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THE EFFECT OF A FAULT ON THE EARTH'S NATURAL ELECTROMAGNETIC FIELD*

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The main purpose of this paper is to study the effect on a natural electromagnetic field of a lateral variation in the physical properties of the ground.

An exact mathematical solution is given for two media of different resistivities in contact along a vertical plane (fault) overlying a horizontal basement that is taken as being either infinitely resistive or infinitely conductive, or at infinite depth. Results are given in the form of curves along profiles perpendicular to the fault. Some practical inferences are drawn from the shape of the curves and from their comparison.

EXTENT OF SURVEYS BY TELLURIC CURRENTS AND THEIR RELATION TO THE MAGNETO-TELLURIC METHOD

Before taking up the study of a particular example of the application of natural electromagnetic fields in prospecting, it should be recalled that these fields have already supplied geophysics with a valuable tool, the telluric current method. This tool has been widely used, often with complete success, particularly in the Eastern hemisphere. The magneto-telluric method is very closely related to certain aspects of the telluric current method, so that the analysis of the experimental and theoretical results obtained in these studies could be quite valuable.

This latter point will be taken up again, but first, a few statistics will be given to show the extent of the work done using the telluric method, which was first employed by the Schlumberger brothers in the "thirties" and which reached its full expansion during and after World War II.

The upper part of Table 1 gives the crew-months, the number of measuring points, and the kilometers of profile with geographic distribution, representing the activity of one French geophysical contractor. As can be seen, the use of the method has reached a high level. During the period under review, crew income exceeded six million dollars.

Although activity in this field has decreased in Western Europe during the last few years, the method is being developed in the Eastern countries, particularly in East Germany, Russia, and Hungary. At the last Geophysical Congress held in Budapest (1959), four papers were devoted to it. The last line of Table 1 shows an activity figure for the U.S.S.R.

Table 1. Exploration activity by telluric method 1941–1955.

	Crew months	Setups	Kilometers of profile
France	230	32,000	25,000
Other European Countries	70	13,000	10,000
North Africa	111	28,000	11,000
Equatorial Africa Madagascar	108	18,000	13,000
U. S. A.	23	2,700	2,100
Venezuela	19	2,000	2,200
Asia	4	300	200
Total	565	96,000	63,500
1955–1957			
U.S.S.R.	312		

Details on the technique of the method or the results obtained will not be presented in this paper. A large number of publications have been devoted to the subject.

The majority of the surveys using this method have been made on the assumption that the telluric field is practically stationary (frequency independent). The validity of this hypothesis was checked experimentally by comparing the results

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given by the use of different frequency components. In the range of the frequencies dealt with, which lies between 1/10 and 1/100 cps, the influence upon the results of these frequency variations were usually found to be negligible. This was in agreement with the theoretical distribution of a time-varying electromagnetic field in a stratified medium, allowing for the order of magnitude of the resistivities involved.

However, in some cases, significant frequency effects were noted. These effects were sometimes greater than could be explained by theory, assuming the strata were horizontal, and this led in part to the present study.

In any event, the results of a study of the variation of the telluric current field as a function of frequency are closely linked to those obtained from a magneto-telluric study. For instance, in the case of cylindrical structures (i.e., structures which have a constant cross-section), it is easily shown that the magnetic field caused by a sheet of current normal to the axis of the structure consists of a single component which is horizontal and parallel to the axis of the structure, and which has the same instantaneous value everywhere at the surface. Therefore, if the profile of the simultaneous values of the telluric field is considered along a cross-section perpendicular to the axis of the structure (values which can also be found by successive measurements, by relating them to a fixed reference base), this profile will be similar, within a constant coefficient, to that which would have been found by the magneto-telluric method. In order to correlate the profiles corresponding to different frequencies, it is necessary to determine the variation of the ratio of the magnetic field to the telluric field as a function of frequency at a single point, either by actual measurement or by calculation. Calculation is feasible if the zone around the point chosen is geologically undisturbed and the distribution of resistivities is known. It follows in this case that the magnetic field is essentially a calibration standard.

