An Analog Model for the Magnetotelluric Effect

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Abstract. A conducting solution in a tank provides the upper layer of a model earth. The magnetic field, which is inductively coupled to this model earth, is generated by a current in a large vertical loop. The effects of finite model dimensions are measured and the range of validity of the model is discussed.

Introduction. There has been a rapidly growing interest in the magnetotelluric method since the comparatively early suggestion by *Tikhonov* [1950] of utilizing the naturally occurring electromagnetic variations measured at the earth's surface as a means of exploring to great depths. The basic concepts of the magnetotelluric effect have been expounded by *Cagniard* [1953] in his definitive paper. Cagniard considers a plane wave impinging on the flat surface of a horizontally stratified earth. The geometry is as shown in Figure 1, with the z direction positive downward and the layers infinitely extended in the x and y directions. According to the theory, the apparent resistivity, $\rho_a = 0.2T|E_x/H_y|^2$, is diagnostic of the subsurface layering. T is the period in seconds, E_x and H_y are the components of the electric and magnetic fields, respectively, as measured in the 'practical' units. According to theory, the phase difference, $\phi(E_x) - \phi(H_y)$, can also make a significant contribution to the analysis of the resistivity structure. The idealization involved in the preceding result is justified in Cagniard's opinion by the global extent of the source of the electromagnetic disturbance and the local nature of field investigations. *Wait* [1954*a*] and *Price* [1962] have indicated the corrections necessary to take into account a finite source. The disagreement between *Cagniard* [1954] and *Wait* [1954*b*] concerns the nature of the source of the magnetotelluric disturbance and not the theory. *Neves* [1957] and *d'Erceville and Kunetz* [1962] have shown how a fault would affect the magnetotelluric ratio. The discrepancy between the results of field measurements and the theory as expressed by Cagniard may be attributed either to geologic structure or to a source effect.

The usefulness of laboratory models in the interpretation of geologic studies has long been recognized. By means of a model it is possible to select simple topographic features and to vary certain parameters in a controlled fashion. In the model described in this paper, the electric and magnetic fields are coupled to the model earth inductively rather than by direct current; hence this is a genuine magnetotelluric

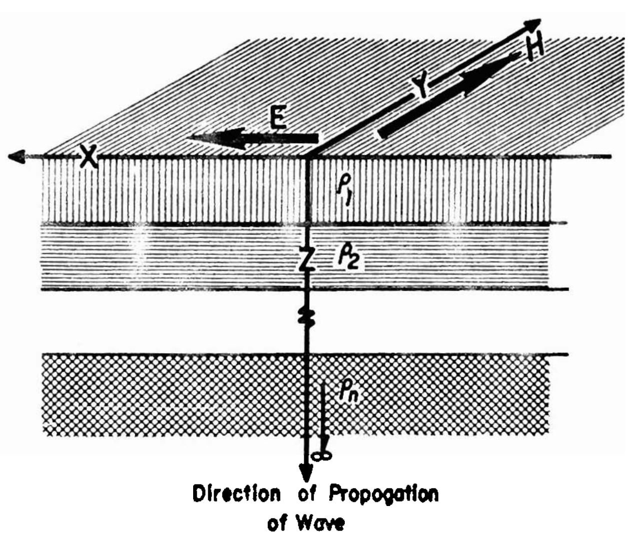

Fig. 1. Schematic diagram of the coordinate system, field vectors, and an n-layered earth according to the Cagniard model.

model, in that the electric and magnetic fields can be measured independently and their relationship can then be studied both as to phase and amplitude. An interesting example of a directly coupled model is discussed by *Westcott and Hessler* [1962].

The problem of scaling in models was well recognized by Galileo, though less clearly by many of his successors until comparatively recent times. *Slichter* [1932] dealt with an electromagnetic model, and *Hubbert* [1937] gave a general theory as applied to geologic problems. *Cagniard* [1953] discussed the magnetotelluric scaling factors specifically and extended the concepts to his most ingenious interpretation technique. In Cagniard's notation, which we will adopt,

$$K_1^2 = K_\rho K_\tau$$

where K_1, K_ρ, and K_τ are the scaling factors for length, resistivity, and time, respectively, when $\epsilon = \epsilon_0$ and $\mu = 1$ in emu.

