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The Perturbations of Alternating Geomagnetic Fields by Conductivity Anomalies

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Summary

The two-dimensional problems of interest in studying the perturbation of alternating electric current by a sharp discontinuity of conductivity in a conductor are considered, and their applicability to geophysical problems discussed. A numerical method has been developed for solving the appropriate differential equations and boundary conditions. The method has been applied to a vertical discontinuity in conductivity such as at a continental-oceanic interface. The two polarization cases are solved, and the fields and current distributions are determined in detail.

1. Introduction

Many observational studies have been made in recent years of the effects of a vertical discontinuity in the electrical conductivity of surface layers of the Earth on geomagnetic variations having periods ranging from those of micropulsations to the daily variations. The effects have been found mainly in the vertical component and have been particularly noticeable at coastal stations, though they occur elsewhere and have then been termed 'conductivity anomalies'. Several mathematical problems in electromagnetic induction have been devised and discussed in attempts to elucidate these effects and to use them to derive information about subterranean conductivity distributions (Truemann 1968).

It has, however, been pointed out by one of us (Price 1964) that the kind of problem we need to consider in this connection is not strictly a problem of evaluating the currents induced by a given varying magnetic field in a given heterogeneous conductor, but rather that of determining the local perturbations of a given alternating system of induced currents by given abrupt changes of conductivity. It is natural to examine first whether there are any two-dimensional problems of this kind that can be usefully discussed. The simplest model conductor that one can take is a semi-infinite conductor occupying $z > 0$, with a vertical plane $y = 0$ of discontinuity, as in Fig. 1. For a two-dimensional problem the field must be independent of x. This model has been previously used by Weaver (1963), but he approached the problem in a different way, and we think that some of his assumptions (for the E polarization case) need reconsidering. We have therefore examined the problem again, and have also developed a new method for determining in detail the magnetic field and the distribution of currents throughout the composite conductor. The results help considerably to elucidate the geomagnetic effects. The method involves considerable use of a computer, and has been developed and applied by one of us (F.W.J.) for several more complex two-dimensional cases. It is believed that the results for these cases will be of considerable value in magnetic and magnetotelluric sounding, and it is intended to publish them in subsequent papers.

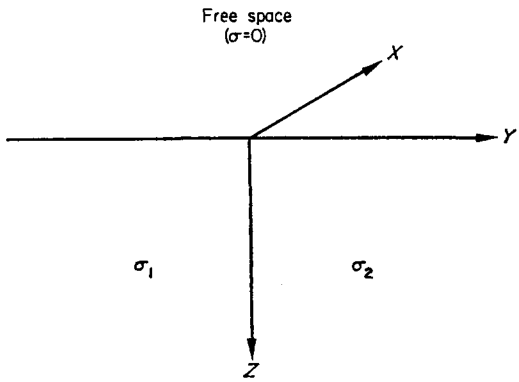

FIG. 1. Conductor model and co-ordinates.

2. Differential equations

The problem is basically that of solving Maxwell's equations in the three regions with suitable boundary conditions, the field being an oscillating one, with period $2\pi/\omega$ sufficiently long to permit displacement currents being ignored. Also, the magnetic permeability is taken as unity. The equations, in electromagnetic units, are therefore

$$\operatorname{curl} \mathbf{H} = 4\pi\sigma\mathbf{E}, \qquad \operatorname{curl} \mathbf{E} = -i\omega\mathbf{H}, \tag{1}$$

where the time factor $\exp(i\omega t)$ is understood in all field quantities, and σ is the conductivity appropriate to each region.

Since all quantities are independent of x, the above equations reduce to

$$\frac{\delta H_z}{\delta y} - \frac{\delta H_y}{\delta z} = 4\pi\sigma E_x, \tag{2a}$$

$$\frac{\delta H_x}{\delta z} = 4\pi\sigma E_y, \tag{2b}$$

$$-\frac{\delta H_x}{\delta y} = 4\pi\sigma E_z, \tag{2c}$$

$$\frac{\delta E_z}{\delta y} - \frac{\delta E_y}{\delta z} = -i\omega H_x, \tag{3a}$$

$$\frac{\delta E_x}{\delta z} = -i\omega H_y, \tag{3b}$$

$$-\frac{\delta E_x}{\delta y} = -i\omega H_z. \tag{3c}$$

These equations are such that only E_x, H_y and H_z are involved in (2a, 3b, 3c) and only H_x, E_y and E_z in (2b, 2c, 3a). Hence we can solve these two separate sets of equations independently. The first set corresponds to E-polarization and the second to H-polarization. In the first, by eliminating H_y and H_z, we get

$$\frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} = i\eta^2 E_x, \tag{4}$$

and in the second, by eliminating E_y and E_z, we have

$$\frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} = i\eta^2 H_x, \tag{5}$$

where

$$\eta^2 = 4\pi\sigma\omega. \tag{6}$$

Further, the equations (2b, 2c) show that the lines of constant H_x are the lines of force of **E**-field, and the equations (3b, 3c) show that the lines of constant E_x are the lines of force of **H**-field.

3. Boundary conditions

At the interface ($y = 0$) between the conductivities and also at the surface $z = 0$, we have the boundary conditions that (i) all components of **H** are continuous, and (ii) the tangential components of **E** are continuous. Further, the normal component of current density must be continuous across $y = 0$ and zero across $z = 0$. This last condition implies that E_z *inside* the conductor is zero at $z = 0$. There are also conditions to be satisfied at $z = \pm\infty$ and $y = \pm\infty$. It is most convenient to consider these conditions separately for the **H** and **E** polarizations.

4. H polarization

For the **H** polarization case, the equations (2b, 2c) show that in the region outside the conductor, where $\sigma = 0$, H_x is independent of y and z. Hence **H** is uniform throughout this region, and the magnetic field immediately above the surface of the conductor is not affected at all by the abrupt change of conductivity of the conductor at $y = 0$. This somewhat surprising result is due to the assumptions inherent in the strictly two-dimensional character of the problem, but it will be noted that it is in accordance with the requirement that the total current flow across all vertical planes parallel to the plane of discontinuity is the same. For the total current flow is given by

$$\int_0^\infty \sigma E_y \, dz = \frac{1}{4\pi} \int_0^\infty \frac{\delta H_x}{\delta z} dz = \frac{1}{4\pi} [H_x]_{z=0}$$

on using equation (2b).

