BLACK HOLE PHYSICS

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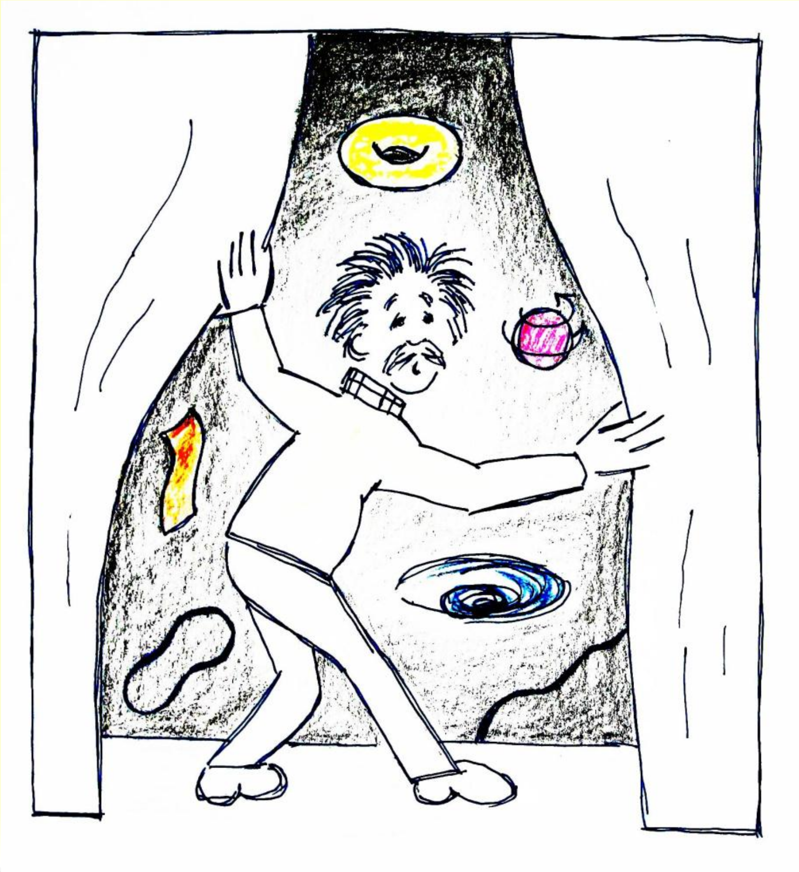
12. Problems and Puzzles of Black Holes



Black Hole Entropy Problem

Entropy scales

Stellar mass black hole $(10M_{\odot}) \Rightarrow S \sim 10^{79}$

Milky Way black hole $(M \sim 4\times10^{6} M_{\odot}) \Rightarrow S \sim 1.7\times10^{90}$

Supermassive black hole $(M \sim 10^{9} M_{\odot}) \Rightarrow S \sim 10^{95}$

All stars in the observable Universe $\Rightarrow S \sim 10^{79}$

Relic gravitons $\Rightarrow S \sim 10^{86}$

CMB photons $\Rightarrow S \sim 10^{88}$

Relic neutrinos $\Rightarrow S \sim 10^{88}$

A single Black Hole in the Milky Way has larger entropy than all visible matter in the observable Universe

