BASIC THEORY OF THE MAGNETO-TELLURIC METHOD OF GEOPHYSICAL PROSPECTING*†‡

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ABSTRACT

From Ampere's Law (for a homogeneous earth) and from Maxwell's equations using the concept of Hertz vectors (for a multilayered earth), solutions are obtained for the horizontal components of the electric and magnetic fields at the surface due to telluric currents in the earth. The ratio of these horizontal components, together with their relative phases, is diagnostic of the structure and true resistivities of subsurface strata. The ratios of certain other pairs of electromagnetic elements are similarly diagnostic.

Normally, a magneto-telluric sounding is represented by curves of the apparent resistivity and the phase difference at a given station plotted as functions of the period of the various telluric current components. Specific formulae are derived for the resistivities, depths to interfaces, etc. in both the two- and three-layer problems.

For two sections which are geometrically similar and whose corresponding resistivities differ only by a linear factor, the phase relationships are the same and the apparent resistivities differ by the same proportionality constant which relates the corresponding true resistivities. This "principle of similitude" greatly simplifies the representation of a master set of curves, such as is given for use in geologic interpretation.

In addition to the usual advantages offered by the use of telluric currents (no need for current sources or long cables, greater depths of investigation, etc.), the magneto-telluric method of prospecting resolves the effects of individual beds better than do conventional resistivity methods. It seems to be an ideal tool for the initial investigation of large sedimentary basins with potential petroleum reserves.

INTRODUCTION

There is no doubt that the first positive success in geophysical prospecting was obtained by electrical methods. These have always appeared promising both for oil and mineral prospecting because one can usually expect large resistivity contrasts in earth materials. Moreover, in the case of horizontal bedding, electrical prospecting can give information at locations where neither magnetic nor gravity anomalies can exist. The equipotential method, which involves the mapping of the equipotential lines on the earth's surface when current is introduced into the ground through two point electrodes, usually failed because of difficulty in analyzing the diagnostic features. In spite of the simplicity of Ohm's law, the theory of current flow in the earth is very complex. One may resort to experiments on scale models, but these preserve many of the shortcomings of the theoretical approach when applied to a practical situation.

In general, petroleum and mining geologists were not satisfied with the am-

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biguous interpretations which geophysicists could offer them on the basis of equipotential data. The use of alternating current is even less desirable in this respect because Maxwell's equations are considerably less manageable than is Ohm's law.

The introduction of resistivity methods was a step in the right direction, chiefly because the "apparent resistivity" of a section whose structure is not too complicated can actually be calculated, or at least estimated, without too much risk of error. However, these new methods, especially with respect to depth determination, have not proved to be as spectacular as they first appeared. Even for the two-layer case, a large amount of labor is involved in developing a master set of curves and one is seldom able to match his experimental curve with any of the curves in his catalogue, extensive as it might be. Moreover, the useful depth of investigation is limited to a few hundred meters in the case of direct current and even less in the case of alternating current, especially at the higher frequencies. In order to investigate to a reasonable depth, it is necessary to use direct current with such great electrode separations that the method no longer has the advantage of being inexpensive.

It is thus evident that electrical sounding, at least in petroleum exploration, originally promised much more than it has realized. However, the relatively recent discovery of the telluric method, although little known and little used outside of France, offers more favorable prospects. Although the principles involved were recognized about 30 years ago by Conrad Schlumberger,* no practical application was made until a few years before World War II. The telluric method has several advantages in that it does away with a current source and the associated long leads, combines flexibility, rapidity, and low cost, and reaches much greater depths of penetration than do ordinary resistivity methods. In spite of its fundamental advantages, however, the telluric method seems to represent only a temporary stage in the development of more advanced methods. The magneto-telluric method, which is the subject of this paper, answers the ever-increasing need for quantitative results. Actually, it is not a strictly electrical method, but rather a combination of telluric and magnetic methods, a combination from which the name of the technique has been derived.

Essentially, the magneto-telluric method involves the comparison, preferably at one and the same place, of the horizontal components of the magnetic and electric fields associated with the flow of telluric currents. The new method offers all the advantages of the telluric method and even improves on it with respect to flexibility, speed, and economy. In addition, it offers the inestimable benefit of making possible, in most cases where the bedding is horizontal, a truly quantitative interpretation. Also, the method can be applied without particular difficulty to submarine prospecting.

* E. G. Leonardon, "Some Observations Upon Telluric Currents and Their Applications to Electrical Prospecting," *Terrestrial Magnetism and Atm. Electr.* 33 (1928), pp. 91–94. A presentation of a report on work dating back to 1921 under the direction of Conrad Schlumberger.

SKIN EFFECT AND ITS CONSEQUENCES. HARMONIC SHEET OF TELLURIC CURRENTS IN AN ELECTRICALLY HOMOGENEOUS EARTH

By way of introduction to the analysis of the magneto-telluric method let us consider a schematic and ideal sheet of telluric current which we shall suppose to be uniform, harmonic, of period T, flowing in a soil electrically homogeneous, of conductivity σ.

During this study, we shall only use electro-magnetic units, both for electric dimensions and magnetic dimensions. Let us choose a rectangular coordinate system o, x, y, z (Fig. 1) such that the origin is on the surface of the ground and oz is the descending vertical. One will notice that on the ground the angle ox, oy is equal to $-(\pi/2)$ for an observer who normally stands with his feet on the ground and his head straight up. It is also useful to remember that, if a current circulates in the ground along ox, oy is at the left of the Amperian man looking up at the sky.

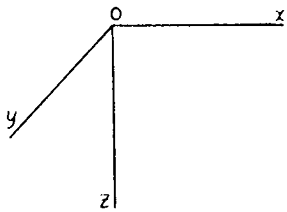

FIG. 1. Coordinate system. xy plane represents earth's surface. z is positive downward.

It is particularly useful when one employs Maxwell's equations and considers a harmonic phenomenon, to bring in the Hertz vector and to make use of imaginary notation. I shall use this approach later, but to handle this first particularly easy case, I prefer to remain as elementary as possible in order to be understood by those who are not familiar with Maxwellian analysis and who are eager to understand the principles of the proposed method.

The term "uniform" when applied to the telluric sheet we want to consider is rather inaccurate. As a matter of fact, there is uniformity only parallel to the surface of the ground, and not along a vertical line. If the density of the current is represented on the surface of the ground, for $z=0$, by

$$I_x = \cos \omega t, \qquad I_y = I_z = 0, \tag{1}$$

the laws of physics show that at depth z one must have

$$I_x = e^{-z\sqrt{2\pi\sigma\omega}} \cos (\omega t - z\sqrt{2\pi\sigma\omega}), \qquad I_y = I_z = 0, \tag{2}$$

e designating the base of natural logarithms. Formula (2) holds for what is called the skin effect. When z increases, one notices an exponential decrease with respect to z at the same time that the phase retardation progressively increases.

Under the conventional name of "depth of penetration" (understood as relating to a layer of conductivity σ and to a telluric sheet of period T) we shall define a term which we are going to use constantly. It designates the depth p when the amplitude is reduced to the fraction $1/e$ of what it is on the surface.

$$p = \frac{1}{\sqrt{2\pi\sigma\omega}} = \frac{1}{2\pi}\sqrt{\frac{T}{\sigma}}. \tag{3}$$

As for the phase, it is retarded one additional radian each time that z is increased by p.

It is obvious that for z infinite, the amplitude of the magnetic field is annulled; otherwise the density of the current could not be zero. At the same time, sym-

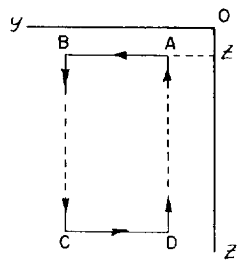

FIG. 2. Path of integration to apply Faraday's Law.

metry requires that the magnetic field be horizontal everywhere, parallel to oy. Let us now apply the theorem of Ampere to a rectangle $ABCD$ with sides AB CD parallel to oy and of unit length, with side AB situated at depth z and with side CD put at infinite depth. It reads

$$H_x = 0,$$

$$H_y(z) = 4\pi\int_z^{+\infty} I_x dz = 2\sqrt{\frac{\pi}{\sigma\omega}}\, e^{-z\sqrt{2\pi\sigma\omega}} \cos\left(\omega t - z\sqrt{2\pi\sigma\omega} - \frac{\pi}{4}\right). \tag{4}$$

In particular, on the surface of the earth, where $z = 0$,

$$H_x = 0,$$

$$H_y = 4\pi\int_0^{+\infty} I_x dz = 2\sqrt{\frac{\pi}{\sigma\omega}} \cos\left(\omega t - \frac{\pi}{4}\right). \tag{5}$$

We shall stress this first result, because it is the key to the proposed method: On the surface of the ground, the magnetic field $\mathcal{H}$ and electric field $\mathcal{E}(E_x = I_x/\sigma)$ are orthogonal. The quotient of the amplitude of the electric field by that of the magnetic field has the value $1/\sqrt{2\sigma T}$. The phase of the magnetic field is retarded by an angle of $\pi/4$ with respect to that of the electric field.

It is well understood that the above result is valid for a telluric sheet flowing in any direction, provided one always chooses the left hand side as positive in measuring the magnetic field. If, for instance, the component of the electric field along oy is of the form

$$E_y = \frac{1}{\sigma} \cos \omega t, \tag{6}$$

it will be necessary to write

$$H_x = -2\sqrt{\frac{\pi}{\sigma\omega}} \cos\left(\omega t - \frac{\pi}{4}\right), \tag{7}$$

with a change of sign relative to the similar formula (5), since the x axis indicates the right hand side when the current flows along the y axis.