Although a model can be used to simplify certain aspects of a problem, or to verify certain theoretical results, it is seldom possible to simulate the desired geometry or physical properties altogether satisfactorily. For example, one can scale down from a very large structure to a suitable model size as accurately as one chooses, provided that the large structure is finite. If, however, one considers an infinitely extended medium, the difficulty is immediate. Scale models will thereby frequently suffer undesirable end effects. Furthermore, one is restricted in the choice of materials by the scaling factor imposed by the choice of geometrical dimensions and frequencies. Conducting solutions are the only readily available material for which a high degree of similitude can be preserved. If one is willing to somewhat relax this restriction, graphite can be shaped and imbedded in the solution to simulate various geologic structures. The two types of difficulties discussed above are common to all models, but in addition electromagnetic models suffer further difficulties in that other objects are coupled to the model through the source and also the model is indiscriminate in its response to outside sources. Indeed, a conducting solution is a most efficient aerial for undesirable radio frequencies from local broadcasting stations as well as power line noise.

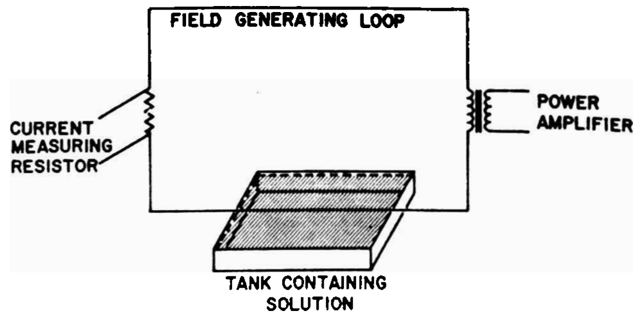

Fig. 2. Diagram of the experimental arrangement.

Methods for overcoming some of these difficulties will be discussed in the next section.

Description of apparatus. In Figure 2 a diagram of the experimental arrangements is shown. The source is a vertical loop of copper bus bar approximately 8 m along the horizontal edge and 4 m along the vertical. These dimensions were determined by the size of the laboratory. The height of the nearest horizontal side of the loop above the surface was approximately 60 cm except where noted (e.g., in Figure 7). This loop was energized, through a transformer with a single-turn secondary, from a 150-watt McIntosh audio power amplifier. Various transformer ratios could be selected in order to match at the different frequencies, the object being to maximize the current and hence the magnetic field. The transformer ratios used were between 1:8 and 1:2. The current was monitored in a noninductive element of the loop and used as a reference in all measurements. The noninductive element consisted of a section of bus bar folded back on itself and inserted into the loop. The conducting solution which acts as the upper layer of the earth was contained in a wooden tank approximately 2.3 m long and 1.1 m wide.

The magnetic field was measured with an air-cored coil, which was previously calibrated in terms of the free space response of the coil to a known magnetic field.

The electric field was derived from the measured voltage difference between two points 2 cm apart. It was thus the average field over this distance which was measured. In actual practice a third probe was introduced which was connected to the center tap of a differential amplifier, in this case the input of a Techtronix 502 oscilloscope. The unbalanced output of the oscilloscope was then fed into a Hewlett-Packard V.T.V.M. The traces on the oscilloscope for both the reference signal and either the electric or magnetic pickup signal provided a convenient nitor on the behavior of the whole system.

The third probe was introduced in order to cancel out common mode noise which affected both probes alike. This function it performed most efficiently; the noise which appeared when an attempt was made to measure the two-point voltage difference was many times larger than the signal, but it could be reduced to a tolerable level when the differential probe system was used, with no loss in signal amplitude. The x component of noise could not be affected by this device and the result was a broadening of the trace on the oscilloscope and a consequent uncertainty in the voltage measurement.

The thickness of the solution was approximately 30 cm, the resistivity was approximately 0.045 ohm-m, and the periods ranged from 9×10^{-6} to $3.6 > 10^{-3}$ sec. If one uses scaling factors $K_1 = 10^4$ and $K_\rho = 10^4$, then $K_T = 10^4$, and this model would correspond to a real layer of thickness 3 km and resistivity 450 ohm-m and the periods would correspond to a range from 0.09 to 36 sec.

