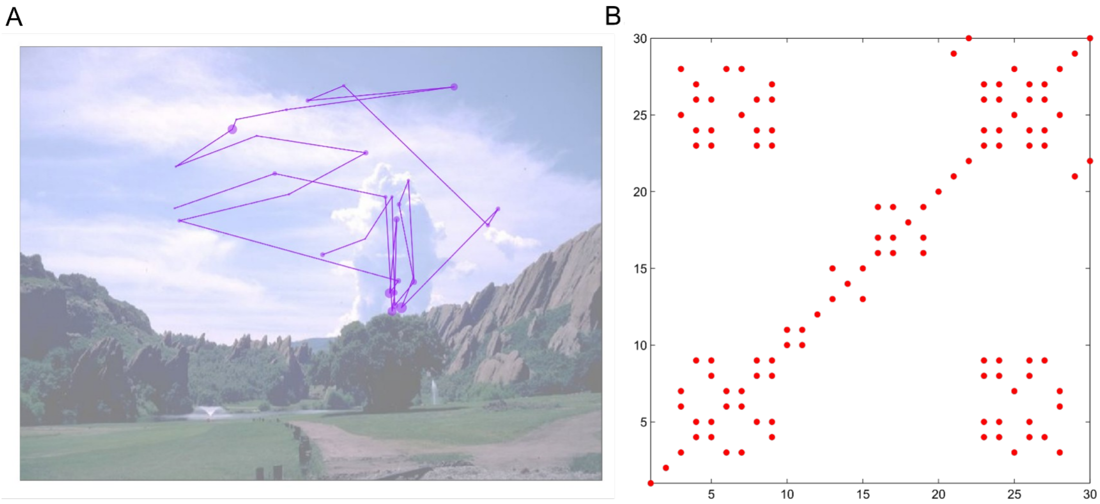


Fig. 11. A Image of a landscape with the scanpath produced by one participant overlaid (the size of the circle at each fixation point represents the duration of the fixation); **B** Recurrence plot of the scanpath in **A**. All elements along the line of incidence (the major diagonal) are recurrent, given that all fixations are recurrent with themselves. Furthermore, the recurrence plot is symmetric, given that all distance metrics are symmetric.

4.5 Recurrence Quantification Measures

The recurrence diagram provides a useful visual representation of the recurrence patterns for a fixation sequence, but it must be complemented by a recurrence quantification analysis for comparison across different fixation sequences, e.g., across different trials, participants and experimental conditions. Here, we describe a subset of RQA measures, those that are particularly useful for the analysis of fixation sequences (see Webber & Zbilut, 2005 and Marwan & Kurths, 2002 for a more complete list of measures). All of these measures describe certain patterns that emerge in the recurrence plot. In Sections 4.5.1 – 4.5.4, we describe each of these measures in detail and explain why they are useful for the interpretation of fixation patterns. Given the symmetry of the recurrence diagram, these quantitative measures are usually extracted from the upper triangle of the recurrence diagram, excluding the line of incidence, which does not add any additional information (recall that the line of incidence indicates that each fixation is recurrent with itself). In general, the length N of a fixation sequence can vary over a very large range, from one (in cases where a trial is terminated after a single fixation) to many thousands (in cases where fixation sequences are observed over very long trial durations). In our experience, RQA is best applicable if N is in a range from about 20 to 200.

First, we give some useful definitions: Given a fixation sequence of length N, f_i, $i=1,\ldots,N$, let R be the sum of recurrences in the upper triangle of the recurrence diagram, i.e., $R=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_{ij}$. Let D_L be the set of diagonals lines, H_L the set of horizontal, and V_L the set of vertical lines, all in the upper triangle, and all with a length of at least L, and let $|\cdot|$ denote cardinality. These definitions are used in the definitions of Sections 4.5.1 to 4.5.4.

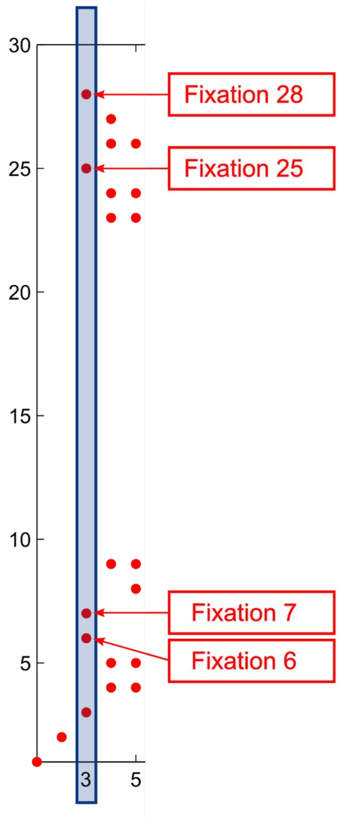
4.5.1 Recurrence

The **recurrence** measure REC is defined as

$$\mathrm{REC}=100\frac{2R}{N(N-1)}$$

For a sequence of N fixations Recurrence represents the percentage of recurrent fixations, i.e., it indicates how often an observer re-fixates previously fixated image positions. As fixations are plotted sequentially, the larger the distance between a recurrent point and the main diagonal, the larger the time interval (in number of fixations) between the original fixation and the re-fixation. In summary, recurrence REC measures how often participants re-fixate previously fixated locations and helps to understand, which image locations are most important for image understanding.

Figure 12 provides an illustration of recurrence. Fixations 6, 7, 25 and 28 in Fig. 11 are all close-by, more specifically within a radius of ρ of fixation 3, hence these fixations are recurrent.



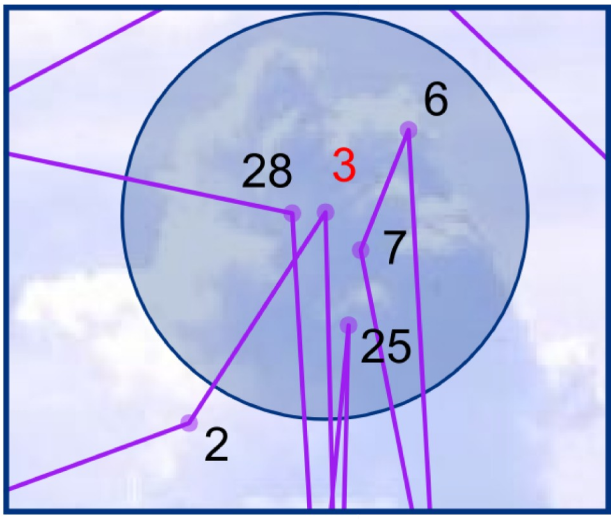


Fig. 12 Detailed view of the recurrence plot and the fixation plot shown in Fig. 11. Fixations 6, 7, 25 and 28 are within the radius ρ of Fixation 3, as indicated by the blue circle, producing the shown recurrences in the recurrence plot.

