

Processing of anisotropic data in the τ - p domain: II — Common-conversion-point sorting

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ABSTRACT

Common-conversion-point (CCP) sorting of P-SV converted-wave data is conventionally done by first sorting data into common asymptotic-conversion-point (CACP) gathers and then computing the involved CCP shifts from analytic approximations. I explore an alternative method where the latter step is replaced by an entirely data-driven approach. Moveout curves of correlated P-P and P-SV reflections in collocated CMP and CACP gathers are first scanned for points of equal slowness. A common-source slowness indicates that the downgoing branches of the P-P and P-SV waves overlap if the conversion occurs at the reflecting interface. The P-SV conversion point is then assumed to be situated underneath the associated P-P wave midpoint. A migration of amplitudes from CACP to CCP gathers is straightforward once the exact CCP position is known. This data-driven approach requires kinematic information only and is exact for laterally homogeneous media with arbitrary strength of anisotropy if horizontal symmetry planes are present at all depths. Both time-offset and τ - p domain implementations are possible, although the latter are preferred.

INTRODUCTION

The last few years have seen a significant increase in the use of multicomponent data, and this trend will undoubtedly continue. In particular, three-component (3C) data are acquired more and more often because they allow us to record the complete elastic wavefield without the need to employ expensive and often ineffective shear vibrators. Some anticipate that effective use of 3C data may even obliterate the need of any shear sources in a foreseeable future (Thomsen, 2001). Unfortunately, this cost-efficient gain of information comes at an expense. Accurate processing of 3C converted P-SV wave data

is considerably more involved than processing of pure-mode P-P data and maybe even of pure-mode SV-SV data.

Some very basic problems that render converted-wave processing more challenging are, for instance, vector fidelity, common-conversion-point (CCP) sorting, and event registration (i.e., corresponding P-P and P-SV reflections need to be identified in order to interpret the data). The issue of vector fidelity is related to the fact that the actual orientations of all components need to be determined before the recorded wavefield can be separated in radial (inline) and transverse (crossline) components. This problem occurs in particular for ocean-bottom data, where twisted and kinked cables are quite common. This problem, however, lies outside the scope of this paper, as does the topic of event identification.

The problem of CCP sorting arises because P-SV raypaths are by their very nature asymmetric (Figure 1). The conversion point does not lie at half the P-SV emergence offset, as does the P-P reflection point, or even at a constant relative distance — even if the medium is laterally homogeneous. It depends both on offset and depth (and thus traveltime) and, in the case of azimuthal anisotropy, on azimuth as well. As a matter of fact, for some rare, strongly anisotropic media, the conversion point may even lie beyond the receiver (that is, the SV-branch bounces back), and it does not necessarily lie within the vertical source-receiver plane but may be offset from it. Such out-of-plane conversion points occur for all azimuthally anisotropic media for wave propagation outside of the symmetry planes without the need to introduce any lateral inhomogeneity (Van der Baan and Kendall, 2003). This happens because of the different azimuthal dependencies of the P- and SV-wave phase velocities.

Several analytic expressions for conversion-point positions have been published. See, for instance, Chung and Corrigan (1985), Tessmer and Behle (1988), and Schneider (2002) for the isotropic case, and Thomsen (1999) and Li and Yuan (2003) for vertically transverse isotropic (VTI) media. Only Van der Baan and Kendall (2003) outline a procedure to compute CCP points for both VTI and HTI (horizontally transverse isotropic) media. In addition, this method is easily

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extended to generally anisotropic media. All published expressions, however, are limited to laterally homogeneous, layered media and have varying degrees of accuracy. In particular, for large offset/depth ratios, their accuracy often dwindles.

Unfortunately, none of these authors discusses in great detail how CCP sorting using their analytic expressions should actually be implemented and applied on real data. In this paper, I describe a simple, novel technique to sort converted-wave data into CCP gathers using a migration-type approach. The technique is entirely data driven and differs from previous methods in that no analytic CCP expressions need to be evaluated. Only a kinematic description of moveout is required. It is exact in 1D, laterally homogeneous media for arbitrary strength of anisotropy if a horizontal symmetry plane is present (i.e., for HTI, VTI, orthorhombic, and monoclinic media, but not for tilted transversely isotropic or triclinic symmetries).

First, I describe the general principle on which the technique is based. I discuss both time-offset and τ - p (intercept time-slowness) implementations and their respective advantages and inconveniences. Finally, I show the performance of the scheme on a synthetic data example, illustrating in particular the importance of CCP sorting for accurate converted-wave amplitude variation with offset (AVO).

CCP SORTING

General principle and t - x domain sorting

The downgoing branches of a P-P and P-SV arrival overlap if their takeoff angles and, thus, source slownesses are identical. Their emergence offsets differ, however (Figure 1). This is true for arbitrarily complex, laterally inhomogeneous, anisotropic media. It is explicitly assumed in this paper that the conversion occurs at the reflection point, which is far from a general truth.

