Overview of moment-tensor inversion of microseismic events

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Abstract

Understanding the source mechanisms of microseismic events is important for understanding the fracturing behavior and evolving stress field within a reservoir, knowledge of which can help to improve production and minimize seismic risk. The most common method for calculating the source mechanisms is moment-tensor inversion, which can provide the magnitudes, modes, and orientations of fractures. An overview of three common methods includes their advantages and limitations: the first-arrival polarity method, amplitude methods, and the full-waveform method. The first-arrival method is the quickest to implement but also the crudest, likely producing the least reliable results. Amplitude methods are also relatively simple but can better constrain the inversion because of the increased number of observations, especially those using S/P amplitude ratios. Full-waveform methods can provide results of very good quality, including source-time functions, but involve much more complex and expensive calculations and rely on accurate seismic-velocity models.

Introduction

Microseismic monitoring is extremely valuable for tracking the performance of hydraulic-fracturing treatments within reservoirs. An improved understanding of the physical processes that govern induced seismicity is important, both for maximizing production and reducing seismic hazard. In particular, seismic-source mechanisms can provide insights into the fracturing behavior of the reservoir and surrounding rocks and an understanding of the evolution of the stress field. Those insights can contribute to an advanced knowledge of fracture type, propagation, and connectivity.

This article reviews the basic concepts of seismic-source inversion. The moment tensor is introduced as a useful description of the seismic source, and various possible seismic sources are described. Then the three main methods of moment-tensor inversion are described, including their advantages and disadvantages. A method for decomposing the moment tensor into practical information about the source mechanism is explained. Finally, the use of source inversion in hydraulic-fracturing monitoring is discussed.

Describing seismic sources

There is a wide variety of seismic sources, both natural and man-made. These include explosions, implosions, shear failure, tensile failure, and single forces. In the cases of natural tectonic and induced seismicity, the sources often are assumed to be pure shear failure, an assumption which is often valid. However, in recent years, it has become apparent that many microseismic events also might include a volumetric component in their source mechanisms, especially if recorded in volcanic environments or during hydraulic-fracturing treatments, in which fluids influence the fracturing behavior. These volumetric components can be large, and it is therefore important to take them into account because a failure to do so can greatly affect interpretations of fracturing behavior and related stress fields (Julian et al., 1998). This makes seismic-source inversion more challenging in environments such as hydraulic fracturing.

The seismic moment tensor is a matrix of nine force couples used to describe the source mechanism (Aki and Richards, 2002):

$$\mathbf{m} = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix}. \quad (1)$$

These force couples are shown figuratively in Figure 1, where 1, 2, and 3 represent x, y, and z, respectively. Because of the conservation of angular and linear momentum, the seismic-moment tensor is considered symmetrical, meaning that only six of the nine components are independent. The off-diagonal elements form balanced double-couples, avoiding net torque or rotation in the tensor. Diagonal elements represent force couples that describe volumetric changes. The moment tensor characterizes the event magnitude, fracture type (e.g., double-couple, tensile), and fracture orientation.

As previously mentioned, most tectonic seismicity is caused by shear faulting and therefore is described by a double-couple source mechanism (DC). Figure 2a shows a schematic diagram of the double-couple mechanism and the far-field P- and S-wave radiation patterns produced. One of the main issues of

