GEOPHYSICS, VOL. XXVII, NO. 5 (OCTOBER, 1962), PP. 666–676, 5 FIGS.

THE MAGNETO TELLURIC EFFECT ON A DIKE*

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The method of d'Erceville and Kunetz (1962) is applied to the solution of the magneto-telluric effect on a vertical dike for the special case of the magnetic vector polarized parallel to the strike. The results for several frequencies have been computed and are presented graphically.

INTRODUCTION

The magneto-telluric method utilizes the naturally occurring electromagnetic disturbances as a source for studying resistivity contrasts in the earth. The measured quantities are the horizontal components of E, the electric field, as deduced from the potential between probes imbedded in the surface, and H, the magnetic field, measured at the surface. The response to these electromagnetic disturbances as measured at the surface of the earth is diagnostic of the interior, with the depth of penetration dependent on the period of the disturbance. The wealth of frequencies available in the spectrum of electromagnetic disturbances, when suitably resolved, provide the possibility of exploring at any depth.

Cagniard (1953, p. 610) gives a table of the depths of penetration corresponding to various frequencies. For a period of 0.01 sec and a resistivity of one ohmmeter, the depth of penetration (skin depth) is 20 meters which is suitable for ground-water studies. For a resistivity of 10 ohmmeters and a period of 30 minutes, the depth of penetration is 67.5 km, which is well into the upper mantle.

Cagniard has discussed the question of the uniformity of the source of the electromagnetic disturbances which are believed to be vast sheets of current flowing in the ionosphere. The simplifying assumption of plane waves incident normally on the surface of the earth would appear to be justified in most cases. The case of nonplanar waves is discussed in an article by Wait (1954) and the discussion is continued in a series of letters by Cagniard and Wait following Wait's paper.

Cagniard in his comprehensive paper outlined the theory of the magneto-telluric effect and presented graphical methods of interpretation of great usefulness. Cagniard's emphasis, however, was directed toward the application of the magneto-telluric method to horizontal strata as in sedimentary basins and, while this case is important in itself, vertical discontinuities such as faults and dikes are also of interest. Neves (1957) and d'Erceville and Kunetz (1962) have presented solutions for the case of a fault in a layered earth. The present paper extends this theory to the case of a dike.

In order to facilitate the reading of this paper as a sequel to the definitive works of Cagniard (1953) and d'Erceville and Kuentz (1962), or for consistent reference back to their work, we employ the electromagnetic system of units in the theoretical discussion.

THEORY

Let us consider a uniform dike infinitely extending in the y-direction, imbedded in an otherwise infinitely extended uniform layer of the earth's crust. The common depth of the dike and layer is denote by h. This structure is underlain by a uniform half-space. The geometrical and physical parameters are as shown in Figure 1.

The resistivity of the medium within the dike is designated by ρ_1, and the resistivity of the medium on either side of the dike by ρ_2. The resistivity of the underlying medium is ρ_s.

We will consider a plane incoming wave polarized with the magnetic vector parallel to the strike and will deal with a single component of angular

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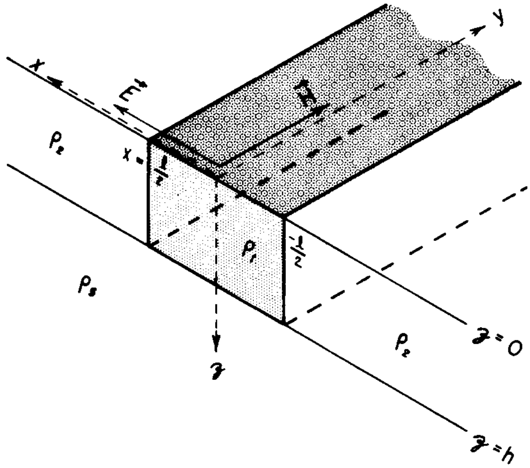

FIG. 1. Dike infinitely extended in the positive and negative y-directions. The surface field vectors are polarized as shown. ρ_1, ρ_2, ρ_3, are the resistivities of medium 1 within the dike, medium 2 on either side of the dike, and the common underlying medium, respectively.

velocity, ω, since in any case the spectrum of frequencies can be resolved in theory at least into Fourier components, and in practice by suitable filtering.

From considerations of symmetry we can set $E_y=0$ and all partial derivatives with respect to y are zero. We will consider only media in which the magnetic permeability is that of free space. Since the time dependence can be expressed in the form $\exp(j\omega t)$, it follows from the relationship:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{H}}{\partial t},$$

which in component form can be written as

$$-j\omega H_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y},$$

and

$$-j\omega H_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z},$$

that

$$H_z = H_x = 0.$$

Within a medium of finite conductivity, displacement currents are negligible and we can write the appropriate equation of Maxwell:

$$\nabla \times \boldsymbol{H} = 4\pi \boldsymbol{i},$$

the z component of which is

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 4\pi i_z,$$

where $\boldsymbol{i}$, i_z are the respective conduction currents. At any horizontal surface on the other side of which the conductivity is zero, we have $i_z=0$, and hence at the upper surface of the earth,

$$\frac{dH_y}{dx} = 0.$$

Thus, H is constant on the upper surface even across the trace of the discontinuity between the media.