Source slowness is retrieved by computing the local slope of traveltimes in common-receiver gathers. Grechka and Tsvankin (2002) calculated in this way the source slownesses of both P-P and P-SV waves. This enabled them to identify P-P and P-SV arrivals with coincident downgoing branches by finding correlated reflections in P-P and P-SV common-

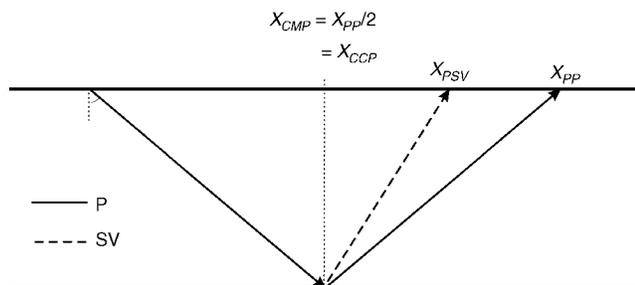


Figure 1. The downgoing branches of a P-P and a P-SV wave overlap if their horizontal source slownesses and, thus, their take-off angles are identical. Their emergence offsets x_{PP} and x_{PSV} are, however, different. The conversion point x_{CCP} can be found underneath the associated common-midpoint (CMP) position $x_{CMP} = x_{PP}/2$ if the medium is laterally homogeneous with horizontal symmetry planes at all depths.

receiver gathers with matching source slownesses at a shared source position. They then continue to compute corresponding pure-mode SV-SV traveltimes. I use their technique as a starting point for building a data-driven CCP sorting scheme.

The PP + PS = SS technique of Grechka and Tsvankin (2002) is exact in arbitrarily complex anisotropic media, but it has two inconveniences: it does not predict where the actual conversion point occurs, and P-P and P-SV traveltimes in common-receiver gathers are not centered around zero offset but are generally moved updip (Levin, 1971; Van der Baan, 2005). It is therefore not possible to extract the moveout curves using a conventional moveout analysis; all traveltimes curves have to be hand picked, which can be prohibitively expensive. I make two simplifying assumptions that lead to a CCP sorting scheme that is exact in laterally homogeneous media with horizontal symmetry planes at all depths (i.e., HTI, VTI, orthorhombic, and monoclinic media, but not tilted transversely isotropic or triclinic symmetries). Keep in mind, however, that conventional analytic expressions used for CCP sorting can handle only isotropic or VTI symmetries and require approximations to deal with multilayer media (Thomsen, 1999; Li and Yuan, 2003).

It is well known that common-midpoint (CMP) sorting of P-P wave data leads to moveout curves that are always centered around zero offset, regardless of the complexity of the medium (Shah, 1973). Van der Baan (2005) proved that common asymptotic-conversion-point (CACP) sorting of converted-wave data also yields moveout curves centered around zero offset for arbitrarily complex, laterally inhomogeneous but isotropic media if the P-to-S wave velocity ratio is constant everywhere. The asymptotic conversion point is the conversion point for near-vertical raypaths, i.e., in the limit of zero offset/depth ratios x/z (Tessmer and Behle, 1988; Thomsen, 1999).

A conventional velocity analysis can therefore detect the P-SV moveout curves after CACP sorting, although positive and negative offsets need to be treated independently since they are characterized by different moveout velocities (Thomsen, 1999; Van der Baan, 2005). Media with varying P-to-S wave velocity ratios and weak anisotropy can still be handled with sufficient accuracy in most cases. Furthermore, converted-wave moveout curves are also centered around zero offset in laterally homogeneous media with arbitrary P-to-S wave velocity ratios if horizontal symmetry planes are present everywhere. The required slopes can thus be computed analytically from the velocities picked using a conventional velocity analysis on the CMP and CACP gathers.

The computed slowness is actually a mixture of the true source and receiver slownesses (Shah, 1973; Van der Baan, 2005), and their difference increases with increasing complexity of the medium. Matching P-P and P-SV slownesses in collocated CMP and CACP gathers thus enables us to recover arrivals with largely overlapping downgoing branches. Source and receiver slownesses are fortunately identical in laterally homogeneous media. This approach is, therefore, exact in laterally homogeneous media with horizontal symmetry planes everywhere while circumventing the need to manually pick traveltimes.

Where, however, is the conversion point located? The reflection point of the P-P wave and the conversion point of the

P-SV wave are collocated if their downgoing branches coincide — even though their emergence offsets differ. The conversion point simply occurs at half the emergence offset of the P-P reflection with identical horizontal slowness if the medium is laterally homogeneous with horizontal symmetry planes everywhere (Figure 1). In laterally inhomogeneous media, the P-P wave reflection point is not underneath the CMP position, which is why we turn to dip moveout (DMO) and migration to correctly position the reflection points afterward. This assumption is well suited as a first approximation. Likewise, P-P wave reflection-point dispersal also occurs in 1D media without horizontal symmetry planes, but this is generally ignored.

Thus, a three-step procedure can successfully sort P-SV data directly in the time-offset domain. In the first step, corresponding P-P and P-SV reflections need to be determined (i.e., event registration performed), and a common ratio $\gamma_{eff} = \gamma_n^2 / \gamma_0$ is defined. The quantity $\gamma_0 = \bar{\alpha}_0 / \bar{\beta}_0$ is the ratio of the average vertical P- and S-wave velocities as determined by the zero-offset traveltimes, and γ_n is the corresponding ratio of the short-spread moveout velocities (Thomsen, 1999). The latter ratios can be obtained from a preliminary velocity analysis. In the second step, data is sorted in CACP and CMP gathers. The CACP position depends on the emergence offset only if a constant ratio γ_{eff} is assumed for all reflectors, in which case CACP gathering then involves sorting of complete traces only. Finally, offsets with identical P-P and P-SV traveltimes slopes are identified at collocated CMP/CACP gathers (Figure 2a and b). The required slownesses are, in practice, directly computed from the picked velocities. The lateral shift δx_{CCP} of the conversion point from the considered CACP position is then computed using

$$\begin{aligned} \delta x_{CCP}(p_x) &= x_{CMP}(p_x) - x_{CACP}(p_x) \\ &= \frac{1}{2} x_{PP}(p_x) \\ &\quad - \frac{\gamma_{eff}}{1 + \gamma_{eff}} x_{PSV}(p_x), \quad (1) \end{aligned}$$

where the effect of the initial CACP sorting is taken into account. The converted-wave amplitude at the point $t(x)$ on the inline x -component can then be put in a new CCP gather located at the computed surface position.

