

tonic belts in eastern Newfoundland like those described by McCartney (1967) and reinterpreted by Hughes and Brückner (1971); (b) an undetected plutonic-metamorphic terrane to the northeast of the Avalon Peninsula now beneath the continental shelf, as supported by Papezik (1972); and (c) locally on the west, the plutonic-metamorphic rocks of the eastern crystalline belt of Jenness (1963), and Kennedy and McGonigal (1972). I suspect that the Cambrian sediments were derived mainly from Hadrynian rocks on the Avalon Peninsula and beneath the continental shelf. During the late Early Ordovician the undetected plutonic-metamorphic terrane became uplifted and supplied quartz-rich sand and silt rich in detrital muscovite to a depressed platform upon which 5000 feet of strata accumulated (Rose 1952).

The plutonic-metamorphic terrane, or more likely the older part of it in the eastern crystalline belt, may indeed extend eastward beneath the Avalon Peninsula and continental shelf as proposed by Papezik and thus be the source of the 'exotic' detrital minerals in the Hadrynian and lower Paleozoic strata. The age of this crystalline terrane supposedly beneath the continental shelf is unknown beyond being pre-latest Precambrian; the 540-m.y. K-Ar date (Wanless *et al.* 1972) on detrital musco-

vite from the Bell Island Lower Ordovician strata is regretfully not helpful. Nevertheless, the lithologic character of these Hadrynian and Paleozoic strata on the Avalon Platform, and the lack of severe deformation, metamorphism, and extensive plutonism, favor the hypothesis that the Hadrynian and Paleozoic strata were laid down upon a continental crust.

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The Perturbation of Alternating Geomagnetic Fields by an Island near a Coastline: Discussion

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The computational results reported by Lines and Jones (1973) are no doubt accurate, but their interpretation can certainly be clarified. The difficulty lies with the effect of regions of inhomogeneous conductivity (σ) on electric fields. The problem has received some attention recently (Hermance 1972; Price and Jones 1972) and not so recently (Lahiri and Price 1939), but the following analysis appears to have the virtue of generality.

In this analysis I show first that any charge on the interface must leak off the interface in a time much less than the period of a geomagnetic disturbance. I then adapt well known EM boundary conditions to the approximations made in geomagnetism to show that these imply that a charge density must nevertheless be resident on the interface. The resolution of this paradox is that current flow supplies charge as fast as it leaks off. The final step is to estimate

the electric fields associated with the charge density and show that these explain the effects observed in the model calculations by Lines and Jones. Their emphasis on displacement currents is an unjustified complication. The problem is easier to understand if they are ignored as they must be in geomagnetic computations.

Any flow of charge (ρ) must satisfy a continuity equation

$$[1] \quad \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

where \mathbf{J} is the current density. If \mathbf{J} is related to the electric field by Ohm's law

$$[2] \quad \mathbf{J} = \sigma \mathbf{E}$$

The continuity equation becomes

$$[3] \quad \mathbf{E} \cdot \nabla \sigma + \sigma \nabla \cdot \mathbf{E} + \frac{\partial \rho}{\partial t} = 0$$

Using exactly the argument in Stratton (1941, p. 15) we get

$$[4] \quad \mathbf{E} \cdot \nabla \sigma + \frac{\sigma}{\epsilon} \rho + \frac{\partial \rho}{\partial t} = 0$$

where σ is the conductivity. For homogeneous media ($\nabla \sigma = 0$) the decay time for ρ is at least 5 orders of magnitude less than the characteristic period of any geomagnetic disturbance. It follows that no *fixed* charges can exist in uniform conducting rocks.

An approximate argument can take account of the term $\mathbf{E} \cdot \nabla \sigma$. Suppose the conductivity change associated with the discontinuity is a finite smooth transition and not abrupt. An abrupt discontinuity can always be considered in the limit. Let E_n be the field normal to the discontinuity and assume a charge distribution ρ exists in the transition.

If ρ is finite in the transition the density of field lines for changes on going through the transition (Fig. 1). Consider an element of area Σ_s on the high conductivity side. The field lines entering Σ_s emerge on the lower conductivity side of the transition in an area Σ_l . Consider the volume V contained by the bounding field lines, Σ_s and Σ_l . Let E_{ns} be the normal field on the high conductivity side and E_{nl} the normal field on the lower conductivity side. Then we have, for small Σ_s , Σ_l with an application of Gauss's theorem

$$[5] \quad E_{ns} \Sigma_s + E_{nl} \Sigma_l = \rho V / \epsilon$$

Let Σ be the average of Σ_s and Σ_l and let d be the local thickness of the transition. When d is small Eq. [5] becomes

$$E_{ns} + E_{nl} = \rho d / \epsilon$$

The gradient in conductivity can be written as

$$|\nabla \sigma| \approx (\sigma_s - \sigma_l) / d$$

where σ_s is the conductivity on the high conductivity side and σ_l on the low conductivity side. An approximate form of $\mathbf{E} \cdot \nabla \sigma$ is thus

$$\mathbf{E} \cdot \nabla \sigma \approx \left(\frac{\sigma_s - \sigma_l}{2\epsilon} \right) \rho$$

and the continuity equation becomes

$$[6] \quad \frac{(\sigma_s - \sigma_l)}{2\epsilon} \rho + \frac{(\sigma_s + \sigma_l)}{2\epsilon} \rho + \frac{\partial \rho}{\partial t} = 0$$

Since Eq. [6] is independent of d the equation for decay of ρ with time becomes

$$\frac{\sigma_s}{\epsilon} \rho + \frac{\partial \rho}{\partial t} = 0 \quad \text{or} \quad \frac{\sigma_l}{\epsilon} \rho + \frac{\partial \rho}{\partial t} = 0$$

and charge will, as might be expected, leak off the interface either with a time constant of the magnitude ϵ/σ_s or ϵ/σ_l . Hence, if no fixed charge exists in the homogeneous media, none can exist in the transition zones.