4.5.2 Determinism

The **determinism** measure *DET* is defined as

$$\mathrm{DET} = 100\frac{|D_L|}{R}$$

Determinism measures the proportion of recurrent points forming diagonal lines in the recurrence plot and represents repeating fixation sequences in the recurrence diagram. This may represent two areas of a scene where one fixation is more likely to follow another, for example, if a person looks first at one person, then another in that same order twice in a trial. It is a small section of a scanpath repeated within a trial. The minimum line length of diagonal line elements is typically set to $L = 2$. The length of the diagonal line element reflects the number of fixations making up the repeated scanpath, and the distance from the diagonal reflects the time (in numbers of fixations) since the scanpath was first followed.

Figure 13 provides an illustration of determinism. Fixations 3 and 4 as well as fixations 25 and 26 follow the same path, defining a deterministic recurrence. In summary, *DET* is used to capture the finer temporal structure of fixation sequences.

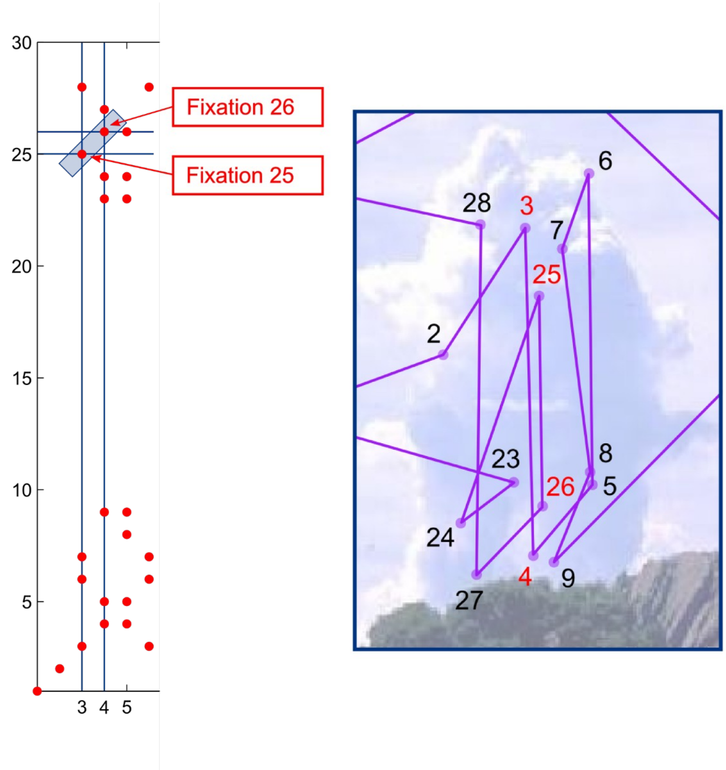


Fig. 13 Detailed view of the recurrence plot and the fixation plot shown in Fig. 11. Fixations 3 and 4 as well as fixations 25 and 26 follow the same path. This creates a diagonal line on the recurrence plot and defines a deterministic recurrence.

4.5.3 Laminarity

The **laminarity** measure *LAM* is defined as

$$\mathrm{LAM} = 100 \frac{|H_L| + |V_L|}{2R}$$

Vertical lines represent areas that were fixated first in a single fixation and then re-scanned in detail over consecutive fixations at a later time (e.g., several fixations later), and horizontal lines represent areas that were first scanned in detail and then re-fixated briefly later in time. Again, we set the minimum line lengths of vertical and horizontal lines to L = 2. We find that the recurrence diagrams sometimes contain recurrence clusters (with horizontal and vertical lines), indicating detailed scanning of an area and nearby locations. In summary, laminarity indicates that specific areas of a scene are repeatedly fixated, for example, when an observer returns to an interesting area of the scene to scan it in more detail.

An example of laminarity is shown with fixations in Fig. 13. The position fixated in fixation 3 is refixated in a more detailed inspection in fixations 6 and 7.

4.5.4 Center of Recurrence Mass

The **center of recurrence mass** *CORM* is defined as the distance of the center of gravity of recurrent points from the line of incidence, normalized such that the maximum possible value is 100.

$$\mathrm{CORM} = 100 \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (j-i) r_{ij}}{(N-1) R}$$

This measure indicates approximately where in time most of the recurrent points are situated. Small *CORM* values indicate that re-fixations tend to occur close in time, i.e. most recurrent points are close to the line of incidence. For example, if an observer sequentially scans three particular areas of a scene in detail and never returns to those areas later in the trial, most of the recurrent points would fall close to the line of incidence. This would be represented by a small *CORM* value. In contrast, large *CORM* values indicate that re-fixations tend to occur widely separated in time, i.e. most recurrent points are close to the upper left and because of redundancy, lower right corners of the recurrence diagram. This occurs, for example, when an observer re-fixates only one scene area, once at the beginning and once at the end of the fixation sequence, but not in between. In summary, *CORM* is also used to characterize the global temporal structure of fixation sequences by characterizing whether re-fixations occur close in time or far apart. The measures introduced in the Sections 4.5.1 to 4.5.4 are based on simple fixations. An extension of these measures that takes into account fixation duration is presented in the Textbox 1.

Textbox 1: RQA and Fixation Duration

Fixation duration can be an important indicator of processing during fixation (Holmqvist et al., 2011, pp. 377ff). RQA can be generalized to take fixation durations into account (Anderson et al., 2013).

