Discussion on "Geomagnetic Effects of Sloping and Shelving Discontinuities of Earth Conductivity," by F. Walter Jones and Albert T. Price (GEOPHYSICS, February 1971, p. 58–66)

In this recent paper, Jones and Price describe the application of finite difference methods to determining the electromagnetic fields near a vertical or inclined contact between materials having different resistivities. In the case of H polarization (the only case discussed in the following comments) they observed that as one approaches the contact from its conductive side the apparent resistivity decreases, which is to say that E horizontal decreases since H horizontal is constant. This behavior has previously been pointed out by d'Erceville and Kunetz (1962) for an analytical calculation. Jones and Price interpret this phenomenon as the effect of surface charges concentrating at the interface due to the impinging current.

The purpose of this communication is to suggest that the presence of a surface charge appears to be incompatible with the boundary conditions the authors applied and furthermore a surface charge is not needed to explain the effect they observe.

The boundary condition under consideration is the continuity of normal current density across electrical discontinuities which can be determined from first principles using Ampere's circuital law from Maxwell's equations in mks units:

$$\nabla \times H = \frac{\partial D}{\partial t} + J. \tag{1}$$

We are interested in the movement and concentration of free charges and neglect polarization effects from bound charges. Applying Stokes's theorem to equation (1) and evaluating the line integration of H for two identical contours parallel and close to the contact but one on either side of it, we obtain

$$\oint_1 H_1 \cdot dl = \iint_{S_1} \left[\frac{\partial D_1}{\partial t} + J_1\right] \cdot \hat{n} ds \tag{2a}$$

$$\oint_2 H_2 \cdot dl = \iint_{S_2} \left[\frac{\partial D_2}{\partial t} + J_2\right] \cdot \hat{n} ds \tag{2b}$$

where the subscripts 1 and 2 denote the two media and S_1 and S_2 are the surfaces of integration. We've implied that $S_1 = S_2$. Since the tangential horizontal field components are continuous across the interface, the left sides of equations (2a) and (2b) are equal and

$$\iint_{S_1} \left[\frac{\partial D_1}{\partial t} + J_1\right] \cdot \hat{n} ds = \iint_{S_2} \left[\frac{\partial D_2}{\partial t} + J_2\right] \cdot \hat{n} ds,$$

or

$$\frac{\partial D_{1n}}{\partial t} + J_{1n} = \frac{\partial D_{2n}}{\partial t} + J_{2n}, \tag{3}$$

the subscript n denoting the normal component of the field vector.

Equation (3) is the general condition on the normal components of the field quantities across the interface between two lossy dielectrics. For restrictive assumptions regarding the nature of the materials or the physical problem, this equation leads to the condition in electrostatics that

$$D_{2n} - D_{1n} = \tau \tag{4}$$

where τ is the surface charge density, or to the condition on slowly varying fields in conductors that

$$J_{1n} = J_{2n}. \tag{5}$$

We see that this equation is compatible with the electrostatic condition in equation (4) if we assume that initially ($t=0$) all quantities are zero and displacement fields are then created by the buildup of charge at the interface by the differential flow of current normal to the interface. By integrating equation (3) with time we obtain

$$D_{2n}(t) - D_{1n}(t) = \int_0^t (J_{1n} - J_{2n}) dt, \tag{6}$$

where the term on the right is just the instantaneous surface charge density τ.

On the other hand if we assume that the normal component of current flow is continuous across the boundary then condition (5) implies the right-hand side of equation (6) is zero, the normal displacement fields must be continuous across the interface, hence there can be no concentration of surface charge at the interface.

