The Perturbation of Alternating Geomagnetic Fields by an Island Near a Coastline

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The numerical method of Lines and Jones for investigating the problem of the perturbation of alternating geomagnetic fields by three-dimensional conductivity inhomogeneities embedded in a layered Earth is extended to include models in which vertical discontinuities may extend to the grid boundaries. The case considered is that in which the electric field vector is parallel to the strike of the discontinuities at such boundaries. A model which includes an island near a coastline is studied and contour plots as well as profiles of the electric and magnetic field component amplitudes at the surface of the Earth are given.

La méthode numérique de Lines and Jones d'investigation du problème de la perturbation des champs géomagnétiques alternatifs au moyen d'inhomogénéités de conductivité tri-dimensionnelles encastrées dans les couches de la Terre a été améliorée pour inclure les modèles dans lesquels les discontinuités verticales peuvent s'étendre jusqu'aux extrémités de la grille. Nous considérons le cas pour lequel le vecteur du champ électrique est parallèle à la direction de la discontinuité à ces extrémités. Un modèle incluant une île près du littoral est étudié et on donne des contours ainsi que des profiles des amplitudes constituant le champ électrique et magnétique à la surface de la Terre.

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Introduction

The perturbation of alternating geomagnetic fields by the abrupt discontinuity associated with a coastline and similar problems of a two-dimensional nature have been considered by several authors (D'Erceville and Kunetz 1962, Rankin 1962, Weaver 1963, Jones and Price 1970, 1971, Jones and Pascoe 1971, and Pascoe and Jones 1972). Jones and Price (1970) have shown the form of the fields near a vertical discontinuity in conductivity and have illustrated the current vortices and wedges associated with the H-polarization and E-polarization cases of the two-dimensional problem. In these two-dimensional problems it is assumed that there is no variation in the fields in one direction, say the y-direction. For example, in the E-polarization case, only the E_y, H_x, and H_z components exist, and these are assumed to be constant in the y-direction along the strike of any discontinuities. Many geophysical problems exhibit a two-dimensional nature, but the assumption of a purely two-dimensional problem is sometimes not realistic, and in such cases it is impractical to assume that there is no variation in the structure in one of the dimensions.

An interesting model to consider is that of a long coastline of unlimited extent, with a nearby island of limited extent. This leads to a situation which is not purely two-dimensional throughout, and so must be considered as a three-dimensional problem. The fields are perturbed by the coastline and the island in such a way as to destroy the two-dimensional nature of the problem.

Jones and Pascoe (1972) have considered the three-dimensional geomagnetic perturbation problem by using a numerical method. Lines and Jones (1973, in press) have considerably extended the preliminary work to allow for a grid of variable mesh dimensions. In the previous work, the models considered were ones with inhomogeneities embedded in a layered conductor.

In the present work, we consider a model in which at a long distance from the three dimensional island, but in the direction of the coastline, the electric field is relatively undisturbed by the island and is parallel to the strike of the coastline. In this way, we may use the solution of the two-dimensional E-polarization problem to provide boundary values for the three dimensional model.

Description of the Model

The model used, shown in Fig. 1, is an island near a continental coastline with a shelf structure. The ocean–continental boundary strikes in the y direction. The conductivities used for the island and continental region, and for the ocean are given in Table 1 together with skin depths for the respective conductivities for a source period of thirty minutes. The model is described over a variable grid consisting of a 25 × 25 × 25 array of points. Table 2 gives the dimensions of the variable grid used. The position of the surface of the conducting region is indicated by the vertical line in the z-direction dimensions. The inner uniform grid portion of the surface plane is shown in Fig. 2, along with the notation used for labelling the profiles. The surface contours and profiles, shown in later sections, are given for the uniform grid portion shown in this figure. The x–z plane perpendicular to the coastline at some distance from the island is shown in Fig. 3, and is the two-dimensional conductivity configuration which is solved to provide the boundary values for the three-dimensional island and coastline model. The electromagnetic fields are assumed to have a sinusoidal time variation with a period of thirty minutes.