The explanation of this result is that the normal component of the current flow sets up a varying surface charge on the plane of discontinuity $y = 0$. The electric field of this surface charge reduces the current flow in the conductor of higher conductivity and increases that in the conductor of lower conductivity, so that the *normal component* of flow is equalized on the two sides. It must be emphasized that this equalization of the normal component of flow is brought about by the *electric field* of the surface charge. The varying current flow required to build up this surface charge has the same order of magnitude as the displacement currents, and we have already seen that displacement currents are negligible for the quasi-steady fields we are considering. More precisely, the magnetic field of the current associated with building up the surface charge is negligible, but the electric field of this surface charge is of the same order of magnitude as that of the other electromotive forces involved.

We now consider the various boundary conditions noted in Section 3. One of these is that **H** is continuous across any boundary. This, together with the result above, implies that H_x is constant, say H_0, everywhere just inside the surface, $z = 0$, of the conductor. It also implies that $\delta H_x/\delta y$ is zero just inside this surface, and hence, from equation (2c), that σE_z is zero. This, of course, is in agreement with the requirement that the normal component of current flow must be zero at the surface. Again the continuity of H_x across the boundary $y = 0$ implies that $\delta H_x/\delta z$ is continuous across this boundary and hence, from equation (2b), that σE_y is continuous, agreeing with the result quoted in the preceding paragraph. Further, the continuity of the tangential component E_z at the boundary $y = 0$ can be ensured by making $\delta H_x/\delta y$ continuous in virtue of equation (2c).

At large distances from the discontinuity in σ we may assume that the field behaves like that for a uniform conductor. Hence as $y \to +\infty$ or $-\infty$, the equation (5) becomes

$$\frac{\delta^2 H_x}{\delta z^2} = i\eta^2 H_x \tag{7}$$

with the appropriate value of σ inserted in η^2.

Also the field tends to zero for large positive values of z. Hence the appropriate solution of (7) is

$$H_x = H_0 \exp\left\{-\frac{1}{\sqrt{2}}(1+i)\eta z\right\}. \tag{8}$$

The actual field quantity H_x is, of course, the real part of the product of the expression (8) with $\exp(i\omega t)$, i.e.

$$H_x = H_0 \exp\left(-\frac{1}{\sqrt{2}}\eta z\right) \cos\left(\frac{1}{\sqrt{2}}\eta z - \omega t\right). \tag{9}$$

It follows that, if we solve the equation (5) for H_x with the appropriate values for η in the two parts of the conductor and with the above boundary conditions on H_x, then the required conditions on $\mathbf{E}$ in the conductor will be automatically satisfied. The differential equations and boundary conditions for H_x are shown diagrammatically in Fig. 2. (For clearness the suffix x in H_x is omitted in the figure.)

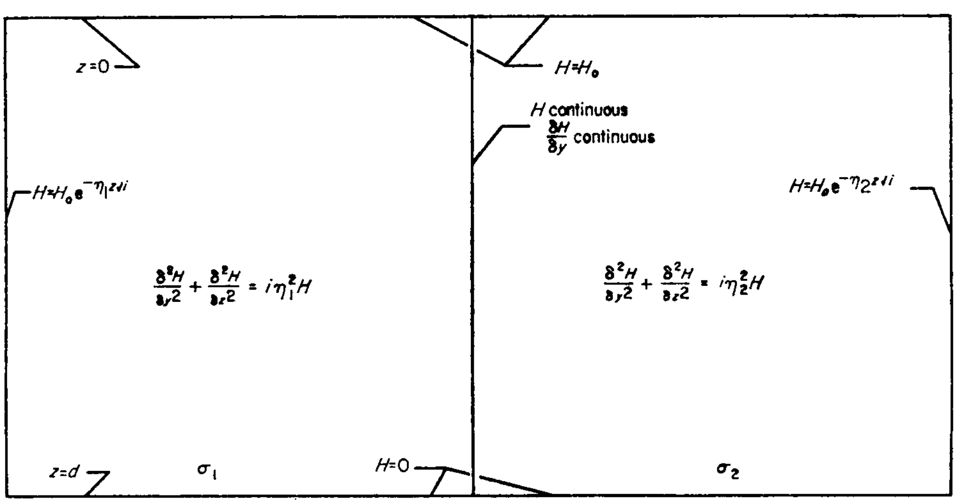

FIG. 2. H polarization. Equations and boundary conditions for H_x.

5. E polarization

In the case of E polarization, the field components E_x, H_y and H_z only are involved, and E_x satisfies equation (4), with the appropriate value of η for each region. Arguments, similar to those used in the last section, now show that, for large positive or negative values of y, E_x within the conductor is of the form

$$E_x = E_0 \exp\left\{-\frac{1}{\sqrt{2}}(1+i)\eta z\right\}, \tag{10}$$

where η depends on σ, and E_0 may have different values at $y = +\infty$ and $y = -\infty$.

We also have the boundary condition that H_z becomes zero for large values of y, and it will be seen from equation (3c) that this condition is satisfied in virtue of the above condition on E_x.

Further, from equations (3b) and (10) we have

$$i\omega H_y = \eta \frac{1}{\sqrt{2}}(1+i)\, E_0 \exp\left\{-\frac{1}{\sqrt{2}}(1+i)\eta z\right\}. \tag{11}$$

In the non-conducting region ($z < 0$), we see from equation (2a) that $\delta H_y/\delta z$ tends to zero for large positive or negative values of y. Hence, for these values of y, H_y must be a constant (H_0 say) with respect to z in $z < 0$ and, since H_y must be continuous at $z = 0$, we have

$$H_0 = \frac{1}{\sqrt{2}}(1-i)\frac{\eta}{\omega} E_0. \tag{12}$$

Hence, from equation (3b) we get, for large positive or negative y in the region $z < 0$,

$$\frac{\delta E_x}{\delta z} = -i\omega H_0 = -\frac{1}{\sqrt{2}}(1+i)\eta E_0 \tag{13}$$

and therefore

$$E_x = E_0\left\{1-\frac{1}{\sqrt{2}}(1+i)\eta z\right\} \tag{14}$$

in $z < 0$ ($|y|$ large), since E_x is continuous across $z = 0$.

Within the conductor, each of the field components vanishes as $z \to \infty$. Equations (3b) and (3c) show that if E_x vanishes at this limit, H_y and H_z also vanish. Other boundary conditions are that E_x, H_y and H_z are all continuous across the boundaries $y = 0$ and $z = 0$. Equations (3b) and (3c) again show that these conditions may all be expressed in terms of conditions on E_x and its normal derivative at each boundary.