For plane waves incident on a homogeneous half-space with the above quoted resistivity, the skin depth, which is given by

$$d = 1/(2\pi\sigma\omega)^{1/2} = (T_\rho/4\pi^2)^{1/2}$$

in emu or

$$d = (10T_\rho/4\pi^2)^{1/2}$$

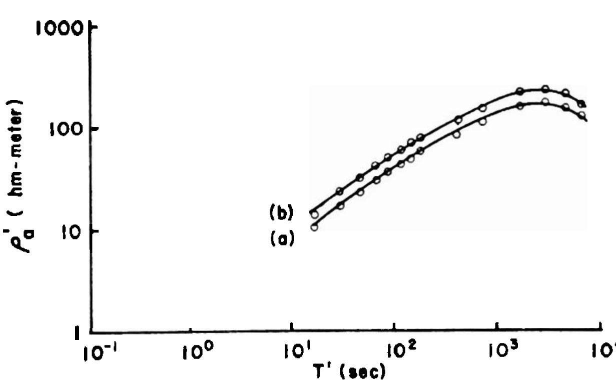

Fig. 3. Frequency analysis: curve (a) corresponds to the tank orientation as in Figure 2 with the longer dimension parallel to the electric vector, curve (b) corresponds to the tank oriented at right angles to the former direction. The scaling factor $K_t = 1.8 \times 10^6$ for the period has been applied.

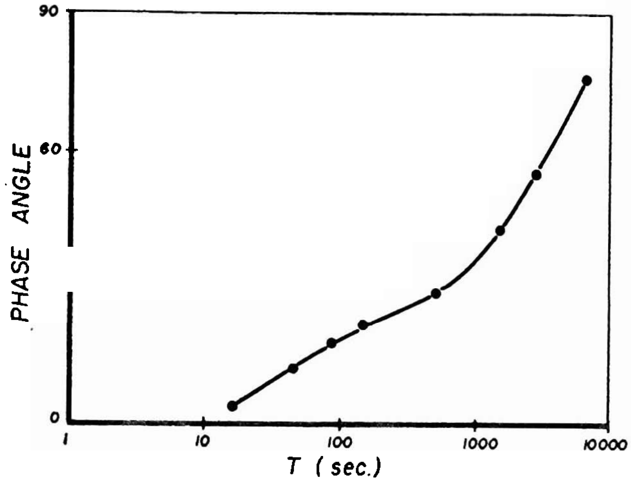

Fig. 4. The phase difference $\varphi(E_x) - \varphi(H_y)$ in degrees with the scaling factor $K_t = 1.8 \times 10^6$ applied to the periods.

in practical units, lies between 3.2 and 64 km.

The model will be used to investigate, among other things, the effect of such geologic structures as faults and dikes. The effect of these structures will be superimposed on the background magnetotelluric ratio which in turn will depend on the finite dimensions of the model as well as the nature of the source, which in this work is far from the plane wave type and is further discussed in the next section.

Results and discussions of results. In Figure 3, $\rho_a = 0.2T|E/H|^2$ is plotted against T on logarithmic scales. For use in later comparisons, the scaling factors, $K_\rho = 100$, $K_1 = 1.33 \times 10^4$, and $K_t = K_1^2/K_\rho \approx 1.8 \times 10^6$, are applied. The 'practical' units of *Cagniard* [1953] are used: E in mv/km, H in γ, and ρ in ohm-m. Curve (a) shows the results when the tank is oriented so that its long dimension is along the x axis, which is the direction of the electric field vector. Curve (b) is for the other orientation, i.e., with the tank at 90° to the former direction. The general result that the magnitude of E/H and hence of ρ was larger for case (b) may be expected if the solution in the tank can be considered as a two-dimensional dike of large lateral extent. The effect of the width of a dike has been discussed by *Rankin* [1962]. The relevance of this effect to certain results measured in the field will be discussed below.

Figure 4 displays the phase difference, $\phi(E_x) - \phi(H_y)$ plotted against the period with a logarithmic abscissa. The periods are scaled by the same factor as in Figure 3, i.e., 1.8×10^6.