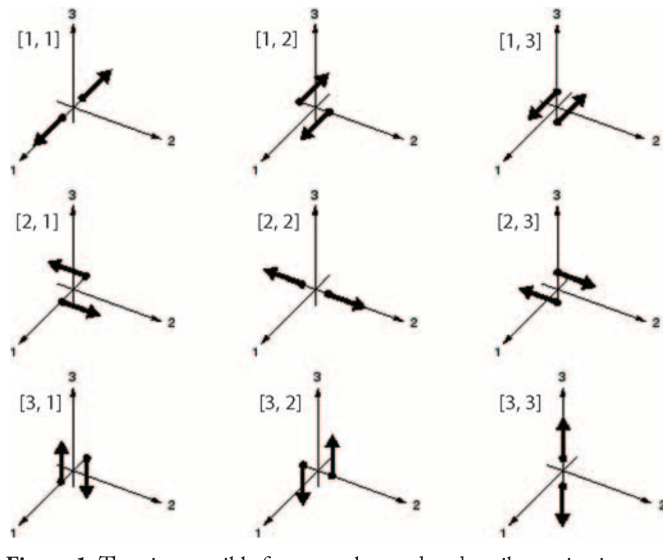

Figure 1. The nine possible force couples used to describe a seismic source (moment-tensor components), where 1, 2, and 3 represent x, y, and z, respectively (e.g., [1, 1] = M_{xx}, [1, 2] = M_{xy}, and so forth). The three diagonal elements describe volumetric changes, whereas the off-diagonal elements describe double-couple forces and thus shearing motion. After Aki and Richards (2002), Figure 3.7. Republished with permission from University Science Books; all rights reserved.

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interpreting double-couple mechanisms is that two nodal planes are present, one which represents the fault plane and the other which represents the auxiliary plane. From the moment tensor alone, it is impossible to distinguish between the two. Other geologic or geophysical evidence is required to further resolve this issue, or studies must bear in mind the two possibilities.

Focal mechanisms often are illustrated graphically using "beach-ball plots." These are lower-hemisphere stereographic projections of the P-wave radiation pattern, simplified into regions of compression (pressure) and dilatation (tension). For double-couple mechanisms, these regions are segmented by the two nodal planes described above. The region of compression is shaded, whereas the region of dilatation is white. Figure 3 shows examples of beach-ball plots for double-couple mechanisms in different major tectonic stress regimes.

However, as previously stated, source mechanisms can be more complex than this, especially in hydraulic-fracturing environments because of the injection of fluids, which can cause volumetric changes within the reservoir. A purely volumetric source is known as an isotropic source (ISO) and is described by a moment tensor that contains equal-valued diagonal elements and zeroes for the off-diagonal elements (Aki and Richards, 2002). Figure 2b shows a schematic diagram of the source mechanism and the far-field P-wave radiation pattern. S-waves are not produced because the source is purely compressional.

Seismic sources also can be described using a compensated linear vector dipole (CLVD), in which no volumetric change or shearing occurs. This describes the situation when one dipole is compensated by the two other dipoles, which are half the magnitude, i.e., the diagonal elements have a ratio of –1:–1:2, whereas the off-diagonal elements are zero (Julian et al., 1998) (Figure 2c). Note that a trace of **M** = 0 indicates absence of volumetric changes in the source; hence CLVD and double-couple sources have no volumetric component (i.e., no dilation).

Care must be taken when interpreting a mechanism described as CLVD by the moment tensor because this also can be described by other possible mechanisms. For example, a CLVD mechanism can be created by two double-couple geometries with different moments of M_0 and $2M_0$. CLVD components also can help to describe the opening or closing of a crack, along with an isotropic component (Julian et al., 1998), in which the diagonal elements form a ratio of 1:1:3 for a Poisson's ratio of 0.25 and 1:1:2 for a Poisson's ratio of 0.35.

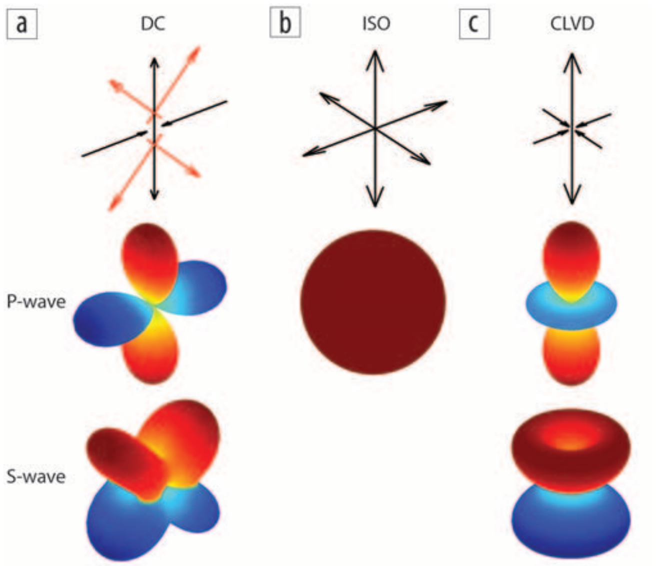

Figure 2. Far-field P-wave and SV-wave radiation patterns (red = compressions, blue = dilatations) of the (a) double-couple (DC), (b) isotropic (ISO), and (c) CLVD sources (for S-waves, the displacement in the θ [angle from z] direction is positive from the positive to the negative z-axis), plotted using the equations of Aki and Richards (2002). Arrows at the top schematically show the corresponding force systems generating the source mechanisms (black) and the shear forces (red).