Following the method of d'Erceville and Kunetz (1962) we will assume that we can write a general solution for H as the sum of two terms:

$$H = H_i(z) + P_i(x, z),$$

where $H_i(z)$ is the solution for the appropriate simple layered earth and $P_i(x, z)$ is a disturbance term which must be added to take account of the discontinuities. H, H_i, and P_i must of course satisfy the appropriate differential equation.

H_i satisfies the equation:

$$\frac{d^2 H_i}{dz^2} = [j4\pi\sigma_i\omega - \epsilon_i\omega^2] H_i,$$

while P_i satisfies the equation:

$$\frac{\partial^2 P_i}{\partial x^2} + \frac{\partial^2 P_i}{\partial z^2} = [j4\pi\sigma_i\omega - \epsilon_i\omega^2]P_i.$$

We will further assume that the disturbance term can be written:

$$P_i(xz) = f_i(x)g_i(z),$$

where $g_i(z)$ can be expanded in a Fourier series.

Case I: Infinitely resistive substratum

We have observed that at the upper surface where $i_z=0$, H is constant even across the trace of the discontinuities. For the same reason, at the lower surface, $z=h$, H is constant.

Reference to the work of d'Erceville and Kunetz (1962) gives the following solution of H_i in the conducting media:

$$H_i = A_i \exp\left(\sqrt{j}\frac{z}{p_i}\right) + B_i \exp\left(-\sqrt{j}\frac{z}{p_i}\right),$$

where

$$A_i = -\frac{H_0 \exp(-\sqrt{j}\,h_i)}{2\sinh(\sqrt{j}\,h_i)},$$

$$B_i = \frac{H_0 \exp(\sqrt{j}\,h_i)}{2\sinh(\sqrt{j}\,h_i)},$$

H_0 is the constant value of H_i at the upper surface, $h/p_i=h_i$, $p_i=d_i/\sqrt{2}$, and d_i is the depth of penetration or skin depth. These relations for A_i and B_i were derived by d'Erceville and Kunetz on the basis of the asssumption that the magnetic field vanishes at the surface of the infinetely resistive substratum. Since this assumption may not be valid, a derivation showing that these expressions are adequate approximations independently of this hypothesis is given in the Appendix.

At remote distances from a discontinuity, $P_i(x, z)$ vanishes, and since H is constant along both the upper and lower surface,

$$H_i(x, 0) = H_0 + P_i(x, 0)$$

and

$$H_i(x, h) = H_h + P_i(x, h),$$

where H_h is the constant value of H at the lower interface. Thus, P_i vanishes at $z=0$ and h and we can write

$$P_i(x, z) = f_i(x)g_i(z),$$

where we can expand $g_i(z)$ in a sine series of argument $n\pi z/h$. We will, for convenience, include the coefficient of sin $n\pi z/h$ in $f_i(x)$ and designate the terms by $f_{in}(x)$. Thus,

$$P_i(x, z) = \sum_{n=1}^{\infty} f_{in}(x) \sin\frac{n\pi z}{h}.$$

Each term of this series must satisfy the same propagation equation:

$$\frac{\partial^2 P_i}{\partial x^2} + \frac{\partial^2 P_i}{\partial z^2} = j4\pi\sigma_i\omega P_i,$$

which implies:

$$\frac{d^2 f_{in}(x)}{dx^2}\sin\frac{n\pi z}{h} = \left[\frac{n^2\pi^2}{h^2} + j4\pi\sigma_i\omega\right] f_{in}(x)\sin\frac{n\pi z}{h}.$$

After canceling the common sine term, we have the solution:

$$f_{in}(x) = a_{in}\exp\left(-\frac{k_{in}x}{h}\right) + b_{in}\exp\left(\frac{k_{in}x}{h}\right),$$

where

$$k_{in} = \sqrt{n^2\pi^2 + jh_i^2}.$$

This solution satisfies the condition that it vanish for $|x| = \infty$ if we take only the a-type terms for positive x and only the b-type terms for negative x both in medium 2, while both positive and negative exponentials are permissible in the finite region of medium 1. Symmetry conditions permit us to write the equation:

$$a_{in} = b_{in},$$

which we observe gives the same value of $H(x, z)$ for the same positive and negative values of x. Employing the symmetry condition is entirely equivalent to using one of the boundaries for solving for continuity of the field components, with only the remaining boundary now being available for solving for the remainder of the coefficients. We write:

$$\sum_{n=1}^{\infty} a_{1n} \exp\left(-\frac{k_{1n}l}{2h}\right) + a_{1n} \exp\left(\frac{k_{1n}l}{2h}\right) - a_{2n} \exp\left(-\frac{k_{2n}l}{2h}\right) = H_2 - H_1.$$

If we now expand H_2-H_1 into a sine series of the same argument:

$$H_2 - H_1 = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi z}{h},$$

we can equate, at the boundary, term by term to obtain:

$$2a_{1n} \cosh\left(\frac{k_{1n}l}{2h}\right) - a_{2n} \exp\left(-\frac{k_{2n}l}{2h}\right) = c_n. \quad (1)$$

