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Some comments on common-asymptotic-conversion-point (CACP) sorting of converted-wave data in isotropic, laterally inhomogeneous media

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ABSTRACT

Common-midpoint (CMP) sorting of pure-mode data in arbitrarily complex isotropic or anisotropic media leads to moveout curves that are symmetric around zero offset. This greatly simplifies velocity determination of pure-mode data. Common-asymptotic-conversion-point (CACP) sorting of converted-wave data, on the other hand, only centers the apexes of all traveltimes around zero offset in arbitrarily complex but isotropic media with a constant P-wave/S-wave velocity ratio everywhere. A depth-varying CACP sorting may therefore be required to position all traveltimes properly around zero offset in structurally complex areas. Moreover, converted-wave moveout is nearly always asymmetric and nonhyperbolic. Thus, positive and negative offsets need to be processed independently in a 2D line, and 3D data volumes are to be divided in common azimuth gathers. All of these factors tend to complicate converted-wave velocity analysis significantly.

INTRODUCTION

Levin (1971) has shown that common-midpoint (CMP) sorting of pure-mode data leads to moveout curves that are symmetric and centered around zero offset if the medium is isotropic and composed of a single dipping layer. Moveout is also hyperbolic but with an increased apparent moveout velocity. This is somewhat surprising since the original traveltime curves of the common shot gathers are shifted updip and the minimum traveltime does not occur at zero offset.

Shah (1973) consequently proved that this is actually true for arbitrarily complex isotropic or anisotropic media for all pure-mode waves because of reciprocity. That is, pure-mode moveout curves are always symmetric and centered around zero offset in a CMP gather.

As a result of the increasing popularity of three-component geophones and thus of converted-wave acquisition, the question arises whether a similar phenomenon also occurs for common-asymptotic-conversion-point (CACP) sorting of converted-wave data. This question has not been conclusively answered despite its practical relevance for velocity analysis of converted waves. This article demonstrates that CACP sorting of converted-wave data in an arbitrarily complex but isotropic medium indeed centers all traveltime curves around zero offset if and only if the P-wave/S-wave velocity ratio is constant everywhere. However, moveout curves are asymmetric and nonhyperbolic. In practice this means that positive and negative offsets in a 2D line need to be processed independently and that 3D data volumes need to be divided in common-azimuth gathers (Thomsen, 1999). In addition, hyperbolic moveout corrections alone almost never are sufficient (Tessmer and Behle, 1988).

CONVERTED-WAVE TRAVELTIME CHARACTERISTICS

The traveltime minimum and thus apex of a pure-mode arrival (e.g., a P-P reflection) in a common-shot gather is positioned updip in an isotropic medium with a single dipping layer (Levin, 1971). Conventional semblance-based velocity analyses are therefore never done on common-shot gathers since they always assume that apexes occur at zero offset. Shah (1973) has proved that CMP sorting of pure-mode arrivals always leads to moveout curves that are symmetric around zero offset. Because of reciprocity, raypaths and traveltimes of pure-mode arrivals are identical if the source and receiver points are interchanged (e.g., Figure 1, top). Hence, positive and negative offsets in pure-mode CMP gathers have identical traveltimes, and moveout is always symmetric around zero offset. This is true for arbitrarily complex isotropic or anisotropic media.

The same shift updip in a common-shot gather can, on the other hand, be used to obtain a rough estimate of the thickness

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and dip of a reflector by computing the ratio of apex position to minimum traveltime for a given P-wave velocity v_p [e.g., Dobrin and Savit (1988, 221–223) and expressions A-11 and A-12]. A conventional velocity analysis does not easily yield such information for dipping structures.

The traveltime minimum of a converted-wave arrival in a common-shot gather is also displaced updip for an isotropic, single-dip medium (equations A-9 and A-10). We can again obtain an estimate of the thickness and dip of a reflector by measuring the same offset/traveltime ratio of a converted-wave arrival, although we now need both the P- and S-wave velocities (equations A-9, A-10, and A-13).

CMP sorting of converted-wave data does not shift the traveltime apex to zero offset if dipping structures are present. Therefore, it is inadvisable to perform converted-wave velocity analysis on either common-shot or common-midpoint gathers. CACP sorting is to be applied to improve the quality of the velocity analysis. The asymptotic conversion point is the conversion point in the limit of zero offset–depth ratios, i.e., for near-normal-incidence raypaths. It is approximately the conversion point for very deep reflectors or small emergence offsets.

In CMP sorting, the source S and receiver R are at equal distances from the gather position Y. That is, the distance $\overline{SY}$ along the surface between the source S and the gather position Y and the distance $\overline{YR}$ from the gather at Y to the receiver R are kept equal (Figure 2). In CACP sorting these distances are different, but the ratio $\overline{SY}/\overline{YR}$ is kept constant. If we denote by $\xi = \overline{SY}/\overline{SR}$ the ratio of the distance $\overline{SY}$ to the total source–receiver distance $\overline{SR}$, then $\xi = 1/2$ indicates CMP sorting. For CACP sorting, however, the sorting ratio ξ_{cacp} is given by

$$\xi_{cacp} = \frac{\gamma}{1+\gamma}, \tag{1}$$

where $\gamma = v_p/v_s$ is the P-wave/S-wave velocity ratio indicated in the following by the ratio of the downgoing/upgoing velocities $\grave{v}/\acute{v}$ for added generality. The sorting ratio ξ_{cacp} was originally derived for a single horizontal layer (Fromm et al., 1985; Tessmer and Behle, 1988), but it is valid for arbitrarily complex isotropic media if the P-wave/S-wave velocity ratio γ is constant everywhere.