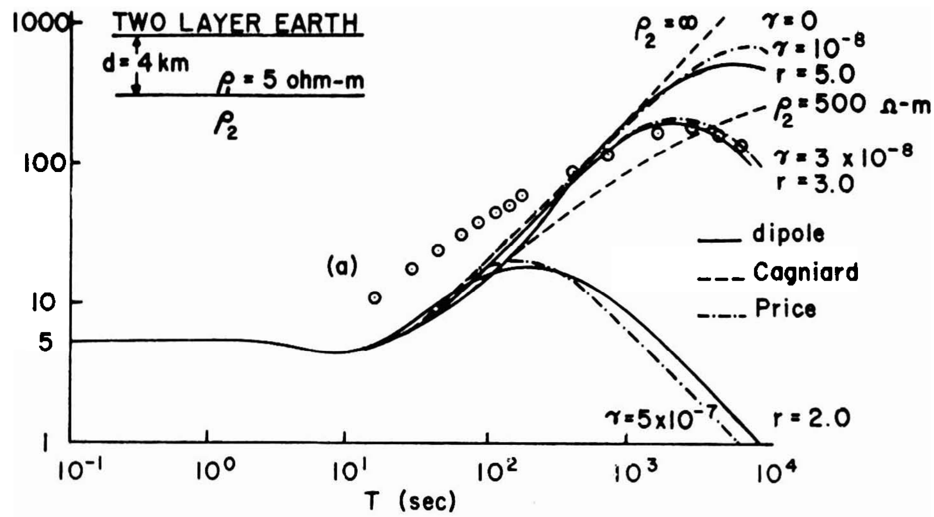

Fig. 5. The experimental results shown as curve (*a*) of Figure 3 are superimposed on the theoretical curves, after *Quon* [1963]. The parameter ρ_2, the substratum resistivity, denotes the Cagniard plane wave source. The parameter γ, the inverse of the source dimensions, denotes Price's more general source. The parameter r denotes the vertical magnetic dipole source as calculated by Quon, r being the ratio of horizontal distance to height from the origin directly below the dipole. The curve for $\gamma = 0$ coincides with that for $\rho_2 = \infty$.

The phase difference is independent of tank orientation.

The variation in the height of the lower edge of the source was from 0.6 to 0.9 m, corresponding to scaled-up values of 80 to 120 km. There was no appreciable variation in the E/H ratio with the height of the loop at the point directly below the center of the loop.

In Figure 5, curve (*a*) of Figure 3 is superimposed on the curves which were calculated by