From

$$\nabla \times \boldsymbol{H} = 4\pi\sigma E$$

we obtain

$$E_z = \frac{1}{4\pi\sigma}\frac{\partial H_y}{\partial x},$$

and equating term by term at $x=l/2$, we obtain

$$\frac{2k_{1n}}{\sigma_1} a_{1n} \sinh\left(\frac{k_{1n}l}{2h}\right) + \frac{k_{2n}}{\sigma_2} a_{2n} \exp\left(-\frac{k_{2n}l}{2h}\right) = 0. \quad (2)$$

Solving for a_{1n} from equations (1) and (2),

$$a_{1n} = \frac{C_n}{2\cosh\left(\frac{k_{1n}l}{2h}\right) + 2\frac{k_{1n}h_2^2}{k_{2n}h_1^2}\sinh\left(\frac{k_{1n}l}{2h}\right)} \quad (3)$$

and

$$a_{2n} = \frac{-C_n \exp\left(\frac{k_{2n}l}{2h}\right)\sinh\left(\frac{k_{1n}l}{2h}\right)}{\frac{k_{2n}h_1^2}{k_{1n}h_2^2}\cosh\left(\frac{k_{1n}l}{2h}\right) + \sinh\left(\frac{k_{1n}l}{2h}\right)}. \quad (4)$$

Following the well-known procedure of Fourier series expansion, the coefficient of $\sin n\pi z/h$ in the expansion of H_2-H_1 is

$$C_n = \frac{2}{h}\int_0^h (H_2 - H_1)\sin\frac{n\pi z}{h}\,dz,$$

where

$$H_2 - H_1 = A_2 \exp\left(\sqrt{j}\frac{z}{p_2}\right) + B_2 \exp\left(-\sqrt{j}\frac{z}{p_2}\right) - A_1 \exp\left(\sqrt{j}\frac{z}{p_1}\right) - B_1 \exp\left(-\sqrt{j}\frac{z}{p_1}\right),$$

from which

$$C_n = j2\pi\frac{nH_0(h_1^2 - h_2^2)}{k_{1n}^2 k_{2n}^2}.$$

Substituting this expression into equations (3) and (4), we obtain

$$a_{1n} = \frac{j2\pi n(h_1^2 - h_2^2)H_0}{k_{1n}^2 k_{2n}^2\left[2\sinh\left(\frac{k_{1n}l}{2h}\right)\frac{k_{1n}h_2^2}{k_{2n}h_1^2} + 2\cosh\left(\frac{k_{1n}l}{2h}\right)\right]}$$

and

$$a_{2n} = \frac{j2\pi n(h_2^2 - h_1^2)H_0 \exp\left(-\frac{k_{2n}l}{2h}\right)\sinh\left(\frac{k_{1n}l}{2h}\right)}{k_{1n}^2 k_{2n}^2\left[\sinh\left(\frac{k_{1n}l}{2h}\right) + \frac{k_{2n}h_1^2}{k_{1n}h_2^2}\cosh\left(\frac{k_{1n}l}{2h}\right)\right]}.$$

In medium 1,

$$H_y = \frac{H_0}{2\sinh(\sqrt{j}\,h_1)}\, 2\sinh\left(\sqrt{j}\,h_i - \sqrt{j}\,\frac{z}{p_1}\right) + \sum a_{1n} 2\cosh\left(\frac{k_{1n}x}{h}\right)\sin\frac{n\pi z}{h}$$

and

$$E_x = \frac{H_0\sqrt{j}\,2\cosh\left(\sqrt{j}\,h_1 - \sqrt{j}\,\frac{z}{p_1}\right)}{4\pi\sigma_1 p_1 2\sinh(\sqrt{j}\,h_1)} + \sum -\frac{n\pi a_{in}}{4\pi\sigma_1 h}\, 2\cosh\left(\frac{k_{1n}x}{h}\right)\cos\frac{n\pi z}{h}\,.$$

At the upper surface, $z=0$, where we measure the magnetic-telluric fields, $H_y=H_0$, and thus

$$\left(\frac{E_x}{H_y}\right)_1 = \sqrt{j}\,\frac{\omega h}{h_1}\coth(\sqrt{j}\,h_1) - 2\pi^2 j\,\frac{\omega h}{h_1^2}(h_1^2 - h_2^2)U_1,$$

where

$$U_1 = \sum_{n=1}^{\infty} \frac{n^2\cosh\left(\frac{k_{1n}x}{h}\right)}{k_{1n}^2 k_{2n}^2\left[\cosh\left(\frac{k_{1n}l}{2h}\right) + \frac{k_{1n}h_2^2}{k_{2n}h_1^2}\sinh\left(\frac{k_{1n}l}{2h}\right)\right]}\,.$$

In medium 2, the constant term is the same as for medium 1, with the subscripts interchanged, but the disturbance term is

$$P(xz) = \sum_{n=1}^{\infty} a_{2n}\exp\left(-\frac{k_{2n}x}{h}\right)\sin\frac{n\pi z}{h}$$

and the disturbance term for E_x is

$$\frac{-1}{\pi 4\sigma_2}\frac{\partial \rho_{(xz)}}{\partial z} = -\frac{1}{4\pi\sigma_2}\sum_{n=1}^{\infty} a_{2n}\frac{n\pi}{h}\cdot\exp\left(-\frac{k_{2n}x}{h}\right)\cos\frac{n\pi z}{h}\,.$$