TABLE 1. Conductivities and skin depths of model

Conductivities	Skin depths for 30 min period
$\sigma = 0$ (air)	infinite
$\sigma_1 = 4 \times 10^{-11}$ emu (ocean)	10.7 km
$\sigma_2 = 1 \times 10^{-14}$ emu (crust)	675.2 km

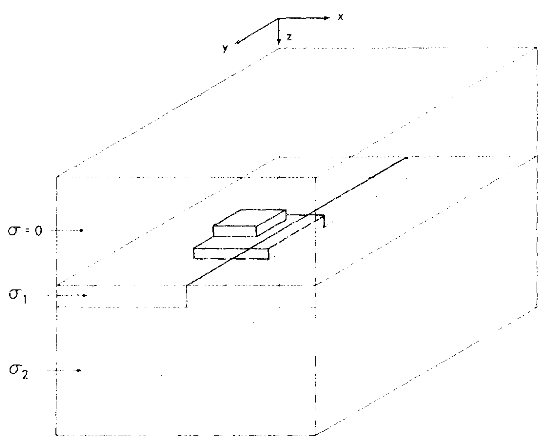

FIG. 1. A diagram of the three dimensional model considered—an island near a coastline.

Equations and Boundary Conditions

If we consider a situation in which the electric field vector of the source is polarized in the y direction (*i.e.* parallel to the coastline), then the equations for the two-dimensional E-polarization case are given by the following (Jones and Price 1970):

$$[1a] \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = 4\pi\sigma E_y$$

$$[1b] \qquad \frac{\partial E_y}{\partial z} = i\omega H_x$$

$$[1c] \qquad \frac{\partial E_y}{\partial x} = -i\omega H_z.$$

The above equations may be combined to give:

$$[2] \qquad \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = i4\pi\sigma\omega E_y,$$

which is the equation to be solved in all regions with the proper value for σ inserted for each region.

The outer planes of the grid are assumed to be sufficiently far from the coastline so that at the right hand and left hand external boundaries of Fig. 3, $\partial E_y/\partial x$ may be assumed to be zero. Also, surface values of H_x are taken as equal at the right and left hand boundaries of Fig. 3, as outlined by Jones and Price (1970). Also, it is assumed that $E_y \to 0$ as $z \to \infty$.

Subject to the above boundary conditions, a solution for E_y satisfying (2) is determined in the x–z plane for the coastline configuration illustrated in Fig. 3. This is then used to provide external boundary values for the E-field, and the three-dimensional perturbation method (Jones and Pascoe 1971, Lines and Jones 1973, in press) is then employed to give a solution for the electric field near the island. The magnetic field is then obtained by taking the curl of the electric field.

Results and Discussion

Contours of the surface amplitudes for the six field components are given in Fig. 4 and

TABLE 2. Spacing of grid points for variable grid (in kilometers)

			X direction				
95.00	45.00	15.00	5.00	3.00	2.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	2.00	3.00	5.00	15.00	45.00	95.00
			Y direction				
95.00	45.00	15.00	5.00	3.00	2.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	2.00	3.00	5.00	15.00	45.00	95.00
			Z direction				
99.00	80.00	40.00	20.00	10.00	5.00	3.00	2.00
1.00	1.00	1.00	2.00	4.00	8.00	16.00	32.00
64.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00

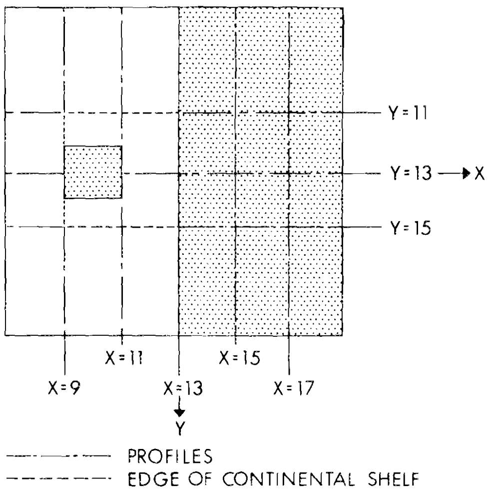

FIG. 2. The surface uniform grid region with some of the profiles considered. This region is 12 by 12 km.