Given a fixation sequence f_i, i = 1, . . . , N, and the associated vector of fixation durations t_i, i = 1, . . . , N, one can redefine recurrence r_{ij}^t as

$$r_{ij}^t = \begin{cases} t_i + t_j, & d(f_i, f_j) \leq \rho \\ 0, & otherwise \end{cases} \qquad ((1)$$

with the (Euclidian) distance metric d and the radius ρ. With the modified recurrence definition of r_{ij}^t, the RQA measures have to be renormalized. Let $R = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} r_{ij}^t$, and $T = \sum_{i=1}^{N} t_i$. Then the revised definitions for *REC, DET, LAM and CORM* are as follows.

$$REC^t = 100 \frac{2R^t}{(N-1)T}$$

$$DET^t = \frac{100}{R^t} \sum_{(i,j) \in D_L} r_{ij}^t$$

$$LAM^t = \frac{100}{2R^t} \left(\sum_{(i,j) \in H_L} r_{ij}^t + \sum_{(i,j) \in V_L} r_{ij}^t \right)$$

$$CORM^t = 100 \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (j-i) r_{ij}^t}{(N-1)^2 T}$$

In summary, the recurrence and corm measures capture the global temporal structure of fixation sequences. They measure how many times given scene areas are re-fixated (recurrence) and whether these re-fixations occur close in time or widely separated in a trial (corm). In contrast, determinism and laminarity measure the finer temporal structure of fixation sequences. Specifically, they indicate sequences of fixations that are repeated (determinism) and points at which detailed inspections of an image area are occurring (laminarity). These measures can then be compared across different types of images, experimental contexts, and participants to assess the dynamic structure of eye movements.

4.6 Recurrence Radius

As explained earlier, two fixations are considered recurrent if they are within a distance ρ of each other, with the radius ρ being a free parameter. The number of recurrences is directly related to the radius. As the radius ρ approaches zero, (off-diagonal) recurrences approach zero, and as ρ approaches the image size, recurrences approach 100%. (see Fig. 14)

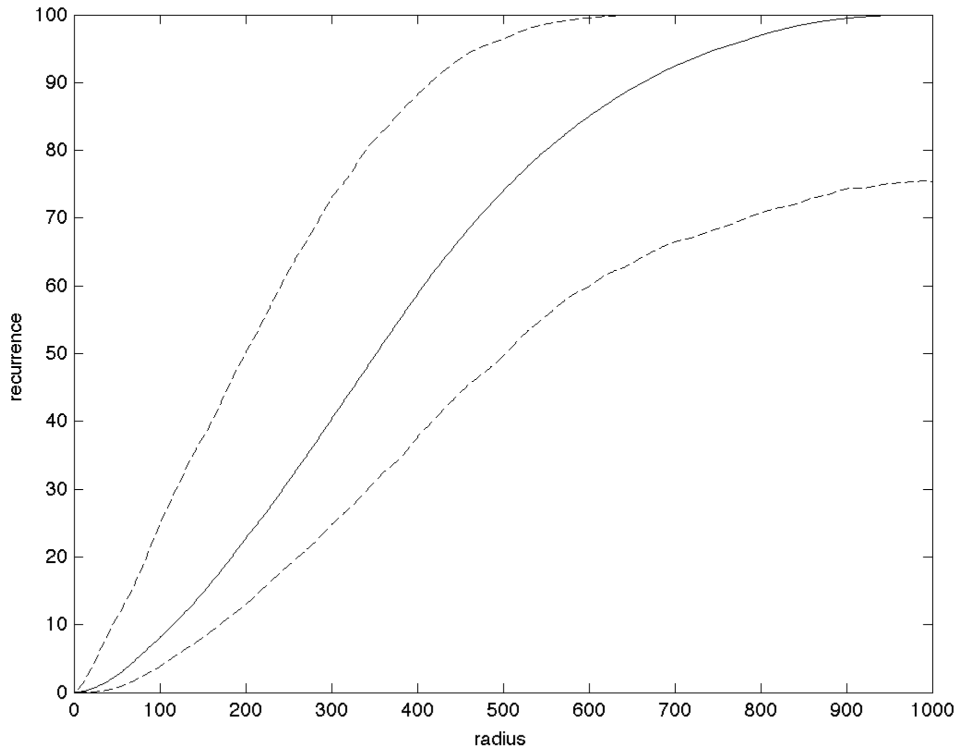


Fig. 14 The median and 95% confidence interval of percent recurrence as a function of radius.

The dependence of recurrence on radius leads to the obvious question of how an appropriate radius for recurrence analysis should be selected. Webber and Zbilut (2005) suggested several guidelines for selecting the proper radius, including the selection of a radius such that percentage of recurrences remains low, for example about 1-2 percent. In the case of eye movements, one can apply more content-oriented criteria. For example, fixations can be considered as recurring if their foveal areas overlap, using a radius size of 1–2 degrees of visual angle. This is discussed further by Anderson et al. (2013). A review of some potential limitations of the RQA method is presented in Textbox 2.

Textbox 2: Potential Limitations of the RQA Method

1. Scope of measurement

RQA captures the dynamic aspects of scanpaths, but it does not directly measure certain eye movement measures (e.g., saccade characteristics). As a result, it is best used in partnership with other measures rather than as a standalone method.

2. Dependence on recurrence

While recurrence can always be computed (and may be zero), some RQA measures, such as determinism, laminarity, and corm, are undefined if there is no recurrence. In practice, one can always compute the RQA measures with larger recurrence radii ρ (see Section 4.3) and possibly with a whole range of recurrence radii. As the recurrence radius ρ increases, so does the number of recurrences (see section 4.6 and Anderson et al., 2013).

3. Applicability to single eye movement studies

RQA requires more than a single fixation to be applied. Of course, this limitation is not unique to RQA and also applies to many other eye movement measures.

4. Computational considerations

The number of cells in the recurrence plot increases with the square of the number of fixations n in a scan path. Consequently, computing RQA measures becomes computationally more and more expensive as N increases. In our own work, we have rarely used RQA for scanpaths with more than 100 fixations.

4.7 Statistical Analysis of Recurrence Measures

In this section, we discuss the distribution of RQA measures as well as their correlations. This is important when RQA measures are used to compare and discriminate between experimental conditions and groups. Fig. 15 shows the histograms of the recurrence, determinism, laminarity and corm measures obtained with 104 participants viewing 1872 images, each for a duration of 10 seconds. As the histograms show, the RQA measures are distributed more or less symmetrically, with the exception of Recurrence, permitting the use of analyses of variance for their statistical analysis.