Microscopic Origin of Black Hole Entropy

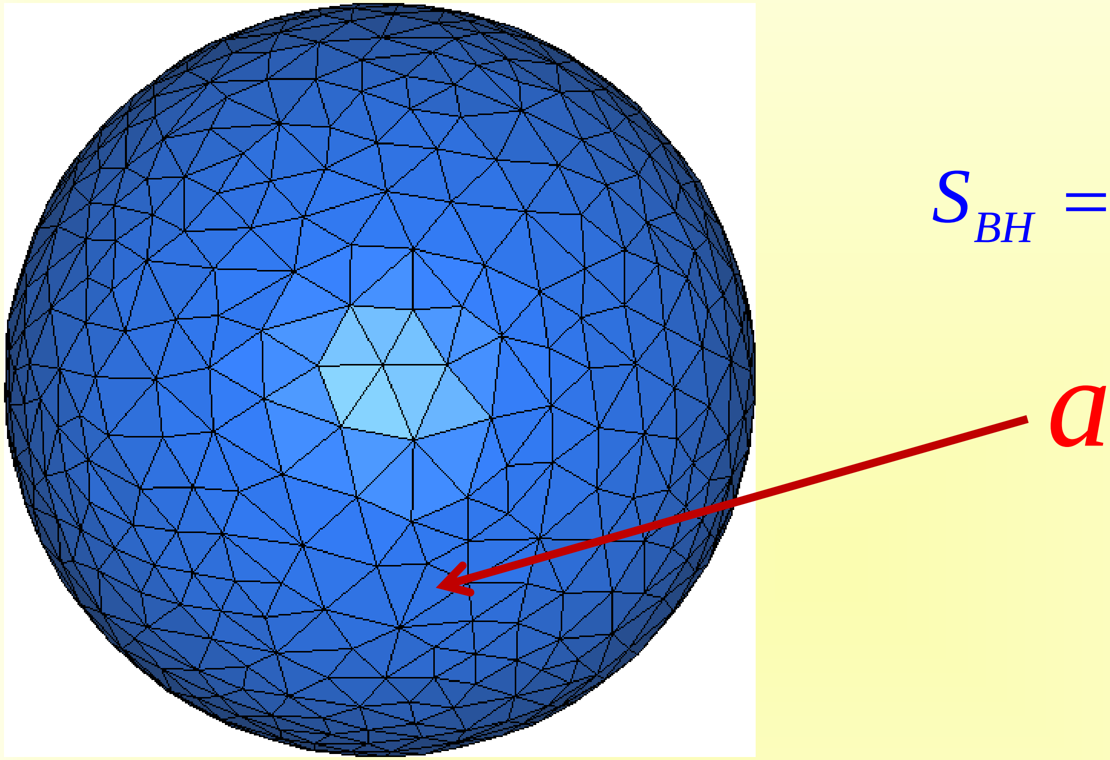
What are microscopical degrees of freedom

responcible for BH entropy $S = \dfrac{A}{4\, l_{Pl}^2}$?

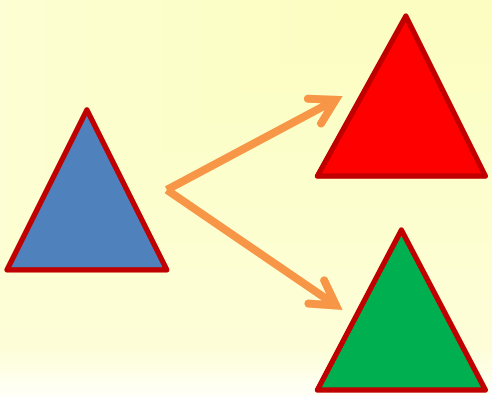
(1) Each $(2l_{Pl})^2$ element of BH surface has 1 bit of entropy $\Rightarrow$ Quantum gravity is required

(2) Universality problem: BH is determined by low energy physics parameters

(3) BH is a ground (vacuum) state of classical gravitational field $\Rightarrow$ gravity should be an emergent phenomenon



$$S_{BH} = \frac{A}{a^2}, \quad a = 2l_{Pl}$$



There exists $\mathbb{Z}=2^N$ 2-colored triangulations of the sphere for N-mono colored one. Entropy S is:

$$S = \ln \mathbb{Z} = N \ln 2$$

Entanglement Entropy

The concept of entanglement was introduced by Schrödinger. He wrote a letter (in German) to Einstein in which he used the word Verschränkung (translated by himself as entanglement). In this letter he discussed was is called Einstein, Podolsky, Rosen paradox (1935).

"If the quantum state of a pair of particles is in a definite superposition, and that superposition cannot be factored out into the product of two states (one for each particle), then that pair is entangled." (Wiki)

Let $\rho_{\mathrm{ab}} = \left|\Psi\right\rangle\left\langle\Psi\right|$ be a density matrix for a pure state of two-particle (a,b) system. Denote $\rho_{\mathrm{a}} = Tr_b\left|\Psi\right\rangle\left\langle\Psi\right|$ and $\rho_{\mathrm{b}} = Tr_a\left|\Psi\right\rangle\left\langle\Psi\right|$. Then entanglement entropy is defined as follows: $S_a = -Tr_a(\rho_{\mathrm{a}} \ln \rho_{\mathrm{a}}) = -Tr_b(\rho_{\mathrm{b}} \ln \rho_{\mathrm{b}}) = S_b$

“Quantum source of entropy for black holes”, Bombelli, Koul, Lee, and Sorkin [Phys.Rev. D34, 373 (1986)]

Abstract

We associate to any quantum field propagating in the background metric of a black hole an effective density matrix whose statistical entropy can be interpreted as a contribution to the total entropy of the black hole. By evaluating this contribution in a simplified case, we show that in general it can be expected to be finite and proportional to the area of the black hole. As a by-product of our calculation we obtain a general expression for the entropy of any real Gaussian density matrix.

Entropy of a vacuum domain with a sharp boundary

$S \sim \dfrac{A}{l^2_{cut-off}}$, A is the surface area of the boundary.

