Nonlinear deconvolution using Markov chain Monte Carlo for sparse reflectivity estimation
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SUMMARY

Deconvolution is a standard step in seismic processing and is usually performed using a linear deconvolution filter; such as the Wiener filter. Unfortunately, since such a filter is obtained from a linear combination of noisy observations it cannot solely recover the underlying reflectivity. Nonlinear deconvolution using Markov Chain Monte Carlo (MCMC) samples directly a statistical distribution for the reflectivity and, therefore, is potentially less noise prone.

A synthetic data example demonstrates the benefit of this nonlinear deconvolution technique over Wiener deconvolution; obtaining a correlation to the true reflectivity of 0.91 compared to 0.61. Furthermore, a real data application performed on a stacked seismic section is presented in which the nonlinear deconvolution technique is benchmarked against a commercially available maximum likelihood deconvolution technique. The results show that both algorithms obtain similar reflectivity estimates that correlate with the well-derived reflectivity; and produce reconstructed traces which have average correlation coefficients in excess of 0.97 with the recorded traces. However, nonlinear deconvolution achieves this without the need for the well controls to constrain the algorithm, rendering it highly suitable for obtaining sparse reflectivity sections in the absence of wells, e.g., in virgin frontier areas.

INTRODUCTION

High-fidelity interpretation of seismic data requires that the data have the highest possible quality and resolution. Seismic deconvolution by means of Wiener filtering is the standard tool for resolution enhancement. It aims to recover the Earth’s reflectivity series. Unfortunately, any linear deconvolution filter (such as a Wiener filter) has to find a compromise between recovery of the reflectivity series and noise amplification. It cannot solely recover the reflectivity series given a linear combination of noisy observations. Nonlinear deconvolution based on Markov chain Monte Carlo (MCMC) does not have this limitation since it does not search for linear combinations of observed values, but samples a function according to an underlying statistical distribution in order to determine the most likely reflectivity values. Nonlinear deconvolution is significantly more computation intensive than its linear counterparts; yet it offers the opportunity to create a step change in current processing strategies for seismic data.

The purpose of this research is two-fold. Firstly, we demonstrate that nonlinear deconvolution using MCMC obtains an estimate of the reflection coefficients that is potentially less noise contaminated than that obtained via traditional Wiener deconvolution. Secondly, estimation of underlying reflectivity series is often a first step in inverting for seismic impedances. The nonlinear deconvolution outcomes are always sparse by construction and, therefore, ideal for creating blocky seismic impedance sections. This raises the question if nonlinear deconvolution by MCMC can obtain reliable acoustic impedance sections with limited to no well control for low-frequency guidance?

In traditional acoustic impedance (AI) inversion an initial AI model derived from well log controls is required. This model provides information about frequencies outside of the seismic bandwidth and, therefore, constrains the AI inversion. Nonlinear deconvolution using MCMC recovers a sparse broadband estimate of the underlying reflection coefficients, without requiring well logs, which may be ideal for AI inversion. This is our ultimate research objective. In this extended abstract we focus on the reliability of obtained reflectivity estimates as an intermediate goal.

Figure 1: Real data application. Top: Original stacked section. Middle: MCMC result. Bottom: \( L_p \)-norm deconvolution result. The solid red line shows schematically the vertical extent of each well.
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Figure 2: Synthetic example. (a) True reflectivity series. (b) Observed trace with added noise. (c) Result after Wiener deconvolution. (d) Result after nonlinear MCMC deconvolution. The MCMC outcome is much closer to the true reflectivity series than the conventional Wiener result; a correlation coefficient of 0.91 versus 0.61.

NONLINEAR DECONVOLUTION BY MCMC

In the deconvolution problem we assume that the observed trace \( x \) is the result of the convolution of the wavelet \( w \) with the reflectivity series \( r \) plus some superposed noise \( n \). That is, \( x(t) = w(t) \ast r(t) + n(t) \), where \( \ast \) indicates convolution and \( t \) represents time.

The best estimate in the mean-squares sense of the reflectivity \( r(t) \) is its conditional expectation given the observations \( x(t) \), i.e., \( E(r|\bar{x}) \), with \( E \) expectation. For a Gaussian reflectivity, this best estimate is known to coincide with the linear least-squares estimate and is provided by the Wiener filter. However, the reflectivity is super-Gaussian (Walden, 1985) and, hence, Wiener filtering is sub-optimal. Unfortunately, solving the conditional expectation \( E(r|\bar{x}) \) analytically is rarely feasible for an arbitrary non-Gaussian reflectivity.

However, one can take recourse to Monte Carlo integration techniques and in particular Markov chain Monte Carlo methods (Gilks et al., 1998). In this approach one builds a Markov chain that samples sequentially all conditional probability densities functions where all but one model parameter are given. This is simpler than direct sampling of the desired distribution. Each iteration cycle provides an estimate of the reflectivity, and the mean of all iteration cycles converges to the desired expectation \( E(r|\bar{x}) \). In order to improve convergence the algorithm performs a number of warm-up cycles which are not included in the final average.

This is the basis of the MCMC deconvolution method devised by Cheng et al. (1996). We adapt their method in two important ways to improve convergence and stability. First, we estimate the source wavelet directly from the data using the method of van der Baan (2008) instead of trying to solve for both the reflectivity and the wavelet simultaneously using MCMC. This enhances convergence and stability since a timing indeterminacy exists between reflectivity and wavelet (i.e., the same trace is observed if one advances the wavelet but delays the reflectivity). Next we initiate the MCMC algorithm using the Wiener filtering outcome. This augments convergence.