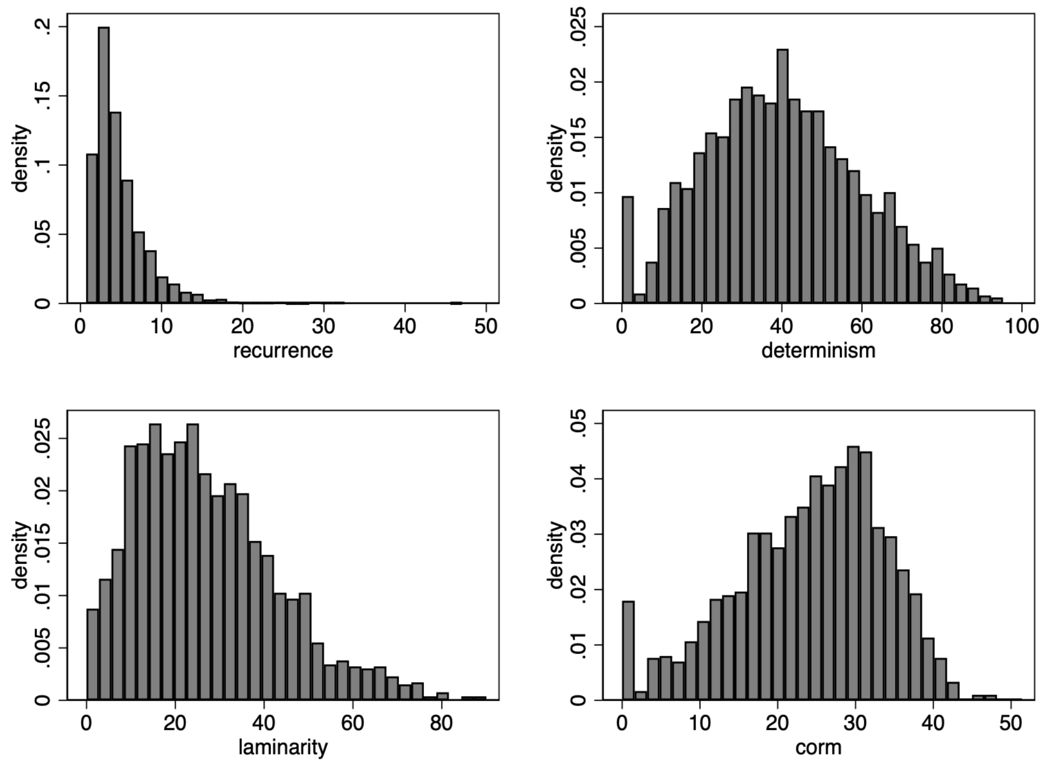


Fig. 15 Histograms of the recurrence, determinism, laminarity and corm measures obtained with 104 participants viewing 1872 images, each for a duration of 10 seconds.

For this reason, it is useful to consider several or all RQA measures for comparing and discriminating different groups of participants or different experimental conditions. In other words, it is advisable to take all RQA measures into account and use discriminant analyses for distinguishing different scanpath patterns, a multivariate analysis of variance, analysis of variance, or canonical correlation in regression-type analysis. This is further described below.

4.8 Discrimination of Gaze Patterns using RQA

The previous sections showed that RQA is a useful method for discriminating eye movement patterns based purely on dynamic characteristics. Each of the RQA measures used (recurrence, determinism, laminarity and corm) can discriminate to a certain extent between experimental groups or different groups of participants. At the same time, however, the measures are not independent. We evaluated how well the

RQA measures discriminate between eye movement patterns of participants who viewed social scenes under natural viewing conditions (Full-View condition) and participants who view the same scenes through a gaze-contingent window of limited spatial extent (Restricted-View condition).

Figure 16 shows histograms for the recurrence, determinism, laminarity and corm measures, for the Full-View and the Restricted-View conditions. For clarity of presentation, the histograms were smoothed using a Gaussian smoothing filter with half-width = 2 percent or σ = 0.849 percent of the total range. An inspection of the four histograms shows that each measure can discriminate to some extent between the two groups, but, at the same time, there is a substantial overlap that varies between RQA measures. Consequently, classification accuracy of the discrimination between the two groups varies accordingly, with accuracy for the recurrence measure at 75%, for the determinism measure of 88.6%, for laminarity at 77.7%, and for corm at 70.8%. In other words, each of the measures can discriminate between the two experimental groups, but discrimination performance is far from perfect.