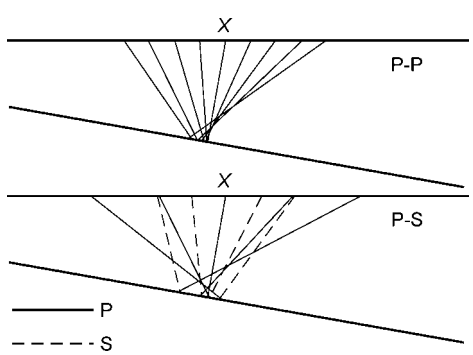

Figure 1. Comparison of reflection-point dispersal and raypaths for pure-mode (top) and converted-wave (bottom) reflections in, respectively, a CMP and CACP gather. Pure-mode reflection points always move updip independent of the shooting direction, and pure-mode raypaths overlap in forward and reversed spreads. Neither phenomenon occurs for converted waves. There is no vertical exaggeration.

Does CACP sorting also lead to converted-wave moveout curves centered around zero offset for arbitrarily complex media and, if so, under what conditions? To answer this question, we could extend the results of Appendix A and compute the converted-wave traveltimes analytically for multilayer media in both shot and CACP gathers. However, this quickly becomes very cumbersome. A simpler approach is available following ideas in Shah (1973).

The traveltime t_ξ minimum in a gather produced for a given sorting ratio ξ occurs at zero offset ($x_\xi = 0$) if

$$\left.\frac{\partial t_\xi}{\partial x_\xi}\right|_{x_\xi=0} = 0, \tag{2}$$

that is, if the apex of the moveout curve is positioned at zero offset. If we denote the source and receiver positions along the line with s and r, respectively, then

$$\begin{aligned}\left.\frac{\partial t_\xi}{\partial x_\xi}\right|_{x_\xi=0} &= \left.\frac{\partial t_\xi}{\partial s}\frac{\partial s}{\partial x_\xi}\right|_{x_\xi=0} + \left.\frac{\partial t_\xi}{\partial r}\frac{\partial r}{\partial x_\xi}\right|_{x_\xi=0} \\ &= [-\xi p_{src} + (1-\xi)p_{rec}]_{x_\xi=0} = 0,\end{aligned} \tag{3}$$

since $s = y_\xi - \xi x_\xi$ and $r = y_\xi + (1-\xi)x_\xi$, with x_ξ the offset in the sorted gather at position y_ξ (Figure 2).

The quantities p_{src} and p_{rec} represent the source and receiver slowness, respectively. The receiver slowness p_{rec} is estimated from the traveltimes in a common-shot gather, while the source slowness p_{src} is computed in a common-receiver gather. Note that p_{src} is computed from the traveltime variations in terms of shot coordinates. If it is computed from the differential moveout as a function of offset, then its sign is flipped. This flip of sign does not occur for p_{rec} if computed from variations in receiver or offset coordinates. The source and receiver slownesses are defined as $p_{src} = \sin\grave{\theta}_0/\grave{v}_0$, and $p_{rec} = \sin\acute{\theta}_0/\acute{v}_0$, respectively, with $\grave{\theta}_0$ and $\acute{\theta}_0$ the take-off and incidence angles at the surface (relative to the surface normal) and $\grave{v}_0$ and $\acute{v}_0$ the corresponding velocities at the surface for the down- and upgoing waves. No sign correction is required to distinguish between down- and upgoing waves. If the down- and upgoing raypaths overlap, then take-off and incidence angles are identical (i.e., $\grave{\theta}_0 = \acute{\theta}_0$).

Moveout curves are therefore centered around zero offset if (equation 3)

$$\xi = \frac{p_{rec}(0)}{p_{src}(0) + p_{rec}(0)}. \tag{4}$$

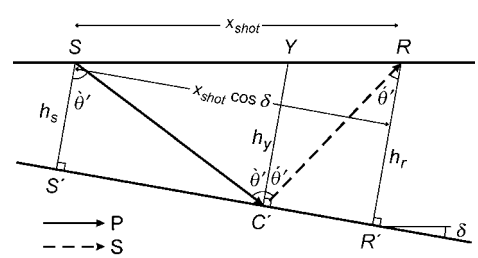

Figure 2. Considered geometry and definition of used variables. A P-wave originates at source position S, travels to C′, converts here to an S-wave, and travels to receiver R. Layer thicknesses are measured perpendicular to the reflecting interface with dip δ. All interface-related quantities are denoted by a prime. Ray angles are denoted by θ.

Source and receiver slownesses vary for different arrivals. Different sorting ratios ξ may therefore be needed for different primary reflections (pure mode or converted). This equation is nonetheless valid for arbitrarily complex isotropic or anisotropic media.