The integral in the second member of relation (5) represents the total intensity of the telluric current through a rectangle, vertical and unlimited, going

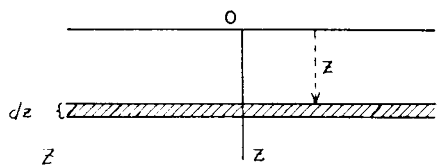

FIG. 3. Section showing horizontal uniform sheet of current.

from the surface, perpendicular to ox, and of unit width. The magnetic field H measures this total intensity within a factor of 4π.

This observation is of great practical importance. *It remains strictly valid for any layered earth, and maintains approximate validity in many cases interesting in exploration.*

Remarks

Assume a horizontal, uniform, extremely thin sheet of direct current of density I, flowing at the depth z between two horizontal planes with sides z and $z+dz$ (Fig. 3). It is well known and easy to show that the magnetic field produced by this horizontal sheet on the surface of the ground is horizontal, that it is directed to the left hand side and that its value is $2\pi I dz$.

For a sheet of direct current, flowing parallel to ox, from the surface of the ground down to depth z, and whose density I_x would be any function of z, one would have

$$H_y = 2\pi \int_0^z I_x(z) dz. \tag{8}$$

Because telluric currents have an extraordinary low frequency, since the length of the wave is enormous relative to p, one might be tempted to apply to them relation (8), assuming their behavior to be that of a direct current, which would lead one to write

$$H_y = 2\pi \int_0^{+\infty} I_x(z) dz, \tag{9}$$

whereas the accurate formula (5) includes the factor 4π, and not the factor 2π.

Units and magnitudes

We measure the magnetic field in γ, the electric field in millivolts/km and the period in seconds. On the other hand, prospectors usually considèr the resistivity ρ rather than conductivity σ. They measure resistivities in ohm-meters.

$$
\begin{aligned}
1\ \gamma &= 10^{-5} \text{ em cgs} \\
1\ \text{mv/km} &= 1 \text{ em cgs} \\
1\ \text{km} &= 10^{5} \text{ em cgs} \\
1\ \Omega\text{m} &= 10^{11} \text{ em cgs}
\end{aligned} \tag{10}
$$

With the new system of units, one obtains:

$$
p = \frac{1}{2\pi}\sqrt{10\rho T}, \qquad \rho = 0.2T\left(\frac{E}{H}\right)^{2}. \tag{11}
$$

In order to become familiar with the order of dimensions, it is useful to consult the two tables of numbers which follow. Table 1 gives the values of p for different values of ρ and T. Table 2 gives, also as function of ρ and of T, the values of H corresponding to an electric field of 1 mv/km.

TABLE 1

DEPTHS OF PENETRATION GIVEN IN KM

ρ, \ T,	1 sec	3 sec	10 sec	30 sec	1 min	2 min	5 min	10 min	30 min
0.2	0.225	0.390	0.712	1.23	1.74	2.47	3.90	5.51	9.54
1	0.503	0.872	1.59	2.76	3.90	5.51	8.72	12.3	21.4
5	1.13	1.95	3.56	6.16	8.72	12.3	19.5	27.6	47.7
10	1.59	2.76	5.03	8.72	12.3	17.4	27.6	39.0	67.5
50	3.56	6.16	11.3	19.5	27.6	39.0	61.6	87.2	151
250	7.95	13.8	25.2	43.6	61.6	87.2	138	195	338
1,000	15.9	27.6	50.3	87.2	123	174	276	390	675
5,000	35.6	61.6	113	195	276	390	616	872	1510

TABLE 2

AMPLITUDES OF THE MAGNETIC FIELD GIVEN IN γ WHEN E IS 1 MV/KM

ρ, \ T,	1 sec	3 sec	10 sec	30 sec	1 min	2 min	5 min	10 min	30 min
0.2	1	1.73	3.16	5.48	7.75	11.0	17.3	24.5	42.4
1	0.447	0.775	1.41	2.45	3.46	4.90	7.75	11.0	19.0
5	0.2	0.346	0.632	1.10	1.55	2.19	3.46	4.90	8.49
10	0.141	0.245	0.447	0.775	1.10	1.55	2.45	3.46	6
50	0.0632	0.110	0.2	0.346	0.490	0.693	1.10	1.55	2.68
250	0.0283	0.049	0.0894	0.155	0.219	0.310	0.490	0.693	1.2
1,000	0.0141	0.0245	0.0447	0.0775	0.110	0.155	0.245	0.346	0.6
5,000	0.00632	0.0110	0.0200	0.0346	0.0490	0.0693	0.110	0.155	0.268

From now on, one will notice the extent to which the depths of penetration are exactly adapted to the needs of petroleum prospecting. One will also notice very large limits between which the ratio of the amplitudes of the electric and magnetic fields may vary, which is, of course, essential when one wants to establish a "precise" method of prospecting in which this ratio is to be measured.

RELATION BETWEEN THE ELECTRIC AND THE MAGNETIC FIELD FOR A NON-HARMONIC TELLURIC SHEET

If the components of the telluric current no longer vary with time according to a sinusoidal law but instead vary in an absolutely arbitrary way, as in natural telluric sheets, the relations obtained above are easily generalized by means of operational calculus. I shall limit myself to give the result, which does not seem to have any great practical interest in connection with prospecting.

$$\begin{aligned} E_y(t) &= -\frac{1}{2\pi\sqrt{\sigma}}\int_{-\infty}^{t} H_x'(u)\frac{du}{\sqrt{t-u}} \\ &= -\frac{1}{2\pi\sqrt{\sigma}}\int_{0}^{+\infty} H_x'(t-u)\frac{du}{\sqrt{u}}\,. \end{aligned} \tag{12}$$

In this expression, $H_x'(t)$ designates the derivative of $H_x(t)$ with respect to t.

GENERALIZATION FOR ANY HORIZONTALLY STRATIFIED SECTION

If the earth is formed by a number of horizontal strata of arbitrary thicknesses and resistivities, we shall start from the equations of Maxwell and we shall preferably use imaginary notation, stipulating that all the alternating quantities depend on time through a factor $e^{-i\omega t}$. From now on, this factor will be understood rather than expressed explicitly.

If the harmonic sheet, assumed uniform, flows along ox, the components of the Hertz vector Π along oy and oz are null. Furthermore, Π_x depends only on z (and on t).

The equations of Maxwell are satisfied if

$$\nabla^2\Pi_x + 4\pi\sigma\omega i\Pi_x = 0. \tag{13}$$

The electric field $\mathcal{E}$ and magnetic field $\mathcal{H}$ are expressed in a general way by

$$\begin{aligned} \mathcal{H} &= 4\pi\sigma \text{ curl } \Pi, \\ \mathcal{E} &= \text{grad div } \Pi - \nabla^2\Pi, \end{aligned} \tag{14}$$

and, specifically, in the actual problem by

$$\begin{aligned} H_y &= 4\pi\sigma\frac{\partial\Pi_x}{\partial z}, \qquad H_x = H_z = 0 \\ E_x &= 4\pi\sigma\omega i\Pi_x, \qquad E_y = E_z = 0. \end{aligned} \tag{15}$$

Because, in this case, E_x is proportional to Π_x, we can choose E_x as Hertz vector, so that

$$\frac{\partial^2 E_x}{\partial z^2} + 4\pi\sigma\omega i E_x = 0,$$

$$H_y = -\frac{i}{\omega}\,\frac{\partial E_x}{\partial z}. \tag{16}$$

Furthermore, we must assure the continuity of E_x and H_y when crossing the different surfaces of separation.

In order to meet condition (16), E_x must be in the form of

$$E_x = Ae^{a\sqrt{\sigma}\,z} + Be^{-a\sqrt{\sigma}\,z}, \tag{17}$$

A and B designating two arbitrary constants and a being defined as

$$a = 2\pi\sqrt{\frac{2}{T}}\,e^{-i\pi/4} = \frac{2\pi}{\sqrt{T}}(1 - i). \tag{18}$$

Let us number from 1 to n the successive formations starting at the surface of the ground. The nth and last one is the lowest stratum. It will be necessary in this layer to put down $A=0$, because the first term becomes infinite at the same time as z. Furthermore, any solution can always be multiplied by a constant complex arbitrary factor. In other words, the problem is only definite as far as the relative amplitudes and the differences of phase are concerned. For this reason, we can assign an arbitrary value to one of the $2n$ constants A and B. We shall assume that it is constant B corresponding to the bottom stratum which is equal to unity.

In all we have $2(n-1)$ arbitrary constants to meet the same number of conditions at the limits. These conditions are the equality of the two fields at each of the $n-1$ surfaces of separation.

The method of calculation being the same no matter what the value of n, we shall only consider the cases of $n=2$ and $n=3$.

It is obvious that these calculations, which do not present any other complications than the resolution of simple algebraic equations of the first degree, are done exclusively by means of addition, multiplication and division and do not necessitate resorting to integrals or series.

SOURCE OF CURRENTS

The above theory does not concern itself with the origin of the currents involved. Whether the source of these currents are internal to the crust of the earth or whether they are ionospheric, whether these sources are natural (actual telluric currents) or whether they are artificial (vagrant currents), the electromagnetic phenomena inside the earth are the same in every case.

In fact, the reasoning depends only on the requirement that the telluric current sheet be sufficiently uniform. But this uniformity is a matter of experience. Telluric prospecting proves that in large sedimentary basins this uniformity extends, in an approximate way, over a considerable expanse, often some ten km in width. Such uniformity should be expected all the more if one only considers the very restricted field that enters into a magneto-telluric comparison. Vagrant currents, because of the relative proximity of the sources which produce them, and because of the poor degree of uniformity of the sheets associated with such artificial currents, are feared by the telluric prospectors. On the contrary, they are looked on as a blessing by magneto-telluric prospectors, because they offer sufficient uniformity to meet the requirements of the new method, and they usefully enlarge the spectrum of frequencies.