At the surface where $H_y=H_0$,

$$\left(\frac{E_x}{H_y}\right)_2 = \sqrt{j}\,\frac{\omega h}{h_2}\coth(\sqrt{j}\,h_2) - 2\pi^2 j\,\frac{\omega h}{h_2^2}(h_2^2 - h_1^2)U_2,$$

where

$$U_2 = \sum_{n=1}^{\infty} \frac{n^2\exp\left(\frac{k_{2n}l}{2h}\right)\sinh\left(\frac{k_{1n}l}{2h}\right)\exp\left(-\frac{k_{2n}x}{h}\right)}{k_{1n}^2 k_{2n}^2\left[\sinh\left(\frac{k_{1n}l}{2h}\right) + \frac{k_{2n}h_1^2}{k_{1n}h_2^2}\cosh\left(\frac{k_{1n}l}{2h}\right)\right]}\,.$$

Case II: Infinitely conducting substratum

The solution for H at remote distances from the fault is once again

$$H_i(z) = A_i\exp\left(\sqrt{j}\,\frac{z}{p_i}\right) + B_i\exp\left(-\sqrt{j}\,\frac{z}{p_i}\right),$$

where in this case from d'Erceville and Kunetz (1962),

$$A_i = \frac{H_0\exp(-\sqrt{j}\,h_i)}{2\cosh(\sqrt{j}\,h_i)}$$

and

$$B_i = \frac{H_0 \exp(\sqrt{j}\, h_i)}{2 \cosh(\sqrt{j}\, h_i)}.$$

Thus, in the general expression:

$$H_i(x, z) = H_i(z) + P_i(x, z),$$

$P_i(x, z)$ must vanish for $z=0$ for the same reason as in the preceding case, while at $z=h$ it must have a finite value determined by the continuity of $H(x, z)$ at $x=\pm l/2$. Thus, neither H_2-H_1 nor P can be represented simply by a sine series in the interval 0 to h. For convenience we will develop these functions in a sine series of argument $n\pi z/2h$, where in order to satisfy the boundary conditions at $z=h$ while vanishing at 0 and $2h$, we will define

$$H_2 - H_1 = F(2h - z), \quad h \le z \le 2h.$$

Proceeding as in the previous example,

$$P_i(x, z) = \sum_{n=1}^{\infty} f_{in}(x) \sin \frac{n\pi z}{2h},$$

each term of which satisfies

$$\frac{\partial^2 H}{\partial x^2}(x, z) + \frac{\partial^2 H}{\partial z^2}(x, z) = 4\pi j\sigma\omega H.$$

This gives

$$\frac{d^2 f_{in}(x)}{dx^2} = \left[\frac{n^2\pi^2}{4h^2} + 4\pi j\sigma_i\omega\right] f_{in}(x),$$

which has solutions:

$$f_{in}(x) = a_{in} \exp\left(-\frac{k_{in}x}{2h}\right) + a_{in} \exp\left(\frac{k_{in}x}{2h}\right),$$

where in this case,

$$k_{in} = \sqrt{n^2\pi^2 + 4jh_i^2}.$$

We have equated $a_{in}=b_{in}$ for the same reasons as in the previous example. The equations of continuity for H_y and E_z at $x=\pm l/2$ result in the same solution for a_{in} in terms of C_n, except that h is replaced by $2h$ to give

$$a_{1n} = \frac{C_n}{2 \cosh\left(\frac{k_{1n}l}{2h}\right) + \frac{k_{1n}h_2^2}{k_{2n}h_1^2} \sinh\left(\frac{k_{1n}l}{4h}\right)}$$

and

$$a_{2n} = -\frac{C_n \exp\left(\frac{k_{2n}l}{4h}\right) \sinh\left(\frac{k_{1n}l}{4h}\right)}{\sinh\left(\frac{k_{1n}l}{4h}\right) + \frac{k_{2n}h_1^2}{h_{1n}h_2^2} \cosh\left(\frac{k_{1n}l}{4h}\right)},$$

where in k_{1n} and k_{2n}, h is also replaced by $2h$.

When solving for C_n we note that

$$C_n = \frac{1}{h}\left[\int_0^h F(z) \sin \frac{n\pi z}{2h}\, dz + \int_h^{2h} F(2h - z) \sin \frac{n\pi z}{2h}\, dz,\right.$$

where

$$F(z) = H_2 - H_1$$

as given above. Thus,

$$C_n = 16\pi j H_0 (h_1^2 - h_2^2) \frac{2n+1}{k_{1n}^2 k_{2n}^2},$$

i.e., only the odd terms of the series appear. Therefore, we must substitute $2n+1$ for n in k_{1n} and k_{2n}, with the understanding that n starts at zero instead of at unity.