The values of E_0 at $y = +\infty$ and $y = -\infty$, and the conditions at a suitable boundary, $z = -h_0$ say, in the non-conducting region $z < 0$, remain to be considered. We recall that in the case of the H-polarization, H was found to be uniform in the region $z < 0$, giving a simple boundary condition along $z = 0$ for the determination of the field within the conductor. But though the roles of H and E are in some ways interchanged when considering the E-polarization, there is no correspondingly simple boundary condition for E_x on $z = 0$. The E-field in the region $z < 0$ must now be taken into consideration, though in the previous case it could be ignored when solving for H and E within the conductor. Nevertheless, there is an important feature common to both cases, namely $|E|$ for $|y|$ large tends to infinity as $z \to -\infty$.

Thus, in the H-polarization case it is easily found from equation (3a) that for large $|y|$

$$E_y \to E_0 + i\omega H_0 z, \tag{15}$$

and for the E-polarization case, from equation (13)

$$E_x \to E_0 - i\omega H_0 z. \tag{16}$$

These results are in accordance with the requirement that there must exist an energy source, on or beyond the plane $z = -h_0$, which maintains the oscillating currents in the conductor via an inducing field in the region $0 > z > -h_0$.

In the E-polarization problem we assume temporarily a condition on the current flow somewhat similar to that found to hold for the H-polarization, namely, that the total (integrated) flow (now in the x direction) is the same for all values of y, even though the values of σ are different for positive and negative values of y. We then have on integrating equation (2a)

$$\int_0^\infty \frac{\delta H_z}{\delta y}\, dz + (H_y)_{z=0} = 4\pi \int_0^\infty \sigma E_x\, dz \tag{17}$$

which we assume is independent of y.

We may note, incidentally, that this assumption of constant total current flow for all y is not consistent with Weaver's assumption of constant H_y along $z = 0$, because the integral of $\delta H_z/\delta y$ cannot always be zero near $y = 0$. However, at large positive or negative values of y, H_z can be taken ultimately as zero, so that the above equation leads to

$$H_{01} = H_{02}, \tag{18}$$

where H_{01} is the limiting surface value of H_y as $y \to -\infty$, and H_{02} the value as $y \to \infty$.

We now examine more closely the assumption that the total current flow is the same for large negative y (conductivity σ_1) as for large positive y (conductivity σ_2). It should first be noted that H_y, H_{01} and H_{02}, appearing in the above equations represent the *total* magnetic field at the surface, i.e. the field of the oscillating currents *plus* the field of the extraneous source inducing those currents. It has been pointed out by Price (1950) that, for a uniform half-space conductor, it is not possible to separate out these two parts, unless the problem is more completely specified. In other words, the two-dimensional problem is not completely determinate unless it is regarded as the limit of a three-dimensional problem, relating to a conductor of given shape and an inducing field of given distribution. Taking the half-space conductor as the limit of a spherical conductor as the radius tends to infinity, and the inducing field as corresponding to a spherical harmonic of degree n, Price showed that the tangential components of the induced and inducing fields are ultimately in the ratio $n/(n+1)$, independent of the conductivity. But *any* surface harmonic of finite degree n will lead to a field that is sensibly uniform over a limited region of a large spherical surface. Hence for the half-space conductor the proportionate contribution to the total horizontal field H_y from the induced currents will lie between $\frac{1}{3}$ and $\frac{1}{2}$, depending on the chosen value of n. Moreover, by taking conductors of other shapes it can be shown that any value between 0 and 1 can be found for this contribution.

However, for our problem, the important result is that the ratio found above is ultimately *independent of the conductivity*, when the radius tends to infinity. Since we assume that the inducing field is of the same intensity and form over the entire composite conductor, we may deduce that the total surface H_y is the same at the extreme values of y, i.e. the equation (18) is a correct boundary condition for our problem. This condition relates to H_y but it is simpler to obtain the solution of the problem in terms of E_x first. Hence we now consider what this condition on H_y implies with regard to E_x.

Using (12) and (18) we find

$$\frac{\eta_1}{\omega} E_{01} = \frac{\eta_2}{\omega} E_{02} \tag{19}$$

and from (16), E_x in the region $z < 0$ with $|y|$ large is given by

$$E_x = E_{01} - i\omega H_0 z \quad \text{with} \quad y \to -\infty \tag{20}$$

$$= E_{02} - i\omega H_0 z \quad \text{with} \quad y \to +\infty. \tag{21}$$

At a sufficiently large (negative) value of z we may assume that the magnetic field of the perturbation due to the discontinuous conductivity along $y = 0$ will tend to zero, so that the total magnetic field will tend to $H_y = H_0$, $H_z = 0$. Equation (2a) would then be automatically satisfied as $z \to -\infty$, equation (3b) would require (20) and (21) to be satisfied for large negative z, and equation (3c) would require

$$\frac{\delta E_x}{\delta y} \to 0 \quad \text{as} \quad z \to -\infty. \tag{22}$$

The last equation is not inconsistent with (20) and (21) although E_{01} differs from E_{02}. This is because

$$\frac{|E_{01} - E_{02}|}{|y_1 - y_2|} \to 0 \quad \text{as} \quad |y_1 - y_2| \to \infty. \tag{23}$$

However, in our numerical method of solution, we have to approximate to the conditions at infinity by taking suitable corresponding conditions at finite boundaries, $z = -h_0$ and $y = \pm k$ say. It is not practicable to take the values of h_0 and k sufficiently large to regard (23) as satisfied when $|y_1 - y_2| = 2k$. It follows that we cannot assume that (22) is satisfied along $z = -h_0$ because this would imply $E_{01} = E_{02}$ which is inconsistent with (19), and it is important that equation (19) should be satisfied in order to satisfy the conditions on the current distribution within the conductor. Since, however, the horizontal component H_y has the same value H_0 at the two extreme values of y for all negative values of z, it is permissible to take H_y constant and equal to H_0 all along the boundary $z = -h_0$, provided this boundary is far enough away to make the *local perturbation* in H negligible there. It should perhaps be emphasized that the abrupt change in continuity at $y = 0$ has two distinct effects. One is the local perturbation in the electromagnetic field near $y = 0$, which will decrease with increasing negative z because it is due to a local concentration of current. The other is the effect on the current distribution and field which extends to infinity in the positive and negative y directions. Insofar as the field in the non-conducting region is concerned, we have shown that there is ultimately no change in the H-field at the extreme values of y, butthere is an important change in the E-field, represented by equations (19), (20) and (21).