“Entropy and Area”, by M. Srednicki, Phys. Rev. Lett. 71, 666 (1993).

Abstract

The ground state density matrix for a massless free field is traced over the degrees of freedom residing inside an imaginary sphere; the resulting entropy is shown to be proportional to the area (and not the volume) of the sphere. Possible connections with the physics of black holes are discussed.

“Dynamical origin of the entropy of a black hole”

by Frolov and Novikov, Phys.Rev. D48, 4545 (1993)

Abstract

Modes of physical fields which are located inside a horizon and which cannot be observed by a distant observer are identified with the dynamical degrees of freedom of a black hole. A new invariant statistical mechanical definition of black-hole entropy is proposed. It is shown that the main contribution to the entropy is given by thermally excited "invisible" modes propagating in the close vicinity of the horizon. A calculation based on the proposed definition yields a value of the entropy which is in good agreement with the usually adopted, the black-hole surface area divided by 4 Planck length squared.

Quantum fluctuations of the horizon as the origin of the ultraviolet cut-off.

$$\rho_+(\varphi_+(x), \varphi_+(x')) = \langle \varphi_+(x) | \hat{\rho}_+ | \varphi_+(x') \rangle$$

$$\hat{\rho}_+ = Z_+^{-1} \exp(-\beta \hat{H}_+),$$

$\hat{H}_+$ is a Hamiltonian of the field Φ in R_+,

$\beta = \kappa / 2\pi$ is the inverse Hawking temperature

(period of the Hawking-Gibbons instanton)

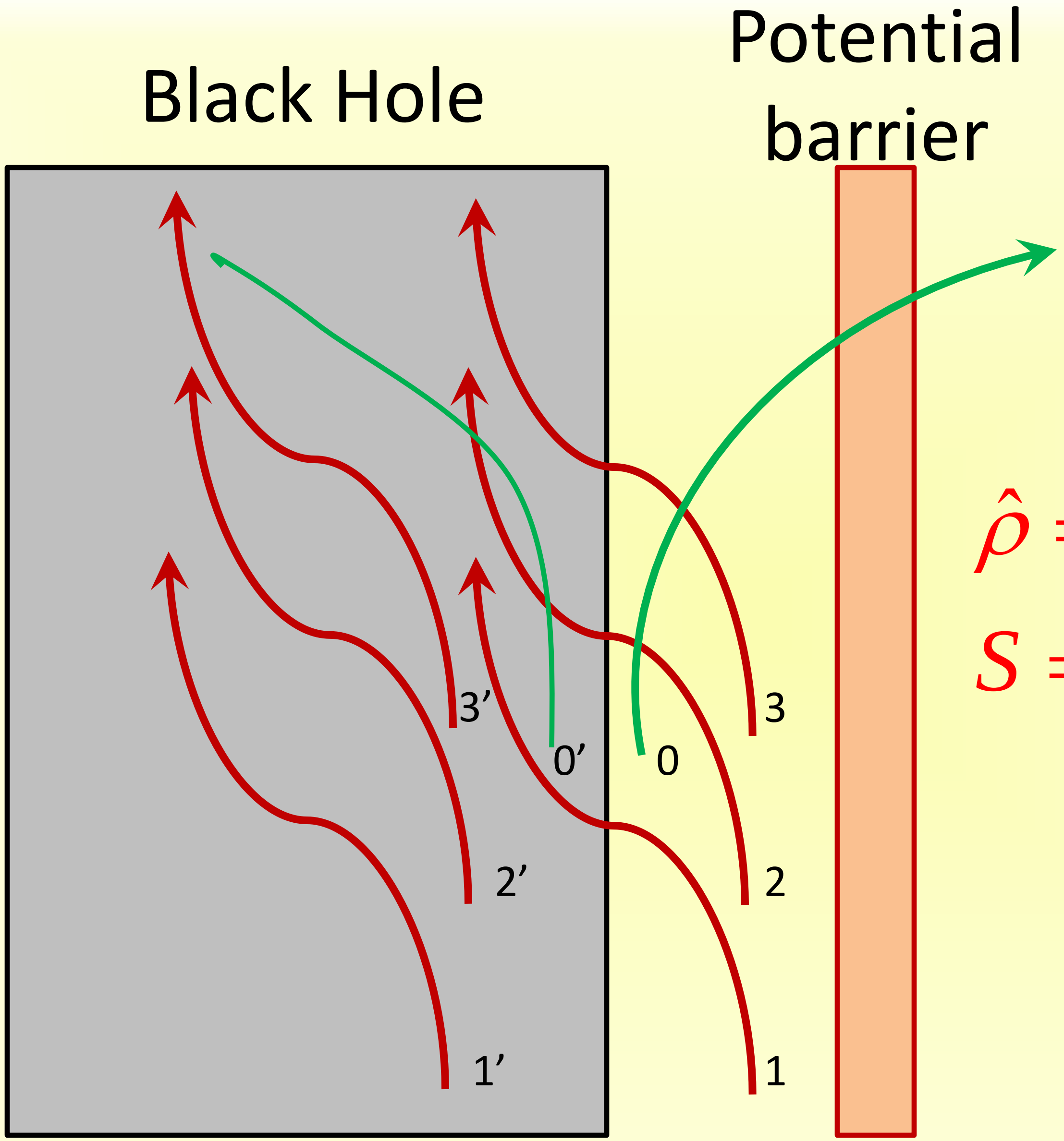
$$S_+ = -Tr(\hat{\rho}_+ \ln \hat{\rho}_+)$$