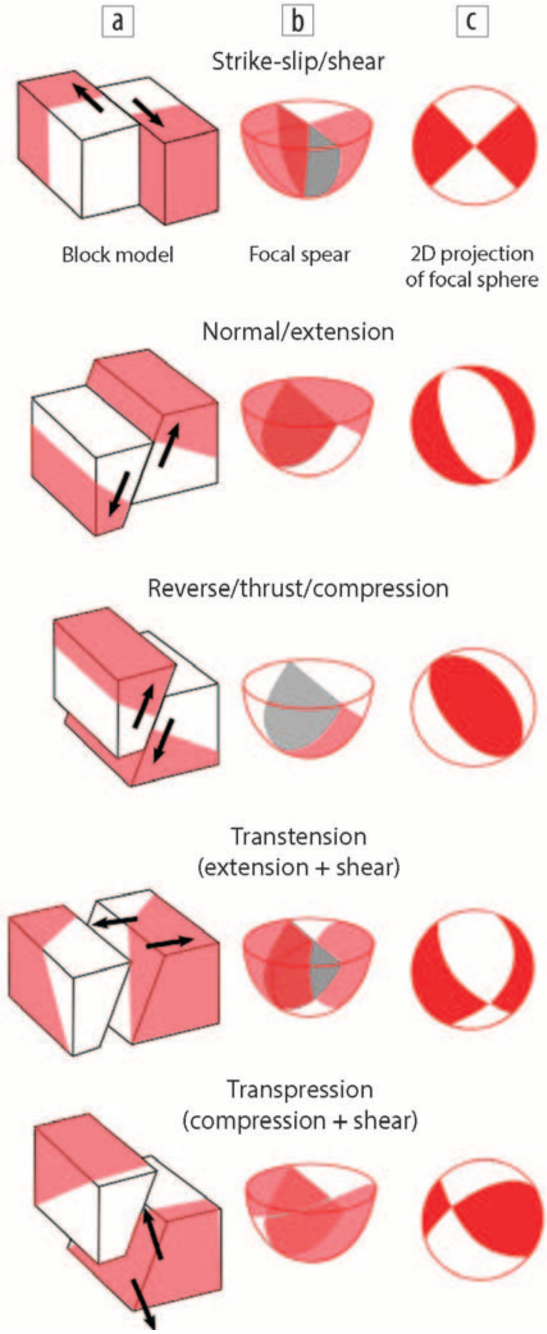

Figure 3. Beach-ball representations of focal mechanisms for common fault types. (a) Simple block models of the first motions of five common types of earthquakes (red = pressure, white = tension). (b) The corresponding lower-hemisphere and (c) 2D beach-ball projections, respectively. After Rowan (2015). Used by permission.

Following this discussion, it is clear that a moment tensor can be decomposed in different ways, resulting in different interpretations of the source mechanism; i.e., the decomposition is nonunique. The total moment-tensor solution consists of an addition of the isotropic, double-couple, and CLVD components. The double-couple and CLVD components often are described collectively as the deviatoric component.

Vavryčuk (2001) details a common method for decomposing the individual components of the moment tensor from the moment tensor and its eigenvalues (characteristic values that can be decomposed from any matrix along with the corresponding characteristic vector [eigenvector]), including calculating the percentages of the components that constitute the moment tensor.

One method of graphically representing the mechanism as described by the moment tensor is to use a plot introduced by Hudson et al. (1989). This uses two parameters T and k (calculated from the eigenvalues of the moment tensor) to characterize the type of deviatoric component in the source and the proportion of volume change component, respectively. A mechanism will plot in different regions of the diagram depending on the proportion of double-couple, CLVD, and isotropic energy. The diagram also distinguishes the sign in the case of the CLVD and isotropic components.