Readers of *Geophysics*, as well as this writer, are mainly concerned with what is underneath their feet and are little interested in what goes on above their

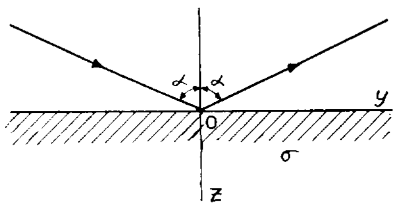

FIG. 4. Plane electromagnetic wave incident upon earth's surface.

heads. However, it may be useful to consider for a few moments longer the nature of the electro-magnetic phenomena as a whole involving the atmosphere.

In the air, where we put down $\sigma=0$, equation (16) becomes $\partial^2 E_x/\partial z^2=0$. E_x appears as a linear function of z, H_y as a constant:

$$\begin{aligned} E_x(z) &= E_x(0) + i\omega z H_y(0); \qquad & E_y &= E_z = 0. \\ H_y(z) &= H_y(0); & H_x &= H_z = 0. \end{aligned} \tag{19}$$

A solution of this kind may surprise the reader. One knows, in particular, that the vertical component of the magnetic field of the earth undergoes quick variations whose correlation with those of the horizontal components of the same field or of the telluric field is evident. But the actual solution shows us that H_z is null.

Let us not forget that, in the expression (13) of the equations of Maxwell, we have, from the start, considered as infinite the speed V of electro-magnetic waves in the ground, as well as the speed c of those waves in empty space. For the real phenomenon of propagation we have substituted from the start a fictitious stationary phenomenon. The approximation was quite sufficient for the calculations we had in mind, but it did not permit an accurate picture of the nature of the physical phenomena involved.

Let us suppose that in the atmosphere, a plane wave spreading in the plane oyz hits the surface of the ground at an angle of incidence α (Fig. 4). In order that the conditions at the limits might be met at the surface of the ground,

it is, first of all, necessary that the expressions for the characteristic vectors of the three waves (incident, reflected and refracted) include, respectively, the following factors:
Incident Wave:

$$e^{-i\omega\left(t-\frac{y\sin\alpha+z\cos\alpha}{c}\right)}$$

Reflected Wave:

$$e^{-i\omega\left(t-\frac{y\sin\alpha-z\cos\alpha}{c}\right)}$$

Refracted Wave:

$$e^{-i\omega\left(t-\frac{y\sin\alpha+Kz}{c}\right)}$$

The constant K is chosen to satisfy the equation

$$\nabla^2\Pi + \Pi\left(4\pi\sigma\omega i + \frac{\omega^2}{V^2}\right) = 0. \qquad (20)$$

It is thus necessary that

$$K^2 = \frac{c^2}{V^2} - \sin^2\alpha + i\,\frac{4\pi\sigma c^2}{\omega}\,. \qquad (21)$$

But, whereas $(c^2/V^2)-\sin^2\alpha$ is at its maximum equal to unity, it happens that the coefficient of i is enormous. For instance, for $\rho = 10\Omega m$ and for $T=30$ sec, it is equal to 5.4×10^{10} so that in practice, and as an excellent approximation, one may write

$$K^2 = 2\sigma c^2 T e^{i\pi/2},$$

$$K = c\sqrt{2\sigma T}\, e^{i\pi/4}, \qquad (22)$$

bearing in mind the fact that the coefficient of i in the imaginary part of K must be positive. Accordingly, we justify in the first place the form itself of the expressions (17) which we have adopted initially as a starting point. After that we notice that an infinity of possible waves in the atmosphere can correspond to a given wave in the ground. Not only is α left completely arbitrary since it does not appear in (22), but the state of polarization of the incident wave remains also totally arbitrary. One is entitled to imagine all kinds of miscellaneous phenomena in the atmosphere, and no particular condition is imposed that the vertical component of the magnetic field must be null or negligible.

SPECIFIC STUDY OF THE TWO LAYER PROBLEM

Let us suppose σ_1 to be the conductivity of the upper formation, and σ_2 that of the lower formation, h being the thickness of the upper one (Fig. 5).

Following the general method sketched above, the general expression for

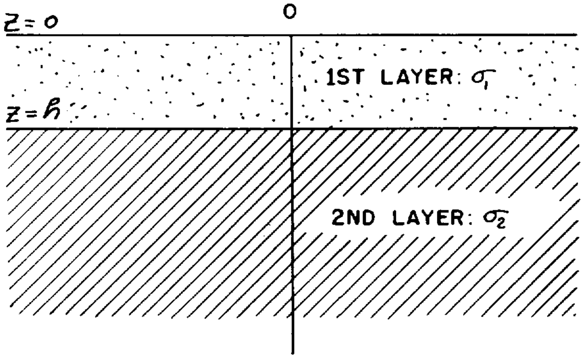

FIG. 5. Two-layer earth section.

the fields will be as follows:

1. In the first formation

$$E_x = Ae^{a\sqrt{\sigma_1}\, z} + Be^{-a\sqrt{\sigma_1}\, z} \tag{23}$$
$$H_y = e^{i\pi/4}\sqrt{2\sigma_1 T}\left[-Ae^{a\sqrt{\sigma_1}\, z} + Be^{-a\sqrt{\sigma_1}\, z}\right].$$

2. In the second formation

$$E_x = e^{-a\sqrt{\sigma_2}\, z} \tag{24}$$
$$H_y = e^{i\pi/4}\sqrt{2\sigma_2 T}\, e^{-a\sqrt{\sigma_2}\, z}.$$

The continuity of E_x and H_y for $z=h$ involves accordingly the two conditions

$$Ae^{a\sqrt{\sigma_1}\, h} + Be^{-a\sqrt{\sigma_1}\, h} = e^{-a\sqrt{\sigma_2}\, h}$$
$$-A\sqrt{\sigma_1}\, e^{a\sqrt{\sigma_1}\, h} + B\sqrt{\sigma_1}\, e^{-a\sqrt{\sigma_1}\, h} = \sqrt{\sigma_2}\, e^{-a\sqrt{\sigma_2}\, h} \tag{25}$$

where

$$A = \frac{\sqrt{\sigma_1} - \sqrt{\sigma_2}}{2\sqrt{\sigma_1}}\, e^{-ah(\sqrt{\sigma_1}+\sqrt{\sigma_2})},$$
$$B = \frac{\sqrt{\sigma_1} + \sqrt{\sigma_2}}{2\sqrt{\sigma_1}}\, e^{ah(\sqrt{\sigma_1}-\sqrt{\sigma_2})}. \tag{26}$$

The result is an expression for the fields on the surface of the ground. In this expression we shall advantageously introduce the depth of penetration p_1 relative to the first formation and we shall be able to set aside a factor common to E_x and H_y, since we are only interested in the relation between those fields. One has then

$$E_x = Me^{-i\phi}$$
$$H_y = \sqrt{2\sigma_1 T}\, Ne^{i(\pi/4-\psi)}, \tag{27}$$

in which:

$$M \cos\phi = \left(\frac{1}{p_1}\cosh\frac{h}{p_1} + \frac{1}{p_2}\sinh\frac{h}{p_1}\right)\cos\frac{h}{p_1},$$
$$M \sin\phi = \left(\frac{1}{p_1}\sinh\frac{h}{p_1} + \frac{1}{p_2}\cosh\frac{h}{p_1}\right)\sin\frac{h}{p_1}, \tag{28}$$

$$N \cos\psi = \left(\frac{1}{p_1}\sinh\frac{h}{p_1} + \frac{1}{p_2}\cosh\frac{h}{p_1}\right)\cos\frac{h}{p_1},$$
$$N \sin\psi = \left(\frac{1}{p_1}\cosh\frac{h}{p_1} + \frac{1}{p_2}\sinh\frac{h}{p_1}\right)\sin\frac{h}{p_1}. \tag{29}$$

Whereupon:

$$\frac{E_x}{H_y} = \frac{1}{\sqrt{2\sigma_1 T}}\frac{M}{N}e^{-i(\pi/4+\phi-\psi)}. \tag{30}$$

The formulas given above relating to the case of a single formation are at once found again if one starts from those more general expressions and puts down: $\sigma_1=\sigma_2=\sigma$ and $p_1=p_2=p$ whereupon

$$M = N = \frac{1}{p}e^{h/p},$$
$$\phi = \psi = \frac{h}{p}, \tag{31}$$
$$\frac{E_x}{H_y} = \frac{1}{\sqrt{2\sigma T}}e^{-i\pi/4},$$

conforming to the previous result.

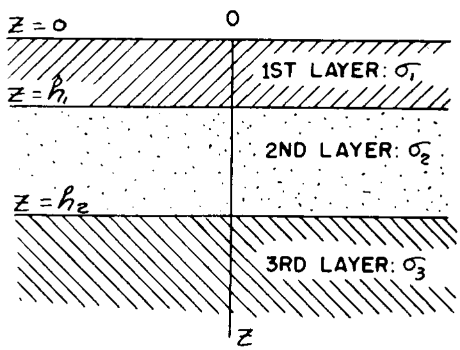

FIG. 6. Three-layer earth section.