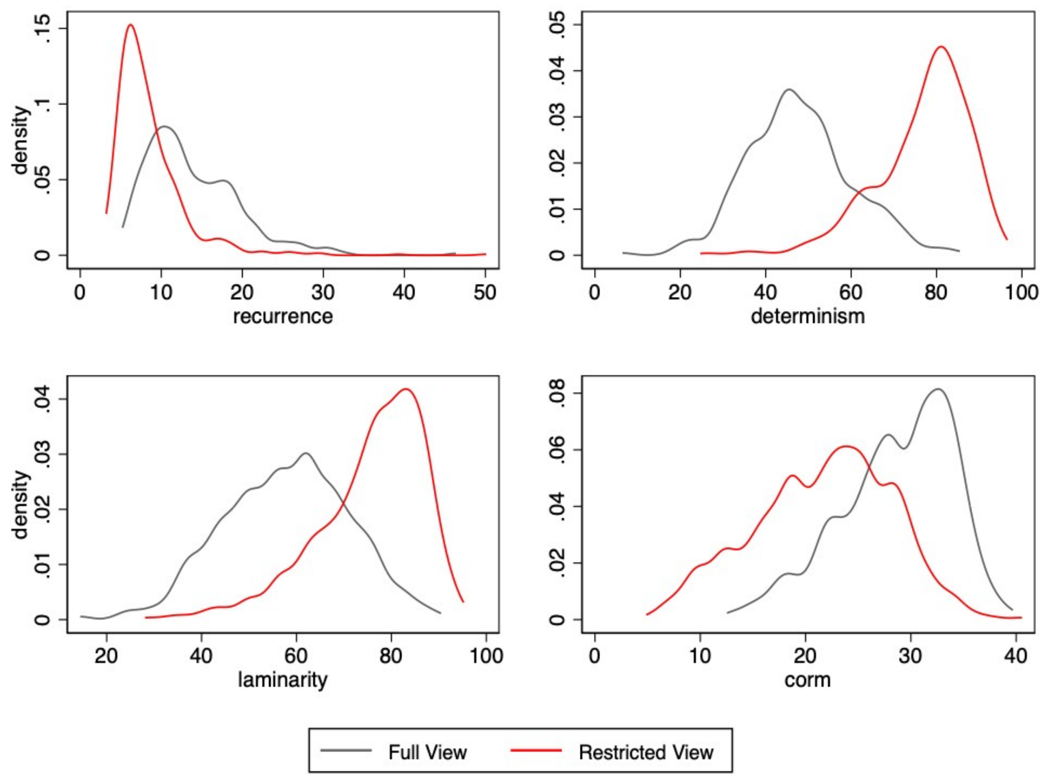


Fig. 16. Smoothed histograms for the recurrence, determinism, laminarity and corm measures for the Full-view (natural viewing) condition and the Restricted-View (viewing through a small, gaze-contingent window). The histograms were smoothed with a Gaussian filter with half-width = 2 percent.

Discrimination between the eye movement patterns of the two groups can be improved substantially using combinations of RQA measures. This is possible because the measures are somewhat, but definitely not perfectly correlated. For example, using the measures recurrence + laminarity + corm improves discrimination accuracy to 85.5%, and the discrimination accuracy using all four measures is 94.5%, a substantial improvement over the use of single measures. The results show clearly that the RQA measures are sensitive to, and useful for discriminating gaze patterns under Full-View and Restricted-View conditions.

TextbBox 3: Running a complete recurrence quantification analysis

This box walks through a complete categorical RQA of a single scanpath, from the raw recording to a table of measures ready for statistics. Each numbered step corresponds to a decision you will make when analysing your own data.

Step 1. Decide what question RQA can answer for you

RQA describes how, and when, gaze returns to locations it has already visited within a single viewing episode. It is well suited to questions such as: Do observers re-inspect the same regions of a scene, and how often (recurrence)? Do they repeat short scanning routines (determinism)? Do they return to an area to examine it in detail (laminarity)? Are revisits clustered early, or spread across the whole trial (center of recurrence mass)? Typical applications include regressions to already-read text in reading research, comparing systematic versus exploratory scanning across groups (e.g., expert vs. novice, clinical vs. control), and contrasting encoding with recognition of the same scene.

Step 2. Check that your dataset has the right qualities

RQA needs an ordered sequence of fixations for each trial, each with a spatial location (x, y) or an area-of-interest label. Good calibration and reliable fixation detection matter more than sampling rate, because the analysis runs on fixations, not raw samples. Sequence length is the main practical constraint: in our experience RQA works best when a trial contains roughly 20 to 200 fixations. Very short sequences give unstable estimates; very long ones can saturate the recurrence plot. Plan trial durations accordingly (for natural scene viewing, a 10 s trial yields a workable sequence).

Step 3. Pre-process the recording into fixation sequences

From the raw gaze stream, detect fixations and saccades (for example with the dispersion threshold IDT algorithm described in Section 2.1) and store, per trial, the ordered list of fixations with their coordinates. For categorical RQA, optionally map each fixation to an AOI label; for spatial RQA, keep the continuous coordinates. The result of this step is one fixation sequence $f1, f2, ..., fN$ per trial.

Step 4. Choose the recurrence radius and minimum line length

Two fixations count as recurrent if they fall within a radius ρ of each other. For eye movements a content-based choice works well: treat fixations as recurrent when their foveal areas overlap, i.e. ρ of about 1 to 2 degrees of visual angle. As a cross-check, Webber and Zbilut (2005) recommend picking ρ so that overall recurrence stays low (about 1 to 2 percent). Set the minimum line length to L = 2 for the diagonal, vertical and horizontal lines used by determinism and laminarity.

Step 5. Build the recurrence plot

Plot a dot at position (i, j) whenever fixations i and j are recurrent. Every fixation is recurrent with itself, so the main diagonal (the line of incidence) is always filled, and because distance is symmetric the plot is symmetric about that diagonal. Inspect the plot before quantifying anything: the measures below are simply summaries of the patterns you can already see here. All measures are read from the upper triangle, excluding the line of incidence.

Step 6. Compute the quantitative measures (and read off their meaning)

Four measures are particularly useful for fixation data. A concrete reading is given for each:

- Recurrence (REC) is the percentage of fixation pairs that recur. High REC means the observer keeps returning to a small set of locations; it flags which scene regions are most important to them. Example: a face in a social scene that is fixated again and again drives REC up.
- Determinism (DET) is the percentage of recurrent points lying on diagonal lines (length >= L). It captures repeated scanning routines, e.g. looking from person A to person B and later repeating that A-to-B sequence. High DET indicates structured, repeatable scanpaths.
- Laminarity (LAM) is the percentage of recurrent points on horizontal or vertical lines. It marks areas first glanced at and later re-scanned in detail (or vice versa), that is, sustained, detailed inspection of a region.
- Center of recurrence mass (CORM) indicates where in time the recurrences sit. Small CORM means revisits happen close together in time; large CORM means an area seen early is revisited only much later in the trial.

Linking to psychological constructs. It is tempting to map each measure onto a single construct, but the mapping is context-dependent and should be made cautiously. Within a specific paradigm such associations can be productive. For instance, Gurtner et al. (2019) related determinism to visuospatial working memory. In general, treat REC and CORM as indices of where and when attention is re-allocated, and DET and LAM as indices of how structured and how detailed the re-inspection is.

Step 7. Assemble one row per trial and run statistics

Collect REC, DET, LAM and CORM into a table with one row per trial (or per participant x condition). With 104 participants viewing 1872 images (as in the example of Section 4.7) these measures are distributed roughly symmetrically (except recurrence), so analysis of variance is appropriate. Because the measures are correlated but not redundant, combining them is powerful: in our scene viewing data, single measures discriminated full-view from restricted-view scanning at 71 to 89 percent, whereas all four together reached 94.5 percent. Use MANOVA, discriminant analysis or canonical correlation when you want to separate conditions or groups on the full measure set.