The identification of corresponding P-P and P-SV reflections is clearly the most challenging part, but this topic needs to be addressed before any CCP sorting can be accomplished, independent of the algorithm chosen. A similar approach can be used for sorting of the P-SH wave recorded on the transverse component.

Only a parametric description of the traveltimes curves is required. That is, any function that accurately describes both the hyperbolic and nonhyperbolic parts of the moveout curves is adequate. Both the shifted-hyperbola equation (Castle, 1994) and third-order or modified Taylor-series expansions

(May and Stratley 1979; Tsvankin and Thomsen, 1994; Thomsen, 1999) are, for instance, appropriate. This is fortunate because significant trade-offs exist between anisotropy parameters and stacking velocities even in VTI media [see, for instance, Alkhalifah (1997)]. Although it is relatively straightforward to detect nonhyperbolic moveout, translating this into the actual underlying anisotropy parameters is much more challenging.

The method is readily extended to handle 3D data. Both the horizontal slownesses p_x and p_y in the inline x - and crossline y -directions should then be matched. The lateral shift δy_{CCP} from the considered CACP position in the crossline direction y is then computed using

$$\delta y_{CCP}(p_x, p_y) = \frac{1}{2} y_{PP}(p_x, p_y) - \frac{\gamma_{eff}}{1 + \gamma_{eff}} y_{PSV}(p_x, p_y). \quad (2)$$

The presence of azimuthal anisotropy generally implies that the conversion point is not located within the source-receiver plane (Van der Baan and Kendall, 2003). Azimuthal anisotropy can, therefore, only accurately be handled if 3D data are available. For a 2D data set, the technique is exact only in isotropic or VTI media because it cannot account for out-of-plane effects since the crossline horizontal slowness p_y is missing.

Practical considerations

There are two primary goals for CCP sorting: either to create P-SV wave stacked traces or to accurately analyze converted-wave amplitude behavior versus offset (AVO). To

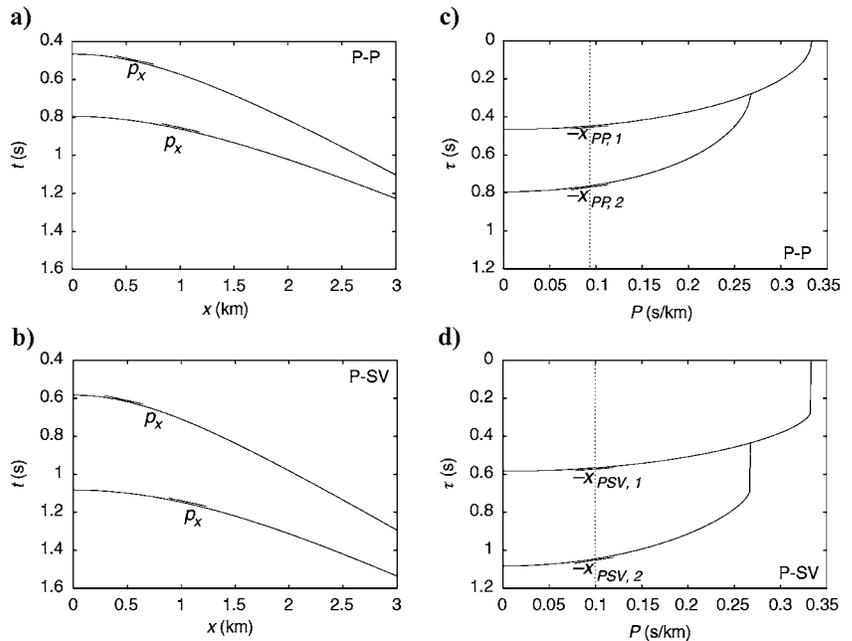


Figure 2. General sorting principle in the t - x and τ - p domains. The conversion point can be estimated by matching the slowness p_x on the $t(x)$ moveout curves of correlated (a) P-P and (b) P-SV reflections. Alternatively, identical slownesses p_x are easily identified in the τ - p domain. The required emergence distance now follows from the local slope of the $\tau(p_x)$ moveout curves of correlated (c) P-P and (d) P-SV reflections.

create stacked traces, we simply migrate moveout-corrected amplitudes from the considered CACP position to the actual CCP position and directly add it to the appropriate zero-offset time (Thomsen, 1999). No offset information, therefore, is required in this case unless a new velocity analysis is to be performed after CCP sorting. If offset information is to be conserved (for instance, for amplitude analysis), then several more practical difficulties are imposed.