FORMULAS FOR THREE FORMATIONS

In the case of three formations of conductivities σ_1, σ_2, and σ_3, when the second one starts at depth h_1 and the third one at depth h_2 (Fig. 6), one uses the following formulas. The ratio between the fields is always in the form

$$\frac{E_x}{H_y} = \frac{1}{\sqrt{2\sigma_1 T}} \frac{M}{N} e^{-i(\pi/4+\phi-\psi)}, \tag{32}$$

by putting down

$$\begin{aligned} h_1\left(\frac{1}{p_1}+\frac{1}{p_2}\right) - \frac{h_2}{p_2} &= u, \\ h_1\left(\frac{1}{p_1}-\frac{1}{p_2}\right) + \frac{h_2}{p_2} &= v, \end{aligned} \tag{33}$$

$$\begin{aligned} M\cos\phi = {} & \left(\frac{1}{p_1}-\frac{1}{p_2}\right)\left(\frac{1}{p_2}\cosh u - \frac{1}{p_3}\sinh u\right)\cos u \\ & + \left(\frac{1}{p_1}+\frac{1}{p_2}\right)\left(\frac{1}{p_2}\cosh v + \frac{1}{p_3}\sinh v\right)\cos v, \end{aligned} \tag{34}$$

$$\begin{aligned} M\sin\phi = {} & \left(\frac{1}{p_1}-\frac{1}{p_2}\right)\left(\frac{1}{p_2}\sinh u - \frac{1}{p_3}\cosh u\right)\sin u \\ & + \left(\frac{1}{p_1}+\frac{1}{p_2}\right)\left(\frac{1}{p_2}\sinh v + \frac{1}{p_3}\cosh v\right)\sin v, \end{aligned} \tag{35}$$

$$\begin{aligned} N\cos\psi = {} & \left(\frac{1}{p_1}-\frac{1}{p_2}\right)\left(\frac{1}{p_2}\sinh u - \frac{1}{p_3}\cosh u\right)\cos u \\ & + \left(\frac{1}{p_1}+\frac{1}{p_2}\right)\left(\frac{1}{p_2}\sinh v + \frac{1}{p_3}\cosh v\right)\cos v, \end{aligned} \tag{36}$$

$$\begin{aligned} N\sin\psi = {} & \left(\frac{1}{p_1}-\frac{1}{p_2}\right)\left(\frac{1}{p_2}\cosh u - \frac{1}{p_3}\sinh u\right)\sin u \\ & + \left(\frac{1}{p_1}+\frac{1}{p_2}\right)\left(\frac{1}{p_2}\cosh v + \frac{1}{p_3}\sinh v\right)\sin v. \end{aligned} \tag{37}$$

APPARENT RESISTIVITY FOR THE CASE OF TWO FORMATIONS

If the comparison of E_x and H_y is made on a ground which is known to be electrically homogeneous, the relation between those two fields allows one to know the true conductivity (or, if one prefers, its reciprocal, the resistivity), of the formation. If the magneto-telluric comparison takes place on any formation, stratified or not, whose structure is not in general known, it will usually happen that the phase of H_y with respect to E_x will not be a retardation of $\pi/4$. This will be the first indication that it is heterogeneous. However, no matter what this phase separation might be, we can agree that the modulus of the ratio is equal to $1/\sqrt{2\sigma_a T}$, in which σ_a would be the conductivity of a homogeneous formation which would give the modulus of the ratio between the fields whose value has been experimentally observed. The quantity σ_a is, by definition, the apparent conductivity and its reciprocal ρ_a is the apparent resistivity.

The apparent resistivity is usually a kind of average of the resistivities one meets in a thickness of ground such that density of the current is not to be neglected with respect to its value along the surface. However, it may happen in exceptional cases that the apparent resistivity might be very slightly less than the smallest of the resistivities of the formations, or on the other hand, very slightly greater than the highest of the resistivities. Actually, one knows that a similar phenomenon occurs for the apparent resistivities that are obtained in the prospecting techniques which use a quadripole of measurement supplied by direct current.

In the case of two formations, the apparent resistivity is easily calculated by means of the formulas established above. In accordance with this definition, one has

$$\frac{1}{\sqrt{2\sigma_a T}} = \frac{M}{N}\,\frac{1}{\sqrt{2\sigma_1 T}}, \tag{38}$$

or

$$\rho_a = \rho_1\left(\frac{M}{N}\right)^2; \tag{39}$$

and

$$\frac{\rho_a}{\rho_1} = 1 + \frac{4\cos\dfrac{2h}{p_1}}{m + \dfrac{1}{m} - 2\cos\dfrac{2h}{p_1}}, \tag{40}$$

if one writes

$$m = \frac{\sqrt{\dfrac{\rho_2}{\rho_1}} + 1}{\sqrt{\dfrac{\rho_2}{\rho_1}} - 1}\, e^{2h/p_1}. \tag{41}$$

The fundamental properties of the apparent resistivity as defined in the technique of electrical sounding, with respect to a certain length of injection line of current, appear again at this point in the apparent resistivity defined now in regard to a certain period or to a certain penetration depth p. Indeed, one immediately establishes that:

1. if $p_1 = 0$, $\rho_a = \rho_1$,

2. if $p_1 = \infty$, $\rho_a = \rho_2$.

EXPRESSION FOR THE RETARDATION OF PHASE IN THE CASE OF TWO FORMATIONS

The other parameter to consider in order to secure interpretation is the phase retardation of the magnetic field with respect to the electric field. In the case of two formations, it is expressed by

$$\theta = \frac{\pi}{4} + \phi - \psi, \qquad (42)$$

with

$$\begin{aligned} \tan \phi &= \frac{m-1}{m+1} \tan \frac{h}{p_1}, \\ \tan \psi &= \frac{m+1}{m-1} \tan \frac{h}{p_1}, \\ \tan(\phi - \psi) &= -\frac{2m}{m^2-1} \sin \frac{2h}{p_1}, \qquad \left(-\frac{\pi}{4} \leqq \phi - \psi \leqq \frac{\pi}{4}\right), \end{aligned} \qquad (43)$$

m having the meaning given previously (equation 41).

SPECIFIC CASE OF A SECTION WITH TWO LAYERS, ONE BEING AN EXTREMELY RESISTIVE OR EXTREMELY CONDUCTIVE SUBSTRATUM

In these specific cases, the above formulas become:
1. Extremely resistive substratum:

$$\left.\begin{aligned} \left|\frac{E_x}{H_y}\right| &= \frac{1}{\sqrt{2\sigma_1 T}} \sqrt{1 + \frac{2 \cos 2h/p_1}{\cosh 2h/p_1 - \cos 2h/p_1}} \\ \theta &= \frac{\pi}{4} - \operatorname{arc\,tan}\left(2 \frac{e^{2h/p_1}}{e^{4h/p_1} - 1} \sin 2h/p_1\right) \end{aligned}\right\}, \qquad (44)$$

The result becomes particularly simple if h is very much smaller than p:

$$\left|\frac{E_x}{H_y}\right| = \frac{1}{2\sqrt{\sigma_1 T}} \frac{p_1}{h}; \qquad \theta = 0. \qquad (45)$$

2. Extremely conducting substratum:

$$\left.\begin{aligned}\left|\frac{E_x}{H_y}\right| &= \frac{1}{\sqrt{2\sigma_1 T}}\sqrt{1 - \frac{2\cos 2h/p_1}{\cosh 2h/p_1 + \cos 2h/p_1}}\\ \theta &= \frac{\pi}{4} + \arctan\left(2\,\frac{e^{2h/p_1}}{e^{4h/p_1} - 1}\sin 2h/p_1\right)\end{aligned}\right\}. \tag{46}$$

The result becomes particularly simple if h is very much smaller than p_1:

$$\left|\frac{E_x}{H_y}\right| = \frac{1}{\sqrt{\sigma_1 T}}\,\frac{h}{p_1}, \qquad \theta = \frac{\pi}{2}\,. \tag{47}$$

LAW OF SIMILITUDE OF THE MAGNETO-TELLURIC SOUNDINGS

It is known that the interpretation of the ordinary electrical soundings is made much easier by the use of logarithmic scales in the construction of theoretical templates on the one hand, and of experimental diagrams on the other. This use of logarithmic scales is based on the laws of similitude (geometric similitude, electric similitude) which are applicable to electrical soundings.

Laws of similitude of the same kind also govern magneto-telluric soundings and will play an important part in their interpretation. Before we explain how to represent the results we have just obtained in the form of master curves, it is necessary to establish these laws of similitude.

Let us consider two structures, as complex as desired, stratified or not, being geometrically similar, the ratio of similitude being K_L. To make it plainer, let us specify that the corresponding parameters of the two structures will be represented by the same letters, respectively primed and unprimed. In this way, L' and L designating corresponding lengths, we shall put down

$$L' = K_L L. \tag{48}$$

At two similar points of the two structures, the resistivities are ρ' and ρ and we postulate electrical similitude

$$\rho' = K_\rho \rho. \tag{49}$$

Finally, if the periods of the electro-magnetic phenomena are T' and T, we require

$$T' = K_T T. \tag{50}$$

If $\Pi'(x', y', z')$ represents a Hertz vector, which is a solution of Maxwell's equations and of the boundary conditions for the primed structure, let us find out the conditions under which

$$\Pi(x, y, z) = \Pi'(x', y', z') \tag{51}$$

is also a solution for the unprimed structure.

It is necessary to consider equation (51) so that $\Pi(x, y, z)$ designates a function of x, y, z obtained when one respectively replaces in $\Pi'(x', y', z')$ the coordinates x', y', z' by $K_L x$, $K_L y$, $K_L z$, which, in other words, makes the same Hertz vector correspond at two similar points of the two structures.

When one has

$$\nabla^2\Pi' = \frac{1}{K_L^2}\nabla^2\Pi; \qquad \sigma' = \frac{\sigma}{K_\rho}; \qquad \omega' = \frac{\omega}{K_T}, \tag{52}$$

the general equation

$$\nabla^2\Pi + 4\pi\sigma\omega i\Pi = 0$$

becomes

$$\nabla^2\Pi' + 4\pi\sigma'\omega' i\Pi' = 0,$$

if

$$K_L^2 = K_\rho K_T. \tag{53}$$

We shall impose this condition.