Step 8. Software you can use

Several tools implement these steps. In R, the crqa package (Coco & Dale, 2014) computes RQA and CRQA and their measures. In Python, pyRQA and PyRQA handle large sequences. In MATLAB, Marwan's Cross Recurrence Plot Toolbox is the longstanding reference implementation. The categorical, fixation-based variant used in this chapter follows Anderson et al. (2013), whose scripts are available at https://barlab.ubc.ca/research. Requirements are modest: the analyses run on ordinary desktop hardware; the main inputs you must supply are the fixation sequences and the radius ρ.

Try it yourself: *Take one scanpath from your own data, set ρ to 1.5 degrees and L = 2, draw the recurrence plot, and compute the four measures. Which scene regions drive recurrence, and do your observers repeat any scanning routines (determinism)?*

5 Cross-Recurrence Quantification Analysis (CRQA)

In this section we present cross-recurrence quantification analysis (CRQA; Dale, Warlaumont & Richardson, 2011) for the comparison of two different event sequences. These include, for example, fixation sequences during the learning and the recognition of images, sequences of gaze and head positions, or fixation sequences of two different observers, and many more. CRQA allows for the quantification of behavioral similarity among people and images. That is, do certain aspects of eye and head movement behaviour overlap across images, or across people when encoding and recognizing a scene? In this section, we begin with a brief description of cross-recurrence and its most important measures, where each measure quantifies an interesting behavioural similarity. Then we present related methods for the analysis of leader – follower relationships between event sequences.

5.1 Formal Definition of Cross-Recurrence

Consider two fixation sequences $\boldsymbol{E} = \boldsymbol{e}_i,\ i=1,\ \ldots,\ N$, with $\boldsymbol{e}_i = <x_i,y_i>$ and $\boldsymbol{R} = \boldsymbol{r}_j,\ j=1,\ \ldots,\ N$, with $\boldsymbol{r}_j = <x_j,y_j>$. For fixation sequences of unequal length, the longer sequence is truncated. Two fixations $\boldsymbol{e}_i$ and $\boldsymbol{r}_j$ are cross-recurrent if they are close together, i.e., we define the cross-recurrence of two fixations c_{ij} as

$$c_{ij} = \begin{cases} 1, & d(e_i,\ r_j) \le \rho \\ 0, & otherwise \end{cases}$$

where d is a distance measure (e.g., Euclidian distance in 2D or the great circle distance in spherical displays) and ρ is a threshold.

5.2 Cross-Recurrence Plot

An example of a cross-recurrence diagram is shown in Fig. 17. Note that, in contrast to recurrence plots (e.g., in Fig. 11B), cross-recurrence plots are not symmetric around the line of incidence. For simplicity, we compare here and in the sections below a sequence of fixations during encoding with a sequence of fixations during recognition, but cross-recurrence works of course for the comparison of arbitrary event sequences.

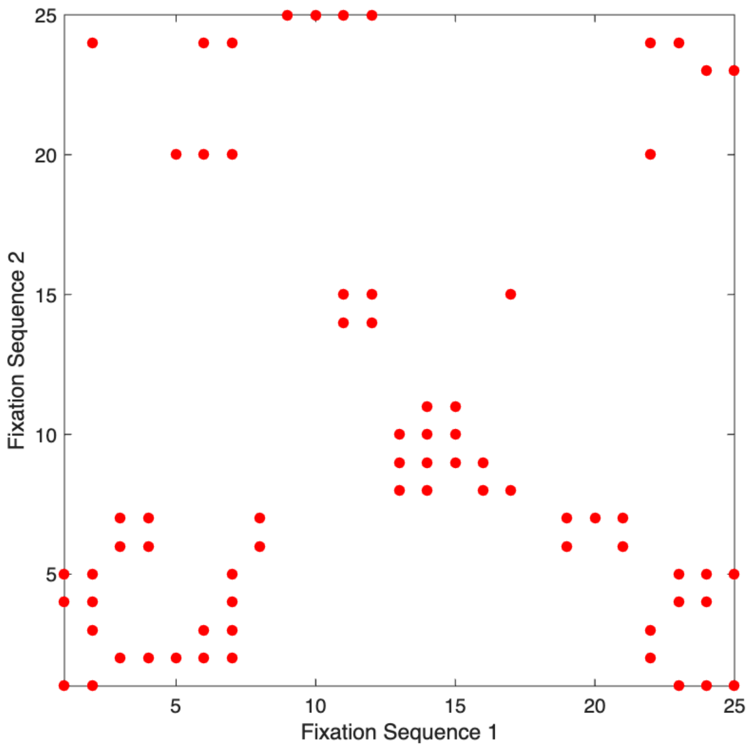


Fig. 17 Two fixation sequences are adjusted to 25 fixations, with dots indicating cross-recurrences. For example, encoding fixation 3 and recognition fixations 2, and 6-7, are cross-recurrent, i.e., their angular distance is below the defined recurrence threshold.

5.3 Cross-Recurrence Measures

Several measures can be used for characterizing cross-recurrence patterns. The measures are extensions of the recurrence measures introduced by Anderson et al. (2013). In the cross-recurrence matrix $\boldsymbol{C} = c_{ij}$, $i=1, \ldots, N, j=1, \ldots, N$; let D be a diagonal line representing a fixation trajectory common to both fixation sequences; let L_H be a horizontal laminate line, representing a single fixation in $\boldsymbol{r}$ being recurrent with a sequence of fixations in $\boldsymbol{e}$; and let L_V be vertical laminate line, representing a single fixation in $\boldsymbol{e}$ being recurrent with a sequence of fixations in $\boldsymbol{r}$. D, L_H and L_V are taken into account only if their lengths are at least $L = 2$. Finally, let C be the sum of recurrences, i.e., $C = \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij}$ and let $|\cdot|$ denote cardinality. These definitions are used in the definitions of Sections 5.3.1 to 5.3.6 below.