Next, we need to establish what general criterion gives rise to a converted wave arriving at zero offset. If and only if the P-wave/S-wave velocity ratio γ is constant everywhere are the up- and downgoing raypaths coincident for a normally incident P-wave at the reflecting/converting interface for otherwise arbitrarily complex isotropic media. This happens since a constant P-wave/S-wave velocity ratio ensures that the ratios of the velocities above and below each interface are identical for both the down- and upgoing waves (i.e., $\grave{v}_{above}/\grave{v}_{below} = \acute{v}_{above}/\acute{v}_{below}$). The interface-parallel slowness $p_{\parallel}$ at the reflecting-converting interface is then zero for the zero-offset arrival.

If the up- and downgoing raypaths are coincident, then the take-off and incidence angles are identical (i.e., $\grave{\theta}_0 = \acute{\theta}_0$) and expression 4 reduces to the sorting ratio ξ_{cacp} given by equation 1. In other words, the source and receiver slownesses at zero offset may differ for each individual arrival, but their ratio remains constant everywhere. CACP sorting of converted-wave data using a single constant sorting ratio ξ_{cacp} therefore centers the apexes of all converted-wave arrivals to zero offset in arbitrarily complex media if and only if the P-wave/S-wave velocity ratio is constant everywhere and if the medium is isotropic.

Are the moveout curves also symmetric around zero offset — likewise, pure-mode moveout curves in a CMP gather? A quick drawing shows that the P-S raypaths do not overlap for an interchanged source and receiver in a single-dip medium, whereas this is the case for a P-P reflection (Figure 1). The P-S and S-P arrivals on the other hand, do overlap in a forward and reversed spread. There is therefore no physical cause for symmetry. The only exception naturally occurs for laterally homogeneous anisotropic media if horizontal symmetry planes are present everywhere (i.e., media are horizontally/vertically transversely isotropic, orthorhombic, or monoclinic but are not tilted transversely isotropic or triclinic). Converted-wave moveout is always symmetric around zero offset in such media, regardless of the sorting ratio ξ.

In conclusion, moveout curves in CACP-sorted P-S gathers are centered around zero offset in arbitrarily complex, laterally inhomogeneous isotropic media with a constant P-wave/S-wave velocity ratio everywhere. This greatly facilitates velocity analysis, but positive and negative offsets generally have different moveout velocities and need to be processed separately. The latter phenomenon is sometimes referred to as converted waves displaying diodic velocities (Thomsen, 1999).

How well are the converted-wave traveltimes described by a hyperbolic moveout term alone, and what is the effect of dip on the apparent moveout velocity? Levin (1971) shows that a dip δ increases the apparent moveout velocity $v_{nmo,pp}$ of a pure-mode arrival in a single-dip, isotropic medium by a factor $1/\cos\delta$. That is,

$$v_{nmo,pp}^2 = \frac{v_p^2}{\cos^2\delta}, \tag{5}$$

where v_p is the actual P-wave velocity of the medium (equation A-19). It turns out that dip has exactly the same influence on the apparent hyperbolic moveout velocity of the converted waves. It is given by

$$v_{nmo,ps}^2 = \frac{v_p v_s}{\cos^2\delta}, \tag{6}$$

with v_s the actual S-wave velocity of the medium (equation A-21).

Approximation 6 for the converted-wave NMO velocity is only valid for a single isotropic layer. Tessmer and Behle (1988) and Thomsen (1999) derive more appropriate expressions for multiple horizontal layers (respectively, their expressions 5 and 23). It is to be expected that the presence of dip still increases the apparent velocity, as it does for pure-mode reflections.

If we are not shooting along the direction of maximum dip but at an angle ϕ with the azimuth of maximum dip of the reflector, then $v_{nmo,ps}$ becomes

$$v_{nmo,ps}^2 = \frac{v_p v_s}{(1 - \sin^2\delta\cos^2\phi)}. \tag{7}$$

The converted-wave NMO velocity $v_{nmo,ps}$ thus increases monotonically from a strike-parallel to a dip-parallel line, as it does for pure-mode waves (Levin, 1971).

Contrary to the pure-mode case, higher order terms do play an important role in the Taylor-series expansion for the converted-wave traveltime t_{cacp}. See, for instance, Tessmer and Behle (1988) for the coefficients for the horizontally layered case. However, the amount of nonhyperbolic moveout is most easily shown using a numerical example instead of deriving these coefficients for a dipping layer. This is done in the example section.

REFLECTION- AND CONVERSION-POINT DISPERSAL

Exact expressions for reflection- and conversion-point dispersal in an isotropic, single-dip medium exist (Levin, 1971; Aldridge, 1992). Therefore, I only briefly allude to the differences in pure-mode and converted-wave dispersal for completeness.

The reflection point is always underneath the CMP location for a pure-mode reflection in a laterally homogeneous, isotropic medium. This remains true even for anisotropic media if a horizontal symmetry plane is present (i.e., vertically/horizontally transversely isotropic, orthorhombic, and monoclinic symmetries but not for tilted transversely isotropic or triclinic media). For a dipping isotropic layer, however, the reflection point moves away updip. The actual reflection-point dispersal is proportional to both the dip and the absolute source–receiver offset (Levin, 1971). However, it is independent of whether we are shooting updip or downdip because the raypaths of a forward and reversed spread overlap.

Converted waves, on the other hand, already display reflection-point dispersal for a single horizontal layer (Fromm et al., 1985; Tessmer and Behle, 1988). For this very reason the term asymptotic conversion point has been coined. Tessmer and Behle (1988) derive the exact dispersal for a single horizontal layer, and Aldridge (1992) extends their results to a single dipping layer. (See also Schneider, 2002.)