Like Cheng et al. (1996) we assume that the reflectivity series can be adequately represented with a Bernoulli-Gaussian model in which the probability that a reflector occurs is given by a Bernoulli law, taking the value 1 with probability \( p \) and 0 with probability \( 1 - p \), and the magnitude of each reflector is described by a Gaussian law. Using the Bernoulli-Gaussian distribution has the advantage that the Markov Chains are easy to implement and it leads automatically to sparse reflectivity series which may be ideal for seismic inversion.

SYNTHETIC EXAMPLE

We first demonstrate the suitability of nonlinear deconvolution by MCMC on a simple synthetic example. Figures 2(a) and 2(b) show respectively the true reflectivity series and the observed trace where noise has been added with a signal-to-noise ratio of 2. The results of Wiener filtering and MCMC deconvolution are displayed in Figures 2(c) and 2(d). The wavelet is assumed known in both cases. The MCMC result uses 100 warm-up cycles and 1000 calculation cycles. It is seen that the
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MCMC reflectivity more closely approximates the true reflectivity. This is because the true reflectivity is super-Gaussian and the wavelet is bandlimited. Hence, Wiener filtering is suboptimal and needs to find a delicate compromise between recovery of the true reflectivity series and noise amplification.

REAL DATA APPLICATION

Figure 1 shows the result of applying the MCMC algorithm to a stacked seismic section. The data are displayed before and after nonlinear MCMC deconvolution and $L_p$-norm deconvolution. $L_p$-norm deconvolution is a maximum likelihood deconvolution technique (Debye and Riel, 1990) and is used to provide a comparison. The most important difference between both deconvolution approaches is that $L_p$-norm deconvolution is predominantly used for seismic inversion and, therefore, requires well logs to recover the low-frequencies outside of the seismic bandwidth. In contrast, no well log information is supplied to the MCMC approach. Only an estimate of the seismic wavelet and noise level are required by the algorithm and these are obtained directly from the input seismic.

A visual inspection of the two deconvolved sections confirms that both algorithms obtain similar results. In particular, reflection coefficients have been estimated for the major reflection horizons at labels (1), (2), (3) and (4). A close inspection of horizon (1) demonstrates that the wavelet sidelobes have been successfully removed by both approaches.

The zoomed sections in Figure 3 demonstrate the increased resolution provided by the deconvolved sections. The red arrows show where onlapping reflectors are more easily identified. The blue arrows highlight an area of reflector termination that is particularly well resolved by the nonlinear MCMC deconvolution. The nonlinear MCMC deconvolution obtains a sparser reflectivity estimate than the $L_p$-norm deconvolution method.

A wavelet is required by both deconvolution algorithms and this was estimated using a kurtosis maximisation technique (van der Baan, 2008). Convolving the estimated wavelet with the recovered $L_p$-norm and MCMC deconvolution results leads to reconstructed traces which strongly correlate with the recorded traces (Figure 4). Figure 5 shows the recorded and reconstructed traces for CDP 100. This correlation result serves to highlight the inherent non-uniqueness of the deconvolution problem, since both approaches lead to a correlation coefficient between reconstructed and recorded traces in excess of 0.98.

In Figure 6 the accuracy of the reflectivity estimates is tested further by a comparison to the two well logs employed to guide the $L_p$-norm deconvolution method. The original well logs are blocked using a 20 m interval to facilitate the comparison. The red dashed line corresponds to reflector (4) in Figure 1. Events on the MCMC reflectivity estimate can be seen to correspond to events on both the well log reflectivity and the $L_p$-norm deconvolution. The MCMC estimate is sparser than the $L_p$-norm estimate; in particular few reflectors are recovered below the large amplitude events occurring around the first, respectively third tick on the horizontal axis. Note that hardly any seismic energy is recorded below this due to its very large impedance contrast with the surrounding sediments. All deeper reflections are weak and discontinuous. The MCMC method does not use any well logs and can, therefore, not recover any reflectors that are not recorded in the seismic data. This is in strong contrast with the $L_p$-norm deconvolution result (Figure 3) which has recovered some of the smaller reflectors, possibly since this techniques uses the well logs for low-frequency guidance.

CONCLUSIONS

MCMC-based nonlinear deconvolution is potentially less biased by the presence of noise than conventional Wiener filtering, in particular if the reflectivity series are strongly non-
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Figure 6: Well log reflectivity comparison. Left: Well log 1; right: Well log 2. Within each well log (a) MCMC deconvolution result, (b) reflectivity estimated from blocked well logs and (c) $L_p$-norm deconvolution result. Both the MCMC and $L_p$-norm deconvolution outcomes compare well with the blocked well logs. However, little energy is recorded below reflector (4) in Figure 1 and highlighted above. Consequently, the MCMC algorithm has recovered few reflectors below the last major event.

Figure 4: Correlation comparison for each CDP. (a) MCMC synthetic. (b) $L_p$-norm synthetic. Both the MCMC and $L_p$-norm reconstructed traces correlate well with the recorded seismic traces.

Gaussian. The MCMC algorithm is able to recover frequencies outside of the seismic bandwidth by making the a priori assumption that true reflectivity series are super-Gaussian and, hence, suitably well described by a Bernoulli-Gaussian distribution. The same a priori assumption leads to sparse deconvolution results, which hold promise for estimating seismic impedance sections with a reduced need for seismic well logs for low-frequency guidance.

A comparison of MCMC deconvolution results for a stacked section with those obtained using the sparse spike $L_p$-norm method and two available well logs shows that MCMC deconvolution recovers the main features in the reflectivity, but not always the finer details - in particular if they are hidden under the noise level. Furthermore, the MCMC reflectivity trace estimates when convolved with the estimated wavelet produce reconstructed traces that strongly correlate with the input seismic data.

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EDITED REFERENCES
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