One practical issue related to converted-wave processing in general is that different optimal stacking-bin sizes exist for sorting of pure-mode P-P waves and converted P-SV waves — even if recorded with the same acquisition geometry. Employing the normal bin size used for CMP sorting of P-P waves leads to highly irregular fold distributions for P-SV converted waves (Eaton and Lawton, 1992; Lawton, 1993; Li and Lu, 1999). This, in turn, affects stacking quality and also has implications for further processing (e.g., migration). Lawton (1993) showed for isotropic data that the optimal converted-wave bin size is proportional to the ratio of the asymptotic conversion point to the emergence distance. This remains true for VTI media. Thus, the optimal bin size $bin_{P_{SV}}$ for converted waves becomes

$$bin_{P_{SV}} = \frac{\gamma_{eff}}{1 + \gamma_{eff}} \min(\delta x_{src}, \delta x_{rec}), \quad (3)$$

with δx_{src} and δx_{rec} the source and receiver spacing, respectively. A similar relation holds for bin sizes in the crossline direction y . For comparison, the normal P-P wave bin size equals $(1/2) \min(\delta x_{src}, \delta x_{rec})$. Thus, the optimal P-SV wave bin size is generally larger than the P-P wave bin size.

The disadvantage of using different bin sizes for P-P and P-SV waves is that CMP and CACP gathers are no longer collocated. P-P and P-SV velocities, therefore, need to be interpolated to common surface positions for any postsorting processing steps that require mutual velocity information. Resulting fold distributions in adjacent CACP gathers are, on the other hand, similar for regularly shot and recorded data.

Once appropriate bin sizes have been computed and data are sorted in CMP and CACP gathers, we can re-sort the converted-wave data into CCP gathers or directly compute CCP stacked traces. In a time-offset implementation, we need to ensure that all relevant time and offset samples in a gather are included. This is most easily done by applying CCP sorting on moveout-corrected data (Thomsen, 1999).

The computed shifts δx_{CCP} and δy_{CCP} will only rarely fall on predefined discrete CACP positions. Thus, P-SV amplitudes need to be spread out between adjacent gathers with weights proportional to one minus the relative distance to each gather. Alternatively, the amplitude can simply be attributed entirely to the closest gather. The latter is similar to binning of traces before stacking. For CCP stacking, the resulting amplitude is then simply added to the gradually forming stacked trace. A new consideration arises, however, if offset information is to be retained.

The use of the optimal bin size for CACP sorting produces gathers with similar folds in adjacent gathers. The actual offset distribution, however, is very different from gather to gather, and the trace spacing is nearly always irregular. The CCP sorting process will thus always attribute some amplitudes to gathers where the considered offset is not present, yielding

very patchy and fragmentary CCP gathers. Even the creation of common-offset stacked CCP supergathers does not completely alleviate the problem. The only true solution is to interpolate all traces in each gather to a common basis of offsets with all due problems. This could be done by applying a forward and inverse τ - p transform. Retaining offset information in a time-offset implementation is, therefore, not a trivial exercise.

Sorting in the τ - p domain

A similar CCP sorting technique can be directly devised in the τ - p domain, where τ represents the vertical delay or intercept time. Identical slownesses are readily identified by transforming both the P-P CMP and P-SV CACP gathers to this domain by means of a slant stack, and by analyzing the resulting $\tau(p_x, p_y)$ moveout curves. We now have direct access to the slownesses, but the associated emergence offsets need to be estimated using the local slope of the $\tau(p_x, p_y)$ curves. The opposite situation occurs for sorting in the time-offset domain. The offsets are known here, but the slownesses need to be computed from the moveout slopes (Figure 2a, b). The emergence offset of the considered event in the τ - p domain equals minus the local slope of the $\tau(p_x, p_y)$ moveout curves (Figure 2c, 2d). That is, in vector form,

$$(x, y)^T = - \left(\frac{\partial \tau}{\partial p_x}, \frac{\partial \tau}{\partial p_y} \right)^T, \quad (4)$$

with T vector transpose (Diebold and Stoffa, 1981).

The P-SV conversion point (x_{CCP}, y_{CCP}) again equals the P-P midpoint (x_{CMP}, y_{CMP}) for identical horizontal slowness (p_x, p_y) if measured from the same shot position (see Figure 1 for the 2D case). The considered $\tau(p_x, p_y)$ sample of the P-SV wave on the radial component can then be put in a new τ - p CCP gather, where the lateral shift $(\delta x_{CCP}, \delta y_{CCP})$ of the CCP point from the CACP/CMP position is again computed using expressions 1 and 2. The P-P and P-SV emergence offsets follow from equation 4 (see also Figure 2).

Again, only a parametric description of the $\tau(p_x, p_y)$ curves is needed. I use the general analytic expressions developed by Van der Baan and Kendall (2002, 2003) to describe the $\tau(p_x, p_y)$ curves in laterally homogeneous media. In a 2D line, we again need to assume that the medium displays radial symmetry around the vertical axis, in which case the crossline slowness p_y vanishes. This is true only for 1D, isotropic, or VTI media.