Another noteworthy difference between pure-mode and converted-wave reflection-point dispersal is that the latter is asymmetric with respect to offset and that the dispersal is only updip if the shooting direction is also updip (Figure 1). Judging from Figure 1, it seems that the absolute converted-wave dispersal in the downdip direction is smaller than or equal to that of the pure-mode waves, whereas the opposite is true updip. This seems to contradict conclusions drawn by Harrison (1992) using an analytic study. Moreover, converted-wave dip moveout (DMO) operators are asymmetric because of this asymmetry in the dispersal (Den Rooijen, 1991; Alfaraj and Larner, 1992; Harrison, 1992); i.e., the converted-wave raypaths in a forward and reversed spread do not overlap.

PRACTICAL CONSIDERATIONS

Is it possible to determine the optimal sorting ratio ξ and thus the velocity ratio γ without any P–P-wave information (e.g., velocities) or, even better, without the need of first performing an event registration (i.e., correlation of related P-P and P-S reflections)? The apexes of the moveout curves are only positioned at zero offset for a correct value of ξ. In other words, a velocity analysis will degrade considerably otherwise. The appropriate value of ξ can therefore be determined by trial and error by selecting those ratios that lead to the sharpest semblance contours and the best-quality stacked sections. Large differences in the quality of the semblance plots should only be seen in areas of complex structure. The appropriate sorting ratio ξ is likely to decrease with depth since the P-wave/S-wave velocity ratio γ also tends to decrease with depth.

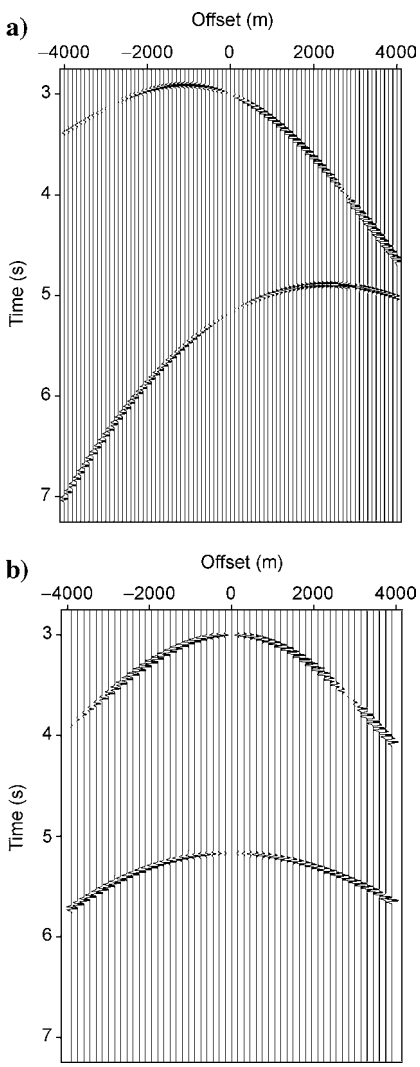

Figure 3. Effect of CACP sorting on converted-wave moveout with a constant P-wave/S-wave velocity ratio. Seismic gathers (x-component) (a) before and (b) after CACP sorting. Both moveout curves are centered around zero offset after CACP sorting, even though the second layer is not homogeneous. The change in polarity at zero offset is caused by the wavefront expanding outward. All other phase rotations are related to critical angles.

Alternatively, the appropriate sorting ratio ξ can be directly estimated from the data. Equation 4 is valid for arbitrarily complex media. The required source and receiver slownesses can be estimated by analyzing the differential moveout near zero offset in, respectively, a common-receiver and a common-source gather located at the same position (i.e., receiver slowness from a common-shot gather). On the other hand, the sum of the source $p_{src}(0)$ and receiver $p_{rec}(0)$ slownesses at zero offset ($x_\xi = 0$) can also be determined directly from the relative dip moveout ($\partial t_\xi / \partial y_\xi$) in a common-offset section (for zero offset). In a similar way as expression 3 was derived, it can be shown that

$$\left.\frac{\partial t_\xi}{\partial y_\xi}\right|_{x_\xi=0} = p_{src}(0) + p_{rec}(0), \tag{8}$$

where t_ξ is the traveltime in a common-offset section and y_ξ is the actual sorting position. In other words, the sum of the source and receiver slownesses can either be derived by analyzing the apparent moveout of the target reflector in a stacked section or by assessing the apparent moveout of the zero-offset traveltimes in adjoining CACP gathers. If required, a local estimate of the P-wave/S-wave velocity ratio can then be obtained by inverting relation 1.

This procedure only works in areas with strong dip and high data quality. In addition, amplitudes of converted-wave arrivals are virtually zero at zero offset. Some interpolations may be required to retrieve the appropriate slownesses. Unfortunately, the repeated divisions (time over offset and division of slownesses) tend to enhance small measurement errors significantly, thus leading to large uncertainties in the final result. All of these factors will limit the practical applicability of this approach to estimate ξ directly from the data.