The question becomes, what does cause the depressed values of E horizontal on the conducting side of the vertical boundary? This can be interpreted as simply the geometric effect of currents readjusting to a different skin depth as they pass through the boundary. Electric current lines, as they pass from a shallow depth of penetration in the good conductor to a larger depth of

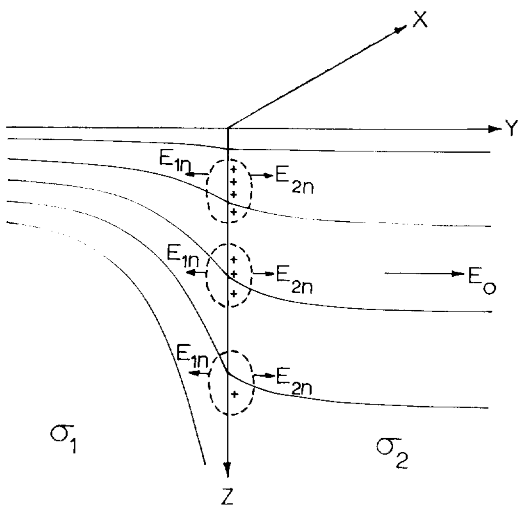

FIG. 8. From Jones and Price (1970).

penetration in the poor conductor, must undergo geometrical spreading. On the conducting side of the contact this implies that current density and, from Ohm's law, the electric field intensity *decreases* as one approaches the contact. On the resistive side of the contact this same effect causes the electric field to *increase* as the contact is approached from the other direction. This is exactly the phenomenon observed by Jones and Price and previously by d'Erceville and Kunetz.

To emphasize that this indeed is a geometric effect we invoke two extreme models, a vertical contact between two infinitely deep conductors and a vertical contact between two thin sheets (sheet thickness$\ll$skin depth) on an infinitely resistive substratum. In the first model currents are unconstrained and adjust to different depths across the boundary. In the second model currents are constrained to flow in the horizontal plane in both conductors. In both cases E horizontal is discontinuous across the boundary by the ratio of resistivities, a result of the boundary condition (5). For the thin layer case the ratio of E horizontal on either side of the contact at large distances away, remains equal to the ratio of resistivities. There is no depression or enhancement of E horizontal at the contact for the constrained case. If indeed a concentration of charge was present at the interface between two thin sheets, the contact should look like a linear charged filament. E horizontal for such a distribution would depend inversely on distance. We simply do not see such an effect. For thin sheets E horizontal is constant in either media though sharply discontinuous across the boundary.

On the other hand, for the unconstrained case of two infinitely deep layers as one progresses away from the contact, the apparent resistivity must approach the resistivity of the material beneath the observer and, since the apparent resistivity in Cagniard's relation is proportional to the square of E horizontal then the ratio of E horizontal at large distances on either side of the contact is equal to the *square root* of the resistivity contrast. Therefore there is considerable change in E horizontal as the contact is approached for the unconstrained case. The comparison of these two models demonstrates the strong effect geometrical spreading can have on E horizontal.

In summary, the boundary condition explicitly used by Jones and Price regarding the continuity of current density normal to the interface precludes the concentration of surface charges. Furthermore the effect that surface charges were invoked to explain is more readily interpreted as the result of geometridal spreading of current lines across the interface.

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Reply by authors to discussion by John F. Hermance

We do not regard the criticism by J. F. Hermance of our interpretation of the results of our H-polarization calculations (nor the alternative "explanation" given by him of our results) as valid, for the following reasons:

1) Only a minute surface charge on the interface between the conductors is needed to produce the electrostatic field, which drives a conduction current in each conductor and leads to the calculated surface effects. This varying surface charge is extracted from the currents impinging on the interface, but it is of the *same order of magnitude* as a displacement current (though not a displacement current itself). This current is of order 10^{-9} (or less) of the conduction current, and therefore, like the displacement current, is negligible in so far as its magnetic effects are concerned. *Nevertheless, the electrostatic field of the resulting (varying) surface charge is of the same order of magnitude as the other electromotive forces in the calculations.* This remarkable fact is perhaps obscured by the different systems of units used in electromagnetic studies, and may not have been sufficiently emphasized in our paper, though it was mentioned in our 1970 paper, quoted by Hermance, and has been pointed out in many other papers, including Lahiri and Price, 1939; Price, 1950; Jones and Price, 1971.