We therefore take

$$H_y = H_0 \quad \text{in the region of} \quad z = -h_0. \tag{24}$$

We then deduce from (2a) that

$$H_z = K \text{ (a constant)} \quad \text{along} \quad z = -h_0. \tag{25}$$

Equation (3c) then gives on integrating

$$E_x = i\omega K y + C. \tag{26}$$

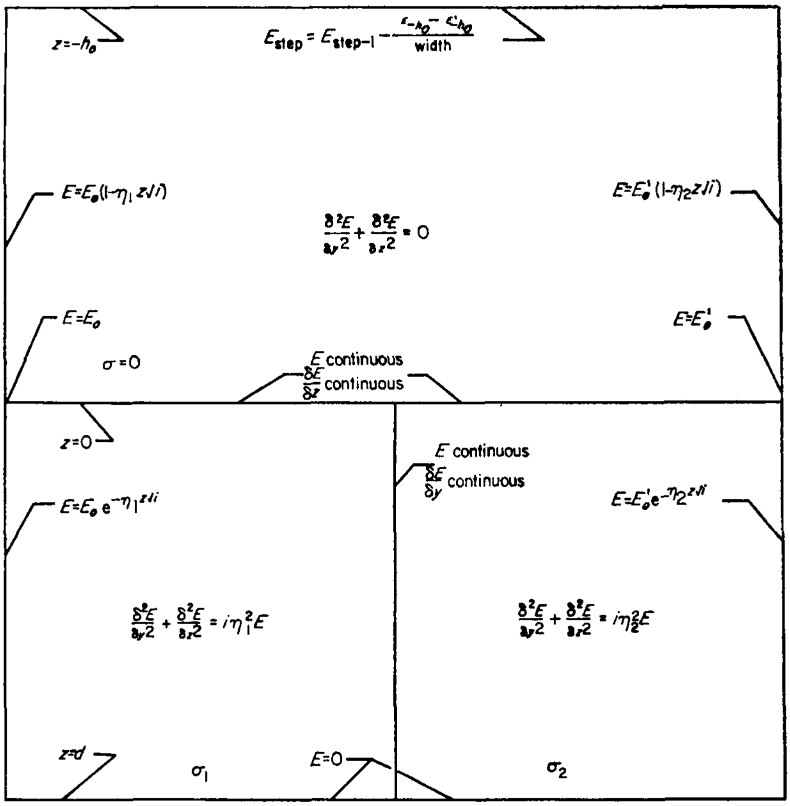

FIG. 3. E polarization. Equations and boundary conditions for E_x.

On inserting E_{01} and E_{02} for E_x at $y = -k$ and $y = k$ respectively, we get

$$E_x = \frac{1}{2k}(E_{02}-E_{01})\,y+\tfrac{1}{2}(E_{02}+E_{01}) \quad \text{on} \quad z = -h_0. \tag{27}$$

We have now obtained all the boundary conditions in terms of E_x. These boundary conditions, and the differential equations for E_x in the different regions, are summarized in Fig. 3.

6. Numerical formulation of the problem

The equations to be solved in all regions for both cases are of the form

$$\nabla^2 F = i\eta^2 F, \tag{28}$$

with the appropriate value of η inserted.
If we let $F = f+ig$, then

$$\nabla^2 F = \nabla^2(f+ig) = \nabla^2 f+i\nabla^2 g$$

and

$$i\eta^2 F = i\eta^2(f+ig) = i\eta^2 f-\eta^2 g.$$

Equating real and imaginary parts, we obtain

$$\nabla^2 f = -\eta^2 g \tag{29}$$

and

$$\nabla^2 g = \eta^2 f. \tag{30}$$

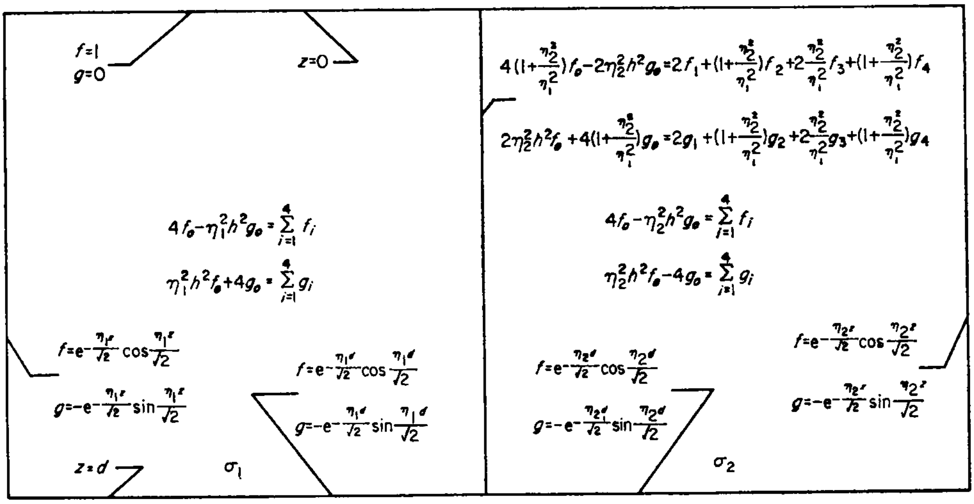

FIG. 4. Finite difference equations for H polarization problem.