NUMERICAL EXAMPLES

To illustrate the behavior of moveout curves before and after CACP sorting with ξ, let's consider a two-layer model with dips of 10° and −15°. The vertical depth to each interface is 2 and 4 km, respectively, and the P-wave velocity is fixed at 2 and 2.5 km/s, respectively, underneath the considered position. Synthetic sections are computed by means of ray tracing (Guest and Kendall, 1993).

First, we investigate the robustness of CACP sorting in a complex medium with a constant P-wave/S-wave velocity ratio γ everywhere. The first layer is kept homogeneous, but the second layer exhibits strong lateral P- and S-wave velocity gradients of 0.2 s^{-1} and 0.1 s^{-1}, respectively. This produces strong ray bending in the second layer. The constant P-wave/S-wave velocity ratio $\gamma = 2$ leads to an optimal sorting ratio $\xi = 2/3$.

Figure 3 shows the resulting shot and CACP gathers. The traveltime minima are shifted updip in the shot gather but are centered at zero offset in the CACP gather, even though the second layer exhibits strong lateral velocity changes. An incorrect estimate of the P–S-wave velocity ratio γ does not lead to a minimum traveltime located at zero offset. The location of the minimum can therefore be used as an independent quality control for CACP sorting.

Next, the same isotropic two-layer model is considered, but the P-wave/S-wave velocity ratio γ changes from 2.5 in the upper layer to 1.7 in the second one. No velocity gradients are imposed in this case. Figure 4 displays the resulting shot and CACP gathers for a constant sorting ratio appropriate for the upper layer ($\xi = 5/7$). The apex of the upper moveout curve is correctly positioned at zero offset after sorting, but the apex of the second reflection shifts to approximately −250 m. This will significantly degrade the velocity analysis for this horizon. It turns out that CMP sorting would have done a much better job for the second layer — at the expense of displacing the upper traveltime curve.

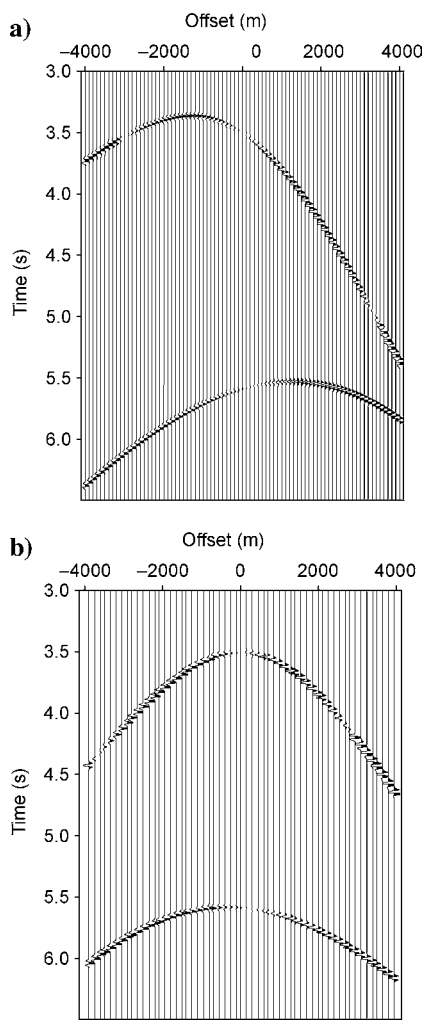

Figure 4. Effect of CACP sorting on converted-wave moveout with a variable P-wave/S-wave velocity ratio. Seismic gathers (x-component) (a) before and (b) after CACP sorting. A constant sorting ratio ξ cannot center both traveltime curves around zero offset if the P–S-wave velocity ratio varies from layer to layer.

Finally, how well does a hyperbolic moveout velocity alone reproduce the actual converted-wave moveout curves in case of dip? Figure 5 displays the effect of dip on the traveltimes in a shot and a CACP gather located at the same position for a simple isotropic single-dip layer with P- and S-wave velocities of 2 and 1 km/s, respectively. The distance to the interface is kept at 2 km. Dips shown are 0°, 10°, and 25°. The traveltimes are computed using expressions A-6, A-8, A-17, and A-18. Also shown are the hyperbolic moveout curves