$S_+ \Leftrightarrow$ (volume contribution)+(surface contribution)

(volume contribution) $\Leftrightarrow$ entropy of thermal radiation
outside the black hole

(surface contribution) $\Leftrightarrow$ entanglement entropy of Rindler
particles near black hole horizon

The latter is the black hole entropy: $\sim \dfrac{\mathrm{A}}{\mathrm{l}_{\mathrm{Pl}}^2}$



$$\hat{\rho} = Tr^{(')}\hat{\rho}_{vacuum};$$

$$S = -Tr(\hat{\rho}\ln\hat{\rho})$$

Entanglement explanation of the black hole entropy is very promising.
However it (and its different modifications) has 2 fundamental difficulties:

(i) For a `sharp' horizon it diverges. Horizon fluctuations may help.

(ii) It is not universal. EE depends on the number of constituents and their properties (spin etc.)

To `solve' the first problem it was proposed to introduce (prescribe) infinite negative etropy of the black hole -$S_{div}$, so that the observed `phisical' entropy is $S_{BH} = S_{ent} - S_{div}$. But this procedure totally breaks the fundumental relation $S_{BH} = -Tr(\rho \ln \rho)$, valid in the entanglement approach.

It seams that both of the problems can be solved simultaneously only if the gravity is an emergent theory.

Selected approaches to BH entropy problem

"Black Hole Thermodynamics and Statistical Mechanics",
by Steven Carlip. Lect.Notes Phys. 769 (2009) 89-123;
e-Print: arXiv:0807.4520

1. **String theory: weakly coupled strings and branes:**

Strominger and Vafa (1996): An extremal supersymmetric (BPS) black hole is uniquely characterized by its charges; in particular, its horizon area can be expressed in terms of these charges. Given such a black hole, one can imaging tuning down the couplings, weakening gravity until the black hole "dissolves" into a gas of weakly coupled strings and branes. In this weakly coupled system, the charges can be expressed in terms of the number of strings and branes and the quantized momentum carried by strings. Furthermore, the states—the excitations of the string-brane system—can be explicitly counted.

2. ADF/CFT correspondence and conformal field theory

Maldacena's AdS/CFT correspondence conjecture asserts a duality between string theory in *d*-dimensional asymptotically anti-de Sitter spacetime and a conformal field theory in a flat (d −1)-dimensional space that can, in a sense, be viewed as the boundary of the AdS spacetime. For asymptotically anti-de Sitter black holes, this correspondence makes it possible to compute entropy by counting states in a dual conformal field theory.

Cardy formula (1986) gives the entropy of (1+1)-dimensional conformal field theory (CFT): $S = 2\pi\sqrt{(c/6)(L_0 - c/24)}$.

Here c and L_0 are parameters, that specify CFT: c is the central charge and L_0 is the product ER of the total energy and radius of system (the Virasoro algebra of the CFT).