The situation is more complicated in the case of structures of arbitrary form. Definite analogies do exist nevertheless. For instance, the two components of the magnetic field are linear functions of the simultaneous components of the telluric field at the same point, and the simultaneous components of the telluric field are linear functions at two different points.

However, experience both in the field and in the laboratory has shown that the hypothesis of "cylindrical" structures is often a very good approximation for anomalies which are even slightly elongated. Moreover, in the present problem, the theoretical study of the effect of cylindrical structures already presents sufficient difficulties.

A particular example of these structures is covered in the following.

EFFECT OF A VERTICAL FAULT ON A NATURAL ELECTROMAGNETIC FIELD

When natural electromagnetic fields are studied, horizontal homogeneous strata are generally assumed. This assumption is not satisfactory in applications to the search for anomalies which deviate from such an arrangement.

Unfortunately, assuming lateral changes in the electromagnetic properties usually results in great mathematical difficulties. For this reason, a particular case (obviously idealized) is considered for which calculations can be carried out without approximation, in order to illustrate the phenomena corresponding to similar field conditions.

A vertical fault of infinite length, separating two formations of different resistivities ρ_1 and ρ_2, is assumed. These two formations rest on the same horizontal substratum of infinite resistivity or infinite conductivity. As a special case, the fault can be extended to an infinite depth. The telluric current is assumed to be horizontal and normal to the fault plane at an infinite distance from the fault.

Under these conditions, the magnetic field on the ground surface is not only horizontal and parallel to the fault, but in addition, its intensity is the same at all points. In contrast, the intensity of the electric field on the surface is a function of the distance between the point of measurement and the fault plane. In the case of a natural electromagnetic field (the intensity of which is highly variable as a function of time), the ratio of the component of the telluric field normal with the fault to the simultaneous component of the magnetic field parallel with the fault is independent of these time variations of intensity. As the magnetic field is invariant in space, variations of this ratio occurring along a profile perpendicular to the fault depend only on the variation of the electric field.

Of course, in addition to the distance to the fault and the geometric and electrical characteristics of the formation, results also depend on the frequency.

METHOD OF SOLUTION[1]

In the foregoing hypotheses, all vertical sections normal to the fault are identical from the electromagnetic point of view, and the magnetic field, which is always normal to these sections, can be represented by a scalar H.

The vertical at one point of the fault is taken as the z axis, and the horizontal normal to the fault as the x axis. Accordingly, the derivatives of the fields with respect to the third co-ordinate axis, y, will be zero.

Disregarding the displacement currents and using the relation Curl $H=4\pi i$ (noting that at the surface i_z is zero), it follows that $\partial H/\partial x=0$, which proves that the magnetic field on the surface is the same at all points.

Considering now a sinusoidal field with angular frequency, ω, the cases of an infinitely resistant, infinitely conductive, and infinitely deep substratum will be discussed.

Substratum of infinite resistivity

The solution for the magnetic field, H, is obtained. From this the electric field on the surface, normal to the fault, E_x, is calculated by means of the formula

$$E_x = \frac{1}{4\pi}\rho\frac{\partial H}{\partial z}.$$

The expressions for the magnetic field are written separately in the two compartments. These expressions have to satisfy boundary conditions at the surface of the earth, at the surface of the substratum, and at infinity (for $x=\pm\infty$), and must satisfy Maxwell's equations. They each comprise a set of arbitrary coefficients which will be determined in turn by the continuity of the magnetic field and that of the vertical component of the electric fields at the fault plane (for $x=0$).

H has been found to be constant on the surface of the earth. In the case of an infinitely resistant substratum, the same must be true on the surface of the substratum. On the other hand, at an infinite distance from the fault, it no longer has any effect, and the variation of the field with depth z should be that which can be calculated for an indefinite horizontal stratum. Take H_1^0 and H_2^0 as the expressions of these undisturbed fields, functions of z, which are different in the two compartments because of the difference in resistivity. In each medium, a field of disturbance due to the fault is superposed on these undisturbed fields. If P_1 and P_2 represent these disturbances, and H_1 and H_2 the total fields, then

$$H_1 = H_1^0 + P_1$$

and

$$H_2 = H_2^0 + P_2.$$

The disturbances P_1 and P_2 are expanded to a Fourier series (as sines), as a function of z, in the interval 0 to h ($h=$depth of the substratum). In this way the disturbance will be zero for $z=0$, and $z=h$, as required by boundary conditions. As to the second factor of Laplace's product, i.e., the function of x, it is determined to within a constant factor by the fact that each term must be a solution of Maxwell's equations and become zero for x infinite.

