The Perturbation of Alternating Geomagnetic Fields by an Island near a Coastline: Reply

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Received August 17, 1973

Accepted for publication August 21, 1973

In his comments Rankin (1973) refers only to the paper by Lines and Jones (1973*a*). However, the original work leading to the general three-dimensional method used in that paper was developed earlier under my direction and preliminary results were published by Jones and Pascoe (1972). Under my supervision L. Lines extended and made more general the original work, and these extensions together with appropriate examples constitute the main part of his M.Sc. thesis (Lines 1972). Some of the results obtained in this work have been published in two papers (Lines and Jones 1973*a*, *b*), the first of which is the one referred to by Rankin. Therefore, as initiator and supervisor of this work I submit a reply.

In the three-dimensional problem the field components cannot be separated in the same way as in the two-dimensional problem (Jones and Price 1970) and must remain coupled. This coupling must be retained for any approximation to the solution. Maxwell's equations in e.m.u. with the usual quasi-static approximation made in geomagnetic studies and assuming a sinusoidal time variation exp $(i\omega t)$ lead to

$$[1] \qquad \nabla^2 \boldsymbol{E} - \nabla(\nabla \cdot \boldsymbol{E}) = i\eta^2 \boldsymbol{E}$$

where $\eta^2 = 4\pi\sigma\omega$.

As pointed out by Lines (1972), in the general three-dimensional problem all components of $\boldsymbol{E}$ vary with x, y, and z. The vector equation [1] may be rewritten as three scalar equations in cartesian coordinates, one of which is that referred to by Rankin:

$$[2] \quad \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) = i\eta^2 E_x.$$

The cross terms such as

$$-\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$

which arise from the $\nabla(\nabla \cdot \boldsymbol{E})$ term in [1] insure the proper coupling of the three field components.

In the numerical method, a pointwise solution to the problem is required. To avoid the increased complication and cost of computing the solution over a double mesh which would allow double-valued functions, the approach taken is to choose the normal component of $\boldsymbol{E}$ at a discontinuity as the average of the normal components on either side of the discontinuity, as discussed by Lines (1972). This is a reasonable approximation, particularly in geophysical situations. If we consider the boundary as a transition zone from one conductivity region to the other as it must be in the geophysical cases we are considering, then $\boldsymbol{J}(=\sigma\boldsymbol{E})$ must be continuous, and if σ varies continuously through the transition, $\boldsymbol{E}$ will vary as we have assumed.

The effect of the above approach is to replace η^2 by $\bar{\eta}^2$, the average of η^2 for all regions surrounding the point of interest, in the finite difference equations used to represent equations like [2]. This implies that the conductivity discontinuity is represented by a transition zone in the region of the discontinuity. It is not apparent, and Rankin (1973) has not shown that "the field component and its derivatives are grossly distorted in the region about the boundary" for geophysically realistic situations. He has not given any evidence to support his assertion that our "methods are in serious error". The procedure followed is a valid approximation to geophysical situations and is adequate for our purpose, which is the description of the behavior of the electric and magnetic fields near such changes in conductivity as we are considering.

As to the comments with respect to the "anti-skin-effect", let me first point out that Fig. 7 (*a* as well as *b*) of our paper (Lines and Jones 1973*a*) is a schematic illustrating how the current flow is deflected from regions of higher resistivity, and is not drawn from computed results, as implied by Rankin. The concept of skin-effect is, strictly speaking, only applicable for uniform conductors, and much more complicated situations arise when laterally non-uniform conductors are considered. In the latter situations the current lines will be distorted by the different conductive regions as well as influenced by the skin effect. A schematic drawn by Cox *et al.* (1970) of currents flowing perpendicular to a shelving coastline (their Fig. 11) shows a similar effect to that shown by us. In fact, results presented in a recent paper by Rankin himself (Reddy and Rankin 1973) support our view that the currents are deflected upward by the shelving discontinuity. In the paper by Reddy and Rankin, Fig. 5(a) shows apparent resistivity profiles calculated over a sloping contact. Curves 1, 2, and 3 are for three different slopes, and curve 4 is for a vertical contact. Since in this H-polarization case, the component of H is constant all along the surface, then these curves represent $|E_\perp|^2$ (from the Cagniard (1953) apparent resistivity equation, $\rho = 2T|E/H|^2$). If Rankin were to interpret his curves in terms of the currents flowing in the conducting region, he would realize that the up-bending of the curves 1, 2, 3 as the lower conducting region is approached from the higher conductivity side is due to the increase of $E_\perp$ on the surface associated with the convergence of current lines over the slope. This up-bending does not occur in the vertical contact case, since the current lines are not 'squeezed' together in that case. When the current lines enter the poorer conductor they then spread down (as illustrated by Cox *et al.* (1970)). If we had drawn current lines entering the poor conductor they would have spread downward, but our intent was to illustrate the deflection of the current by the shelf and not the well known skin-effect.

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