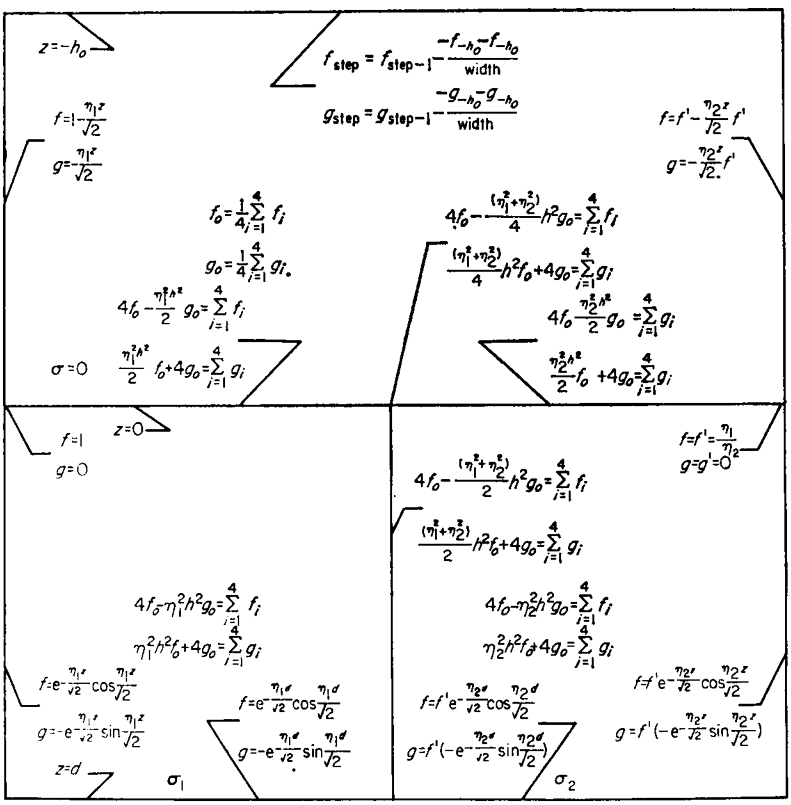

FIG. 5. Finite difference equations for E polarization problem.

These two equations are replaced by corresponding finite difference equations, which are then solved simultaneously for each point on a rectangular mesh by finite difference methods, which also take into account the boundary conditions already considered and now expressed in finite difference form. Figs 4 and 5 give the finite difference equations for the two polarizations in the interior regions and on the boundaries (where h is the mesh size.) The equations were solved by the Gauss–Seidel iterative method.

7. Numerical solution for H-polarization case

The numerical solution has been obtained for a composite conductor in which

$$\sigma_1 = 10\sigma_2 = 10^{-13}\,\text{e.m.u.},$$

and the period of oscillation $(2\pi/\omega)$ is 1 second. Since ω always appears in the equations in combination with σ_1 or σ_2 as a product $\sigma\omega$, it follows that the same solution will apply if both conductivities and the period are all increased in the same ratio. For example, it will apply if $\sigma_1 = 4{\cdot}10^{-11}$ e.m.u. (approximately the conductivity of sea water), and the period is 400 s. Also the 'skin depth' for each conductor remains the same, being 5·03 km for the conductor σ_1 and 15·91 km for the conductor σ_2. These values determine the linear scale of the solution.

The solution is exhibited in the nine diagrams of Figs 6 and 7, which show the contours of equal magnitude of H_x for equal steps of time during one-half of the oscillation period. These contours are also the lines of force of the E-field and consequently also the lines of flow of the electric current. Noteworthy features of the diagrams are the strong refraction of the lines of current flow at the discontinuity, and the formation of current vortices in the corner of the conductor σ_1, these vortices migrating inwards into the conductor with time. Two vortices of opposite senses are formed during each complete oscillation.

In spite of these remarkable effects on the distribution of currents within the conductor, the magnetic field *outside* remains uniform and quite unaffected by them, as we have already noted in Section 4. This implies that the total current system can be divided into two parts, one consisting of sheets of current flowing parallel to the surface and contributing to the uniform field outside, and the other consisting of toroidal current systems whose magnetic field is contained entirely within the conductor.

Applying the above results to actual geomagnetic field variations, we may conclude that the component of the magnetic field normal to an extended line of abrupt conductivity change, e.g. a long coast line, will not be appreciably affected by the induced electric earth currents, and therefore a study of this component is unlikely to reveal much about the Earth's conductivity. It should, however, be noted that this idealized two-dimensional problem cannot exactly represent any real geophysical situation. In most real situations the electric currents in the higher conductivity region flowing towards the interface with the lower conductivity region would be able, at least partially, to leak away laterally as well as vertically, and the charge built up on the interface would be smaller than in the ideal two-dimensional case. Hence the above theoretical result that the external magnetic field is unaffected by the local perturbation of earth currents may not always be applicable to actual geomagnetic problems that appear to satisfy the right *local* conditions.

8. Numerical solution for E-polarization case

We take the same numerical values for the conductivities and period of oscillation as in the previous case, and the same remarks apply about the solution being useful for other conductivities and suitably adjusted periods. The 'skin depths'

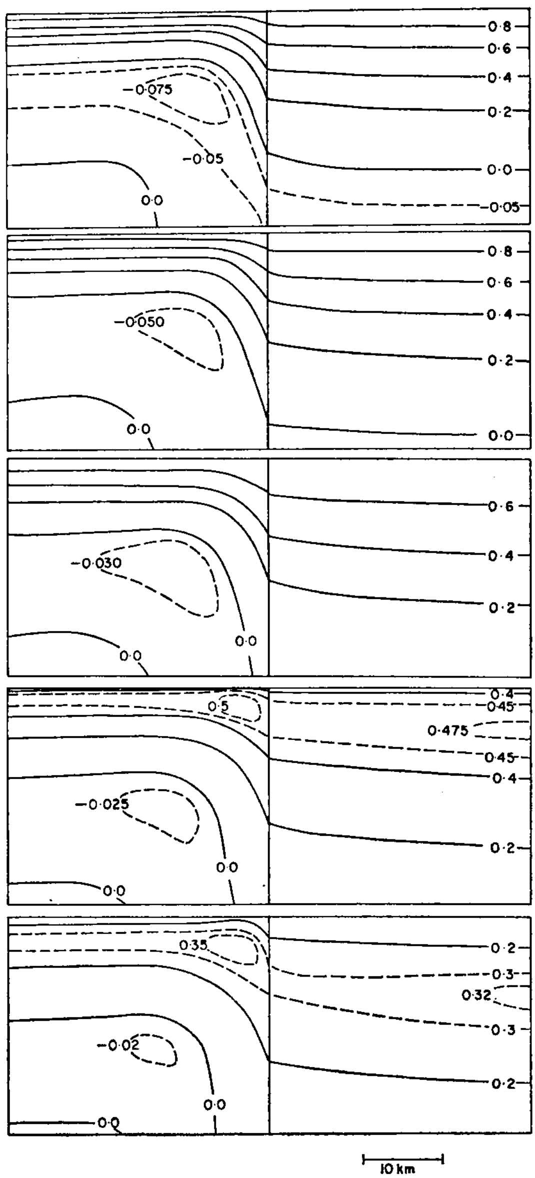

FIG. 6. H-polarization. Lines of force of E-field (≡ contours of equal H_x). Line diagrams give successive intervals of field at equal intervals of one-eighth of the period.