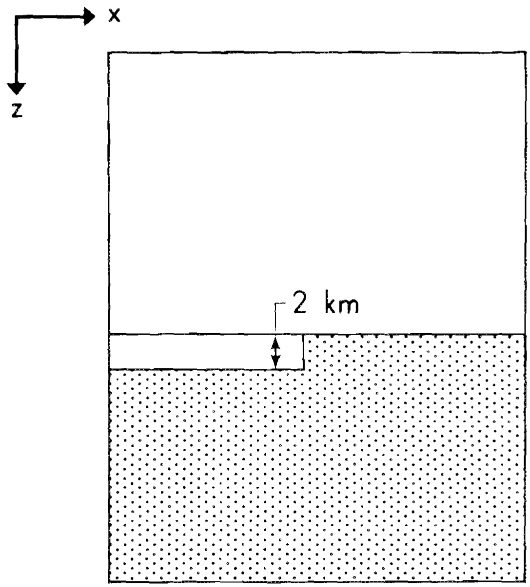

FIG. 3. The coastline configuration shown in the x–z plane.

profiles across the region illustrated in Fig. 2 are shown in Figs. 5 and 6. The amplitude contours and profiles of $|E_y|$ shown in Figs. 4 and 5 show that the island perturbs the E_y values associated with the coastline discontinuity. Over the island, the E_y amplitudes increase due to the presence of poorly conducting crustal material, causing contours of higher E_y values on the continent to be distorted toward the island. The E_x amplitude increases near the seaward corners of the island, indicating that at these positions, electric currents are bending around the island. Three-dimensional graphs of the phase of the electric field components are given in Fig. 8. The graph for the phase of E_x exhibits shifts of π radians giving the expected sign differences in the E_x components at any given time for currents bending around the island. These sign changes are expected from examining the current flow diagram of Fig. 7(a).

Contours of the amplitude of E_z shown in Fig. 4 display two maxima near the edges of the island-shelf structure. It should be noted that the values of E_z are related to the behavior of the vertical component of subsurface currents shown in Fig. 7(b). An explanation for the behavior of the E_z component in our results has been given by Price in a private communication (1972). Also, the explanation has been discussed previously by Lahiri and Price (1939) and Price (1950, 1962, 1967).

The normal component of current flow sets up a varying surface charge on the plane of discontinuity $z = 0$. This charge is given by