Besides, one has

$$\mathcal{H}' = \frac{\mathcal{H}}{K_\rho K_L}, \qquad \mathcal{E}' = \frac{\mathcal{E}}{K_L^2}; \tag{54}$$

so that the conditions of continuity supposed to be met in one of the structures are also met in the other one.

The ratio E'/H' of an electrical component to a magnetic component is equal to the corresponding ratio with a factor of proportionality, *which is real.* The phase separation between those components is, consequently, the same in both structures.

On the other hand, the ratio ρ_a'/ρ_a of the apparent resistivities has the value

$$\frac{\rho_a'}{\rho_a} = \left(\frac{E'}{E}\right)^2\left(\frac{H}{H'}\right)^2\frac{T'}{T} = \frac{1}{K_L^4}\cdot K_\rho^2\cdot K_L^2\cdot K_T = K_\rho, \tag{55}$$

if we take (53) into consideration. In other words, when one goes from one structure to the other, the apparent resistivities are modified in the same ratio as the real resistivities, which moreover might seem obvious enough on the basis of the principles we have considered.

To sum up the preceding, when one knows the apparent resistivity relative to a certain structure and a certain period T, one deduces at once from this *one* apparent resistivity relative to another structure deduced from the first one by geometrical similitude (ratio K_L) and by electrical similitude (ratio K_ρ). The new apparent resistivity is equal to the former one multiplied by the ratio of electrical similitude and it is relative to a period such that

$$K_T = \frac{K_L^2}{K_\rho} \cdot \tag{56}$$

CONSTRUCTION AND DESCRIPTION OF MASTER CURVES FOR TWO FORMATIONS

A magneto-telluric sounding (in order to abbreviate we shall from now on say MT sounding) will be represented by means of two curves, namely those indicating ρ_a and θ as functions of T. In the preparation of master sets of curves for the case of two formations, it is necessary to consider three arbitrary parameters, namely two resistivities and one thickness, each of which may vary from zero to infinity.

The value of the law of similitude lies in the fact that, in order to represent the whole of the MT-soundings, for two formations, it is sufficient to limit one's self to the specific case of $\rho_1 = 1$ and $h = 1$. In this way there only remains one single arbitrary parameter, namely the resistivity ρ_2 of the substratum, so that the totality of MT-soundings is represented by means of two systems of curves.

Indeed, when, in a more general way, the resistivities of the two present formations will be $\rho_1' \neq 1$ and ρ_2' and when the thickness of the first formation will be $h' \neq 1$, in order to obtain the curve $\rho_a' = \rho_a'(T')$ it will be sufficient to multiply

1. by ρ_1' the ordinates of that one of the curves $\rho_a = \rho_a(T)$ characterized by the ratio ρ_2'/ρ_1' equal in magnitude to the value of the parameter ρ_2.
2. by h'^2/ρ_1' the abscissas of this same curve.

Furthermore, in order to obtain the curve $\theta' = \theta'(T')$, it will be sufficient to multiply by the same factor h'^2/ρ_1' the abscissas of that of the curves $\theta = \theta(T)$ characterized by the value ρ_2'/ρ_1'. There will be no reason to modify the ordinates.

Rather than to carry out these multiplications, it is obviously much easier to choose for each of the systems ρ_a and θ the logarithmic abscissas representing the logarithm of $\sqrt{T}$. Furthermore, for the system ρ_a, the ordinates will represent the logarithm of ρ_a. The two sets of curves reproduced here were constructed in this way, with scales as indicated in Figures 7 and 8.

With the help of these logarithmic master curves, the expansions of the abscissas and of the ordinates described at the beginning of this section will amount from now on to a simple translation. A translation will be carried out parallel to the axis of the abscissas for curve ρ_a, as well as for curve θ, *and this translation will be of the same amplitude in both cases*. Furthermore, in the case of curve ρ_a a second translation will be carried out parallel to the axis of the ordinates.

The whole of the curves of system ρ_a, corresponding to the changing values of ρ_2, have an infinity of points in common, defined by

$$\rho_a = 1, \qquad \cos\frac{2}{p_1} = 0.$$

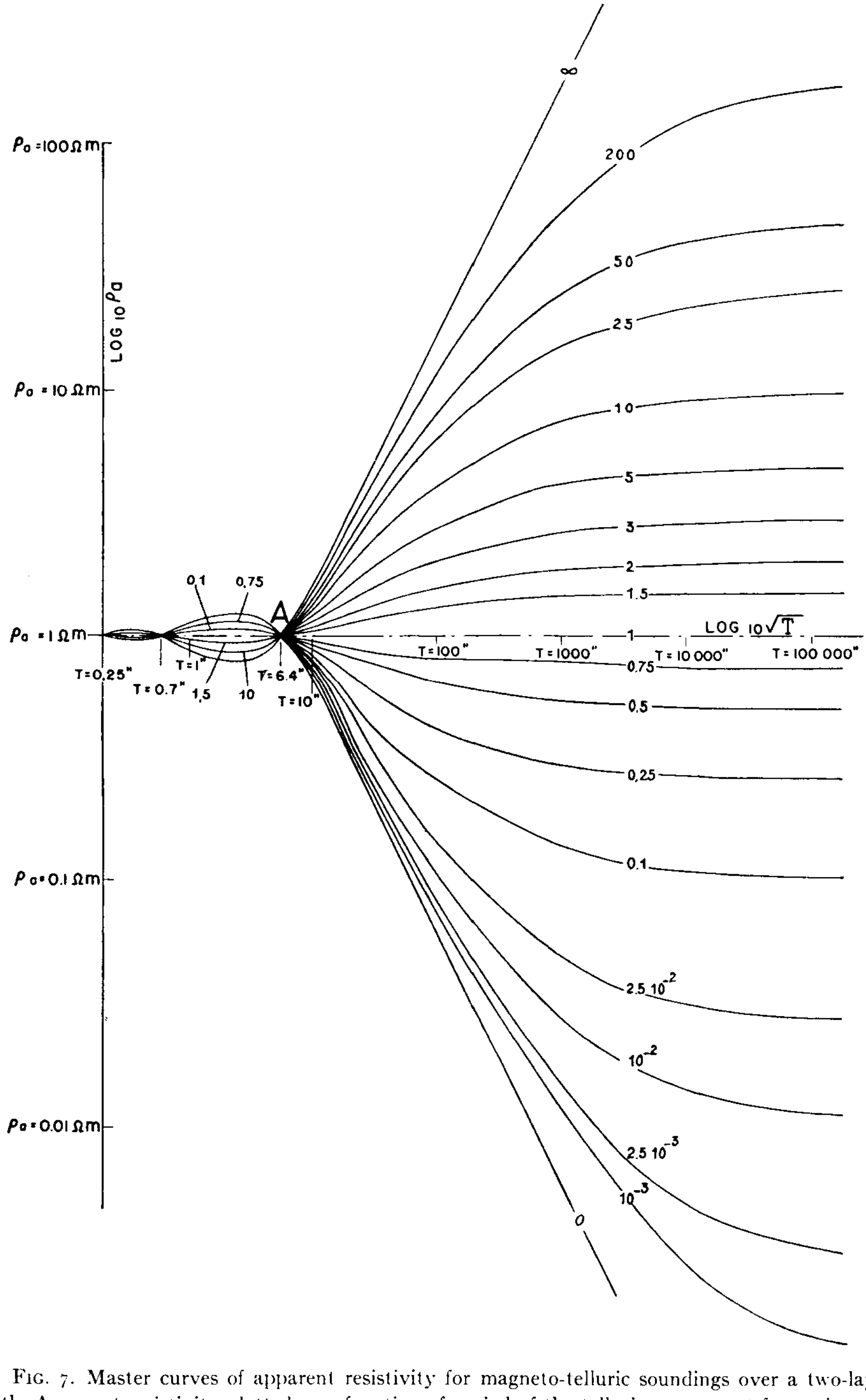

FIG. 7. Master curves of apparent resistivity for magneto-telluric soundings over a two-layer earth. Apparent resistivity plotted as a function of period of the telluric component for various resistivity contrasts. Numbers on the curves show the resistivity of the lower medium in ohm-meters. Resistivity of the upper layer is always 1 ohm-meter.

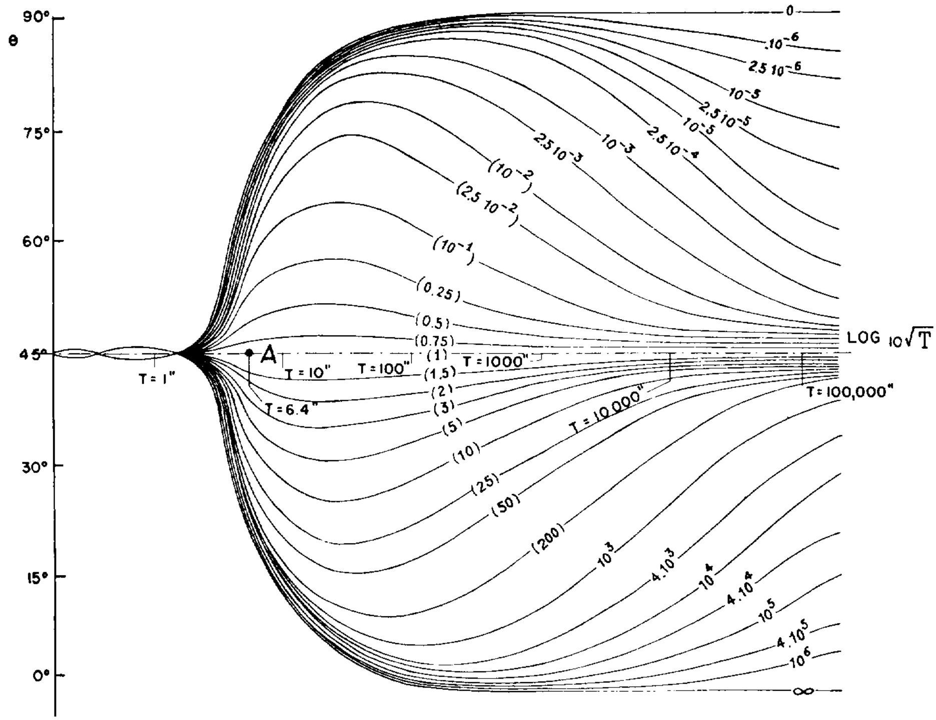

FIG. 8. Master curves of phase differences versus the period of the telluric component for various resistivity contrasts in a two-layer earth. Numbers on the curves show the resistivity of the lower medium in ohm-meters. Resistivity of the upper layer is always 1 ohm-meter.