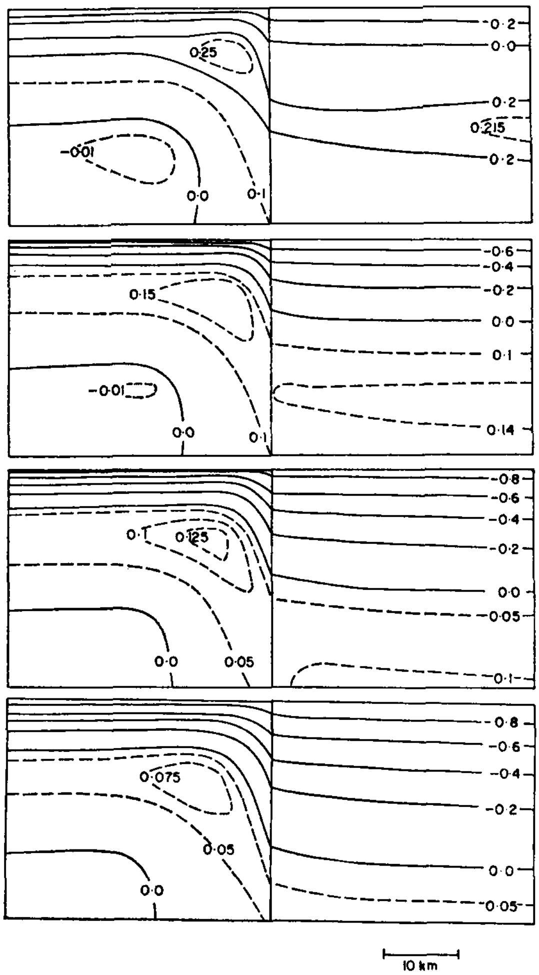

FIG. 7. H-polarization. Lines of force of E-field ($\equiv$ contours of equal H_x). Line diagrams give successive intervals of field at equal intervals of one-eighth of the period.

and the linear scale of the solution are also unchanged. However, the E-field and therefore also the current flow are now everywhere parallel to the x-direction. An important difference between this problem and the previous one is that the magnetic field outside the conductor is no longer uniform and has to be calculated.

The nine diagrams of Figs 8–10 show the contours of equal magnitude of E_x for nine epochs at equidistant intervals of time during one half of the period of oscillation. These E_x contours are also the lines of force of the magnetic field. With regard to the currents in the conductor, the E_x contours are also the contours of current density, but since this is proportional to σ, the current density in the σ_1 half is ten times that in the σ_2 half for the same value of E_x.

In the first diagram, it will be seen that a wedge of high current density (flowing perpendicular to the plane of the diagram in the positive x direction) is present at the corner of the higher conductivity part. The current density decreases with depth and ultimately changes sign, the magnetic field lines forming closed curves in this region of negative current. The current density in the part of lower conductivity

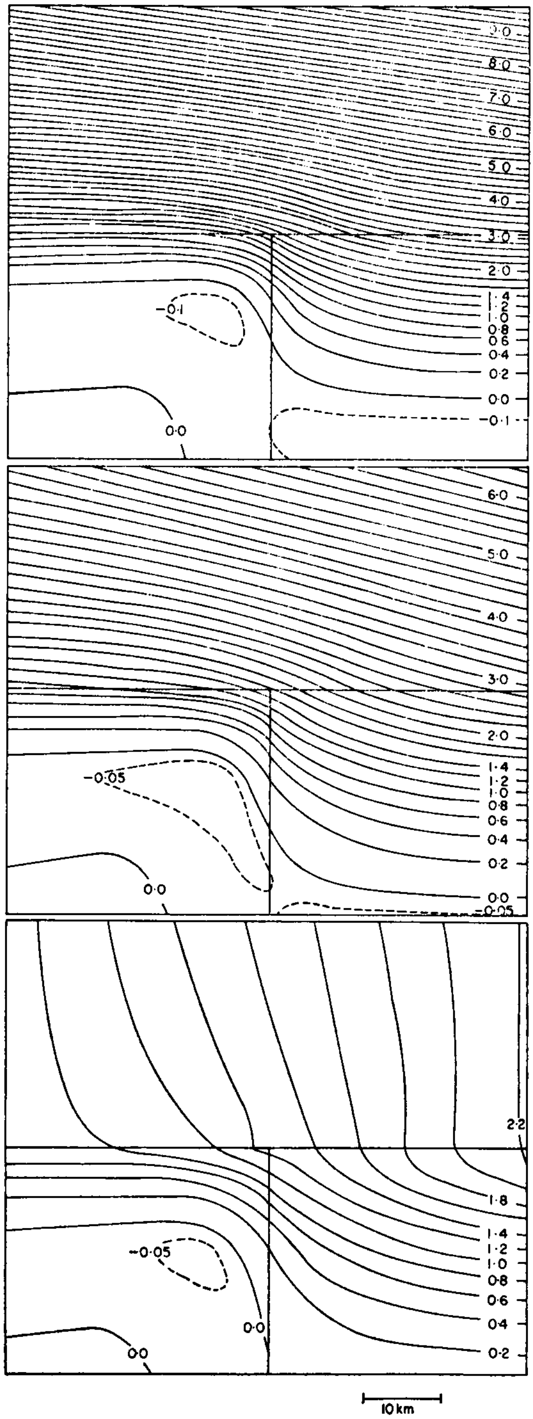

FIG. 8. E-polarization. Lines of force of H-field ($\equiv$ contours of equal E_x). Line diagrams give successive intervals of field at equal intervals of one-eighth of the period.