5.3.1 Cross-Recurrence

The **cross-recurrence** measure *REC* of two fixation sequences defined as

$$REC = 100 \cdot \frac{C}{N^2}.$$

It represents the percentage of cross-recurrent fixations, i.e., the percentage of fixations that are close between the two fixation sequences. It thus indicates the amount of spatial overlap between the two fixation sequences. For example, if the spatial overlap between the two fixation sequences is high, the percentage of cross-recurrent points is high. In summary, *REC* measures the spatial overlap of two fixation sequences and thus characterizes their global similarity.

5.3.2 Determinism

The **determinism** measure *DET* is defined as

$DET = 100 \cdot \frac{|D|}{C}$.It is the percentage of cross-recurrent points that form diagonal lines and represents the percentage of fixation trajectories common to both fixation sequences. That is, it quantifies the overlap of a specific sequence of fixations, preserving the sequential information. A high percentage of deterministic fixations means that there are fixation trajectories that are common to both fixation sequences. For example, sequences of fixations in the first sequence that are then exactly repeated in the second sequence. In summary, *DET* quantifies fixation trajectories common to both fixation sequences and thus characterizes local similarities between the two fixation sequences.

5.3.3 Vertical Laminarity and Vertical Trapping Time

The **vertical laminarity** measure *vLAM* is defined as

$\mathrm{vLAM} = 100 \cdot \frac{|L_V|}{C}$. It is the proportion of fixations in vertical laminate lines L_V and represents the percentage of fixation that were fixated in detail in ***r***, but only fixated briefly in ***e***.

The **vertical trapping time** *vTT* is defined as the average length, in number of fixations, of all vertical laminate lines.

Vertical Laminarity and *Vertical Trapping Time* quantify the percentage of recurrent points that form vertical lines in the cross-recurrence diagram, with *vLAM* indicating the proportion of vertical lines and *vTT* indicating their average length. They capture for example, the behaviour where an individual briefly scans a particular region during encoding and then inspects it in detail during recognition

5.3.4 Horizontal Laminarity and Horizontal Trapping Time

The **horizontal laminarity** measure *hLAM* is defined as

$\mathrm{hLAM} = 100 \cdot \frac{|L_H|}{C}$. It is the proportion of fixations in horizontal laminate lines L_H and represents the percentage of fixation that were fixated in detail in ***e***, but only fixated briefly in ***r***.

The **horizontal trapping time** *hTT* is defined as the average length, in number of fixations, of all horizontal laminate lines.

Horizontal Laminarity and *Horizontal Trapping Time* quantify the percentage of recurrent points that form horizontal lines in the cross-recurrence diagram, with *hLAM* indicating the proportion of horizontal lines and *hTT* indicating their average length. In summary, they capture the behaviour where an individual scans a region in detail during encoding and then only briefly inspects it during recognition.

5.3.5 Center of Recurrence Mass

The **center of recurrence mass** *CORM* is defined as the distance of the center of gravity from the main diagonal, normalized such that the maximum possible value is 100,

$$CORM = 100 \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} (j - i) r_{ij}}{(N - 1) C}.$$

The *CORM* measure indicates the dominant lag of cross-recurrences. Small corm values indicate that the same fixations in both fixation sequences tend to occur close in time, whereas large corm values indicate that cross-recurrences tend to occur with either a large positive or negative lag. *CORM* values are positive if the fixation sequence ***e*** is leading the fixation sequence ***r***, and it is negative if the fixation sequence ***r*** is leading the fixation sequence ***e***.

The *CORM* measure quantifies where on the cross recurrence plot the majority of recurrences are. A large positive value indicates that during recognition, observers spend more time at locations looked at earlier on during encoding. A large negative corm value indicates that during recognition, observers spend more time at locations looked at later during encoding. In summary, this measure essentially quantifies primacy (positive corm) and recency (negative corm) effects.

5.3.6 Clusters

A **cluster** in the cross-recurrence matrix is defined by the following connectivity criterion: If the cross-recurrence *c* is in a cluster, then the cross-recurrences in the 8-connected region of *c* are also in the cluster ("Pixel connectivity", n.d.). Clusters with less than 8 cross-recurrences are ignored. The *CLUST* measure is defined as the percentage of cross-recurrent fixations that are part of a cluster. The presence of clusters in the cross-recurrence diagram expresses the tendency to focus on the same spatio-temporal subsets of locations in the two event sequences.

5.4 Cross-Recurrence Radius

The relation between encoding and recognition scanpaths were analyzed using cross-recurrence analysis. The CRQA measures depend on the recurrence radius, and the same criteria discussed under recurrence radius (see Section 4.5) also apply to cross-recurrence.

5.5 Diagonal Cross-Recurrence Profiles

The previous section introduced cross-recurrence plots and cross-recurrence analysis. On this basis, Richardson and Dale (2005), Dale, Kirkham & Richardson (2011) and Dale, Warlaumont & Richardson (2011) introduced diagonal cross-recurrence profiles (DCRP), which focus on a leader-follower analysis. DCRP is closely related to the *CORM* measure introduced in the previous section, but is limited to a narrow band around the incidence. This is illustrated in Fig. 18: Fig.18A shows a small cross-recurrence plot, and Fig. 18B shows the corresponding DCRP plot. The dashed lines in the cross-recurrence plot (labeled +2, 0, and -2) indicate sets of points that are summed and normalized to produce DCRP probabilities at lags +2, 0 (line of synchrony) and -2. In the example of the Fig. 18, cross recurrence was calculated between gaze points and head points of a single trial in one of our studies. The DCRP function reaches a peak at lag +1, indicating that, on average, gaze is leading head by one fixation (see also Fig. 7B).

Consistent with this result, the cross-correlation *CORM* value is 3.36, also indicating that gaze is leading head. The difference between the two analyses is that the Corm value of the cross-recurrence plot takes into account all recurrences of the whole plot, whereas DCRP considers only points close to the line of incidence (in this case with lags in the range [-10 10]) and thus considers only recurrences that are temporally close.