If we assume that the medium is composed of horizontal, constant-velocity layers, then the P-P and P-SV emergence offsets associated with each intercept time τ within layer i can be obtained by linear interpolation between the emergence offsets occurring respectively at the top and bottom of the considered layer. This happens because the interval intercept time $\Delta \tau_i(p_x, p_y)$ is linearly proportional to the layer thickness, and the total intercept time $\tau_i(p_x, p_y)$ consists simply of a summation over all interval intercept times $\Delta \tau_i(p_x, p_y)$ up to layer i (Van der Baan and Kendall, 2002, 2003). The emergence offset, therefore, linearly increases in a one-layer medium from zero to its position at the bottom of the layer

and for multilayered media from the emergence offsets occurring at the top and bottom of each layer.

Again, CMP and CCP gathers only exist at predefined positions. The same amplitude-weighting scheme can be used as that for a time-offset implementation. Hence, the amplitude is distributed linearly between the surrounding gathers with the principal part being attributed to the closest gather. This simple scheme leads to perfect reconstructions for a regularly spaced data set composed of identical sections (as should occur for a perfect 1D layered earth without noise).

One advantage of a τ - p over a time-offset implementation is that it is straightforward to conserve slowness information after CCP sorting. Thus, it is no more demanding to create gathers suitable for amplitude-versus-slowness analysis than it is to create CCP stacked traces. This is because the τ - p gathers will share a common slowness basis — even if the time-offset input CACP gathers are irregularly distributed with varying folds. This prevents the occurrence of very patchy and fragmentary CCP prestack gathers and allows for a renewed velocity analysis after CCP sorting to determine the optimal P-SV moveout corrections.

Stacked traces can be created by either applying an inverse τ - p transform and consequently stacking over offset, or by directly stacking data in the τ - p domain (Van der Baan, 2004). The latter approach saves considerable computation time since the inverse τ - p transforms become redundant. A similar three-step procedure, therefore, is needed to sort the CCP gathers in the τ - p domain as that needed for the time-offset case, with the added benefit that slowness information is readily retained.

SYNTHETIC EXAMPLE

To illustrate the CCP sorting scheme, I created a synthetic data set for a two-layer model over a half-space and analyzed the amplitudes in the resulting gathers. The upper layer is isotropic with a P-wave velocity v_p of 3 km/s, an S-wave velocity v_s of 2 km/s, a density ρ of 2000 kg/m³, and a thickness of 700 m. The second layer is a 500-m-thick strongly anisotropic shale (see Table 1 for specifications), and the lower half-space is again isotropic with a horizontal P-wave velocity gradient dv_p/dx of -0.4 s^{-1} , a constant v_p/v_s ratio of 1.5, and a horizontal density gradient $d\rho/dx$ of $-0.110^{-3} \text{ kg/m}^4$. The minus sign indicates that the velocities and density decrease in the shooting direction. At the target location, $v_p = 2 \text{ km/s}$ and $\rho = 2250 \text{ kg/m}^3$ in the lower half-space. This particular model was chosen for its interesting AVO effect for the bottom reflection; a polarity reversal is present at an offset of 900 m or, equivalently, at a horizontal slowness of 0.11 s/km. It therefore provides a good illustration of the effect of the different sorting schemes on the amplitudes.

The synthetic data set is created by means of ray tracing (Guest and Kendall, 1993) and consists of end-spread shooting with a source and receiver spacing of 25 m with a maximum offset of 3 km. Thomsen's parameter γ_{eff} for the effective v_p/v_s ratio changes from 1.5 for the top reflection to 0.858 for the bottom reflection, which leads to different optimal P-SV bin sizes $bin_{p_{SV}}$ of 15 m and 11.5 m, respectively (equation 3). For comparison, the CMP bin size is 12.5 m. Although it is

not unusual for v_p/v_s ratios (and thus γ_{eff}) to decrease with depth, the change here is quite dramatic and entirely due to the presence of the highly anisotropic shale. Only the latter value for γ_{eff} is retained since the bottom reflection is considered to be the target.

To test whether the CCP-sorting performance critically depends on the specified value for the effective v_p/v_s ratio γ_{eff} , the P-SV data were sorted in CMP instead of CACP gathers, transformed to the τ - p domain, and subsequently CCP sorted. Effectively, this means that γ_{eff} was set to one during CACP sorting to compute the relevant shift δx_{CCP} of the conversion point from its asymptotic value (equation 1). The shot gather at the target CDP location was also extracted and transformed to the τ - p domain for comparison. In addition, a reference gather was created in the medium entirely composed of homogeneous layers (including the half-space). The latter gather represents the desired best response after CCP sorting.

Figure 3 shows the stacked sections obtained after CMP and subsequent CCP sorting. The stacked sections were directly created in the τ - p domain (Van der Baan, 2004). Only the second reflection is shown. Both stacked sections are identical at first view; however, a closer inspection reveals that the position of the polarity reversal has shifted more than 125 m

Table 1. Elastic parameters of the shale used in the numerical examples. All values are taken from Thomsen (1986).

