1. Let \( a, b > 0 \). Determine the area of the ellipse

\[
E := \left\{ (x, y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \right\}.
\]

2. Let \( D \) in spherical coordinates be given as the solid lying between the spheres given by \( r = 2 \) and \( r = 4 \), above the \( xy \)-plane and below the cone given by the angle \( \theta = \frac{\pi}{3} \). Evaluate the integral \( \int_D xyz \).

3. Let \( K \subset \mathbb{R}^2 \) be the triangle with vertices \((0,0), (1,3), \) and \((0,3)\). Evaluate the line integral

\[
\int_{\partial K} x^2 y^2 \, dx + 4xy^3 \, dy
\]

where \( \partial K \) is oriented counterclockwise.

4. Let \( \emptyset \neq U \subset \mathbb{R}^3 \) be open, and let \( f, g : U \to \mathbb{R} \) be twice continuously partially differentiable. Show that \( \text{div}(\nabla f \times \nabla g) = 0 \) on \( U \), where \( \times \) denotes the cross product in \( \mathbb{R}^3 \).

5. Let

\[
f : \mathbb{R}^3 \to \mathbb{R}^3, \quad (x, y, z) \mapsto (x \cos^2 y + \arctan(yz), (y + e^z), z \sin^2 y).\]

Evaluate \( \int_S f \cdot n \, d\sigma \) where \( S \) is the sphere with radius \( r > 0 \) centered at the origin, and \( n \) is the outward pointing normal unit vector.

6*. Let \( D \subset \mathbb{R}^2 \) be the trapeze with vertices \((1,0), (2,0), (0,-2), \) and \((0,-1)\). Evaluate \( \int_D \exp \left( \frac{x+y}{x-y} \right) \). (Hint: Consider

\[
\phi : \mathbb{R}^2 \to \mathbb{R}^2, \quad (u, v) \mapsto \left( \frac{1}{2}(u + v), \frac{1}{2}(u - v) \right)
\]

and apply Change of Variables.)

Due Wednesday, December 2, 2020, at 10:00 a.m.; no late assignments.