The complete result for an infinitely conducting substratum is

$$\left(\frac{E_x}{H_y}\right)_1 = \sqrt{j}\, \frac{\omega h}{h_1} \tanh(\sqrt{j}\, h_1) - 8\pi^2 j \frac{\omega h}{h_1^2} (h_1^2 - h_2^2) U_1,$$

where

$$U_1 = \sum_{n=1}^{\infty} \frac{(2n+1)^2 \cosh\left(\frac{k_{1n}x}{2h}\right)}{k_{1n}^2 k_{2n}^2 \left[\cosh\left(\frac{k_{1n}l}{4h}\right) + \frac{k_{1n}h_2^2}{k_{2n}h_1^2} \sinh\left(\frac{k_{1n}l}{4h}\right)\right]}.$$

Similarly,

$$\left(\frac{E_x}{H_y}\right)_2 = \sqrt{j}\,\frac{\omega h}{h_2}\tanh\left(\sqrt{j}\,h_2\right) - 8\pi^2 j\,\frac{\omega h}{h_2^2}\left(h_2^2 - h_1^2\right)U_2,$$

where

$$U_2 = \sum_{n=0}^{\infty} \frac{(2n+1)^2 \exp\left(\frac{k_{2n}l}{4h}\right)\sinh\left(\frac{k_{1n}l}{4h}\right)\exp\left(-\frac{k_{2n}x}{2h}\right)}{k_{1n}^2 k_{2n}^2\left[\sinh\left(\frac{k_{1n}l}{4h}\right) + \frac{k_{2n}h_1^2}{k_{1n}h_2^2}\cosh\left(\frac{k_{1n}l}{4h}\right)\right]}$$

and

$$k_{in} = \sqrt{(2n+1)^2 + 4jh_i^2}.$$

RESULTS AND CONCLUSIONS

The results as they appear in Figures 2-5 are plotted in the so-called "practical" units:

H in gammas,
E in mv/km,
lengths in km,
Ω in ohm-meters,
$1\gamma = 10^{-5}$ gauss,
1 mv/km = 1 em unit,
1 km = 10^5 cm., and
1Ω-m = 10^{11} em units.

While these are mixed units, they are indeed the practical units of geophysics. It should be noted that the skin depth as given in each case is for the

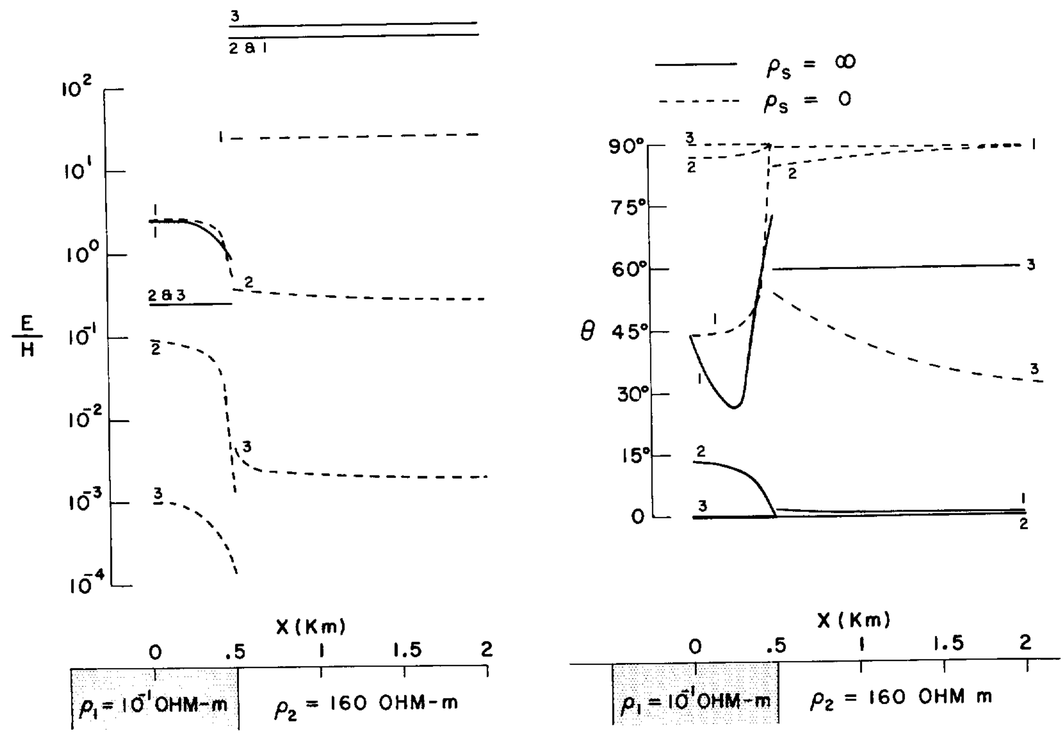

FIG. 2. *Conducting dike*—depth of layer $h=1$ km; width of dike $l=1$ km. ρ_s is the substratum resistivity. 1. Period 0.78 sec; skin depth $d=0.14\,h$ in the more conducting medium. 2. Period 78 sec; skin depth $d=1.4\,h$ in the more conducting medium. 3. Period 2 hr 10 min; skin depth $d=14\,h$ in the more conducting medium.