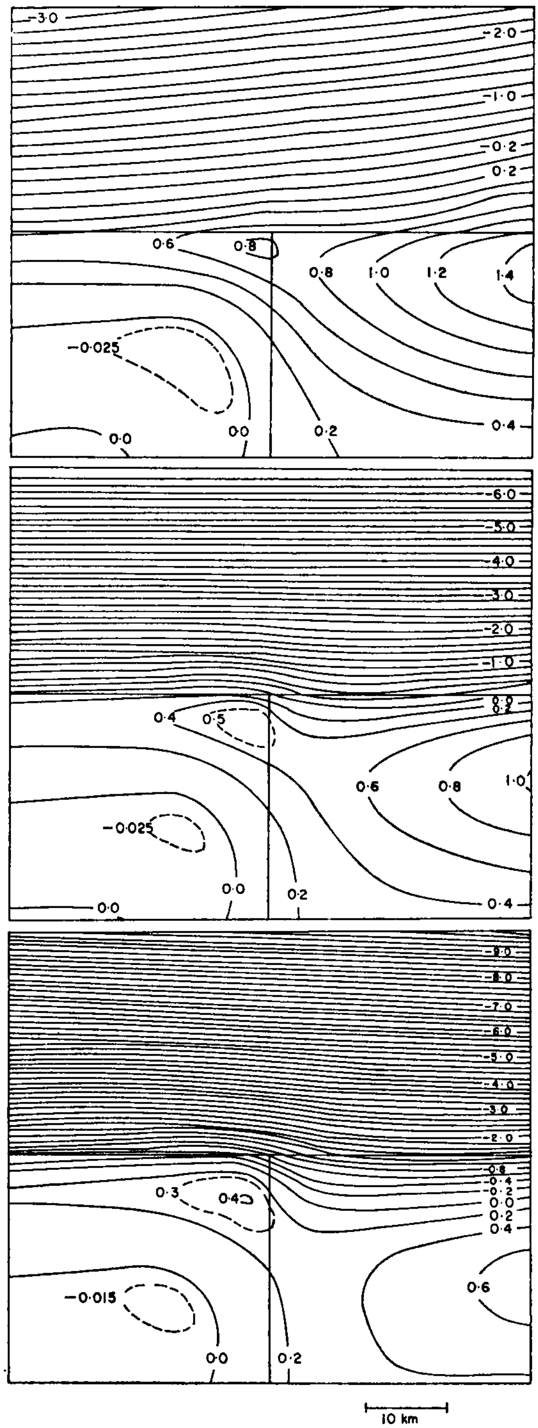

FIG. 9. E-polarization. Lines of force of H-field (≡ contours of equal E_x). Line diagrams give successive intervals of field at equal intervals of one-eighth of the period.

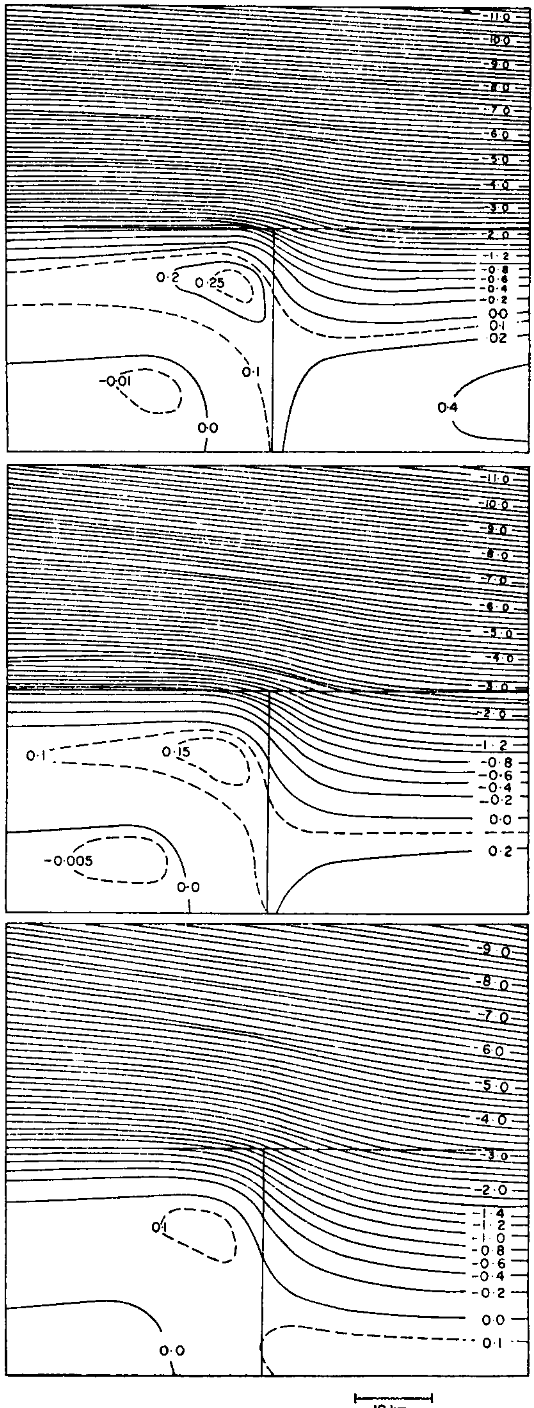

FIG. 10. E-polarization. Lines of force of H-field (≡ contours of equal E_x). Line diagrams give successive intervals of field at equal intervals of one-eighth of the period.

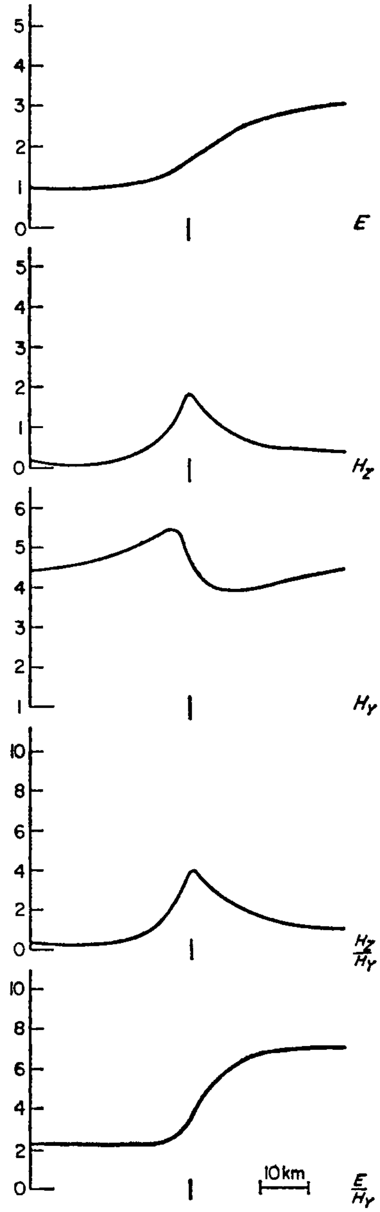

FIG. 11. E-polarization. Changes in amplitude during one-half the period on passing across the surface over the discontinuity of E (e.m.u.), H_z ($\times 10^{-2}$ gammas), H_y ($\times 10^{-2}$ gammas), H_z/H_y ($\times 10^{-1}$) and E/H_y ($\times 10^{6}$).

is much weaker, and it changes more slowly with increasing depth, corresponding to the fact that the skin depth is greater.