Whereupon

$$\frac{2}{p_1} = (2n + 1)\frac{\pi}{2}; \qquad \sqrt{T} = \frac{8}{2n + 1},$$

n being an integer.

Of their common points, the one which is situated the most to the right, and which is marked A on the chart, is consequently defined by

$$\rho_a = 1, \qquad \sqrt{T} = 8.$$

The curves of system θ also have an infinite number of points in common which are defined by:

$$\theta = \frac{\pi}{4}, \qquad \sqrt{T} = \frac{4}{n},$$

n being an integer.

The coordinates of the point which is situated the farthest to the right, are consequently

$$\theta = \frac{\pi}{4}, \qquad \sqrt{T} = 4.$$

In order to make use of the master curves easier, we have marked on the set for θ the point A, having the coordinates

$$\theta = \frac{\pi}{4}, \qquad \sqrt{T} = 8,$$

which means the point having the same abscissa as point A of the curves for ρ_a.

An examination of system ρ_a shows that the apparent resistivity, equal to unity for $T=0$ approaches ρ_2 when T becomes infinite. The general configuration of system ρ_a is, consequently, the same as that of the abacus for two formations in classical electrical soundings, which we shall designate from now on as E-soundings in order to abbreviate. Let us notice that when T approaches zero, the apparent resistivity only approaches unity by indefinite oscillation on both sides of its limit. In this way, it is sometimes possible to observe apparent resistivities which are *very slightly greater* than the greatest real resistivities of the formations present, or which are, on the other hand, very slightly smaller than the smallest of those resistivities. This phenomenon, a little paradoxical, is also observed, as one knows, in E-soundings, but only starting with three formations.

The examination of system θ shows that θ is equal to $\pi/4$, as well for $T=0$ as for T infinite. This set of curves, which has no equivalent in E-soundings, is evidently going to provide one of the most useful means of control in MT-soundings.

PRACTICAL USE OF MASTER CURVES FOR TWO FORMATIONS FOR THE INTERPRETATION OF MT-SOUNDINGS

All the calculations and *theoretical* formulas developed in this memorandum imply the use of electro-magnetic units, which may be of any sort providing they are consistent: cgs for instance. We have said previously which electro-magnetic units we should use in the expression of the experimental results (Formula 10). Those units are very practical, but they are neither classical nor self-consistent.

Therefore, it is necessary to specify now that we no longer want to consider our theoretical master curves as relating to the cases of two formations with resistivities 1 and ρ_2. The resistivities in question are $1\Omega m$ and $\rho_2\Omega m$. The depth of the stratum is not 1 but 1 Km. The abscissa of point A is not 8 but $(8/\sqrt{10})$ $(\text{sec})^{1/2}$.

This being established, when we represent graphically the *experimental* results of a real MT-sounding we shall plot as our abscissas the logarithms of the square root of the period expressed in seconds. The ordinates of the curve will be logarithms of the numerical value of the apparent resistivities expressed in Ωm.

In addition to this, we shall adopt the same scales as for the theoretical curves. It is convenient to draw the experimental curves on commercial tracing paper on which cross-section lines are printed. The master curves, on the contrary, are drawn on *plain* Bristol board.

In order to know if the two experimental curves ρ_a and θ are characteristic of a subsurface involving two formations, and in order to know the thickness of the first one, or in other words to carry out an interpretation, one must try, by suitable translations, to bring the two experimental curves into coincidence, on the one hand with curve ρ_a, on the other hand with curve θ, of the theoretical set of curves.

If we are to be entitled to consider the result as satisfactory, it is necessary to insure that the two theoretical curves with which we compare the respective experimental curves correspond to the same value of the parameter ρ_2. Furthermore, the two translations which are to be executed parallel to the axis of the abscissas must be identical. From then on, we shall be able to calculate the resistivities ρ_1' and ρ_2' of the two formations at the same time as the depth h' of the second one.

Point A of family ρ_a, as seen through the transparent tracing paper on which we plot the experimental data, has itself an ordinate whose numerical value is the logarithm of $\rho_1'\Omega m$. Likewise, the asymptote of the theoretical curve ρ_a, considered as sufficient, has *on the tracing paper* an ordinate whose numerical value is the logarithm of $\rho_2'\Omega m$. In other words, the value of ρ_1' and ρ_2' can be read at once on the tracing paper if one does not care for a precision of expression which, in this case, has the inconvenience of making things which are very plain look extremely complicated.

The depth h' remains to be determined. Point A of the one or the other abacus, seen through transparent tracing paper, has an abscissa whose numerical value is $X(\sec)^{1/2}$. Conformably to the laws of similitude, one finds, consequently,

$$K_T = \frac{10}{64} X^2; \qquad K_\rho = \rho_1'.$$

Whereupon

$$K_L{}^2 = \frac{10}{64} X^2 \rho_1'; \qquad h' = \frac{X}{8} \sqrt{10\rho_1'} \text{ km}.$$

INTERPRETATION IN THE CASE OF ANY STRATIFIED EARTH.
RESOLVING POWER OF MT-SOUNDINGS

Let us now supose that one has to deal with three formations, of resistivities ρ_1, ρ_2, and ρ_3. The depth of the second one is h_1 and that of the third formation or substratum is h_2. If the ratio h_2/h_1 is sufficiently great, the influence of the sub-

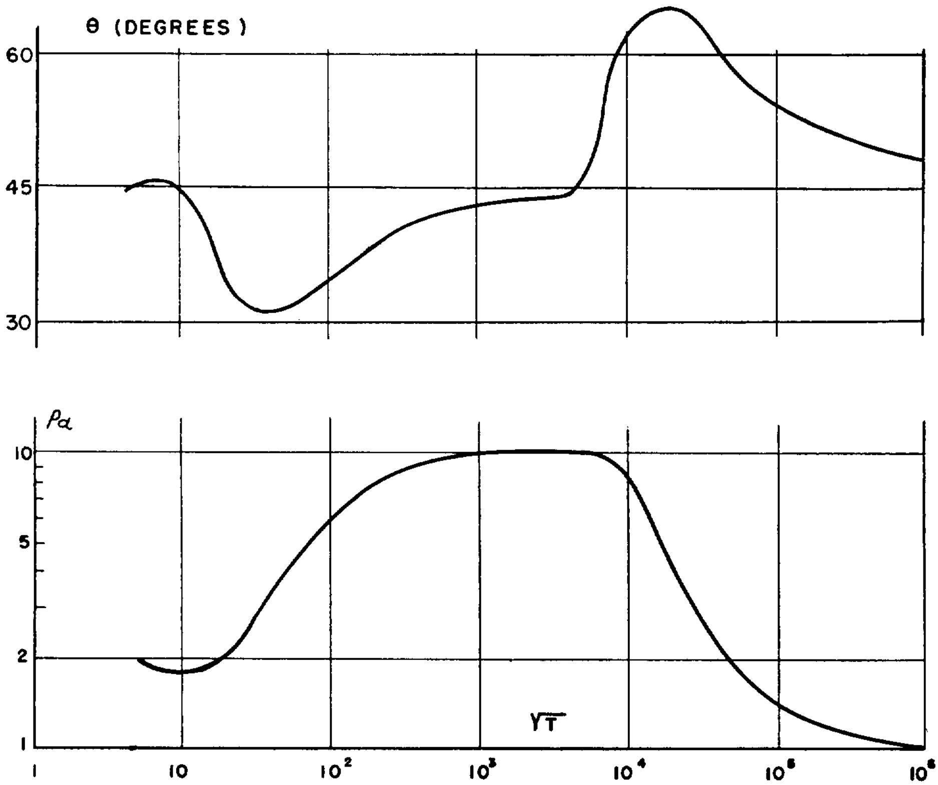

FIG. 9. Computed curves for hypothetical magneto-telluric sounding over three layers in which thickness of second layer is 900 times that of first and in which ρ_1, ρ_2, and ρ_3 are in the ratio of 2:10:1.

stratum starts to be appreciable only for such large periods that the apparent resistivity is already practically equal to ρ_2, while θ has already regained, to a close approximation, its initial value, $\pi/4$. In other words, the influence of the third formation only starts to make itself felt for such periods that the influence of the first formation may be neglected. In order to determine the termination of a graph for three formations of this kind, one is simply led to construct two graphs (ρ_a or θ) for two formations, one after the other. In the second of the graphs for two formations, the formation which is from now on to be known as the first one has the resistivity ρ_2 and the thickness h_2, while the formation from now on to be known as the second one possesses the resistivity ρ_3.

An example of this kind is furnished by Figure 9, in which the ratio h_2/h_1 is supposed to have the value of 900, while the resistivities ρ_1, ρ_2, and ρ_3 are proportional to the numbers 2, 10, and 1.

This highly favorable circumstance in which the master curves for two formations at once allow the interpretation of a sounding carried out over a section involving more than two formations does not occur if one is dealing with strata of insufficient thickness, either for E-soundings or for MT-soundings.