Generic name	α_0 (km/s)	β_0 (km/s)	ε	δ	η	σ	ρ (g/cm ³)
Shale (5000)	3.048	1.490	0.255	-0.050	0.339	1.276	2.000

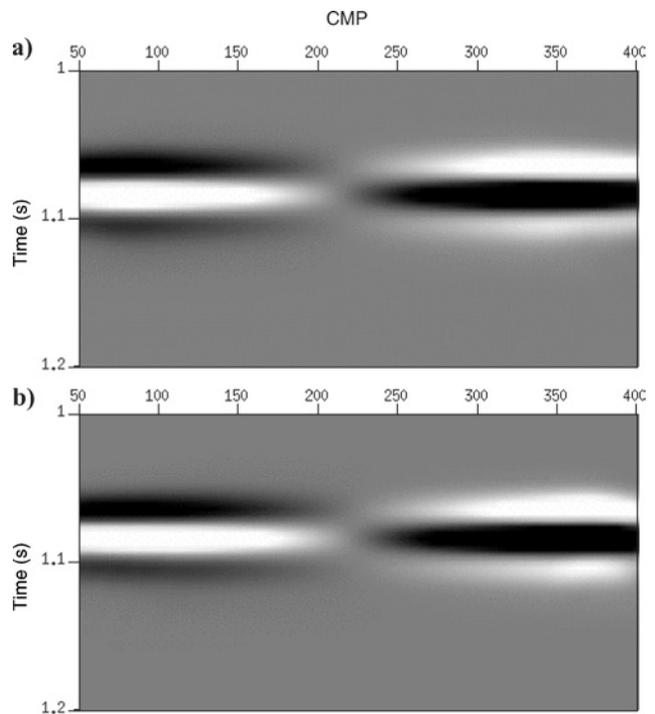


Figure 3. Stacked sections (a) before and (b) after CCP sorting. Only the second reflection is shown. Both stacked sections are very similar except that the polarity reversal is shifted more than 10 CMP positions (i.e., 125 m) to the right after CCP sorting.

(i.e., 10 CMP positions). This is a relatively subtle feature. On the other hand, the actual position of polarity reversals and structural discontinuities is often used to assess whether the P-to-S wave velocity ratio is correctly determined (Frasier and Winterstein, 1990), and such a shift, therefore, potentially affects such quality-control procedures. The effect of CCP sorting on the resulting converted-wave AVO signatures, however, is much more pronounced.

Figure 4 displays the envelopes (i.e., absolute value of the analytic signals) of four τ - p gathers before and after CCP sorting, and Figure 5 shows the extracted amplitudes along the τ (p) moveout curves. Refer to Figure 2 for displays of the ray-theoretical $t(x)$ and $\tau(p_x)$ moveout curves for both the top and bottom reflections. Unsurprisingly, the amplitudes of

the bottom reflection in the shot gather for the inhomogeneous medium are clearly too large (Figure 4b and long dashes in Figure 5). CMP sorting leads, in this case, to amplitudes that are too small, but it does show the amplitude reversal (Figure 4c). Most of the amplitudes have been recovered after CCP sorting except for the largest amplitudes (Figure 4d).

The imperfect reconstruction of the largest amplitudes is not caused by inaccuracies in the description of the P-SV moveout curves but by the finite spread length of the shot gathers. A slowness of 0.235 s/km corresponds to an emergence offset of 3 km. It will, therefore, be very difficult to correctly reconstruct amplitudes around and beyond this slowness. The reference gather (Figure 4a) was actually created using a spread length of 4 km instead of 3 km.

The abrupt termination of the reconstructed CCP moveout curves at a slowness of 0.27 s/km happens because slownesses beyond this value are theoretically impossible according to ray theory and, thus, cannot be reconstructed. Energy beyond this value in the other gathers is caused partly by the fact that the employed zeroth-order ray theory does not lead to exact amplitudes along the $t(x)$ moveout curves, which consequently introduces artifacts after τ - p transformation.

Figure 5 also displays the amplitudes found by applying proper CACP sorting using $\gamma_{eff} = 0.858$ and a bin size of 11.5 m. The polarity reversal has again been recovered, but all amplitudes are too small. CCP sorting starting with CACP gathers leads to CCP gathers nearly identical to those shown in Figure 4d. The performance of the CCP sorting scheme depends to a much larger extent on a correct kinematic description of the P-P and P-SV moveout curves (and a correct event registration) than on an accurate estimation of the effective v_p/v_s ratio γ_{eff} .

Finally, Figure 6 shows the actual distances involved in the various sorting schemes, both as a function of the P-SV emergence distance x_{PSV} (lower axis) and the horizontal slowness p_x (upper axis). It also displays the exact CCP offset computed using the approach outlined in Van der Baan and Kendall (2003). Differences in the final and exact CCP offsets are the result of small inaccuracies in the kinematic description of the P-P and P-SV moveout curves. In this case, it turns out that CMP sorting leads to slightly better results than CACP sorting. This is, naturally, an exception rather than a general rule. Adding the CCP shifts δx_{CCP} onto the CMP distances yields the final CCP offsets.

Figure 6 also explains why both CMP and CACP sorting reproduce the polarity reversal. Both sorting distances are good approximations to the actual CCP distance