Moment-tensor inversion

The principal method for calculating seismic-source mechanisms is moment-tensor inversion, which uses the seismic radiation pattern to calculate the seismic moment tensor (see the section above, "Describing seismic sources"). There are three main techniques which will be described: the first-arrival polarity method, amplitude methods, and the full-waveform method.

Before discussing the methodology, it should be noted that an important factor when applying any of the moment-tensor inversion methods, especially when the mechanism is not assumed to be double-couple, is distribution of seismic sensors. Results of the inversion will be more reliable if sensor locations allow for a good sampling of the focal sphere. Therefore, ideally, as many sensors as possible should be deployed, surrounding the region in which the events occur.

Of course, this is not usually possible in practice, but it is desirable to deploy the sensors in a configuration as similar to this as possible. If the focal sphere is not sampled adequately, there are very few constraints on the mechanism, and therefore the results can be meaningless. Assumptions for the source mechanism are necessary in those cases (e.g., assume a double-couple mechanism), but the assumptions might be invalid, and because of poor sampling, orientation of the mechanism might be defined poorly.

First-arrival polarity method. The simplest method with which to calculate moment tensors is the first-arrival polarity method. In this method, the mechanism often is assumed to be double-couple. Figure 4 illustrates the theory of the method. The polarities of the first arrivals (i.e., the P-wave) at each sensor reflect the radiation pattern at the source. Because the P-wave radiation pattern for double-couple mechanisms is known (Figure 2), the orientation of the mechanism can be determined by the polarities and the locations of the sensors at which they were recorded with respect to the source. When recorded at the surface, upward first motions correspond to compressional first arrivals, and downward first motions correspond to dilatational first arrivals. Sometimes no apparent P-wave can be recorded because the sensor is on a nodal plane of the radiation pattern.

Again, sensor distribution is vital. For example, if all the sensors are southeast of the source location, it is plausible for the P-wave polarity to be identical for all sensors, and therefore it is impossible to determine the mechanism orientation with any accuracy. Even with a large number of sensors, nodal planes might not be well constrained with a range of possible orientations that fit the data, especially if no stations are close to the nodal planes (Figure 5).