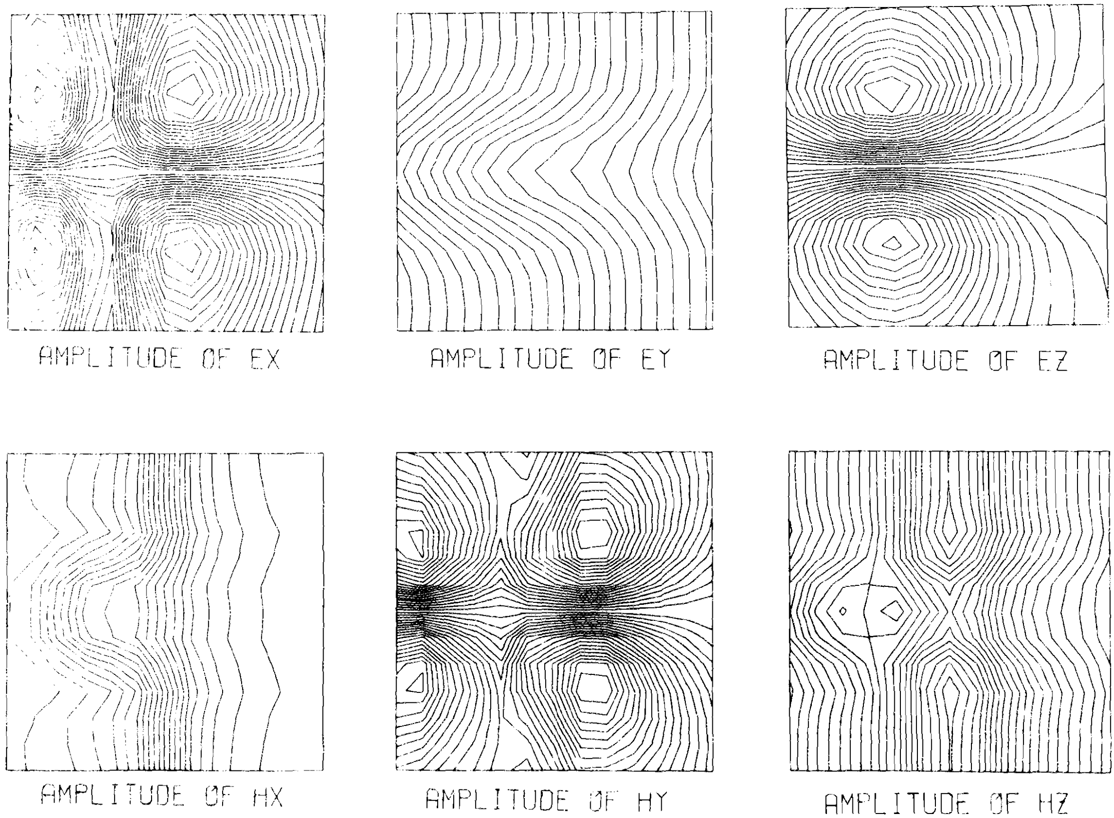

FIG. 4. Top: The amplitude contours of E_x, E_y, E_z in the uniform grid region at the surface. Bottom: The amplitude contours of H_x, H_y, H_z in the uniform grid region at the surface.

the surface boundary condition on the normal component of D, the electric displacement vector;

$$D_z'' - D_z' = 4\pi\beta,$$

or

$$\varepsilon'' E_z'' - \varepsilon' E_z' = 4\pi\beta.$$

In the above, β is the surface charge, the primed quantities refer to the medium just outside the conductor, and the double primed quantities refer to the medium just inside the conductor.

By the equation of continuity,

$$J_z'' = -\frac{\partial\beta}{\partial t},$$

where J_z'' is the current density impinging on the surface from within the conducting ocean. But,

$$J_z'' = \sigma E_z'' = -\frac{\partial\beta}{\partial t},$$

so that in substituting for E_z'' in the previous boundary condition on the normal component of $D(=\varepsilon E)$, we obtain the following equation in β:

$$\frac{\varepsilon''}{\sigma}\frac{\partial\beta}{\partial t} + 4\pi\beta = -\varepsilon' E_z'.$$

Since the time variation of the electric field is sinusoidal,

$$\frac{\partial\beta}{\partial t} = i\omega\beta.$$

Also, since displacement currents have been neglected under the condition that $\omega << \sigma/\epsilon''$, the term $(\epsilon''/\sigma)(\partial\beta/\partial t)$ in the preceding equation is negligible compared to the term $4\pi\beta$, and so

$$\beta \cong -\frac{\varepsilon'}{4\pi} E_z'.$$

Hence, the surface charge distribution, β, is approximately of order $-c^{-2}E_z'$.