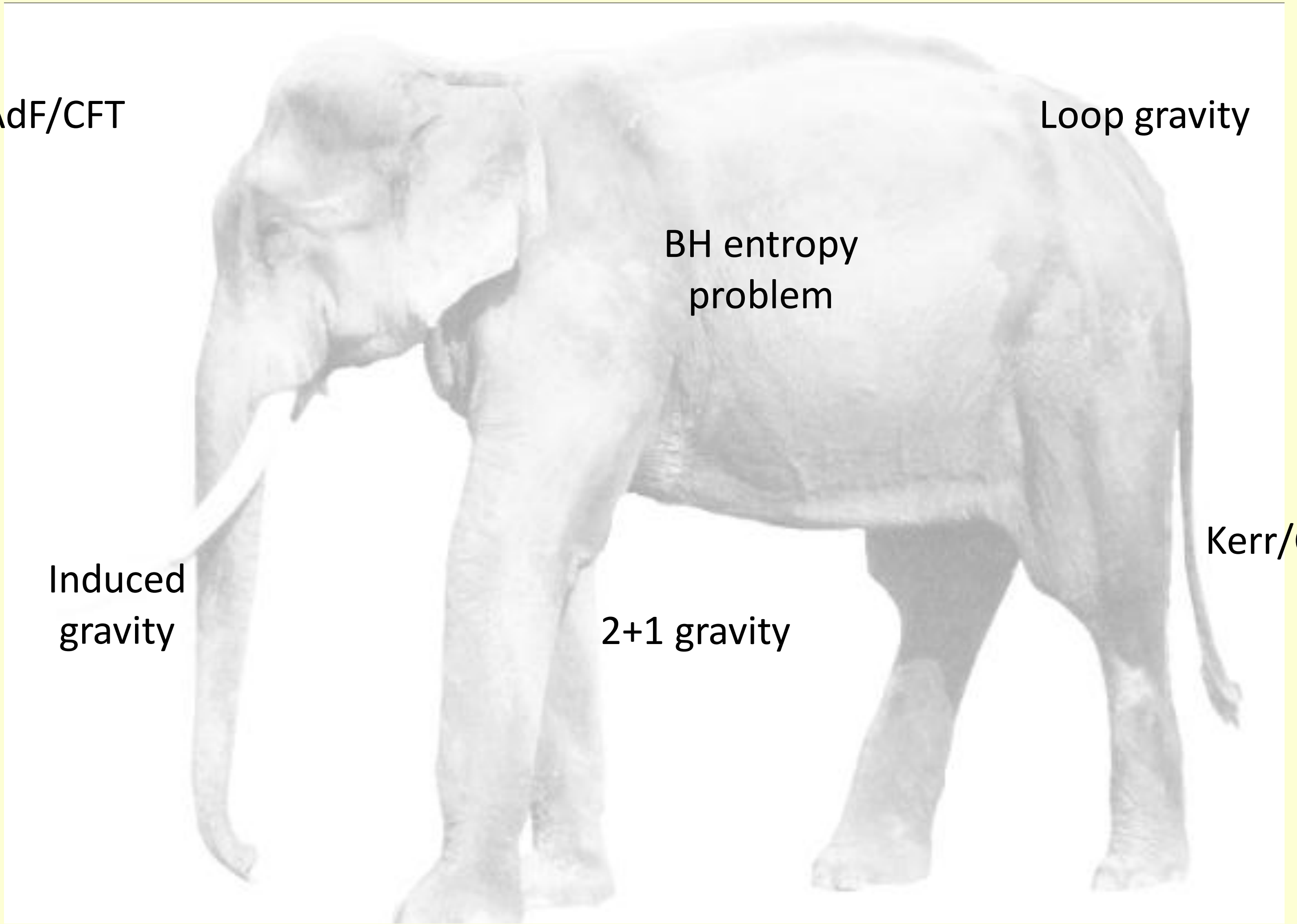
3. Loop quantum gravity (Ashtekar)

Space is described as a network of finite size loops, called spin network. BH entropy is related with counting the logarithm of number of different spin structures crossing the horizon.

4. Induced gravity approach (Frolov, Fursaev and Zelnikov (1996).

In Sakharov's induced gravity (1967) one starts with a theory of heavy fields (constituents) propagating in a curved spacetime. The gravity becomes dynamical as a result of their quantization. This approach resembles the well-known approach in the solid state physics when instead of variables describing the oscillations of the atoms of the lattice one uses new collective variables describing the phonon field. It is important that the thermodynamical characteristics of a solid state in the low energy regime, say at the low temperatures, can be expressed by using only the spectrum of the phonon excitations. In SIG-approach the BH entropy is related to the counting of the degrees of freedom of the heavy constituents [see V.F. and Fursaev Class.Quant.Grav. 15 (1998)]

String theory



Information loss paradox

In the process of the black hole evaporation, its energy is radiated away and its mass reduces. During the Hawking process the initially pure quantum state (initial vacuum), is transformed into entangled pure state. In this state part of the particles, forming the Hawking radiation is located outside the black hole, while the other part of the particles is located inside the black hole. There exists strong correlation between these two subsystems of spatially separated particles, which makes a state of a complete system, formed by these two subsystems, pure. After the complete evaporation of the black hole it disappears. In such a process the initial pure quantum mechanical system would be transformed into a mixed state of outer particles, described by a (non-pure) density matrix.