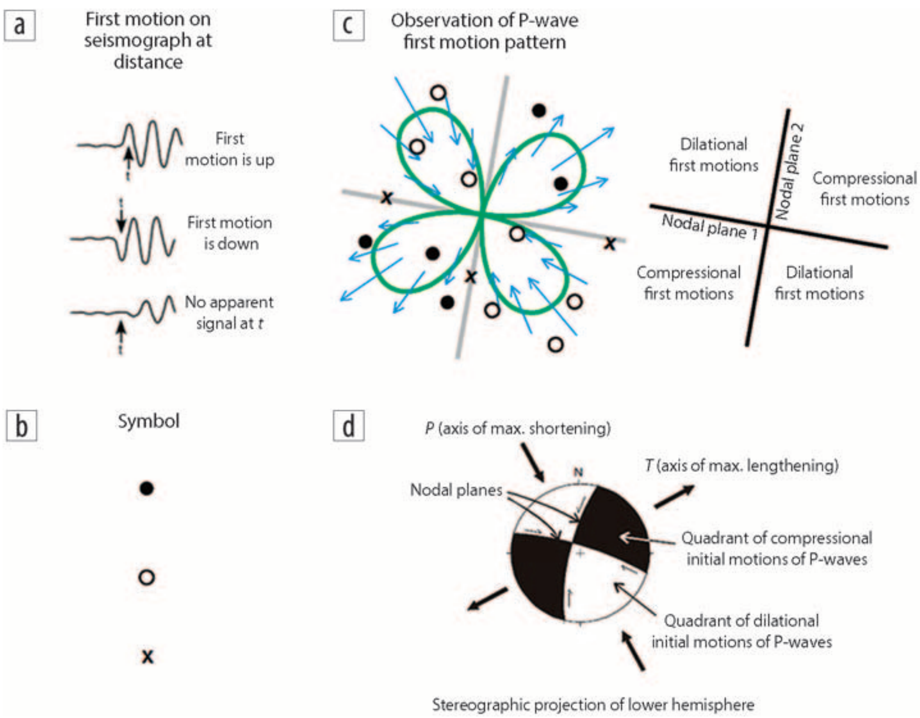

Figure 4. Construction of beach-ball focal-mechanism representations using first-arrival polarities. (a) Example vertical seismographs and (b) the corresponding symbols for stations recording positive, negative, and no apparent first motions. (c) Representation of how the first motions recorded at several seismometers can be used to determine orientations of the nodal planes expected because of the P-wave radiation pattern of double-couple events (right). These divide sensor locations (symbols) where first motions are upward and downward. Hence, the P-wave radiation pattern itself can be estimated (green). All this information then can be used to construct (d) the lower-hemisphere stereographic projection (beach ball). After Barth et al. (2008), Figure 3. Used by permission.

The advantages of this method are that it is the simplest, easiest, and quickest to implement. However, this results in the disadvantage that it is therefore the crudest method, with the least constraints on the orientation of the mechanism because of the binary nature of the data, and therefore it can produce many possible results that equally fit the data. Another major disadvantage is that it is difficult to resolve for mechanisms that are more complex than the basic double-couple or explosion/implosion mechanisms. It is also possible that P-waves might be difficult to identify, especially when we work with microseismic events.

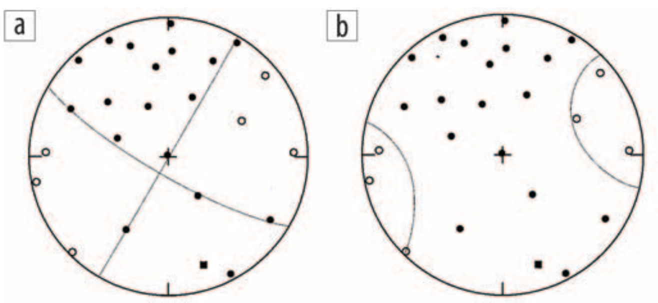

Figure 5. Limitation of the first-arrival method. Examples of two focal mechanisms that fit a high-quality data set of first polarity observations from the Hengill geothermal area, Iceland. Lower focal hemispheres are shown in equal area projection. Open and solid circles indicate inward and outward motions, respectively. (a) Pure double-couple mechanism. (b) Mechanism with a large isotropic component that is believed to be more reliable because it was obtained using an amplitude-based method. After Julian et al. (1998), Figure 5. Used by permission.

Amplitude methods. Amplitude methods are an extension of the first-arrival polarity method. Often, P- and S-wave amplitudes are used to better constrain the orientation of the P- and S-wave radiation patterns. However, the optimal method that incorporates amplitudes uses S/P amplitude ratios (Julian et al., 1998). Systematic variations in the ratio are expected because P-wave amplitudes are large near the pressure (P) and tension (T) axes and smallest near the nodal planes, whereas S-wave amplitudes are the opposite (Figure 2).

The advantage of this method over the polarity method is that the amplitudes are not binary and have a range of values, which can help to better constrain the mechanism. Another advantage is the increase in the number of observations if S-waves are used. S/P amplitude ratios also contain additional information. For example, Eaton et al. (2014) demonstrate that S/P amplitude ratios of less than 5 indicate that tensile failure is most likely.

Disadvantages of the simple amplitude methods are that the amplitudes are influenced by several other factors as well as the radiation pattern, including geometric spreading, attenuation, and station site effects, and these must be taken into account. However, the amplitude-ratio method simplifies this so that only site effects and the difference between P- and S-wave attenuation need to be considered. Another disadvantage is the possible difficulty of picking the P and S arrivals. Care also must be taken with how to filter the data and measure the amplitudes.