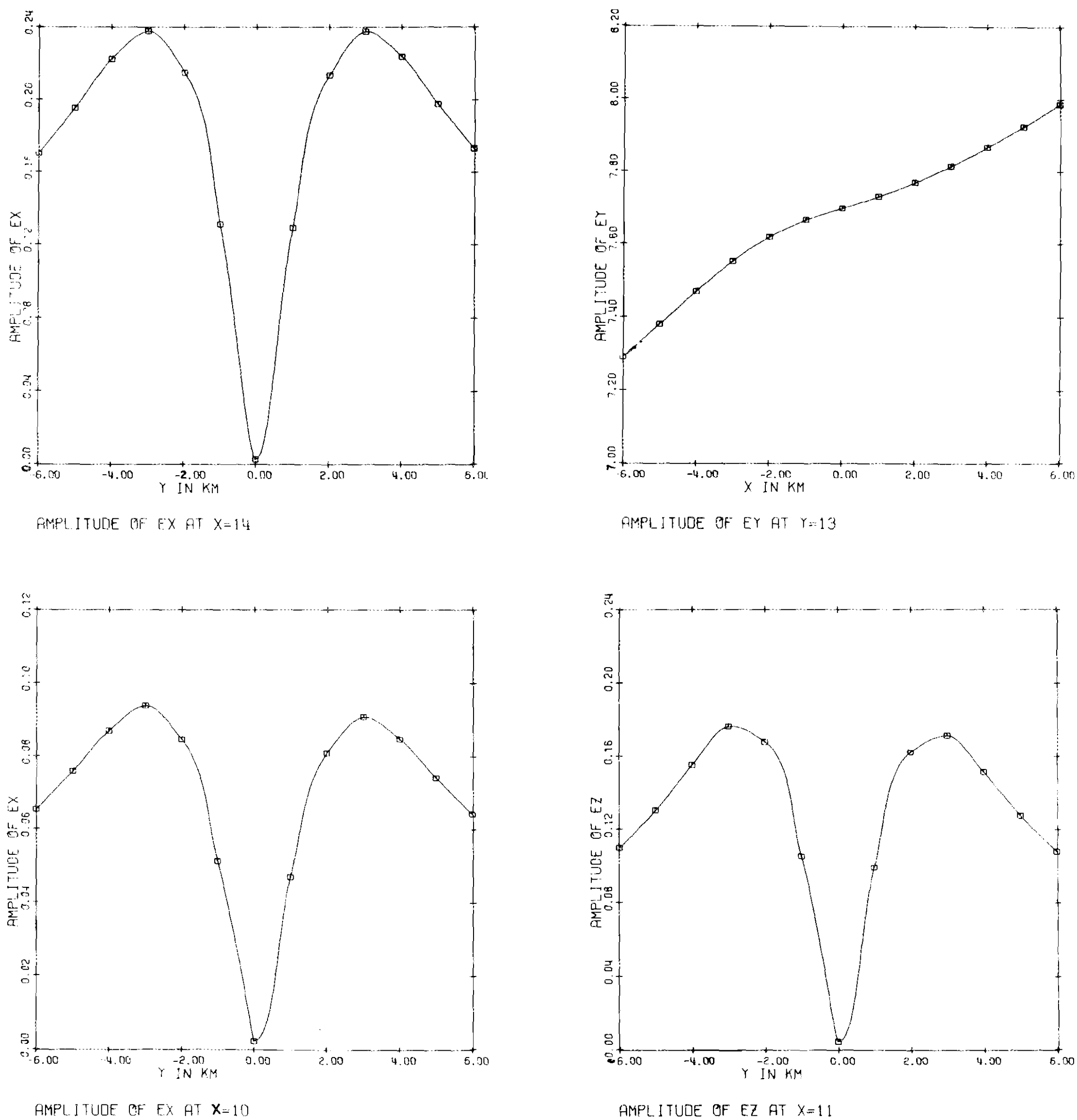

FIG. 5. Surface amplitude profiles of E_x, E_y, E_z.

Since $E_z'' = -(1/\sigma)(\partial\beta/\partial t)$ and

$$|(\epsilon''/\sigma)(\partial\beta/\partial t| << |4\pi\beta|$$

then

$$|E_z''| \ll \left|\frac{4\pi\beta}{\varepsilon''}\right|$$

or

$$|E_z''| \ll \frac{\varepsilon'}{\varepsilon''}|E_z'|.$$

The vertical component of the electric field just inside the conductor is negligible when compared to the vertical component of the electric field just outside the surface.