Although charge leaks off the interface very rapidly it remains to show to what extent the normal component of current restores this charge. In order to do that we adapt an argument found in Stratton (1941, p. 483).

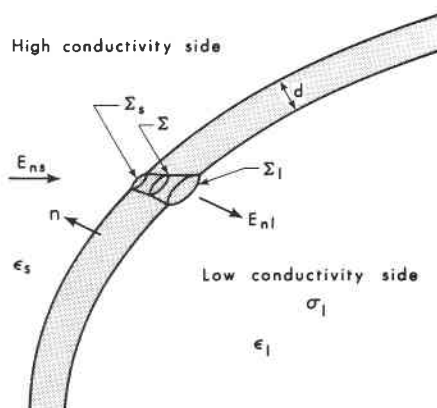


FIG. 1. A typical element of transition from high conductivity to low conductivity material. The dielectric constants ϵ_s and ϵ_l are approximately equal.

Any EM field, not just the restricted problem considered by Lines and Jones must satisfy 3 boundary conditions

$$[7] \quad \mathbf{n} \times \mathbf{E}_s - \mathbf{n} \times \mathbf{E}_1 = 0$$

$$[8] \quad \epsilon_s \mathbf{n} \cdot \mathbf{E}_s - \epsilon_1 \mathbf{n} \cdot \mathbf{E}_1 = \beta$$

$$[9] \quad \mathbf{n} \cdot \mathbf{J}_s - \mathbf{n} \cdot \mathbf{J}_1 = -\frac{\partial \beta}{\partial t}$$

\mathbf{n} is the normal to the interface, β is the charge density, if any, on the surface. Equations [8] and [9] can be written

$$[10] \quad \begin{aligned} \epsilon_s E_{ns} - \epsilon_1 E_{n1} &= \beta \\ \sigma_s E_{ns} - \sigma_1 E_{n1} &= +i\omega\beta \end{aligned}$$

in which a trigonometric time dependence $\exp(-i\omega t)$ is assumed. Only if $\epsilon_s \sigma_1 - \epsilon_1 \sigma_s = 0$ can solutions exist for the case $\beta = 0$. In spite of the high leakage rate, current flow will usually restore the transition layer charges. Solving Eq. [10] we get

$$\begin{aligned} E_{ns} &= -\frac{\begin{vmatrix} 1 & -\epsilon_1 \\ +i\beta\omega & -\sigma_1 \end{vmatrix}}{(\epsilon_s \sigma_1 - \epsilon_1 \sigma_s)} = \frac{(\sigma_1 - i\omega\epsilon_1)\beta}{\epsilon_s \sigma_1 - \epsilon_1 \sigma_s} \\ E_{n1} &= -\frac{\begin{vmatrix} \epsilon_s & 1 \\ \sigma_s & +i\beta\omega \end{vmatrix}}{\epsilon_s \sigma_1 - \epsilon_1 \sigma_s} = \frac{(\sigma_s - i\omega\epsilon_s)\beta}{\epsilon_s \sigma_1 - \epsilon_1 \sigma_s} \end{aligned}$$

The geomagnetic approximation must be applied to the boundary conditions. This approximation is the neglect of displacement currents and is valid when

$$\omega^2 \epsilon \mu \ll \omega \sigma \mu \quad \text{or}$$

$$\omega \epsilon \ll \sigma \quad \text{where } \epsilon_1 \approx \epsilon_s \approx \epsilon$$

With this condition the electromagnetic wave equation becomes a diffusion equation. Within this approximation and the condition $\sigma_1/\sigma_s \ll 1$

$$E_{ns} \approx \frac{\sigma_1 \beta}{\sigma_s \epsilon} \quad E_{n1} \approx +\frac{\beta}{\epsilon}$$

Eliminating β between these equations

$$E_{ns} \sigma_s = E_{n1} \sigma_1$$

This is the condition on the normal component of current. Within the approximations the boundary condition on \mathbf{D} is ignored by Lines and Jones. This is possible because current \mathbf{J}_s

flowing through the transition from the high conductivity side to the low conductivity side leaves a charge $+(J_s/\sigma_1)\epsilon$ on the transition. This charge is restored by the current as fast as it leaks off.

Although the charge is very small, it must exist. The geomagnetic approximations are applied after the boundary conditions are solved, not before. The magnitude of the field near such a charge is *not negligible*. It is routine to show that a field $\mathbf{E} = +\mathbf{J}_s/\sigma_1$ affects the normal component of \mathbf{E} at distances far removed from the edges of, or changes in the transition zone, but near the interface itself. The anomalous vertical field at the surface, near the edge of the interface, must be of this magnitude at least.

The preceding argument is essentially the one used by Lines and Jones (1973) and Jones and Price (1972). It differs only in what I feel is a clearer analysis of the place of the geomagnetic approximation in the boundary conditions. The problem is also clearly explained in a rather well known text (Stratton 1941).

Once the magnitude of the interface charge is established and estimates are made of its effect on the electric field, such things as spreading of field lines become a simple consequence of the existence of charge on the interface. Such charging is intimately associated with current flow through media of continuously variable conductivity as well. In this case it is easy to show that terms involving σ appear where one would expect sources in the EM field equations. Sources of EM fields can only be charges.

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