Full-waveform method. A more computationally expensive method for calculating source mechanisms is 3D full-waveform moment-tensor inversion (e.g., Eyre et al., 2015). As the name suggests, in this method, full-waveform data recorded on all components at each of the stations are inverted to calculate the seismic moment tensor. The data consist of both the source contribution and a path (propagation) contribution. The propagation effects can be removed by modeling the propagation of seismic waves between source and receiver locations as accurately as possible, producing Green's functions. Green's functions are the displacement responses recorded at the receivers when an impulse force (or moment) function is applied at the source position in a viscoelastic earth (i.e., the medium response). The nth component of the displacement u, recorded at position $\mathbf{x}$ and time t, can be written as

$$u_n(\mathbf{x}, t) = M_{pq}(t) * G_{np,q}(\mathbf{x}, t), \quad n, p, q = x, y, z, \tag{2}$$

where M_{pq} is the force couple in the direction pq, and $G_{np,q}$ are the spatial derivatives of the nth components of the Green's functions generated by the moment M_{pq}. The asterisk indicates convolution, and the summation convention applies. The Green's functions can be calculated using several methods such as ray tracing and full-wavefield simulations, and for this, the seismic-velocity structure should be modeled as accurately as possible.

In the frequency domain, equation 2 is linear because the convolution becomes a multiplication. Therefore, the inversion often is performed in the frequency domain and can be solved separately for each frequency. This can be represented in matrix form and solved through the weighted least-squares method. The weighting matrix can play an important role in the inversion procedure if noise varies significantly between stations. The quality of the inversion results can be tested through evaluation of the misfit between calculated and observed data.

For a better understanding of source mechanisms, moment tensors can be decomposed into their principal components by using a method based on the singular value decomposition of the six time-dependent moment-tensor components. This leads to an estimation of a common source-time function and its contribution to each component of the moment tensor, thus giving a source-time history of the source process and its mechanism. The eigenvalues of the scalar moment tensor give the source mechanism, and the eigenvectors give the orientation of the principal axes.

Figure 6 shows an example of calculated moment-tensor results using the full-waveform method for a long-period seismic event recorded at Turrialba volcano in Costa Rica. Figure 6a displays the waveforms of the six moment-tensor components, and Figure 6b shows the eigenvectors of the solution. Figure 6c shows the fits between the calculated moment-tensor components and the results of the singular value decomposition. Figure 6d shows the waveform fits between the real data and the reconstructed data. The misfit provides a measure of the quality of the inversion.

The advantages of this method are that as long as care is taken to model the Green's functions as accurately as possible, the mechanism should be well constrained, and as well as the scalar moment tensor, the time-dependent moment tensor also is determined, giving the source waveform.

The disadvantages of the full-waveform method are that a good knowledge of the velocity structure is necessary, computing the Green's functions can be computationally expensive, and the method performs better for low-frequency data because it can become unstable at higher frequencies. Station site effects also must be taken into account before performing the inversion.