Therefore, the same minute time-varying surface charge which causes a non-zero electric field (E_z') outside the conductor reduces the vertical component of the electric field just inside the conductor to a negligible value, so that currents just inside the conductor essentially flow parallel to the surface. Price (1967) emphasizes that the current required to set up

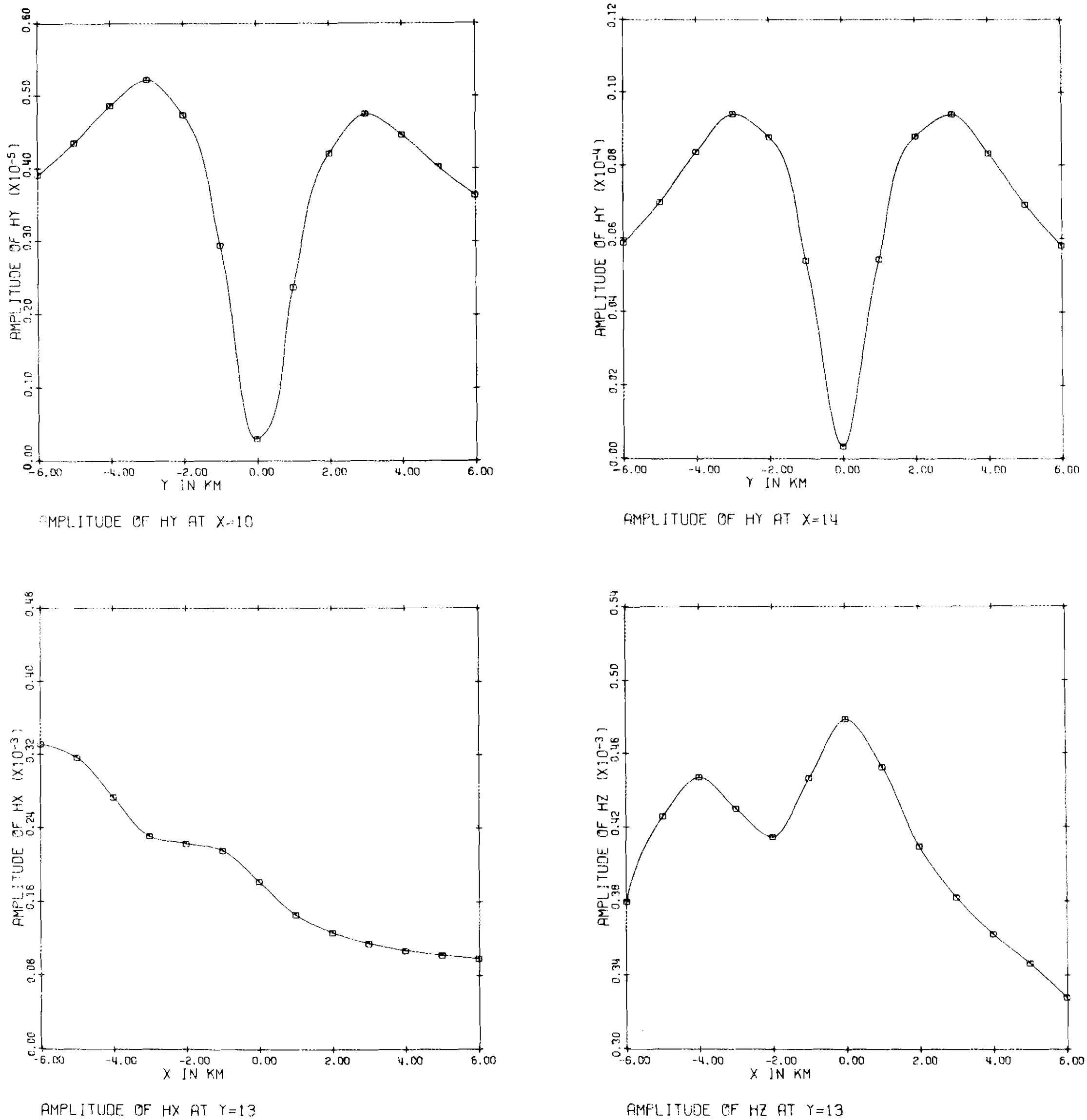

FIG. 6. Surface amplitude profiles of H_x, H_y, H_z.

the surface charge has a negligible magnetic effect.