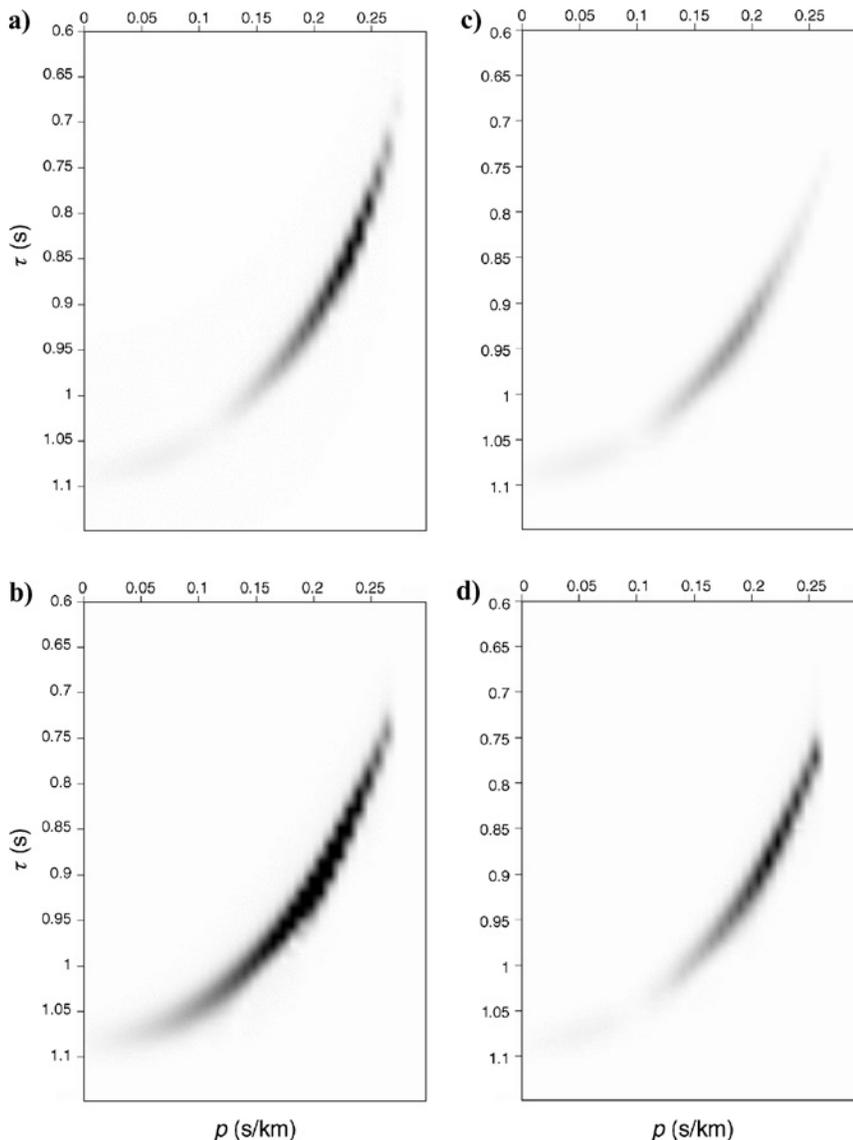


Figure 4. Envelopes of various $\tau(p_x)$ gathers for the bottom reflection. (a) Reference gather created in a medium composed of entirely homogeneous layers. (b) Shot gather at the target location in the model with an inhomogeneous lower half-space and resulting gathers after (c) CMP and (d) CCP sorting. Maximum amplitudes change slightly from gather to gather, but only the CCP gather resembles the reference gather well. Figure 5 displays the extracted amplitudes along all moveout curves.

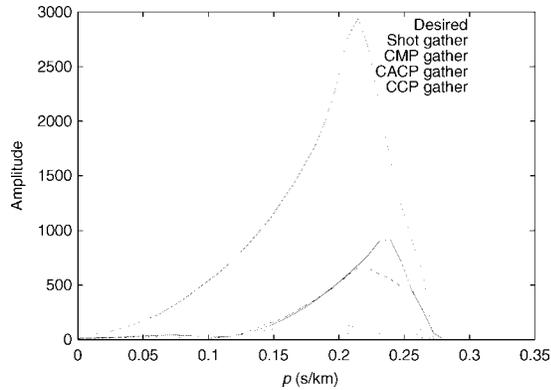


Figure 5. Associated amplitudes along the $\tau(p_x)$ moveout curves in Figure 4 for the bottom reflection. Amplitudes for the shot gather at the target location are much too large (long dashes). Both CMP (short dashes) and CACP (dots) sorting greatly underestimate the amplitudes. Only CCP sorting (dots-dashes) reconstructs the desired amplitude response except for the largest slownesses. Longer offsets are required for a better reconstruction.

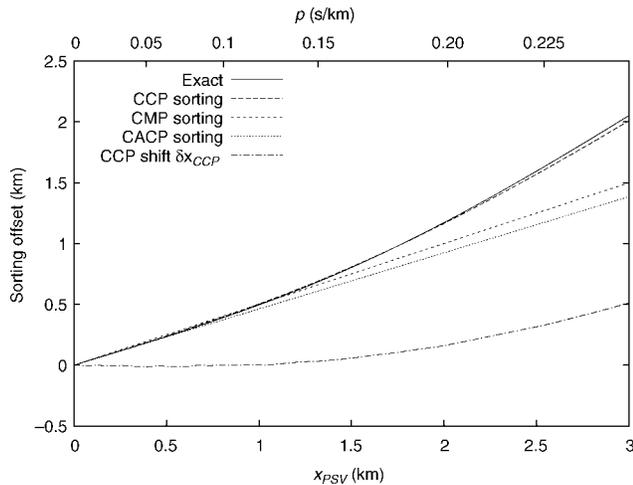


Figure 6. Sorting offsets involved as a function of the P-SV emergence distance $x_{p,SV}$ (lower axis) and horizontal slowness p_x (upper axis). Both CMP (short dashes) and CACP (dots) sorting approximate the exact CCP offset (solid line) well for offset/depth ratios less than one (i.e., 1.2 km). Only offsets computed by the CCP sorting scheme (long dashes) are accurate for larger offset/depth ratios. The CCP offsets are obtained by adding the CCP shift δx_{CCP} (dots-dashes) onto the CMP distance because CMP instead of CACP sorting was the starting point for CCP sorting (γ_{eff} was set to one).

for slownesses less than 0.12 s/km (i.e., offsets less than 1.1 km), thereby including the polarity reversal. However, larger slownesses (offsets) are not well approximated, thus leading to underestimated amplitudes in the resulting gathers.