In the second diagram, the wedge of current in the corner has slightly decreased in intensity. The negative current flow has also decreased and moved further into the conductor.

In the third diagram, the field and current flow at great distances becomes small, being theoretically zero at infinity, so that the field then existing is purely the perturba-

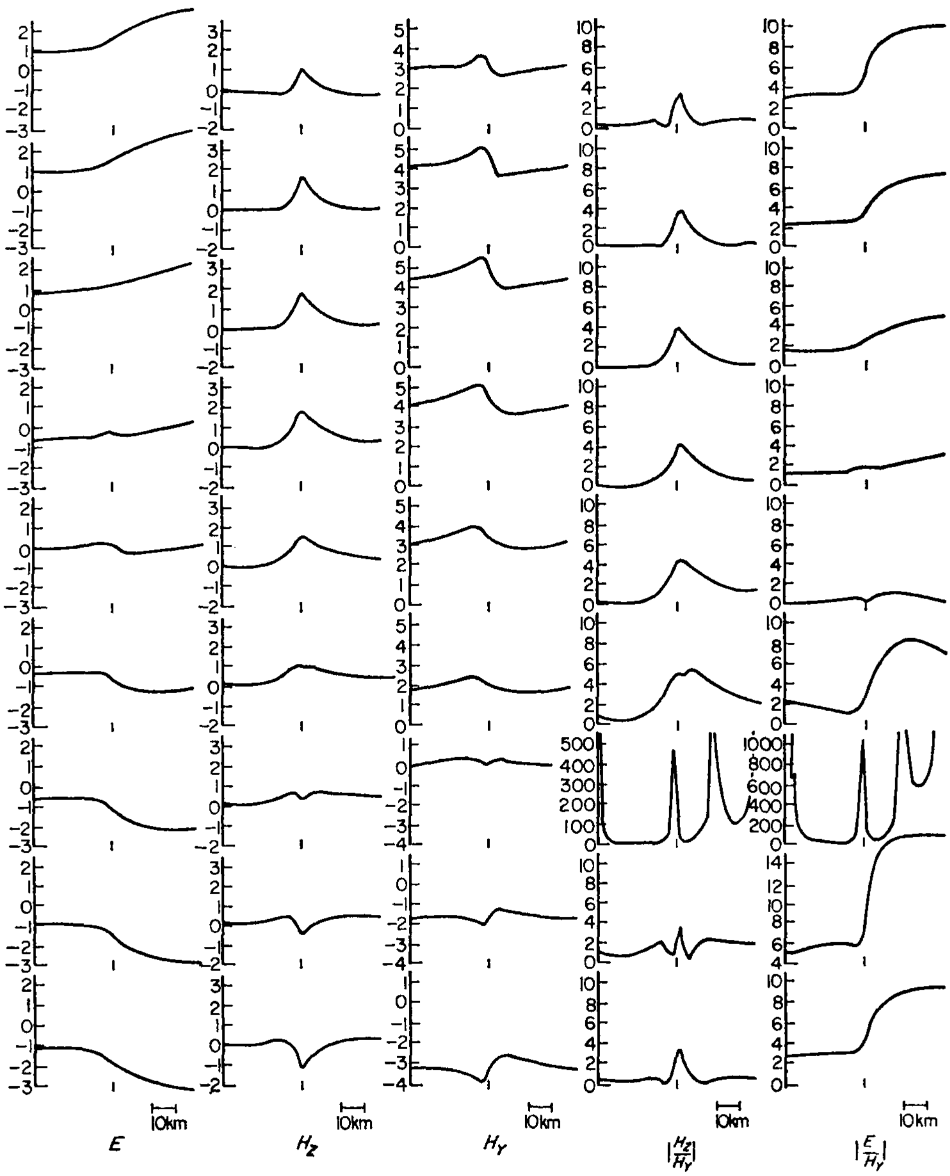

FIG. 12. E-polarization. Changes in E (e.m.u.), H_z ($\times 10^{-2}$ gammas), H_y ($\times 10^{-2}$ gammas), $|H_z/H_y|$ ($\times 10^{-1}$) and $|E/H_y|$ ($\times 10^{6}$) on passing across the surface over the discontinuity at equal intervals of one-eighth of the period.

tion field at that instant. It will be seen that the magnetic field outside the conductor is nearly vertical over most of the surface of the conductor, though there is a significant horizontal component at a place on the more highly conducting side some distance from the discontinuity. It will also be noticed that the wedge of high positive current density is showing some signs of getting detached from the corner of the good conductor.

In the fourth diagram, the positive current concentration is definitely detached from the corner, and magnetic field lines surround it. The negative current system is weaker and still further inside the conductor.

The remaining diagrams clearly show that current concentrations are being continually formed at the corner of the more highly conducting half, and move inwards into the conductor. The successive concentrations are of opposite sign, and two are formed in each complete oscillation. It is these concentrations of current that are mainly responsible for the local perturbation of the electromagnetic field. There are also, of course, changes in the current distribution within the half conductor of lower conductivity, but these make a much smaller contribution to the perturbation.

The changes in amplitude during one-half period of E, H_z, H_y and of the ratios H_z/H_y and E/H_y on passing across the surface over the discontinuity are shown in Fig. 11. It will be seen that H_y is the same at the extreme ends, but varies as the discontinuity in conductivity is crossed. This does not agree with Weaver's assumption that H_y is constant along $z = 0$. Also, H_z and the ratio H_z/H_y are enhanced near the discontinuity with a steeper decline in this enhancement with distance from the discontinuity over the higher conducting region. This is in agreement with observed measurements near the continental-oceanic interface.

Fig. 12 shows the changes in the instantaneous values of E, H_z, H_y, $|H_z/H_y|$ and $|E/H_y|$ on passing across the surface over the discontinuity for the nine time steps taken during the half period. It will be noted that the large variations of $|H_z/H_y|$ and $|E/H_y|$ at the seventh stage are due to the small value of H_y at this time.

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