The effect of a non-zero E_z' is shown in our results. Furthermore, since the size of the surface charge needed to cancel E_z just inside the surface of the conducting ocean depends on the strength of subsurface currents, the amplitude of E_z in our results is largest over regions where the subsurface currents are largest. Also, the sign of E_z changes with the sign of the subsurface currents, as is shown from the phase plots. It should be recognized that the amplitude of E_y is more than one order of magnitude greater than $|E_z|$.

The H_z component is found from the values of E_x and E_y by using

$$H_z = -\frac{1}{i\omega}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right).$$

The profiles and contours of Figs. 4 and 6 show a definite increase in $|H_z|$ near the coastline. A smaller maximum in H_z, shown in the $y = 13$ amplitude profile, is caused by the presence of the island. The island interrupts the flow of oceanic currents and causes variations in the H_z component. The effect of an isolated island on H_z has been described by Mason (1963), Price (1967), and Klein (1971), and was illustrated in a previous model by Lines and Jones (1973, in press).

However, the dominant effect on H_z in the present results is due to the coastline effect. The effect of a coastline on surface values of $|H_z|$ has been discussed by several authors including Rikitake (1964), Schmucker (1964), Roden (1964), Price (1967), and Cox *et al.* (1970).

Schmucker (1964) observed an increase in H_z amplitudes for profiles transverse to the California coastline. He considered variations in period between 30 minutes and 2 hours and suggested that the $|H_z|$ increase may have been caused by concentrations of electric currents in the ocean flowing parallel to the coastline.

Cox *et al.* (1970) give a physical explanation for the increase of $|H_z|$ near a coastline. Mutual repulsion of oceanic current lines which are parallel to the shoreline cause an increased current density near the edge of the continent. This results in a subsequent increase in $|H_z|$ at the surface.

By using theoretical calculations and laboratory model studies, Roden (1964) concluded that ocean currents in the Pacific Ocean contributed to anomalous magnetic variations in Japan.

It should be noted that observations of H_z variations along a coastline may not be entirely due to the effect of currents in the ocean. A high concentration of currents in good conducting material shallowly buried beneath the ocean may also cause the increase in $|H_z|$. Rikitake's studies in Japan suggest that the anomalous H_z behavior may be due primarily to subcrustal conductivity structures. Investigations by Parkinson (1964) of the preferred planes of magnetic variations along the Australian coast also implied that oceanic currents were not the only cause of magnetic variations near a coastline. Another possible cause of the variations is the difference between the mantle structure under the continents, and the oceans, as was pointed out by Price (1967).

The amplitude of H_x depends on current flow in the y–z plane. This current flow is less through the island and the continent, causing a decrease in the H_x component over these regions. This is shown in the amplitude contours and profiles of $|H_x|$ of Figs. 4 and 6.

The behavior of H_y agrees with the application of the right hand rule to currents in the x–z plane. The amplitudes of H_y are shown in a contour plot in Fig. 4 and in two profiles in Fig. 6. The correct sign relationship between H_y components at any given time is shown by the three-dimensional phase plots which indicate phase differences of π radians where appropriate.