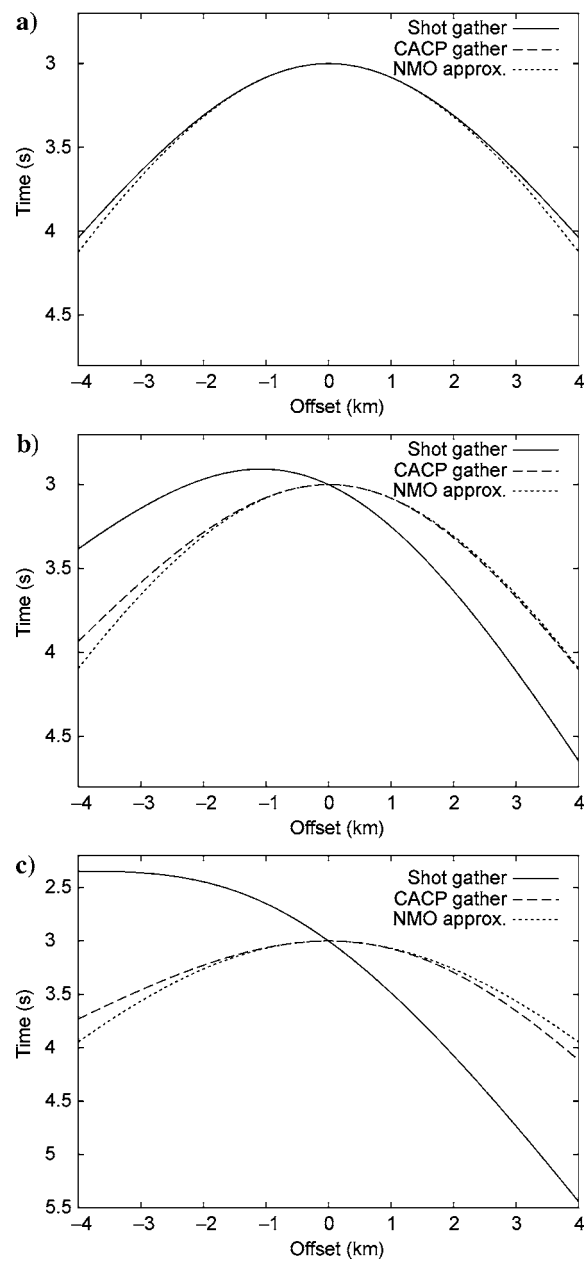

Figure 5. Moveout curves in an isotropic, single-layer model with a dip of (a) 0°, (b) 10°, and (c) 25°. Converted-wave traveltimes in a common-shot gather are moved updip (solid line). They are centered around zero offset in a CACP gather (dashed line) but are nearly always nonhyperbolic, as shown by the theoretical short-spread moveout velocity (dots). Positive and negative moveout curves have different moveout velocities if a dip is present. Note the change of time scale in (c).

predicted by the theoretical NMO velocity (equation 6). The accuracy of the predicted hyperbolic moveout curve depends on the actual dip in an unexpected way. For a horizontal layer, it seems to be sufficiently accurate up to an offset/depth ratio of one. For a 10° dip, the positive offsets are nearly perfectly explained for all offset/depth ratios, whereas the traveltimes are too slow for the negative offsets. Finally, for a 25° dip, the moveout of neither the positive nor the negative offsets is well approximated. Most importantly, however, Figure 5 shows that positive and negative offsets in a converted-wave gather should be processed independently since they are characterized by different apparent velocities if a small amount of lateral inhomogeneity is present.

DISCUSSION

An important question is whether the apexes of all converted-wave moveout curves can be shifted to zero offset using a single CACP sorting ratio ξ if the medium is anisotropic and under what conditions. The requirement of a constant P-wave/S-wave velocity ratio γ everywhere leads to coincident up- and downgoing raypaths for the zero-offset ray and, thereby, to a constant ratio between the source and receiver slownesses.

In anisotropic media with depth-varying properties, it is highly unlikely that the zero-offset down- and upgoing raypaths are coincident, since this would require the group-velocity angles of the P- and S-waves to be identical at every point in space. Thus, ξ shifts all converted-wave traveltime curves to zero offset only under exceptional circumstances (e.g., an anisotropic single-dip medium). Different sorting ratios ξ_{cacp} are therefore required at different depths for anisotropic, laterally inhomogeneous media. Unfortunately, this is in strong contrast to the pure-mode case, where reciprocity always enforces identical up- and downgoing zero-offset raypaths irrespective of the complexity of the medium and type of anisotropy present.

A last comment about common-conversion-point (CCP) sorting versus dipmoveout (DMO) corrections. Several authors have published approximations and techniques to account for the conversion-point dispersal in laterally homogeneous layered media (e.g., Thomsen, 1999; Li and Yuan, 2003; van der Baan and Kendall, 2003; and references therein). Their approximations have varying degrees of accuracy and are limited to horizontal layering but can handle arbitrary changes of velocity with depth and simple types of anisotropy.

Converted-wave DMO, on the other hand, tries to account for the total conversion-point dispersal resulting from both horizontal layering and dip (Den Rooijen, 1991; Alfaraj and Larner, 1992; Harrison, 1992). Unfortunately, most algorithms assume constant P- and S-wave velocities and, more importantly, a constant P-wave/S-wave velocity ratio. The latter assumption is particularly dubious since the P-wave/S-wave velocity ratio generally decreases with depth. It may therefore be advantageous to develop converted-wave DMO algorithms that account for only the conversion-point dispersal from dip and therefore are to be applied after CCP sorting. This could increase the applicability and performance of converted-wave DMO techniques.

Applying DMO after CCP sorting thus overcorrects for reflection-point dispersal. Not taking reflection-point dispersal into account leads in all cases to strong smearing of horizons in the final sections.

CONCLUSIONS

Depth-independent CACP sorting of converted-wave data leads to moveout curves that are centered around zero offset in isotropic, laterally inhomogeneous media if and only if the P-wave/S-wave velocity ratio is constant everywhere. The presence of anisotropy or a varying P-wave/S-wave velocity ratio thus induces significant problems in velocity estimation in structurally complex areas, since it requires a depth-varying CACP sorting to shift the apexes of all converted-wave moveout curves to zero offset. In other words, significant structure in shallow parts may cause dramatic decreases in the semblance analysis of the deeper parts if a single CACP sorting ratio ξ was chosen to optimize the near-surface analysis, and vice versa.