Let us imagine, for instance, a subsoil of three formations, such that h_2/h_1 is equal to 10, while the resistivities ρ_1, ρ_2, and ρ_3 are proportional to 9, 1, and ∞. Figure 10 represents the corresponding E-sounding, while Figure 11 represents the two curves for the MT-sounding.

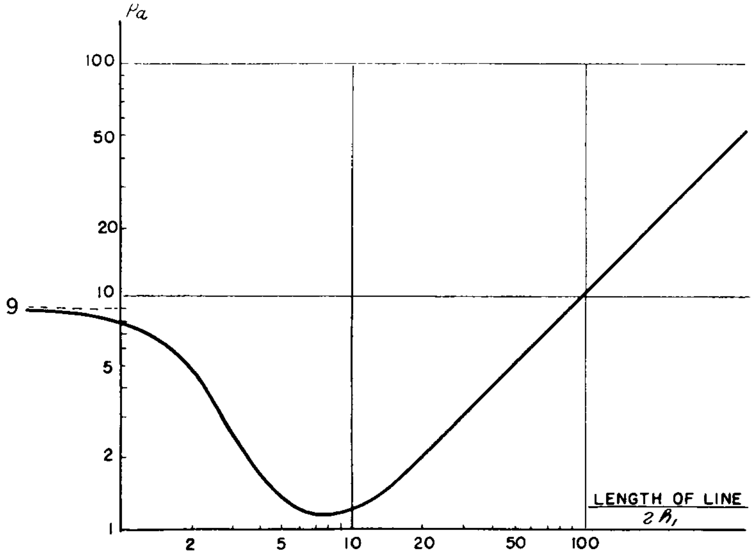

FIG. 10. Computed curve for hypothetical resistivity survey of conventional type over three layers in which thickness of second layer is 10 times that of first and in which ρ_1, ρ_2, and ρ_3 are in the ratio of 9:1: and ∞.

On each of those diagrams, the apparent resistivity, equal to 9 for the short lengths of line (E-sounding) or the small periods (MT-soundings), decreases at first when one increases the length of the line or the period, reaches a minimum, and increases indefinitely afterwards. This minimum is not equal to 1, either on the E-sounding or on the MT-sounding. One will notice however, that while it is practically equal to 1 in the case of the MT-sounding,* it is only equal to 1.25 in the case of the E-sounding. In order to obtain, in the case of the E-sounding, with the same resistivities, a minimum practically equal to 1, it would be necessary that the ratio be at least 25.

We shall conclude from this, at first, that the MT-sounding separates the individual effects of the different strata of the subsoil better than the E-sounding, and that its resolving power is almost two and a half times higher. Also bearing in mind the additional information furnished by the phase curves, it is consequently

* And even slightly less than unity, because of the somewhat paradoxical phenomenon pointed out when we described the master curve for two formations.

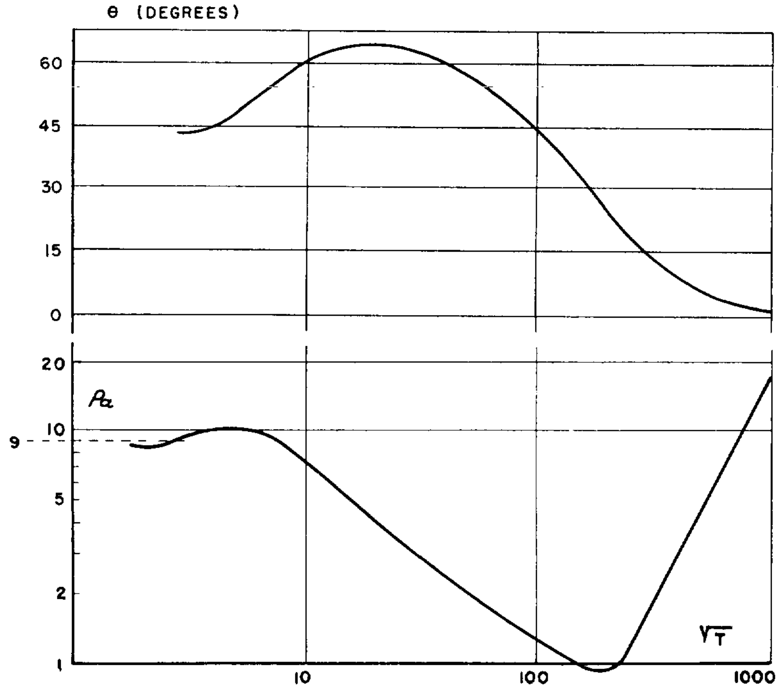

FIG. 11. Computed curves for hypothetical magneto-telluric sounding over three layer configuration of Figure 10.

already very obvious that the MT-sounding allows one to arrive at more precise conclusions than the E-sounding, even if one is satisfied with semi-qualitative information.

But still, in the case of MT-soundings, when the problem calls for it, there is nothing to keep us from submitting the semi-qualitative hypothesis we are referring to here to the test of exact calculation. When one has suspected the existence of a certain number of strata, when one has been able to estimate approximately the order of magnitude of their thicknesses and of their resistivities, one can perform the complete calculation of the results that one would obtain if the subsoil presented exactly the supposed structure. If there is disagreement between calculation and experience, one will alter the values formerly assumed for the resistivities and the thicknesses so as to obtain an entirely satisfactory result by a method of successive approximations.

In other words, the MT-sounding can be analyzed by the same method of interpretation one can apply in gravimetry and in magnetism which is so satisfactory for the prospector, but without fear of the disastrous consequences of the fundamental ambiguity which characterizes those last two methods.

Indeed, the calculation in question does not involve integrals nor series, as we have seen. It can be readily carried out when the general formulas for four, five, or more formations have been established in advance in algebraic form, as we have demonstrated here in the case of two and three formations.

However, it is possible to do much better and to save much time by use of an almost exclusively graphical method which is based on the results which will be obtained in the following paragraph.

APPARENT RESISTIVITIES AND PHASES AT THE DIFFERENT LEVELS INSIDE A HOMOGENEOUS FORMATION

Within a stratified section the complex relation E_x/H_y has a specific value at each level of depth z. We are going to obtain a formula particularly important

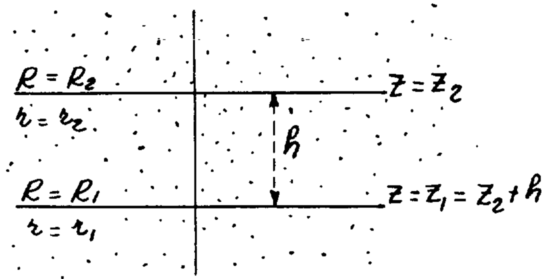

FIG. 12. Geometry for computing relationships between two levels in same medium.

in practice by considering two levels z_1 and z_2 at a distance h, inside the *same* formation of conductivity σ (Figure 12). We shall put down

$$R = -\frac{ia\sqrt{\sigma}}{\omega}\frac{E_x}{H_y} = -i\tan r. \tag{57}$$

The complex numbers R and r are functions of z which represent respectively the values R_1 and r_1 for $z=z_1$ and R_2 and r_2 for $z=z_2$.

A and B designating two constants, we have learned that the expressions for the fields (formulas 17, 18) are of the form

$$\left.\begin{aligned} E_x &= Ae^{a\sqrt{\sigma}\,z} + Be^{-a\sqrt{\sigma}\,z}, \\ H_y &= -\frac{i}{\omega}a\sqrt{\sigma}(Ae^{a\sqrt{\sigma}\,z} - Be^{-a\sqrt{\sigma}\,z}) \end{aligned}\right\}. \tag{58}$$

One deduces from these that:

$$R = \frac{1 + \lambda e^{-2a\sqrt{\sigma}\,z}}{1 - \lambda e^{-2a\sqrt{\sigma}\,z}}; \qquad \lambda = \frac{B}{A} = C^r, \tag{59}$$

or in another form

$$\frac{R-1}{R+1}\,e^{2a\sqrt{\sigma}\,z} = \lambda = C^r. \tag{60}$$

Consequently, one calculates at once R_2 as a function of R_1:

$$R_2 = \frac{R_1(1+e^{2a\sqrt{\sigma}\,h}) + (1-e^{2a\sqrt{\sigma}\,h})}{R_1(1-e^{2a\sqrt{\sigma}\,h}) + (1+e^{2a\sqrt{\sigma}\,h})}, \tag{61}$$

and, afterwards, r_2 as a function of r_1:

$$\tan r_2 = \frac{\tan r_1 - \tan(ia\sqrt{\sigma}\,h)}{1+\tan r_1 \tan(ia\sqrt{\sigma}\,h)} = \tan(r_1 - ia\sqrt{\sigma}\,h), \tag{62}$$

from which finally

$$r_2 = r_1 - ia\sqrt{\sigma}\,h = r_1 + \sqrt{2}\,(h/p)e^{-3i\pi/4}. \tag{63}$$

The indeterminacy of the argument r does not concern us, since we are only interested in the value of R.

It is easy now to go back to the apparent resistivities ρ_a and to the phases θ defined by

$$\frac{E_x}{H_y} = \sqrt{\frac{\rho_a}{2T}}\,e^{-i\theta}. \tag{64}$$

Consequently one has

$$\sqrt{\frac{\rho_a}{\rho}}\,e^{-i(\theta+\pi/4)} = \tan r, \tag{65}$$

and finally

$$\frac{(\rho_a)_2}{(\rho_a)_1}\,e^{-2i(\theta_2-\theta_1)} = \left(\frac{\tan r_2}{\tan r_1}\right)^2. \tag{66}$$

Since the calculations of the prospectors are not usually carried out to 20 decimal places, a simple chart of the complex values of the tangents of a complex argument allows one to calculate an MT-sounding very quickly for $n+1$ formations starting from a sounding for n formations when the $(n+1)$st formation is situated on top of the nth one.

