1. Show that \( \mathbb{Z} \) is closed in \( \mathbb{R} \), but not open, and that \( \mathbb{Q} \subset \mathbb{R} \) is neither open nor closed.

2. Let \( a_1, b_1, \ldots, a_N, b_N \in \mathbb{R} \) such that \( a_j < b_j \) for \( j = 1, \ldots, n \). Show that \( (a_1, b_1) \times \cdots \times (a_N, b_N) \) is open and that \( [a_1, b_1] \times \cdots \times [a_N, b_N] \) is closed in \( \mathbb{R}^N \).

3. Let \( \emptyset \neq S \subset \mathbb{R}^N \) be arbitrary, and let \( \emptyset \neq U \subset \mathbb{R}^N \) be open. Show that

\[
S + U := \{x + y : x \in S, y \in U\}
\]

is open.

4. Let \( S \subset \mathbb{R}^N \). Show that \( x \in \mathbb{R}^N \) is a cluster point of \( S \) if and only if each neighbourhood of \( x \) contains an infinite number of points in \( S \).

5. Let \( S \subset \mathbb{R}^N \) be any set. Show that \( \partial S \) is closed.

6*. For \( j = 1, \ldots, N \), let \( I_j = [a_j, b_j] \) with \( a_j < b_j \), and let \( I := I_1 \times \cdots \times I_N \). Determine \( \partial I \). (Hint: Draw a sketch for \( N = 2 \) or \( N = 3 \).)

Due Monday, October 1, 2018, at 10:00 a.m.; no late assignments.