Furthermore, even if the moveout curves after CACP sorting are centered around zero offset, they are nearly always asymmetric. As a consequence, positive and negative offsets need to be processed independently in a 2D converted-wave gather and, more generally, 3D data volumes are to be separated in common-azimuth gathers. In addition, moveout curves are nearly always nonhyperbolic, even for small offset/depth ratios.

Both the required depth-varying sorting and the asymmetry in the converted-wave moveout are in strong contrast to the behavior of pure-mode waves in arbitrarily complex, anisotropic media since reciprocity ensures that pure-mode traveltimes are symmetric around zero offset after CMP sorting. All of these factors tend to significantly complicate converted-wave velocity analysis.

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APPENDIX A

SINGLE-DIP MEDIUM

Common-shot gathers

Figure 2 displays the considered geometry for a medium composed of a single dipping layer. A receiver R and source S are separated by a distance $x_{shot} = \overline{SR}$, and a conversion plus reflection occurs at point C′ on an interface with dip δ. The dip is defined to be clockwise positive. Quantities on the interface are denoted by a prime. The interface has a thickness $h_s = \overline{SS'}$ and $h_r = \overline{RR'}$ underneath, respectively, the source and the receiver. Thicknesses are measured perpendicular to the interface. The medium further has a P-wave velocity $\grave{v}$, an S-wave velocity $\acute{v}$, and a P-wave/S-wave velocity ratio $\gamma = \grave{v}/\acute{v}$. A wave travels as a P-wave along $\overline{SC'}$, converts to an S-wave at C′, and then travels along $\overline{C'R}$ with a total two-way traveltime t_c given by

$$t_c = \frac{\overline{SC'}}{\grave{v}} + \frac{\gamma\overline{C'R}}{\grave{v}}. \tag{A-1}$$

The incidence $\grave{\theta}'$ and conversion $\acute{\theta}'$ angles at the interface are determined by the interface-parallel slowness $p'_{\parallel}$. Snell's law states that the latter is conserved. Thus,

$$p'_{\parallel} = \frac{\sin\grave{\theta}'}{\grave{v}} = \frac{\gamma\sin\acute{\theta}'}{\grave{v}}. \tag{A-2}$$

All computed quantities are expressed in terms of the interface-parallel slowness $p'_{\parallel}$. For brevity, its prime is omitted henceforth.

The thickness h_y of the interface at an arbitrary point Y on the surface at a distance $\overline{SY}$ away from the source is given by

$$h_y = h_s + \xi x_{shot}\sin\delta. \tag{A-3}$$

The ratio $\xi = \overline{SY}/\overline{SR}$ represents the ratio of the distance $\overline{SY}$ to the total source-receiver offset $\overline{SR}$. The latter is also denoted by x_{shot} if measured from a common-shot position. The lengths of the paths $\overline{SC'}$ and $\overline{C'R}$ are related to the angles $\grave{\theta}'$ and $\acute{\theta}'$ at the interface by

$$\overline{SC'} = \frac{h_s}{\cos\grave{\theta}'} \tag{A-4}$$

and

$$\overline{C'R} = \frac{h_r}{\cos\acute{\theta}'}. \tag{A-5}$$

Combining expressions A-1–A-5 leads to the traveltime t_{shot} from a fixed-source position, i.e.,

$$t_{shot} = \frac{h_s}{\grave{v}}\left[\frac{1}{B_1^{1/2}} + \frac{\gamma^2}{B_\gamma^{1/2}}\right] + \frac{\gamma^2 x_{shot}\sin\delta}{\grave{v}B_\gamma^{1/2}}, \tag{A-6}$$

where $B_1 = 1 - p_{\parallel}^2\grave{v}^2$ and $B_\gamma = \gamma^2 - p_{\parallel}^2\grave{v}^2$.

The offset x_{shot} still needs to be specified in terms of the interface-parallel slowness $p_{\parallel}$. Again, from Figure 2 it follows that

$$x_{shot}\cos\delta = \overline{S'C'} + \overline{C'R'} = h_s\tan\grave{\theta}' + h_r\tan\acute{\theta}' \tag{A-7}$$

Thus,

$$x_{shot} = \frac{h_s p_{\parallel}\grave{v}\left(B_1^{1/2} + B_\gamma^{1/2}\right)}{B_1^{1/2}\left(B_\gamma^{1/2}\cos\delta - p_{\parallel}\grave{v}\sin\delta\right)}. \tag{A-8}$$

The minimum traveltime occurs if $dt_{shot}/dp_{\parallel} = 0$. The derivative of equation A-6 with respect to slowness is unwieldy. Its root, however, can be established with the help of a symbolic package. This leads to the slowness $p_{shot,min} = -\gamma\sin\delta/\grave{v}$, for which the minimum traveltime $t_{shot,min}$ in a common-shot gather occurs. Substituting this slowness in expressions A-6 and A-8 produces the desired minimum traveltime $t_{shot,min}$ and the offset position $x_{shot,min}$, yielding

$$t_{shot,min} = \frac{h_s}{\grave{v}}[(1-\gamma^2\sin^2\delta)^{1/2} + \gamma\cos\delta] \tag{A-9}$$

and

$$x_{shot,min} = -h_s\sin\delta\left[1 + \frac{\gamma\cos\delta}{(1-\gamma^2\sin^2\delta)^{1/2}}\right]. \tag{A-10}$$