The values of $|E_y|$ are normalized to 1.0 on the far left hand side of the model shown in

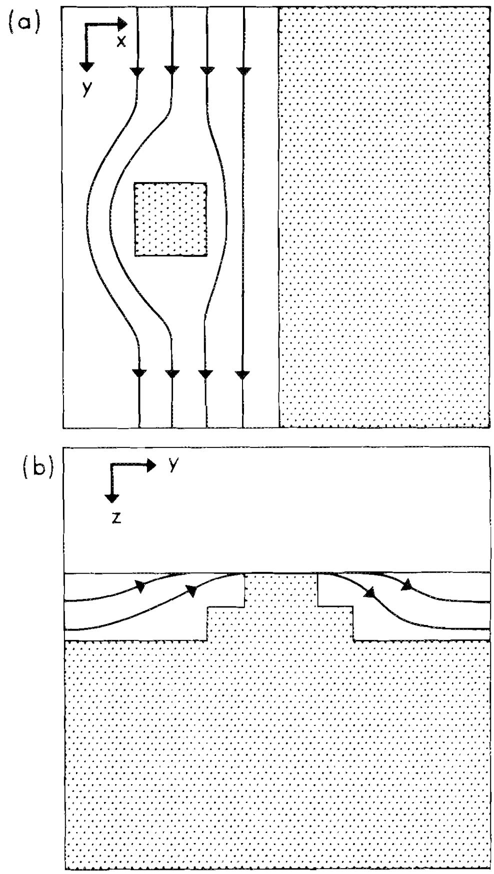

FIG. 7. a) Flow of electric current lines in the x–y plane near the coastline.
b) Flow of electric current lines around the island in the y–z plane.

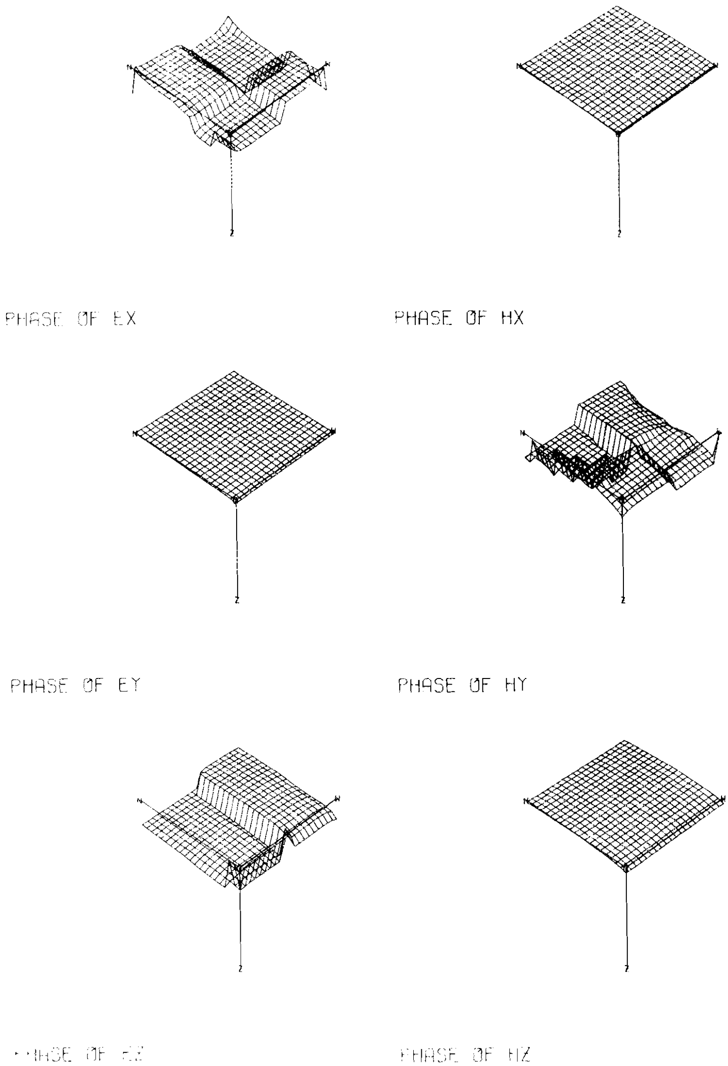

FIG. 8. Three dimensional plots of the phases of the electric and magnetic fields. $N \equiv X$ direction, $W \equiv Y$ direction. The inner 21×21 surface grid region is plotted.

Fig. 3. All other values of the E and H components are scaled accordingly.

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