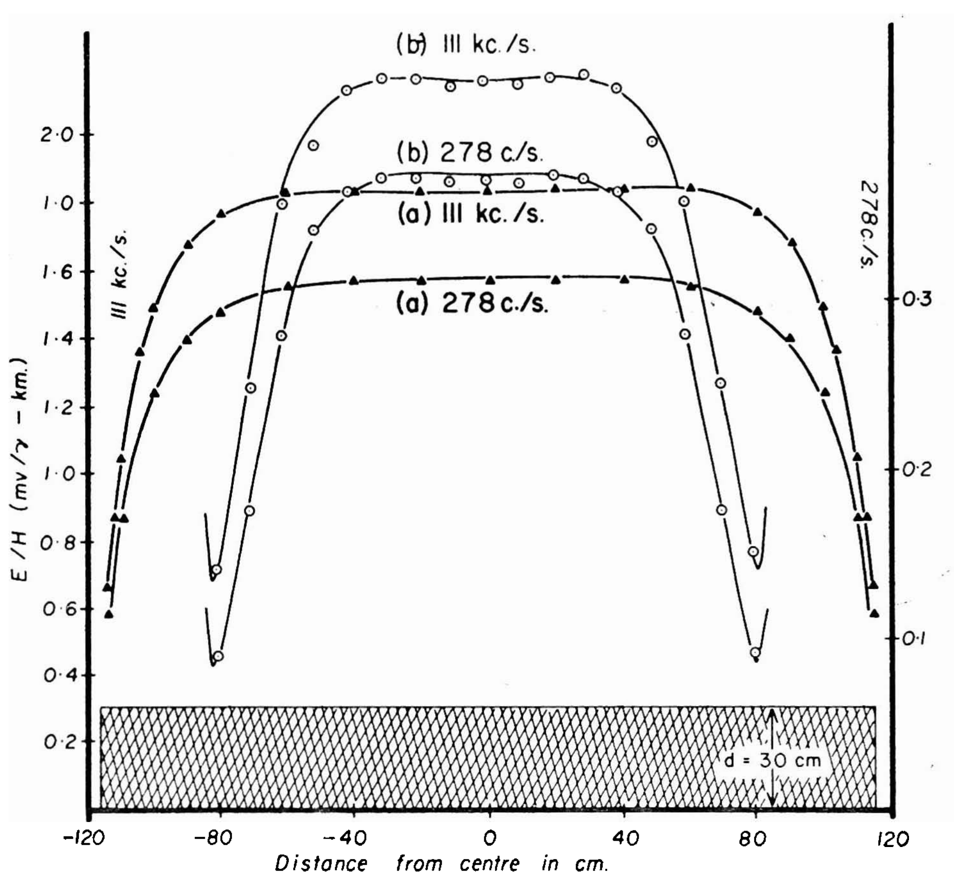

Fig. 6. The magnetotelluric ratio for traverses (*a*) in the direction of the electric vector and (*b*) in the direction of the magnetic vector.

Quon [1963] for the various sources as proposed by *Cagniard* [1953], *Price* [1962], and *Quon* (*ibid.*). The scaling factors for the results own in Figure 3 were chosen to agree with the calculated results of Figure 5. The decrease at the long-period end of the experimental curves is attributed to the finite source.

In Figure 6 are plotted the results of traverses in the x and y direction for the highest and lowest frequencies used in this work. The magnetotelluric ratio $|E/H|$ is plotted in place of the apparent resistivity which contains $|E/H|^2$. The curves marked (a) are in the x direction, which is that of the electric vector, and the curves marked (b) are in the y direction, which is that of the magnetic vector. The x and y directions are parallel to and perpendicular to the plane of the source loop, respectively, and it should be further noted that $E_y = H_x = 0$ at all times. In both cases the tank was oriented to make its long dimension in the direction of the traverse. In curve (a) there is a considerable region of flatness in the middle of the tank, and it would seem reasonable that lengthening the tank would lengthen this flat region. It is not obvious that increasing the lateral dimensions of the model will produce a flattening in case (b). The paper by *Law and Fannin* [1961] indicates the expected response for a line source of the magnetotelluric ratio. As is demonstrated in Figure 7, variations in the height of the lower side of the source loop will produce noticeable fluctuations in the magnetotelluric ratio, and it was fortuitous that the height chosen produced the flat response as shown. Only the highest-frequency results are shown since the results for the lowest frequency are similar.

Traverses in the x direction were normally made in the plane of the source loop, where the vertical component of the magnetic field was zero. In the y-direction traverse, the vertical component is not zero except at the origin, taken directly below the loop. In Figure 8 are plotted the results of calculations of the horizontal field H, the vertical field V, and the ratio