The fields of disturbances will thus be developed in series of terms such as

$$a_{in}\sin\frac{n\pi\cdot z}{h}\exp\left[\pm x\sqrt{\frac{n^2\pi^2}{h^2}+\frac{4\pi j\omega}{\rho_i}}\right] \qquad (i = 1 \text{ or } 2),$$

the sign being taken in such a way that the exponent will be negative.

All that now remains to be done is to determine the two series of constants a_{1n} and a_{2n}. This is done by stating that H and E_z must be continuous for $x=0$ at the point where the fault is crossed.

To express equality of the magnetic fields, the undisturbed fields H_1^0 and H_2^0 are also expanded into Fourier series. Then, after adding this to the series representing the disturbances, the two expressions obtained for $x=0$ are identified, term by term on both sides of the fault. This gives a first series of linear equations (not homogeneous) for the series of constants a_{1n} and a_{2n}.

To ensure continuity of the vertical electrical field, it suffices to state the equality of the dis-

[1] The detailed mathematical derivation of the solution, along with its complete analytical expressions, are given in the Appendix.

turbing fields, the other fields being horizontal at all points. This equality is expressed as

$$\rho_1 \frac{\partial P_1}{\partial x} = \rho_2 \frac{\partial P_2}{\partial x},$$

and gives the second series of linear equations which, with those resulting from the equality of the magnetic fields, permits calculating the series of the a_{1n} and a_{2n} pairs.

The magnetic field H is thus completely determined and from this the electric field E, which must be zero on the surface of the substratum.

Substratum of infinite conductivity

Calculation in this case is similar to the preceding one, taking into account that this time it is the horizontal component of the electric field, E_x, which must be zero on the surface of the substratum.

Now, examination of the formulae relating to the insulating substratum shows that the above condition is satisfied in this case for $z=h/2$, for all uneven terms of the series giving the disturbances. In these terms, if h is replaced by $2h$ (and x by $x/2$), the disturbances relating to the conductive substratum are obtained.

Substratum of infinite depth

This case is the common limit of the two cases considered above, when h tends towards infinity. The solution could be obtained by passing to the limit on the results obtained, but it is easier to revert to the calculations, substituting Fourier integrals (in sines) for the expansions into Fourier series.

RESULTS OBTAINED

In view of their length, the analytical expressions of the solution in the various cases are not presented here (they will be found in the Appendix), but a few results of numerical calculations in the form of curves are presented.

Values of the parameters involved

First, it will be observed that, as was to be expected, results depend only on dimensionless quantities:

$$h\sqrt{\frac{\omega}{\rho_1}},\quad h\sqrt{\frac{\omega}{\rho_2}},\quad \text{and}\quad \frac{x}{h}\,.$$

In the case of faults of finite depth (h, finite), profiles perpendicular to the fault have been represented by showing, as abscissae on a linear scale, the ratio x/h of the distance, x, between the point of measurement and the fault plane to the depth, h, of the latter, and as ordinates, first, the ratio of the moduli (on a logarithmic scale), and second, the phase difference of the electric field, E_x, and of the magnetic field, H, in radians and in degrees on a linear scale.

The two other parameters adopted are: the ratio of resistivity of the two compartments, ρ_2/ρ_1, to which ratio have been given the numerical values 9 and 100; and the ratio d_1/h of the "depth of penetration"

$$\left(d_1 = \sqrt{\frac{\rho_1}{2\pi\omega}}\right)$$

in the medium of greatest conductivity to the depth of the fault, h. The values $\sqrt{2}$, $\sqrt{2}/2$, $\sqrt{2}/4$ have been assigned to this latter ratio.

These two parameters are kept constant on each curve and the curves relating to a single resistivity ratio have been grouped on the same graph. Moreover, to relate the graphs one to another, it has been assumed that the resistivity ρ_2 of the most resistive compartment remains the same for both ratios ρ_2/ρ_1.