For reference, for a pure-mode reflection we have $\grave{v} = \acute{v} = v$ and $\gamma = 1$. Expressions A-10 and A-9 then reduce to the well-known values [e.g., Dobrin and Savit (1988), 221–223]

$$t_{shot,min}(\gamma = 1) = \frac{2h_s\cos\delta}{v} \tag{A-11}$$

and

$$x_{shot,min}(\gamma = 1) = -2h_s\sin\delta. \tag{A-12}$$

Equations A-11 and A-12 are sometimes used to estimate the thickness and dip of the reflector using the position of the traveltime minimum in the shot gather. Their ratio yields the dip for known velocity v; thereafter, the thickness follows from either equation (Dobrin and Savit, 1988). A similar trick can be done using the ratio $x_{shot,min}/t_{shot,min}$ of the converted-wave arrival. However, we now need knowledge of both the P- and S-wave velocities since

$$\frac{x_{shot,min}}{t_{shot,min}} = -\frac{\grave{v}\sin\delta}{(1-\gamma^2\sin^2\delta)^{1/2}}. \tag{A-13}$$

CACP gathers

Expressions A-6 and A-8 determine the traveltime and emergence offset in a common-shot gather for a specific interface-parallel slowness $p_{\parallel}$. However, we wish to re-sort our data around a point Y on the surface such that ξ is kept constant. We therefore need to express equations A-6 and A-8 in terms of the thickness h_y underneath Y and the sorting ratio ξ. Rewriting equations A-6 and A-8, by expressing h_s in terms of h_y using expression A-3 yields the traveltime t_ξ of a converted-wave reflection in a gather at point Y for a given slowness $p_{\parallel}$, arriving at offset x_ξ. That is,

$$t_\xi = \frac{h_y}{\grave{v}}\left[\frac{1}{B_1^{1/2}} + \frac{\gamma^2}{B_\gamma^{1/2}}\right] + \frac{x_\xi\sin\delta}{\grave{v}}\left[-\frac{\xi}{B_1^{1/2}} + \frac{\gamma^2(1-\xi)}{B_\gamma^{1/2}}\right], \tag{A-14}$$

with

$$x_\xi = \frac{h_y C}{1 + \xi C\sin\delta} \tag{A-15}$$

and

$$C = \frac{p_{\parallel}\grave{v}\left(B_1^{1/2} + B_\gamma^{1/2}\right)}{B_1^{1/2}\left(B_\gamma^{1/2}\cos\delta - p_{\parallel}\grave{v}\sin\delta\right)}. \tag{A-16}$$

Expressions A-14–A-16 can, for instance, be used to determine analytically where the traveltime minimum will occur for a given ratio ξ and a variety of dips and P–S-wave velocity ratios γ. It is thus possible to forward predict where the traveltime minimum will be positioned in, for example, a CMP gather ($\xi = 1/2$).

Inserting the CACP sorting ratio ξ_{cacp} (equation 1), in expressions A-14 and A-16 leads to our final equations for the traveltimes t_{cacp} and emergence distances x_{cacp} in a CACP gather located at point Y for a given interface-parallel slowness p. They are given by

$$t_{cacp} = \frac{h_y}{\grave{v}}\left[\frac{1}{B_1^{1/2}} + \frac{\gamma^2}{B_\gamma^{1/2}}\right] + \frac{x_{cacp}\gamma\sin\delta}{(1+\gamma)\grave{v}}\left[-\frac{1}{B_1^{1/2}} + \frac{\gamma}{B_\gamma^{1/2}}\right] \tag{A-17}$$

and

$$x_{cacp} = \frac{(1+\gamma)h_y C}{1+\gamma+\gamma C \sin\delta}, \tag{A-18}$$

where C is again given by expression A-16.

What is the effect of dip on the apparent moveout velocity? To answer this question, we need to expand t^2_{cacp} in a Taylor series and compare it with the hyperbolic moveout equation for pure-mode data in a CMP gather. The latter is given by (Levin, 1971)

$$t^2_{cmp}(\gamma = 1) = \frac{4h_y^2}{v^2} + \frac{x^2_{cmp}\cos^2\delta}{v^2}. \tag{A-19}$$

The traveltimes in the CACP gather can also be expanded in a Taylor series around $x = 0$ (Taner and Koehler, 1969). That is,

$$t^2_{cacp} = t_0^2 + A_2 x^2_{cacp} + O\left(x^3_{cacp}\right), \tag{A-20}$$

where $t_0 = h_y(1+\gamma)/\grave{v}$ is equal to the minimum traveltime in a CACP gather and

$$A_2 = v_{nmo}^{-2} = \left.\frac{d(t^2)}{d\left(x^2_{cacp}\right)}\right|_{x_{cacp}=0} = \left.\frac{t}{x_{cacp}}\frac{dt}{dx_{cacp}}\right|_{x_{cacp}=0}$$
$$= \left.\frac{t}{x_{cacp}}\frac{dt}{dp_\parallel}\left[\frac{dx_{cacp}}{dp_\parallel}\right]^{-1}\right|_{p_\parallel=0} = \frac{\gamma\cos^2\delta}{\grave{v}^2} = \frac{\cos^2\delta}{\grave{v}\acute{v}}. \tag{A-21}$$

The last limit is obtained using a symbolic package.

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