Information Loss Puzzle

$$\begin{pmatrix}\text{Pure}\\ \text{state}\end{pmatrix} \overset{\text{Collapse}}{\Rightarrow} \begin{pmatrix}\text{Black}\\ \text{hole}\end{pmatrix} \overset{\text{Evaporation}}{\Rightarrow} \begin{pmatrix}\text{Thermal}\\ \text{radiation}\end{pmatrix}$$

If an initial pure state evolves into a mixed state, described by the density matrix, the unitarity of quantum evolution is lost. The unitarity is a basic principle of the quantum mechanics. Does it mean that

Gravity + Quantum Mechanics= Inconsistent theory ?

Main ways out:

1. Quantum mechanics is still valid on a causal evolution of the initial Cauchy surface, but after evaporation an external observer has access only to a part of the system;
2. Information gradually leaks out during the evaporation process;
3. Information is collected in a small mass remnant;
4. Entanglement is immediately broken between the in-falling particle and the outgoing particle. A falling observer will see a `firewall', when he/she crosses the horizon. This "resolution" requires a violation of Einstein's equivalence principle,

5. No event horizon but only apparent horizon. Information returns `back' after apparent horizon disappears.

V.F. and G.Vilkovisky "Spherically symmetric collapse in quantum gravity" , Trieste preprint (1979), Phys.Lett. B106, 307 (1981)