Variation of the field for various depths of penetration

Figure 1 shows these results for a substratum of infinite resistivity, while Figure 2 gives those corresponding to a substratum of infinite conductivity. A comparison of the curves relating to these two extreme cases permits an approximate estimation of the outline of the profiles which would be obtained with a substratum of finite resistivity, especially since it is easy to calculate the asymptotes of these profiles (for $x=\pm\infty$ and $-\infty$) by using the formulae relating to infinite horizontal strata.

For given rock characteristics (resistivities and depth), the various curves on a single graph correspond to different frequencies of the field. According to whether the strata involved are thin and resistive or, on the contrary, thick and conductive, the values adopted can correspond to frequencies of an entirely different order of magnitude. Thus, for strata 100 meters thick, and for

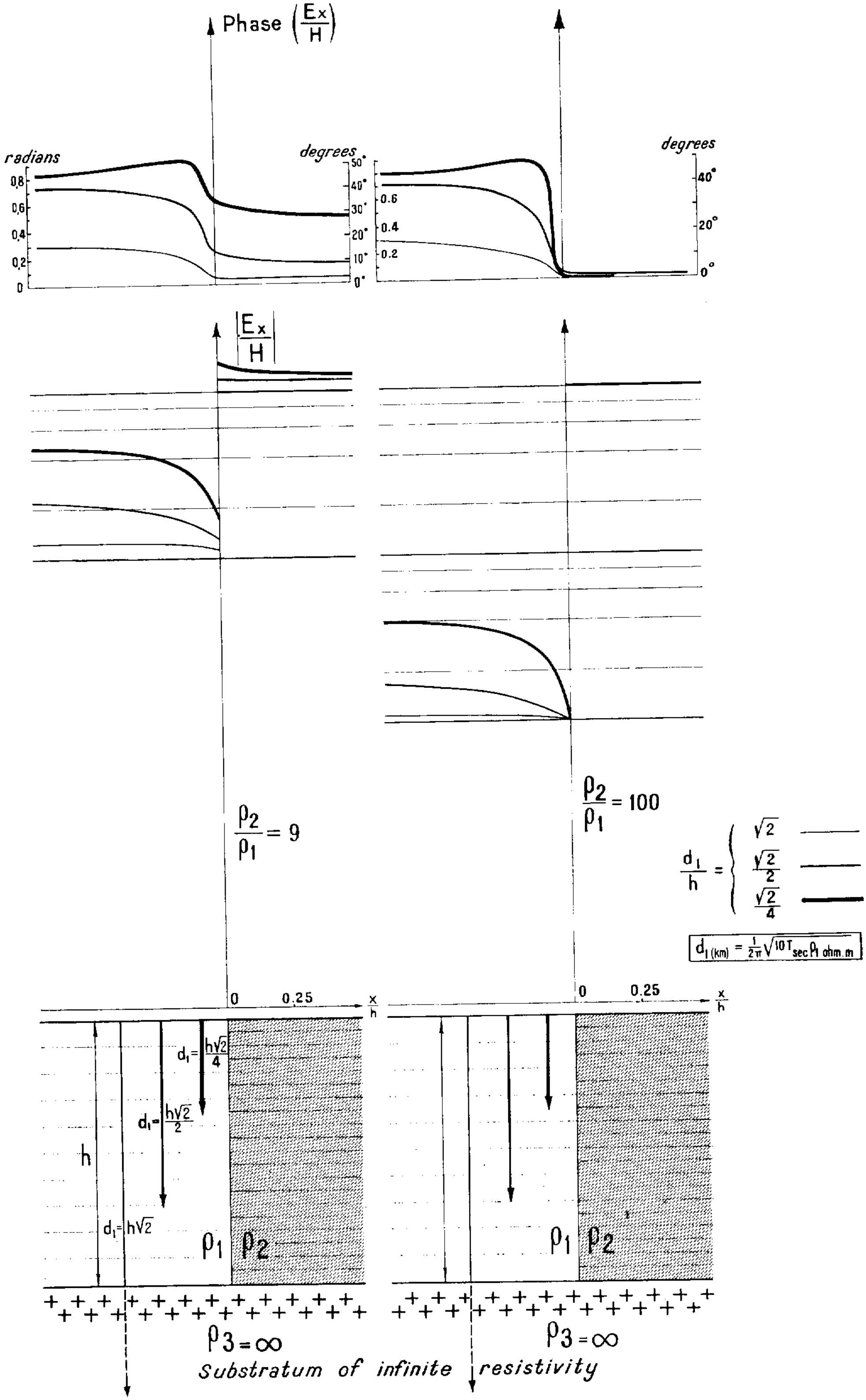

FIG. 1. Variation of the field for various depths of penetration.

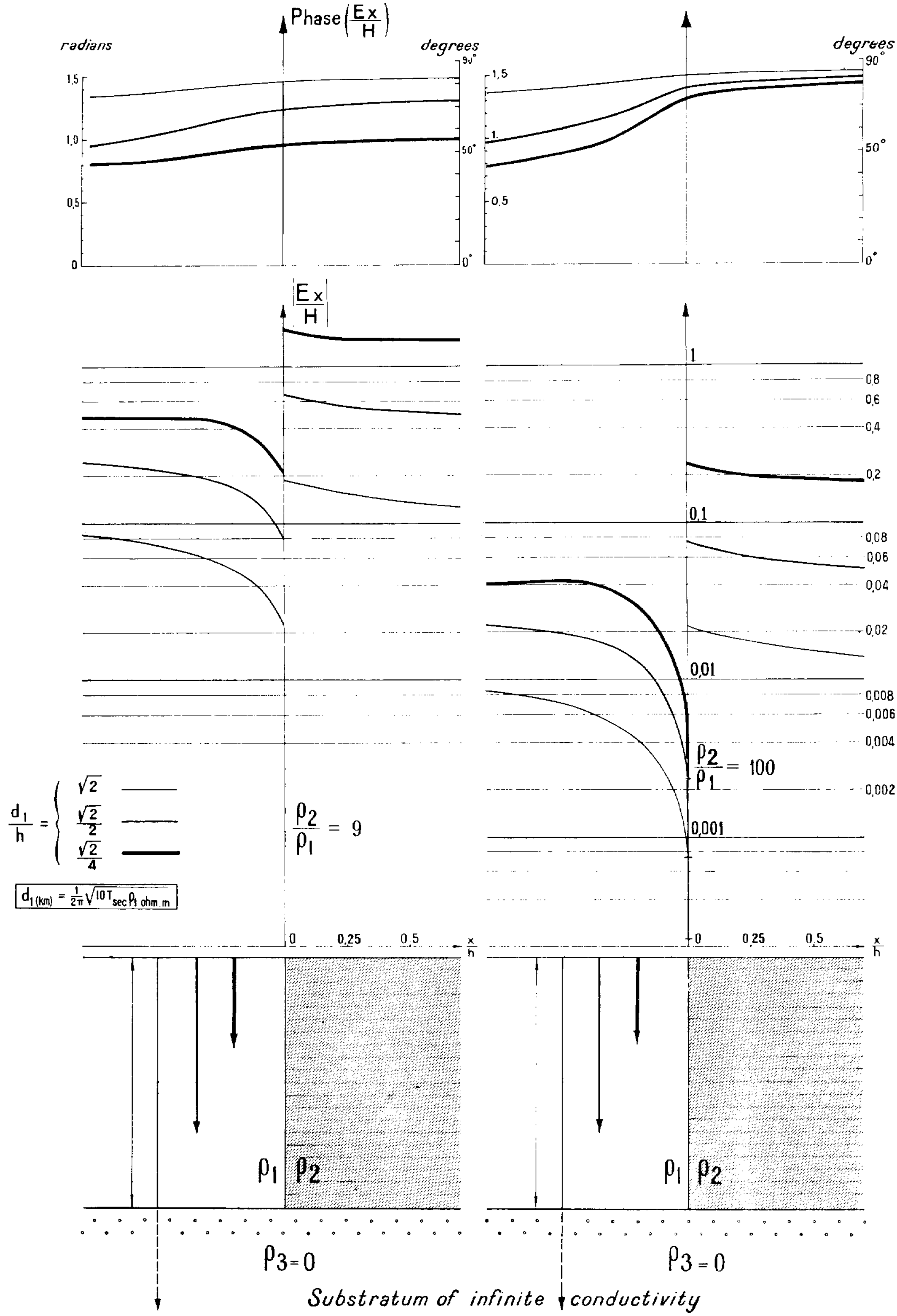

FIG. 2. Variation of the field for various depths of penetration.