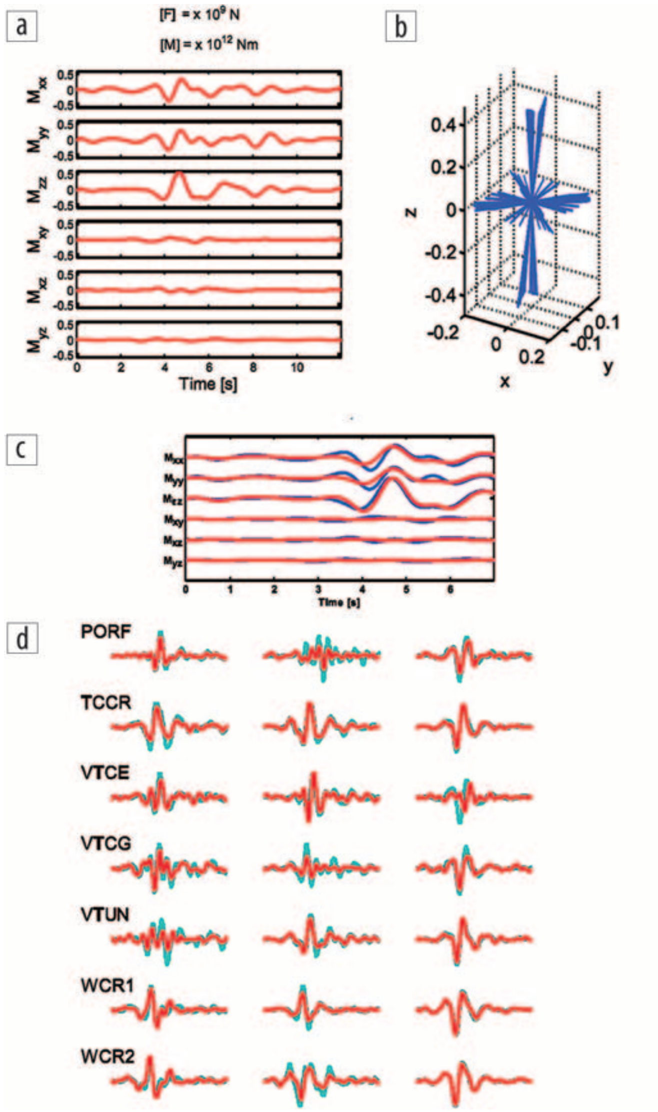

Figure 6. Results of the full-waveform moment-tensor inversion for a long-period seismic event recorded on Turrialba volcano in Costa Rica. (a) Time-dependent moment-tensor solution and (b) corresponding eigenvectors, sampled every 0.002 s. (c) The fits between the calculated moment tensors (blue) and the results from the singular value decomposition used in the calculation of the source mechanisms (red). (d) Normalized waveform fits between the real (blue) and reconstructed (red) data, used to test the quality of the full-waveform inversion. Each row represents a different station labeled with the station name, and the columns represent the x, y, and z components, respectively. Traces are 12 s long (Eyre et al., 2015).

A similar but simpler method for calculating the scalar moment tensor is to ignore the waveforms and to invert using only the amplitudes of the phases (P- and S-waves) picked from the data and picked from the Green's functions. This method uses the same theory as the full-waveform method but overcomes some of the issues with using high-frequency data.

Decomposition of the moment tensor

Once a scalar moment-tensor solution is obtained, further analysis can be carried out to better facilitate interpretation of the results in terms of the source-failure mechanism. For each method, the solution can be decomposed into the percentage of isotropic, CLVD, and double-couple source mechanisms. One method for implementing this is that of Vavryčuk (2001), which was introduced above in the section titled "Describing seismic sources," and the percentages of the individual components that constitute the moment tensor can be calculated.

Figure 7 shows an example of decomposed results for the full-waveform inversion of 107 long-period events recorded at Turrialba volcano. It can be seen that the mechanisms appear to be complex, with high isotropic components and small but stable CLVD and double-couple components. An isotropic and CLVD mechanism combined is interpreted as a crack mechanism (see "Describing seismic sources" above). With a small contribution of double-couple included, the mechanisms could be interpreted as a crack mechanism with a small amount of shearing, akin to a transtensional or transpressional crack mechanism.

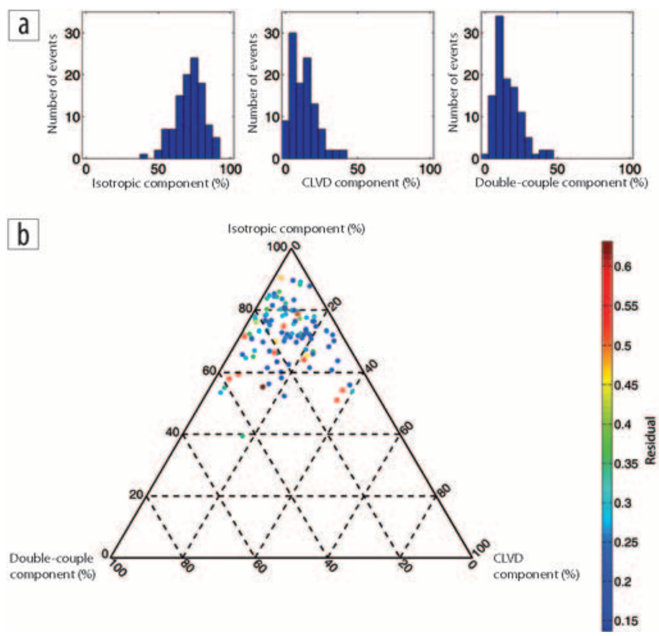

Figure 7. Collective results from the full-waveform moment-tensor inversion of 107 long-period events recorded at Turrialba volcano, showing results of the principal component analysis from Vavryčuk (2001), with full-waveform moment tensors decomposed into their proportions of isotropic, CLVD, and double-couple components. (a) Histograms and (b) triangle graph that includes residuals obtained for each inversion represented by the color scale. After Eyre et al. (2015), Figure 6. Used by permission.