DISCUSSION

The developed CCP sorting technique can be implemented in both the time-offset and τ - p domain. Both approaches are exact in laterally homogeneous anisotropic media if a horizontal symmetry plane is present in each layer, whereas existing conventional CCP sorting schemes, based on the evaluation of analytic expressions, can only deal approximately with lay-

ered, isotropic, or VTI media (Thomsen, 1999; Li and Yuan, 2003). The sorting technique probably still provides good results even if such a symmetry plane does not exist (e.g., if the medium is transversely isotropic with a tilted symmetry axis or, worse, triclinic). Anisotropic P-P wave reflection-point dispersal in such media is always ignored during processing. Ignoring the associated conversion-point dispersal, therefore, does not lead to worse assumptions.

Both implementations break down simultaneously in the presence of moderate to strong lateral velocity changes and complex structures because of the increasing difference between the source and receiver slownesses and the increasing reflection/conversion-point dispersal from the estimated CMP/CCP position. It is for this very reason that we should appeal to DMO and migration after P-P wave CMP sorting or P-SV wave CCP sorting.

What, then, is the advantage of one implementation over the other? One of the most evident practical advantages of a τ - p domain implementation is that slowness information is easily kept after sorting, and it can handle irregularly spaced offset gathers. A time-offset implementation requires trace interpolation to ensure a common set of offsets in all input gathers. This, however, is not a problem if the only desire is to directly create CCP-stacked traces from moveout-corrected CACP gathers since no offset information is then required. A τ - p implementation thus offers the possibility of an amplitude-versus-slowness analysis or a renewed velocity analysis after CCP sorting.

Stacked traces after τ - p domain CCP sorting can be obtained by either transforming the CCP gathers back to the time-offset domain or by directly stacking data over slowness after τ - p moveout corrections (Van der Baan, 2004). The latter approach has the double advantage of not only eliminating the need for an inverse τ - p transform but also the possible simultaneous removal of the geometric spreading of all recorded wave modes and types if a proper τ - p transform is used. Proper τ - p transforms are plane-wave decompositions, and horizontally propagating plane waves in laterally homogeneous media are not subject to geometric spreading; therefore, no a priori knowledge of the underlying velocity field is required to remove the geometric spreading.

An additional advantage of processing data in the τ - p domain is that primary P-P wave reflections do not cross here in laterally homogeneous media, and neither do primary P-SV converted-wave reflections. Furthermore, both multiple removal and predictive deconvolution are known to yield good results in the τ - p domain, and it can be shown that even head waves (refractions) can have a constructive influence on stacked sections. The τ - p transform acts, in addition, as a dip filter, which may result in an increase in data quality since some noise components will separate from the reflection signals. The interested reader is referred to Van der Baan (2004) for more background on geometric spreading and moveout corrections of anisotropic data in the τ - p domain.

When, then, does a time-offset implementation hold advantages over a τ - p -based approach? The main advantage occurs for low-fold data (e.g., at the edges of the acquisition geometries) because it then becomes difficult to transform the gathers to the τ - p domain, or any other domain for that matter (e.g., the f - k domain). On the other hand, it is still possible to directly create CCP-stacked traces if we can get reliable

estimates of the velocities involved (i.e., a good kinematic description of the moveout curves). Seismic experiments are generally designed such that a high coverage occurs in the area of interest. Thus, this may only be a minor advantage in practice.

CONCLUSIONS

In this paper, I develop a migration-type algorithm for CCP sorting of anisotropic converted-wave data. Correlated P-P and P-SV reflections in collocated CMP and CACP gathers are scanned for points of common slowness, indicating that the downgoing branches of the two waves are approximately coincident. The conversion point is then assumed to occur underneath the P-P wave midpoint. The technique is exact for laterally homogeneous media with arbitrary strength of anisotropy if horizontal symmetry planes are present at all depths.

The advantage of this approach over conventional techniques is that no analytic expressions for the conversion points are evaluated, and it can handle azimuthal anisotropy in layered media exactly. The quality of the final results depends mostly on the accuracy of the kinematic description of the P-P and P-SV moveout curves and the quality of the event registration (i.e., the accuracy of the correlation of P-P and P-SV reflections). The technique is applicable to both 2D and 3D data sets. Only in the latter case can it handle conversion points that are not contained within the vertical source-receiver plane. Such out-of-plane conversions occur for all azimuthally anisotropic media.

CCP sorting can have a large influence on the behavior of converted-wave amplitudes even if the difference between stacked sections obtained after CACP and CCP sorting at first glance seems to be limited. The sorting technique is preferably implemented in the τ - p domain instead of the time-offset domain since it is then much simpler to retain slowness and, thus, offset information after sorting for a converted-wave amplitude analysis or a renewed velocity analysis.

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