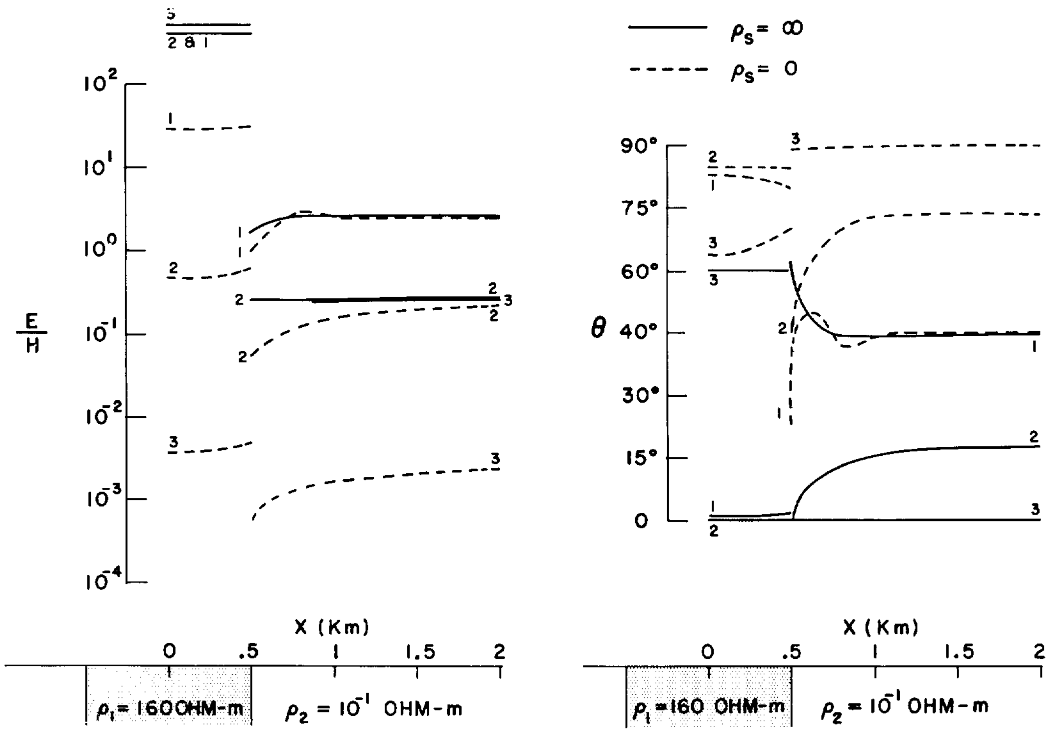

FIG. 3. *Resistive dike*—depth of layer $h=1$ km; width of dike $l=1$ km. ρ_s is the substratum resistivity. 1. Period 0.78 sec; skin depth $d=0.14\ h$ in the more conducting medium. 2. Period 78 sec; skin depth $d=1.4\ h$ in the more conducting medium 3. Period 2 hr 10 min; skin depth $d=14\ h$ in the more conducting medium.

more conducting medium, with the skin depth in the more resistive medium being 40 times greater in all cases.

It may be noted that as one moves away from the discontinuity, the values of E/H and the phase approach the value appropriate to a uniform two-layered system. In the region close to or within the dike, the resulting values are affected by both surfaces of discontinuity, but the effect of the more remote surface is significant only for the higher frequencies. For a conducting dike, the disturbance in the external region is smaller than for the resistive dike, with the disturbance diminishing as the width of the conducting dike diminishes and as the frequency increases. This latter point is of importance in a discussion of a model for the magneto-telluric effect, which is the proposed subject of a future paper.

Even for low frequencies, where the upper media have negligible results in the absence of discontinuities, measurable effects are apparent in the presence of vertical discontinuities. This is of considerable value for exploration purposes.

The case of electric polarization parallel to the strike is of equal importance, but the simplyifying fact that H_0 is continuous across the structural discontinuity is no longer valid. Further theoretical analysis of these problems is at present being carried out.