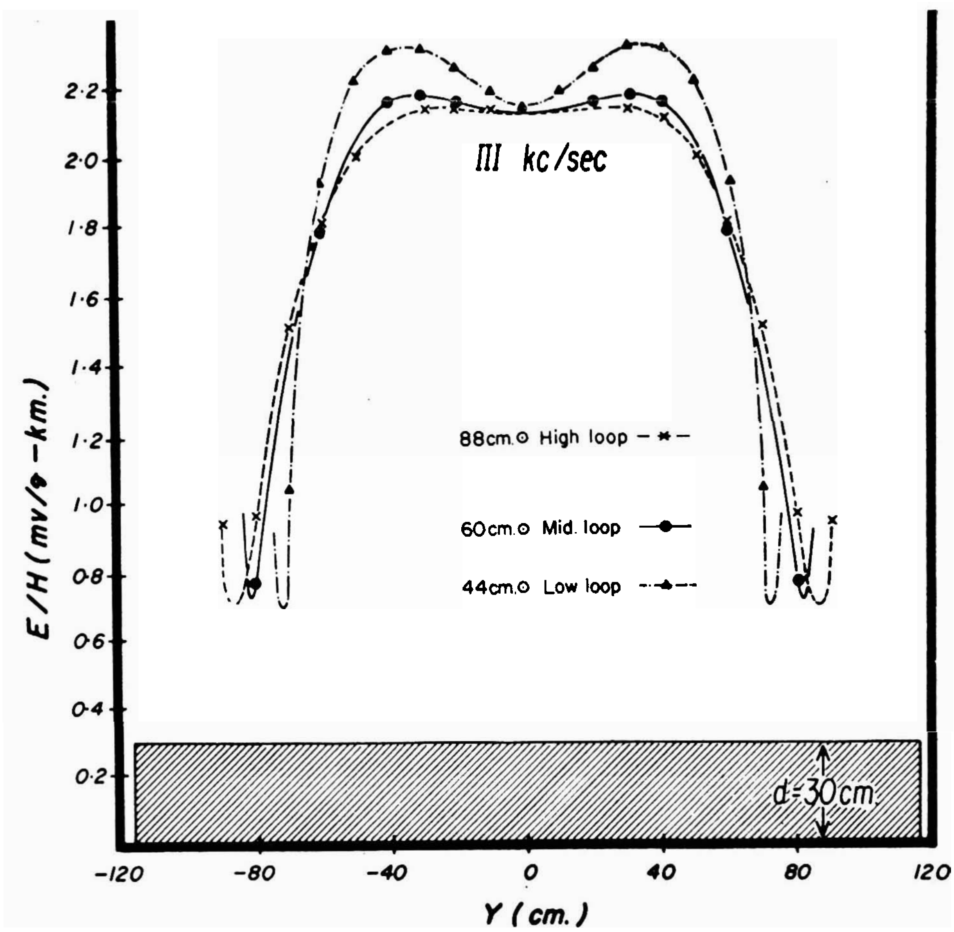

Fig. 7. The effect of the height of the source on the magnetotelluric traverses in the magnetic vector direction.