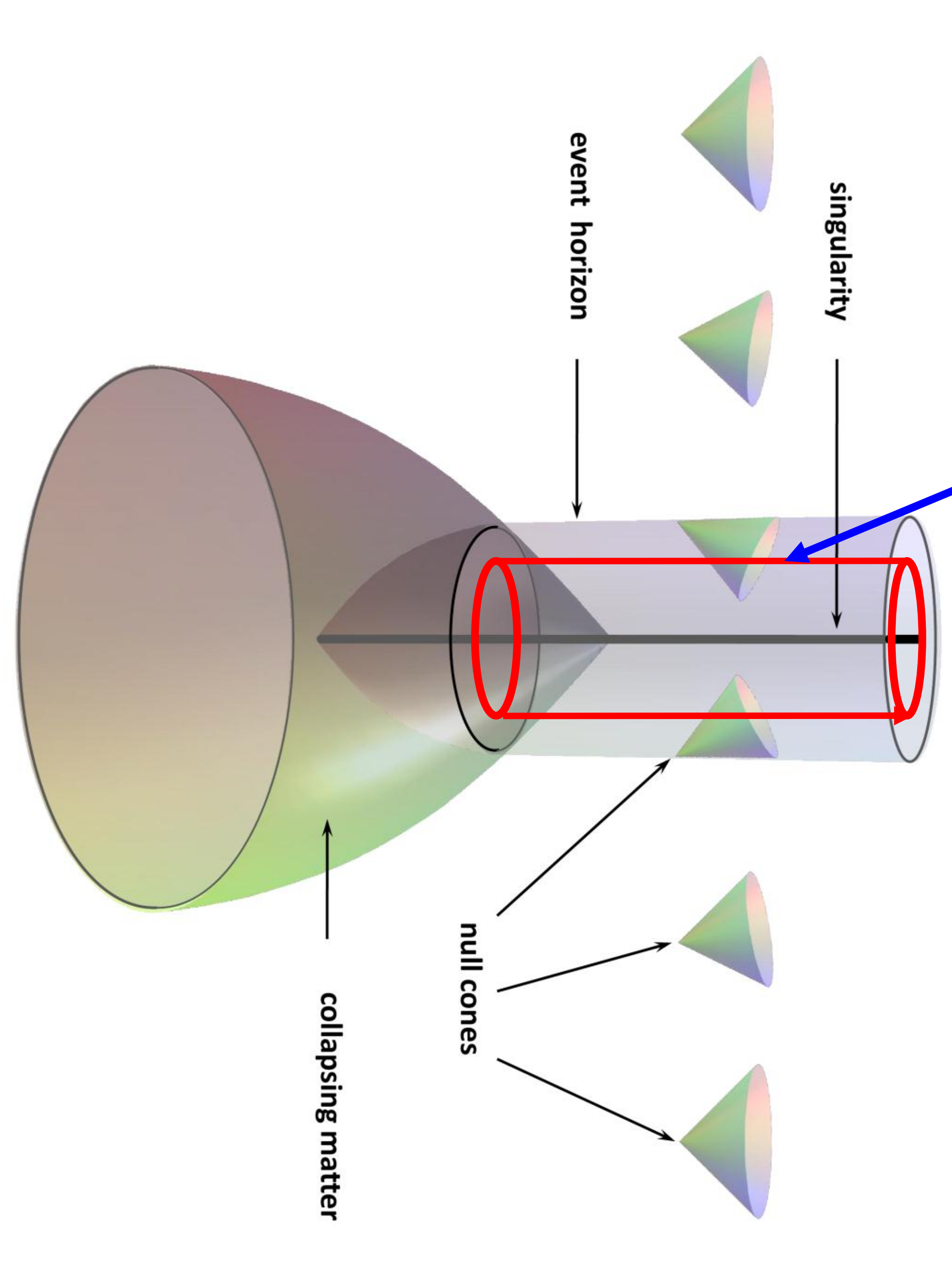
S.Hawking, January 22, 2014

Black Hole Interior Puzzle

Gravitational radius $r_g = \dfrac{2GM}{c^2}$

"The spatial volume inside the black hole is $\sim r_g^3$".

Is this right? No, this is completely wrong!