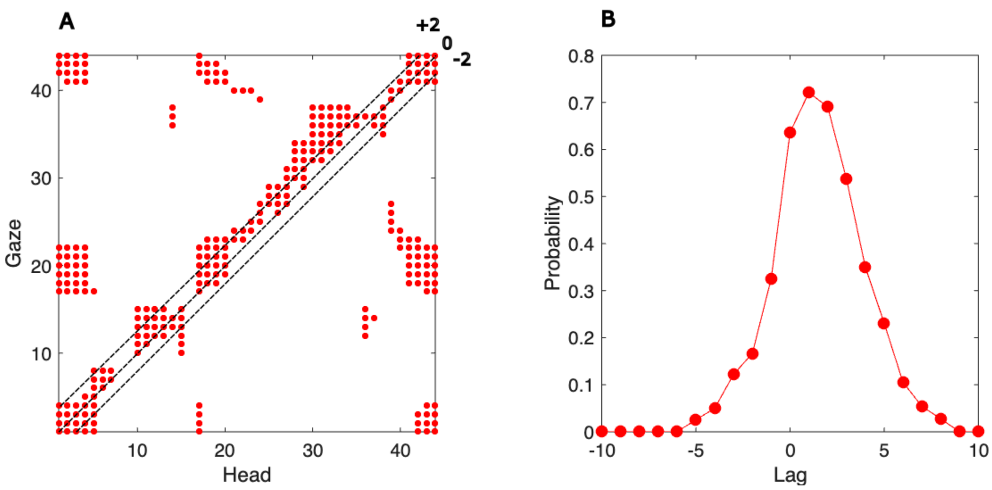


Fig. 18. A Cross-recurrence plot of two sequences of length $N = 44$ of head positions and gaze positions. Each dot indicates a recurrence of head position and a gaze position, with a distance of gaze position and head position less than 30°. The three dashed lines indicate sets of recurrences with a lag of +2, 0 (line of incidence) and -2. **B** Diagonal cross-recurrence plot corresponding to the cross-recurrence diagram on the left, for lags in the range [10 10]. The probabilities for a given lag are obtained by dividing the number of recurrences for that lag by the number of possible recurrences, i.e., N for the line of incidence, N -1 for lags +1 and -1, and so on.

5.6 Multidimensional RQA

Cross-recurrence analysis can be generalized to multivariate time series, for example, the temporal variation of hand and face movements (Louwerse et al., 2012; Wallot, 2019; Wallot et al., 2016) or sets of physiological measurements, such as heart rate, skin conductance, and electromyography of multiple persons (Mønster et al., 2016; Wallot, 2019). Tomashin et al. (2024) extended multidimensional RQA to lagged multidimensional RQA for analyzing leader-follower relationships. It can be applied to group data, for example, facial expression in groups or heart beats per minute in groups.

6 General Summary

In the preceding sections, we first reviewed traditional measures for characterizing eye movements, starting with basic fixation and saccade measures and continuing with area-of-interest (AOI) analyses. Then we examined temporal analyses of eye movements and of the relation between eye and head movements. Taken together, the reviews showed that the temporal analyses were able to capture crucial temporal characteristics of these two movement systems.

We then introduced recurrence quantification analysis (RQA) as a method for measuring the dynamic characteristics of eye movements. Critically, the RQA measures we introduced capture important and interpretable aspects of this pattern. We showed that RQA is suitable for discriminating eye movement patterns independent of the spatial structure of stimuli by focusing on temporal aspects exclusively. For this reason, it is ideally suited for the analysis of eye movements in more complex and dynamic situations.

Finally, we introduced cross-recurrence analysis (CRQA) to measure the temporal coordination of two event sequences, for example, of eye and head movements, or of eye movements during the encoding and the recognition of stimuli. These methods for capturing the temporal nature of multiple movements sequences prove useful in quantifying more complex behavior. For instance, Haraped et al. (2024, 2025) have applied CRQA to understand how partners coordinate attention while looking, pointing, and speaking together.

7 Suggested Readings

Anderson, N. C., Bischof, W. F., Laidlaw, K. E., Risko, E. F., & Kingstone, A. (2013). Recurrence quantification analysis of eye movements. Behavior research methods, 45(3), 842–856.

This paper introduces recurrence quantification analysis for the analysis of eye movements. Much of the material in section 4 is based on this paper.

Bischof, W. F., Anderson, N. C., & Kingstone, A. (2024). A Tutorial: Analysing Eye and Head Movements in Virtual Reality. *Behavior Research Methods*, 56, 8396-8421.

This paper discusses the spatial and temporal relations between eyes and head movements. Much of the material in section 3 is based on this paper.

Dale, R., Warlaumont, A. S., & Richardson, D. C. (2011b). Nominal cross recurrence as a generalized lag sequential analysis for behavioral streams. International Journal of Bifurcation and Chaos, 21, 1153–1161. doi:10.1142/S0218127411028970.

Recurrence analysis can be generalized to categorical data. This paper introduces categorical cross-recurrence analysis for analyzing the coordination of gaze patterns between individuals (see section 4.1).

Holmqvist, K., Nyström, M., Andersson, R., Dewhurst, R., Jarodzka, H., & van de Weijer, J. (2011). Eye tracking: A comprehensive guide to methods and measures. Oxford, U.K.: Oxford University Press.

This handbook is a comprehensive guide to the methods and measures for eye tracking. We recommend that researchers in the area of eye movement analysis consult this handbook. A new edition is scheduled for 2016 or 2017.

Marwan, N., & Kurths, J. (2002). Nonlinear analysis of bivariate data with cross recurrence plots. Physics Letters A, 302, 299–307.

This paper introduces recurrence analysis as a tool for describing complex dynamic systems. The works of Dale and colleagues and of Anderson and colleagues are applications of, and extensions to this paper.

8 Questions to Students

a. What are the characteristics of stimuli for which area-of-interest analyses are useful, and when are they less useful or not at all useful?
b. What are the fundamental advantages of recurrence analysis over the other spatial and temporal methods presented in this chapter?
c. What aspect of the recurrence patterns are captured by the measures recurrence, determinism, laminarity and corm?
d. How would you analyze the leader-follower relationship between two event sequences?
e. What kinds of research questions, in psychological or methodological terms, can be addressed using recurrence quantification analysis? Give an example from a domain that interests you, such as reading, scene memory, expertise, or social attention.
f. Following the worked example in Box 1, what data would you need to collect, and what processing steps would you take, to run a recurrence quantification analysis on a study of your own?
g. Why is it often advantageous to combine several recurrence measures rather than relying on a single measure when discriminating between experimental conditions or groups?

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