Vavryčuk (2001) develops a method of describing this type of source. A tensile earthquake (an earthquake with tensile faulting or combining shear faulting and tensile faulting) can be described using a slip vector that is not restricted to orient within the fault plane and deviates from the fault plane, causing its opening or closing. This slip vector is labeled $[\bar{u}]$ and has an angle from the fault plane (labeled Σ) of α (Figure 8), which is 0° for pure double-couple and 90° for pure tensile events. The angles that describe the fault plane, slip vector, and *P*- and *T*-axes can be calculated by using equations detailed in Vavryčuk (2001).

Discussion

Moment-tensor inversion is an extremely useful tool for monitoring microseismic events in hydraulic-fracturing environments. Results include the magnitude, source mechanism, and mechanism orientation. In the instance of nondouble-couple mechanisms, detailed decomposition of the moment tensor is crucial for a better understanding of the mechanism. This information can be used to determine how the fluid injection is stimulating the reservoir and can help to monitor the efficiency of the stimulation because it contributes to a greater understanding of fracture type (whether tensile or pure shear and normal, reverse, or strike-slip), propagation, and connectivity.

Seismic moment tensors also can be used to calculate stress tensors, which can be used to analyze temporal and spatial stress variations within a reservoir. Such a method has been used successfully to calculate stress tensors for induced seismicity in a geothermal field (Martínez-Garzón et al., 2013). It therefore has good potential for use in hydraulic-fracturing environments, especially when combined with other information such as *b*-value variations and geotechnical information.

Conclusion

Moment-tensor inversion can be used to constrain the source mechanisms of microseismic events, which can be useful in microseismic monitoring. Methods using P- and S-arrivals can be used, but the more computationally expensive full-waveform inversion (and similar amplitude method) is likely to give more accurate solutions, especially for fluid-induced seismicity in which seismic sources might contain volumetric components. However, if the velocity model is constrained poorly, the S/P method is preferred. For accurate determination of seismic-source mechanisms, monitoring networks should be designed to maximize the sampling of the seismic radiation pattern. TLE

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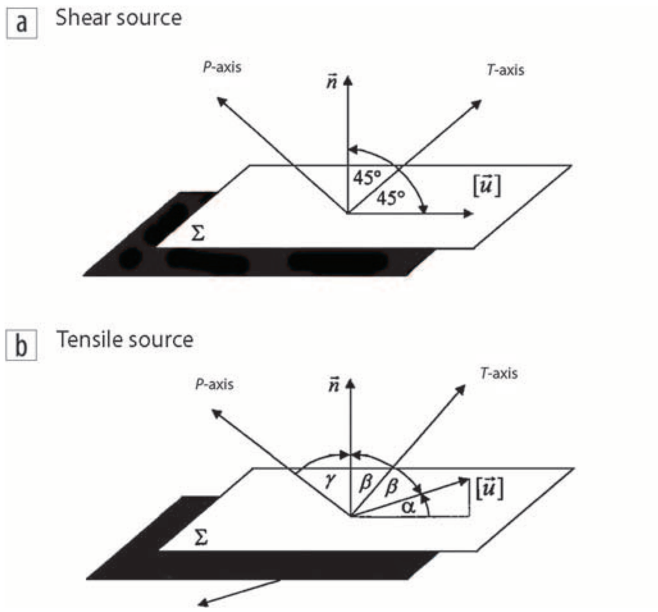

Figure 8. Models for (a) pure shear and (b) tensile fractures. Σ is the fault plane, $\bar{n}$ is the normal to the fault, $[\bar{u}]$ is the slip vector at the fault, and α is the inclination of the slip vector from the fault. The relationship to *P*- and *T*-axes is shown for each case. After Vavryčuk (2001), Figure 5. Used by permission.

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