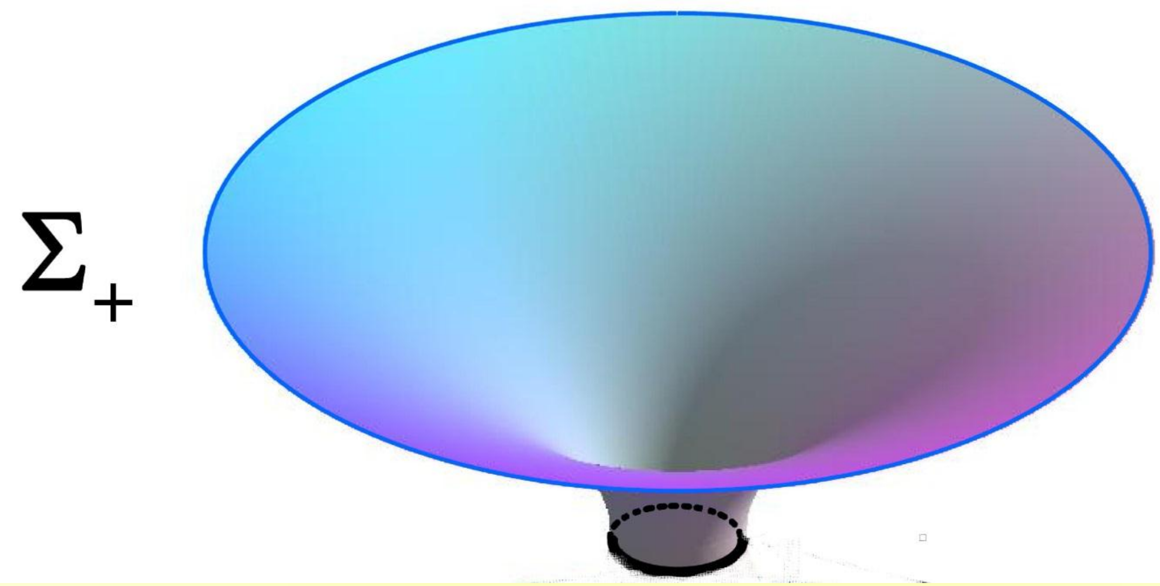


r plays the role of time inside BH

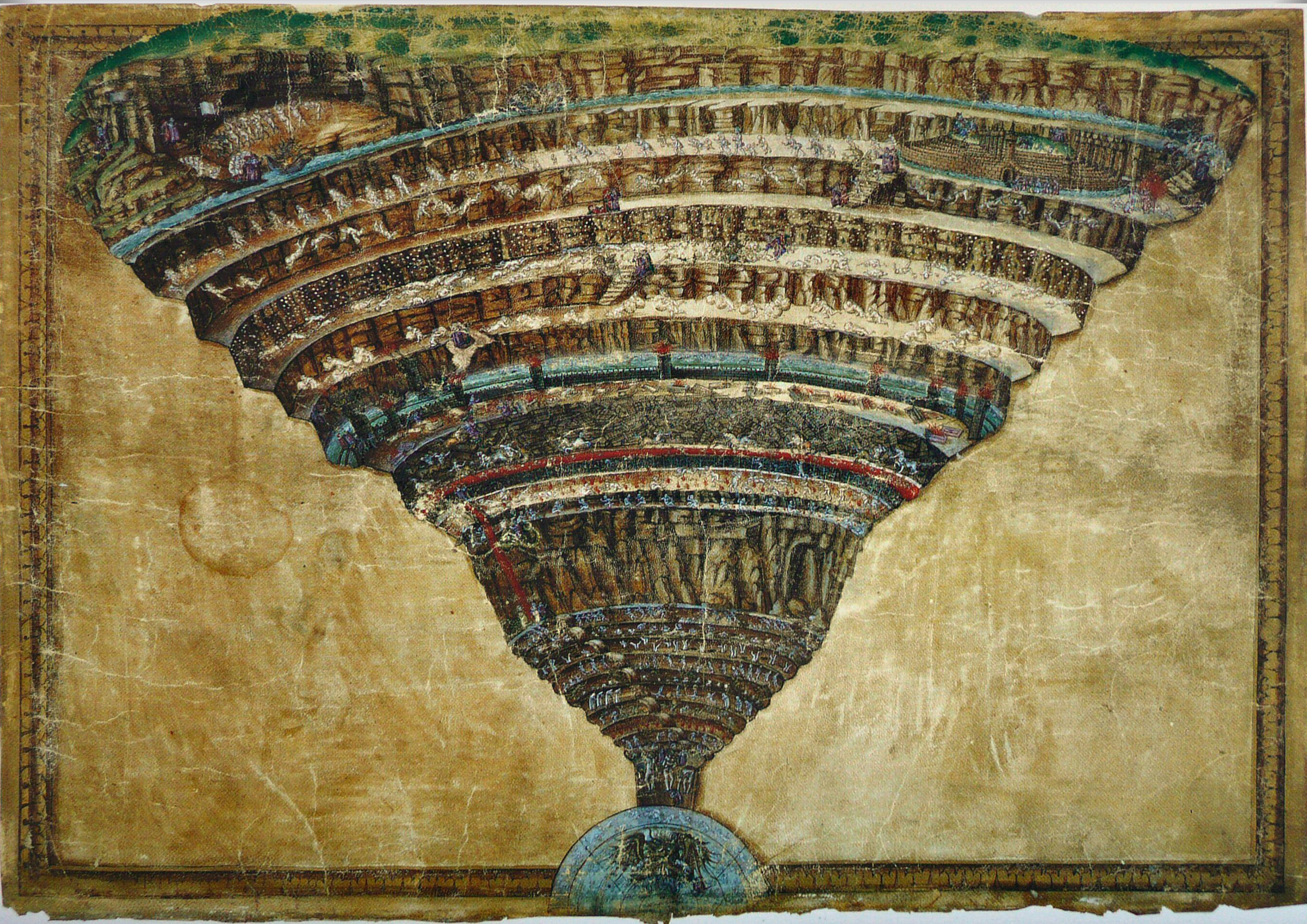
Slice $r = \text{const}$ has topology $S^2 \times R$

Spatial volume $\sim r_g^2 \times (t / c) >>> r_g^3$

Contracting anisotropic universe



Sandro Botticelli (1480) “La Carte de l'Enfer”
(“The Abyss of Hell’). An illustration to Dante’s “The Divine Comedy”



“If the Hell exists the best place for it is in the Black Hole Interior”

V.F., Markov, Mukhanov:
"Through A Black Hole Into A New Universe?" Phys.Lett. B216 (1989) 272-276;
"Black Holes As Possible Sources Of Closed and Semiclosed Worlds" , IC/88/91. May 1988. Phys.Rev. D41 (1990) 383**;**
Barrabes and V. F., "How many new worlds are inside a black hole?" Phys.Rev. D53 (1996) 3215

Buonanno, Damour, Veneziano, "Pre- big bang bubbles from the gravitational instability of generic string vacua", Nucl.Phys. B543 (1999) 275-320:
"The picture is therefore that inside each black hole, the regions near the singularity … will blister off … as many separate pre-Big Bangs. These inflating patches are surrounded by non-inflating, or deflating (decreases ϕ) patches, and therefore globally look approximately closed Friedmann-Lemaˆıtre hot universes.

Smolin, *"The Life of the Cosmos"* , 1997:
Wiki: The theory surmises that a collapsing black hole causes the emergence of a new universe on the "other side", whose fundamental constant parameters may differ slightly from those of the universe where the black hole collapsed. Each universe thus gives rise to as many new universes as it has black holes.

`VERY BIG' PICTURE

Birth: Matter and its structure in our world
is a result vacuum fluctuations amplification
in the early inflating Universe

Life: Supernova explosions -> heavy elements
-> planets and life

Death: Black holes are `matter graves' in our Universe

Reincarnation: Black holes as origin of new
universes: `Life after death' ???????????

Brief Summary

1. Evolution of the black hole concept: formal solutions of Einstein equations-> physical objects with amazing properties-> important elements of the Universe;

2. The most powerful sources of energy;

3. `Rosetta stones' providing relations between different areas of physics;

4. Universal probes of new physics;

5. An `arena' for interesting applications of modern mathematics;

6. Source of fundamental puzzles and problems;

7. Philosophical problems;

8. Role in the `very big picture' of our Universe.