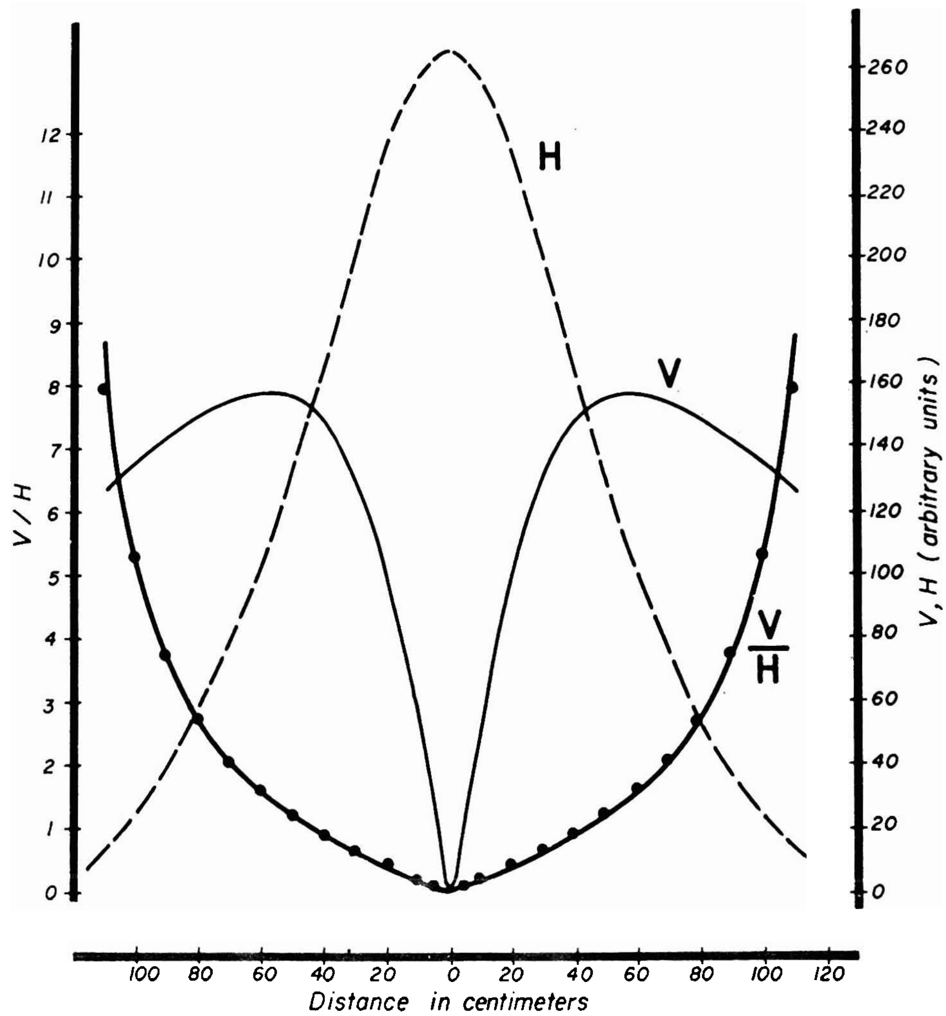

Fig. 8. The ratio of the vertical to horizontal magnetic field from a quasi-static source loop.

V/H. In these calculations the quasi-static field of the actual source was assumed. Measured values of the V/H ratio are shown as solid circles. The lower-frequency results only are shown. The measured results begin to show a departure as the edges of the model are approached; this departure is more pronounced at the higher frequency. The calculated values are for free space and the measured values are made with no solution in the tank.

The analysis of field measurements by many workers indicates the complexity of the problem. *Vozoff et al.* [1963] and *Srivastava* [1963] have attributed some of the discrepancy in the apparent resistivity when the orthogonal pairs of electric and magnetic components are interchanged to anisotropy in resistivity or minor structure below the station. The results of the present work as shown in Figure 3 indicate that the distant truncation of a uniform layer, which may be considered either as a sedimentary basin or an extended dike, can produce effects which are similar to local anisotropy. Further effects of dimensions are illustrated in Figure 9, in which all four curves are for traverses in the x directions for the orientations of the tank as indicated in the hatched portion at the bottom of the diagram. The curves (a) are the same as those marked (a) in Figure 6.

Although it would be advantageous to increase the dimensions of the model, a sufficient region of flatness exists to display the effect of structure on the magnetotelluric response in the case of traverses. Although the V/H ratio shows much more curvature, the effect of structure can also be observed in this case. Results of measurements on both two- and three-dimensional structures will be reported on in a forthcoming paper. It should be emphasized that, although the source used in this model is probably not representative of a real ionospheric source, the wavelengths are very much larger

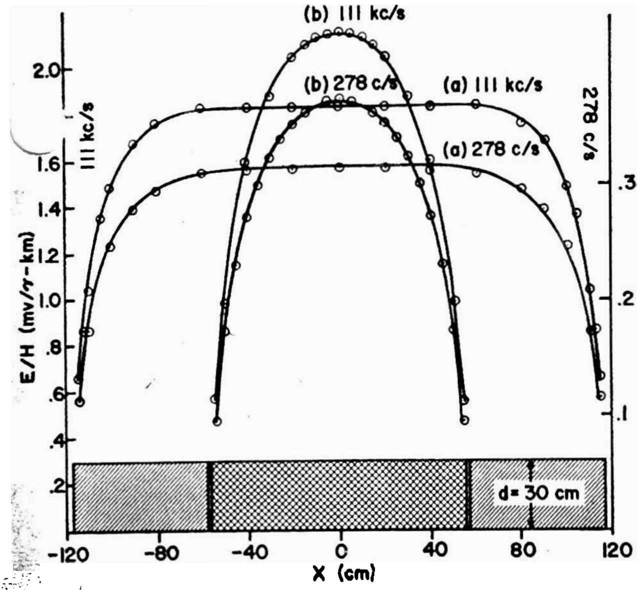

Fig. 9. The effect of tank dimensions on the magnetotelluric ratio.

than the extent of the model, and thus the results of surveys over structure will represent the effect of the structures themselves and not the finite dimensions of the source. Flattening into an approximately plane wave front can be achieved by a suitable array of conductors, but was not attempted in this work.

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