REMARKS

1. In the calculation of a theoretical MT-sounding by an operation of successive approximations, the geophysicist, by constructing his theoretical section through the stacking of strata laid down one on top of the other, proceeds exactly in the same way as nature did when the real strata of the ground were laid down by successive processes of sedimentation.

2. At two stations over a sedimentary basin, the section only differs, in principle and as a first approximation, through the addition—or through the subtraction—of a certain number of superficial

strata. Consequently, it will often be convenient as a first working hypothesis to calculate the complex ratio of the complex quotients E_x/H_y, obtained experimentally at the two stations, a ratio whose interpretation involves only the thickness of the superficial layer, which is different for the two stations.

3. When, for one reason or another, one knows with certainty the resistivities of the ground to a certain depth, it may be easy to omit, through calculation, the influence of this known part of the ground and limit the interpretation only to the unknown subjacent portion.

4. This circumstance occurs in particular when one performs a MT-sounding over a body of water for which the depth and conductivity are known. The former calculation allows one in such a case to correct the MT-sounding for the influence of the sea; in other words, it allows one to obtain, through a very accurate calculation, the diagrams for the MT-sounding that one could have determined experimentally if the water were to have been drained away.

VARIATIONS APPLICABLE TO MT-SOUNDINGS PERFORMED UNDER THE SEA

The measurement of the electrical field at sea does not present any specific technical difficulty. The line of measurement is maintained on the surface of the water through the use of floaters, in the same way as fishermen do with their nets. Moreover, there is no difficulty whatsoever in carrying out a correct galvanometric recording on board of a ship tossed about by the waves. One need only be suspicious if one observes phenomena which have a period the same either as that of the marine currents or as that of the swell, since the electrical currents induced by motion of the conducting water in the magnetic field of the earth do not meet the requirements of the theory we have set forth.

The measurement of the magnetic field offers more serious technical difficulties if one is not willing or able to install a self-recording magnetometer on a series of piles forming a foundation or in an immersed box on the bottom of the sea.

One way of avoiding the difficulty consists in registering the magnetic field *on the ground* and the electric field *in the sea*. The daily experience of prospectors who use the telluric method, has shown, indeed, that in sedimentary beds, the line of the telluric current keeps an almost constant direction over expanses as large as 20–70 km. Besides, this direction would be strictly uniform in a precisely stratified earth.

Now the telluric current, even if it is a variable current, is, approximately conservative because of its very low frequency. As I have already pointed out, the magnetic field is very approximately the same at two stations not too distant from each other on the same straight line ox, since it represents, except for a factor 4π, the total intensity of the telluric current through a stratum of unit width starting from the surface normal to ox.

The argument essentially implies that the two stations are situated on the same straight line ox, perpendicular to the magnetic component one is considering. Consequently, it is very advisable to adhere to this condition if possible. However, experience shows that, in practice, this requirement is not always strictly binding.

These observations will not come as a surprise to observatory geophysicists. Through experience, they are well convinced of the fact that the meaning of their magnetic data does not depend particularly on the electrical resistivity of the geological strata in the vicinity of their observatory. But, for over a hundred years, since the first observation of telluric current, it has been found that in telluric registrations, on the contrary, one has had to be greatly concerned with the local geologic structure.

Another way to avoid the difficulty consists in observing that in the homogeneous medium formed by the sea water, where a telluric sheet is flowing parallel to ox, the relation between *any* two electro-magnetic dimensions, depending linearly on the Hertz vector, is expressed as a function of parameter λ only (Equation 59). The ratio E_x/H_y is also expressed as a function of λ. In other

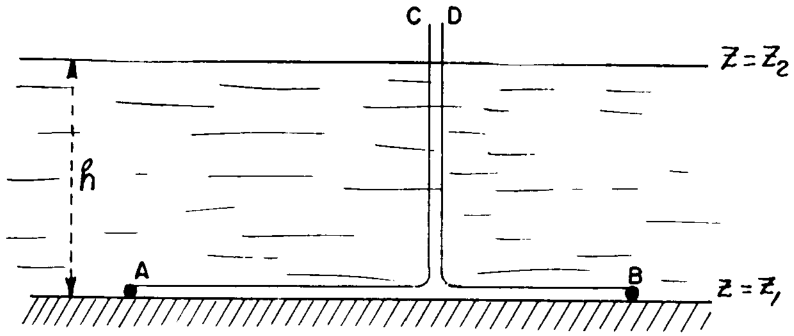

FIG. 13. Configuration of electrodes on water bottom for submarine MT measurements.

words, the study of the relation of any two electro-magnetic quantities is absolutely equivalent to that of the ratio E_x/H_y.

One can, for instance, substitute for the measurement of Hy, the measurement of the electromotive force induced in a large vertical ring parallel to ox, this ring being constructed much more easily on the sea than on the ground. Yet one knows that, if the vertical height of this ring is small so that the magnetic component H_y inside it is almost uniform, the measurement of the induced electromotive force is a classical way of measuring H_y.

It may be easier to substitute for the measurement of the magnetic field H_y that of a second electric field. Let us go back to Figure 11, supposing this time that level z_1 represents the horizontal sea bottom, level z_2 the surface of the water (or, in a more general way, any level between the bottom and the surface of the sea). It is easy to measure the field E_x on the bottom of the sea by means of two immersed electrodes A and B connected with recording equipment on the boat by the two lines AC and BD (Fig. 13).

It can be shown that

$$\sqrt{\rho_a}\, e^{-i\theta} = \frac{2\pi p}{\sqrt{T}} e^{-i\pi/4} \frac{\dfrac{(E_x)_1}{(E_x)_2} \sinh\left(\sqrt{2}\, e^{-i\pi/4} \dfrac{h}{p}\right)}{1 - \dfrac{(E_x)_1}{(E_x)_2} \cosh\left(\sqrt{2}\, e^{-i\pi/4} \dfrac{h}{p}\right)}, \tag{67}$$

in which ρ_a and θ have reference to the apparent resistivity and to the phase relative to level z_1, the level of the bottom of the sea; that is to say, the parameters of an MT-sounding that could be performed on the sea bottom if drained.

CONCLUSION. FIELDS OF PRACTICAL APPLICATION FOR THE MAGNETO-TELLURIC METHOD

It follows from the above that the ideal way to apply the magneto-telluric method consists in performing an MT-sounding as described. When the subsoil is approximatively tabular, the harmonic analysis of the telluric and magnetic diagrams makes it possible to conduct a careful quantitative interpretation which gives us the thickness and the resistivity of the various strata.

The periods higher than one second are exactly adapted to the study of large sedimentary beds and their petroliferous structure. Besides, their recording does not involve serious technical difficulties.

The study of the shortest periods, less than one second, seems technically difficult at the present stage of the art. However, it is less urgent in the light of present needs in geophysical prospecting. It should eventually allow us to adapt the magneto-telluric method to various applications requiring detail of the kind involved in civil engineering studies, in mineral prospecting, and in the search for underground water.

We want to draw attention to the fact that an isolated MT-sounding carried on in the center of a large unknown area can present information similar to that given by a wildcat well in a large scale reconnaissance. For instance, the measurement of the number of kilometers thickness of sediment in the center of a large basin presents a problem which cannot be solved even partially by any geophysical method up until now. The magneto-telluric method should be able to solve the problem by use of only a single station.

The discarding of the base station, which is indispensable in the telluric method, gives the operator more freedom of movement and improves the organization of his survey. He is no longer compelled to proceed slowly. He can afford to operate in a more rational way by setting up his initial stations at some distance from each other. Later on, he can locate stations with a closer spacing, but only to the extent required by continuity.

Consequently, one can lay out the survey of a large sedimentary basin by performing at the start a small number of MT-soundings far removed from one another, but with a great depth of investigation. In the second step, one will intercalate stations closer together, and at these he will perform MT-soundings with a more moderate depth of penetration. Finally, the continuity between the stations will be assured either by soundings with a relatively small depth of investigation, or, once in a while, by simple, quick determinations of the apparent resistivity summarily evaluated through a very simplified analysis of the magneto-telluric data. It is unnecessary to add that the magneto-telluric method will be particularly appreciated every time that a deep petroliferous structure appears in complete disharmony with the structure on the surface. In this case it is essential for the prospector to penetrate to a great depth of investigation.

If it should happen that the earth is not even approximately stratified, the quantitative interpretation of the MT-soundings must be just about ruled out. A knowledge of the apparent resistivities and of their variation as a function of the period and of the direction of the line provides, nevertheless, certain indications which can be diagnostic in special cases, even though they are to a large degree qualitative.

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The theoretical work reported in this paper was done some time ago and has been mentioned in applications for patents which have been made in several countries to protect the new prospecting method involved. Because of the potential practical applications I have had to postpone any publication related to magneto-telluric phenomena for many years.

Meanwhile, the Russian scientist Tikhonov, and the Japanese scientists Kato, Kikuchi and Rikitake had also recognized the existence of such an effect. To my knowledge, they have not pointed out the possibility disclosed by my work of applying these results to practical geophysical exploration. They have, however, paid attention to their possible use for investigating the electrical conductivities of very deep regions in the Earth's crust.

It is therefore a real pleasure for me to give these scientists proper credit and to list the papers that, to my knowledge, they have published on this subject:

Kato, Y. and Kikuchi, T. (1950) *Sci. Rep. Tôhoku Univ. Ser. V, Geophysics*, Vol. 2, p. 139.

Rikitake, T. (1950) *Bull. Earthquake Res., Inst. Tokyo Univ.*, Vol. 28, p. 45, 219. Also (1951) Vol. 29, pp. 61, 271.

Tikhonov, A. N. (1950) *Dokl. Akad. Nauk S.S.S.R.* 73, 295.