a resistivity of 10 ohmmeters of the more conductive compartment, the three curves worked out correspond to frequencies of approximately 125, 500, and 2,000 cps, whereas if $h=2{,}000$ meters and $\rho=1$ ohmmeter, they correspond to frequencies 4,000 times less, or to periods of 32, 8, and 2 sec, respectively.

The curves also make it possible to estimate to what extent a "magneto-telluric sounding," *i.e.*, the curve of the values of E/H at a definite point, as a function of frequency, will be influenced by the proximity of a fault. To determine this, it suffices to note the values of the ordinates of the successive curves for a single abscissa. It will be found that the disturbance is definitely greater in the compartment of lowest resistivity, and decreases quickly as the distance from the fault plane is increased. At a distance of about half the depth of the substratum, the influence becomes very slight, and is practically zero at a distance equal to h.

Variation of the field for various depths of the basement

Results relating to a substratum of infinite depth cannot be shown in the same way, since h, being infinite, cannot be taken as a unit of length.

A second method of presentation is therefore adopted in which the "depth of penetration", d_1, (in the more conductive medium) is taken as the unit of length (Figures 3 and 4). By way of comparison, the results relating to finite h have been shown in the same way, still grouping the profiles relating to the same value of ρ_2/ρ_1 on a single graph. However, with this second presentation of results, the different profiles shown on a given graph no longer correspond to a single cross-section of rock (or to different frequencies), but to the same frequency and the same resistivities, and to different depths of the substratum.

A comparison of the curves relating to $h=\infty$ with the other curves shows that the phenomenon is far from evolving monotonically with the variations in depth of the substratum, and not only in reference to the phase differences (of which it seems difficult to take practical advantage), but also in reference to the field moduli ratio.

An example of a magneto-telluric sounding curve

The magneto-telluric sounding curve for $h=\infty$ is given this time by the points of the curve relating to $d_1/h=0$ itself, as these points can be considered to correspond to constant values of x and ρ, and to variable values of d_1, and therefore of frequency. To obtain the usual representation of these curves, it is sufficient to adopt a logarithmic scale for the abscissae, as shown on Figure 5.

Additional results in the case of an infinitely deep basement

Finally, in Figure 6, curves corresponding to an infinitely deep substratum, and a more complete set of resistivity ratios are given. The same results are presented in the form of numerical values in Table 2.

An apparent paradox

When the case of a fault of infinite depth and of a sinusoidal field is considered, the electric field ratio at an infinite distance from the fault is equal to $\sqrt{\rho_1/\rho_2}$, and is independent of the frequency of the field. But if, from the beginning, the case of a direct current is considered, ρ_1/ρ_2 is found.

The foregoing results clear up this apparent paradox. On the fault itself, the field ratio is always equal to ρ_1/ρ_2. As the distance from the fault is increased indefinitely, on either side, the ratio tends toward $\sqrt{\rho_1/\rho_2}$, but the lower the frequency, the slower this tendency becomes. When the frequency becomes zero, the asymptote can no longer be reached.

In other words, the asymptotic value of this ratio depends on the order in which the parameters involved approach their limit. If, for a fixed frequency, however low, the points of measurement are moved further and further from the fault, on either side, the field ratio tends toward $\sqrt{\rho_2/\rho_1}$. If, for two fixed points of measurement, however far from the fault, the frequency is caused to tend toward zero, the field ratio will approach ρ_2/ρ_1.