2) The "geometric effect" quoted by Hermance is certainly associated with the surface field effects and the resulting effect on the Cagniard apparent resistivity, but it does not constitute a physical explanation of these effects. In fact, the geometric effect itself can be attributed to the charge distribution found by us on the interface.

To confirm the above arguments in detail, we discuss the case of the vertical contact between two conductors discussed by Hermance, and we adopt the notation and the mks units used by him.

We agree of course with all the equations (1–6) given by him. His equation (3) can be written

$$J_{1n} - J_{2n} = -\frac{\partial}{\partial t}(D_{1n} - D_{2n}).$$

The right-hand side of this equation is of the same order of magnitude as the displacement current, which, as usual, we have ignored as negligibly small for fluctuations in the range of frequencies of interest in these problems. It is in this sense only that the equation (5) ($J_{1n}=J_{2n}$) is true in the present application. It is a very good approximation, but [$\tau=D_{2n}-D_{1n}$, equation (4)] is not *exactly* zero, and the corresponding electrostatic field is significant. We have, in fact,

$$E_{1n} = \frac{1}{\epsilon_1} D_{1n}, \quad E_{2n} = \frac{1}{\epsilon_2} D_{2n},$$

where ϵ_1 and ϵ_2 are each of order 10^{-9} or less (ϵ in vacuo $=10^{-9}/36\pi$ in mks units). Thus E_{1n} and E_{2n} are of the same order of magnitude as the other electromotive forces in the conductors and contribute conduction currents of the same order of magnitude. We have

$$J_{1n} = \sigma_1(E_0 + E_{1n}), \quad J_{2n} = \sigma_2(E_0 + E_{2n}),$$

where E_0 is the applied electric field normal to the interface. Also, by symmetry,

$$D_{1n} = -D_{2n}.$$

Hence E_{1n} and E_{2n} are of opposite signs.

The normal electric field is thus decreased in the good conductor σ_1 and increased in the poor conductor, as indicated in the figure. This accounts for one aspect of the geometric spreading of the electric field, referred to by Hermance.

We now examine the precise *physical* mechanism by which this geometric spreading is brought about. To obtain the spread of the E lines and the corresponding spread of the current lines, the lines must be bent and refracted as in the figure. [For more details obtained from actual calculations, see Figure 8, p. 329 of Jones and Price (1970).] The down bending lines in the better conductor (σ_1) show that there is a net electric force E_z in the downward z direction. This net force E_z arises from the nonuniform distribution of surface charge on the interface. The same electrostatic force acts in the conductor σ_2 but its effect on the current lines is much less because of the lower conductivity. Alternatively, if one thinks in terms of lines of electric force, the vertical force E_z at a given depth is the same in both conductors, but the net horizontal force is greater in the σ_2 conductor, so that the lines are bent less out of the horizontal and are therefore refracted at the interface. The surface charge engendered on the interface is thus the physical cause of the geometric spreading of the lines and the concomitant surface field effects.

The above argument refers to Hermance's first model. With regard to his second model, the adjacent thin sheets, there is certainly a concentration of charge along the dividing line of the two conducting sheets, but this does *not* look like a linear charged filament in free space as he implies; this is because all the currents and therefore also all the lines of electric force are confined to the horizontal sheet and do not radiate all round from the filament. Consequently the field does *not* decrease inversely with distance, but remains constant at all distances.

In conclusion, we should like to emphasise again that

1) The minute current extracted from the currents impinging on the inteface, though having the order of magnitude of a displacement current is not itself a displacement current. Its magnetic field is negligible, but the electrostatic field of the surface charge it builds up is significant.

2) The currents arising from this electrostatic field are true *conduction* currents, proportional to the conductivity of the conductor.

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Discussion on "Radius of Investigation in dc Resistivity Well Logging," by A. Roy and R. L. Dhar (Geophysics, August 1971, p. 754–760)

This recent article continues the approach taken by the authors in an earlier article (Roy and Dhar, 1970) on induction systems. As in that earlier article, the authors confine their attention to homogeneous media.