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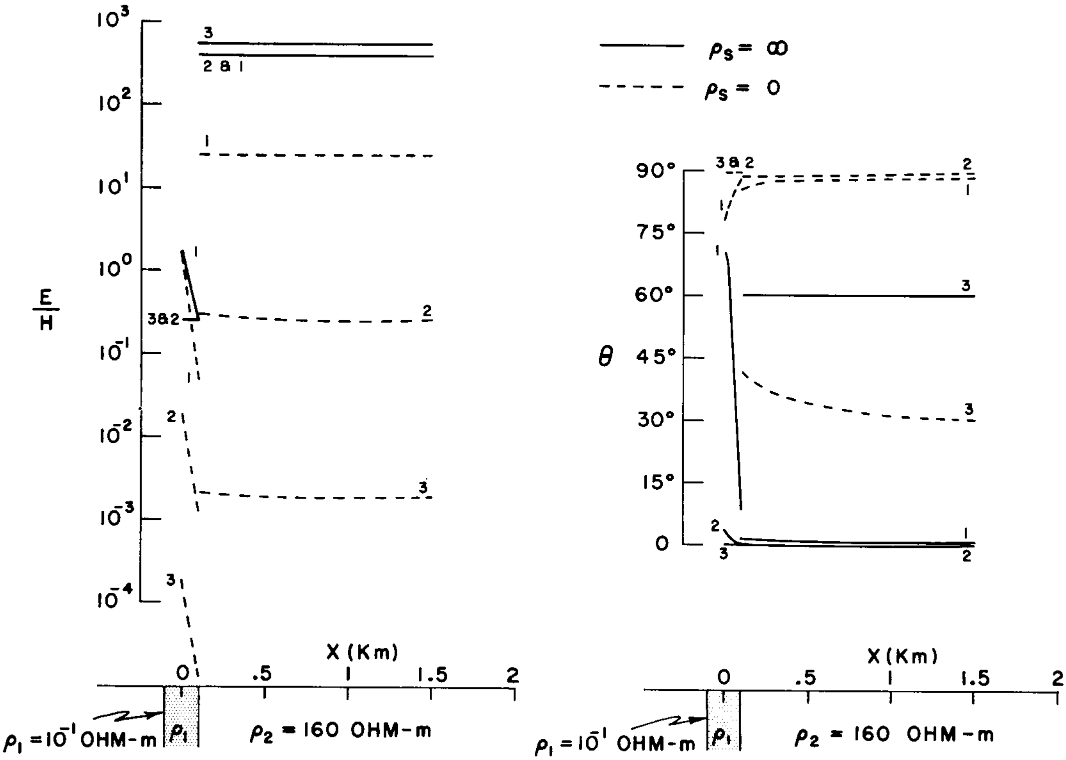

FIG. 4. *Conducting dike*—depth of layer $h=1$ km; width of dike $l=0.2$ km. ρ_s is the substratum resistivity. 1. Period 0.78 sec; skin depth $d=0.14\ h$ in the more conducting medium. 2. Period 78 sec; skin depth $d=1.4\ h$ in the more conducting medium. 3. Period 2 hr 10 min; skin depth $d=14\ h$ in the more conducting medium.

REFERENCES

Cagniard, L., 1953, Basic theory of the magneto-telluric method: Geophysics, v. 8, pp. 605–635.

d'Erceville, I., and Kunetz, G., 1962, Some observations regarding naturally occurring electromagnetic fields in applied geophysics: Geophysics, v. 27, pp. 651.

Neves, A. S., 1957, The magneto telluric method in two-dimensional structures: Ph.D. Thesis, M.I.T.

Wait, J. R., 1954, On the relations between telluric currents and the earth's magnetic field: Geophysics, v. 19, pp. 281–289.

APPENDIX

NOTE ON THE CALCULATIONS OF A_i AND B_i FOR AN INFINITELY RESISTIVE SUBSTRATUM

In the theory presented under Case I, the following expressions for A_i and B_i were used from the paper by d'Erceville and Kunetz (1962) for the case of a simple two-layered earth where the lower layer is infinitely extended downward:

$$A_i = -\frac{H_0 \exp(-\sqrt{j}\, h_i)}{2 \sinh(\sqrt{j}\, h_i)}$$

and

$$B_i = \frac{H_0 \exp(\sqrt{j}\, h_i)}{2 \sinh(\sqrt{j}\, h_i)}.$$

The result is derived on the assumption that the magnetic field, H, vanishes at the surface of the infinitely resistive substratum. This assumption is neither necessary nor necessarily valid. However, as will be shown below, these expressions for A_i and B_i are acceptable, and can be obtained as approximations which are generally valid in the magneto-telluric effect.

We will study solutions of the differential equation:

$$\frac{d^2H}{dz^2} = [j4\pi\sigma\omega - \epsilon\omega^2]H,$$

in a simple layered system remote from any vertical discontinuity. These solutions are of the form:

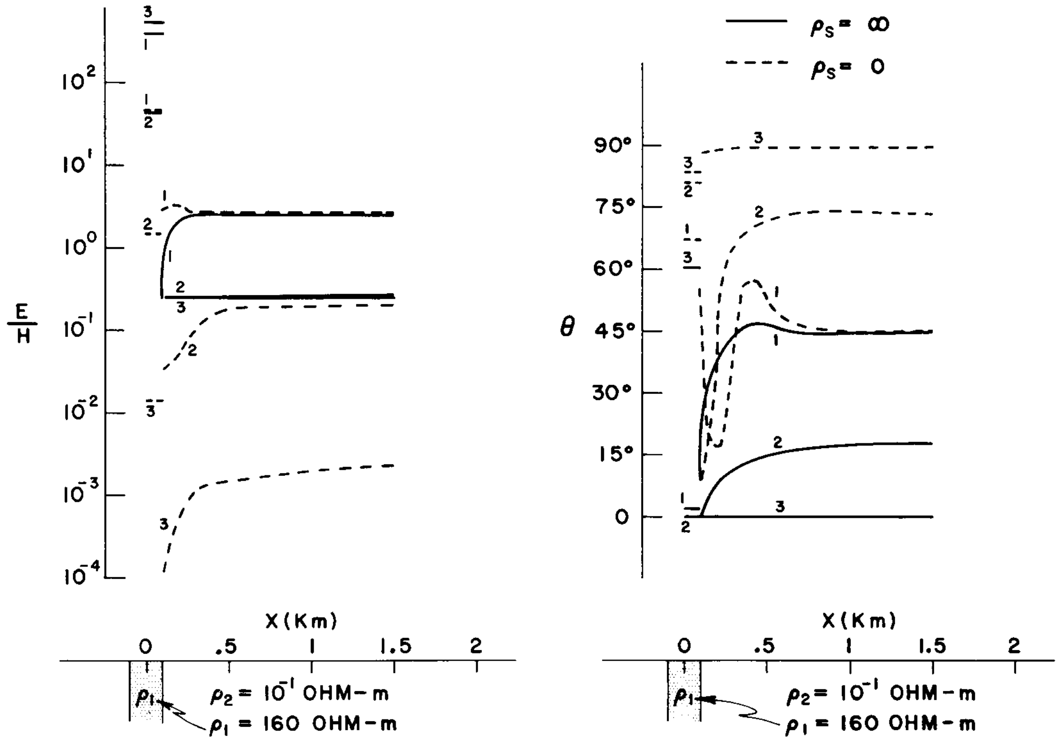

FIG. 5. *Resistive dike*—depth of layer $h=1$ km; width of dike $l=0.2$ km. ρ_s is the substratum resistivity. 1. Period 0.78 sec; skin depth $d=0.14\ h$ in the more conducting medium. 2. Period 78 sec; skin depth $d=1.4\ h$ in the more conducting medium. 3. Period 2 hr 10 min; skin depth $d=14\ h$ in the more conducting medium.

$$H = A \exp\left[(j4\pi\sigma\omega - \epsilon\omega^2)^{1/2}z\right] + B \exp\left[-(j4\pi\sigma\omega - \epsilon\omega^2)^{1/2}z\right].$$

In medium (1) with a finite conductivity, σ_1, the displacement current can be neglected. Hence,

$$H^{(1)} = A_1 \exp\left(\sqrt{j}\,\frac{z}{p_1}\right) + B_1 \exp\left(-\sqrt{j}\,\frac{z}{p_1}\right).$$

In medium (2), where the conductivity is zero,

$$H^{(2)} = B_2 \exp\left(-j\sqrt{\epsilon\omega^2}\,z\right).$$

At the boundary, $z=h$,

$$A_1 \exp(\sqrt{j}\,h_1) + B_1 \exp(-\sqrt{j}\,h_1) = B_2 \exp(-j\sqrt{\epsilon}\,\omega h). \quad (1)$$

Consider now the electric field defined by:

$$\boldsymbol{E} = \boldsymbol{E}(z) \exp(j\omega t)$$

and

$$\nabla \times \boldsymbol{H} = 4\pi\sigma\boldsymbol{E} + \epsilon\,\frac{\partial \boldsymbol{E}}{\partial t},$$

where since only the x-component is nonzero,

$$-\frac{dH}{dz} = (4\pi\sigma + j\epsilon\omega)E.$$

In medium (1),

$$E^{(1)} = -\frac{1}{4\pi\sigma_1}\,\frac{dH^{(1)}}{dz}.$$

In medium (2),

$$E^{(2)} = \frac{j}{\epsilon\omega}\,\frac{dH^{(2)}}{dz}.$$

Thus we obtain

$$E^{(1)} = \frac{-\sqrt{j}}{p_1 4\pi\sigma_1}\left[A_1 \exp\left(\sqrt{j}\,\frac{z}{p_1}\right) - B_1 \exp\left(-\sqrt{j}\,\frac{z}{p_1}\right)\right]$$

and

$$E^{(2)} = \frac{B_2}{\sqrt{\epsilon}} \exp\left(-j\sqrt{\epsilon}\,\omega z\right),$$

and at the boundary, $z=h$,

$$\omega p_i\sqrt{j\epsilon}\left[B_1 \exp\left(-\sqrt{j}\,h_1\right) - A_1 \exp\left(\sqrt{j}\,h_1\right)\right] = B_2 \exp\left(-j\sqrt{\epsilon}\,\omega h\right). \quad (2)$$

Eliminating B_2 between equations (1) and (2),

$$A_1 \exp\left(\sqrt{j}\,h_1\right)\left[1 + \sqrt{j}\,\omega p_1\sqrt{\epsilon}\right] = B_1 \exp\left(-\sqrt{j}\,h_1\right)\left[\sqrt{j}\,\omega p_1\sqrt{\epsilon} - 1\right],$$

or

$$B_1 = A_1 \exp\left(2\sqrt{j}\,h_1\right)\left[\frac{1 + \sqrt{j\,\frac{\epsilon\omega}{4\pi\sigma_1}}}{\sqrt{j\,\frac{\epsilon\omega}{4\pi\sigma_1}} - 1}\right]$$

If the resistivity of the conducting upper medium is as high as 10^4 ohm-meters (10^{15} em units), which corresponds approximately to the resistivity of dry sand and gravel, then for a dielectric constant of approximately unity, and frequencies of one cps,

$$B_1 = -A_1 \exp\left(2\sqrt{j}\,h_1\right)$$

is a valid approximation. This is an extreme case with respect to the magneto-telluric effect since resistivities several orders of magnitude smaller will normally be dealt with. The results, as given previously, follow from the boundary conditions at the upper surface, $z=0$.