APPENDIX

Let a vertical fault be considered infinite in horizontal direction. It separates two formations with resistivities ρ_1 and ρ_2, resting on the same horizontal substratum at a depth, h. A normal to the fault at the surface will be taken as the x-axis, the vertical along the fault as the z-axis, and the trace of this fault at the surface as the y-axis. Considering now a sinusoidal telluric current uniform at infinity and perpendicular to this fault with pulsation ω, by reason of symmetry, we will have $E_y=0$ and all the other factors will be independent of y. Taking into account the relation curl

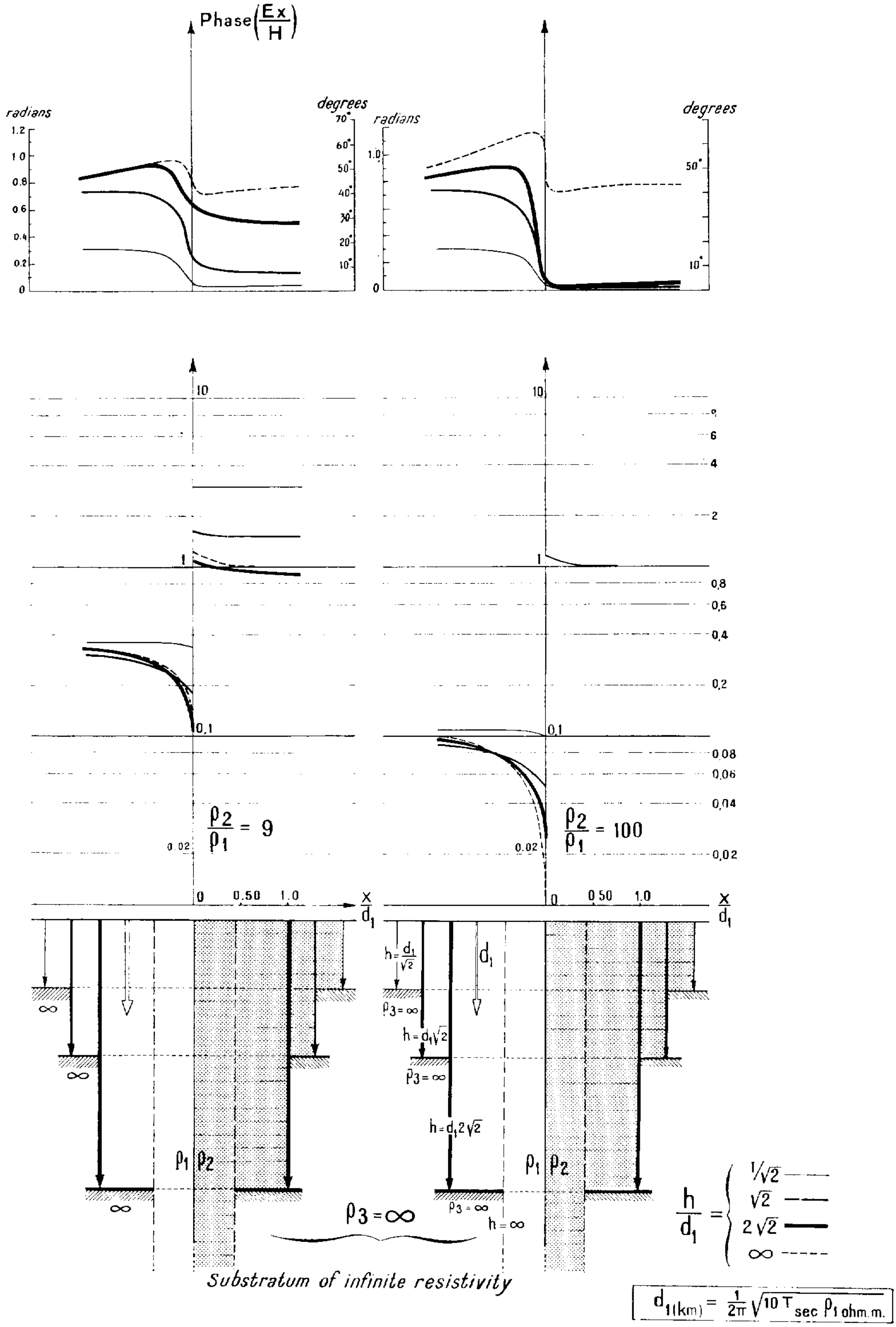

FIG. 3. Variation of the field for various depths of the basement.

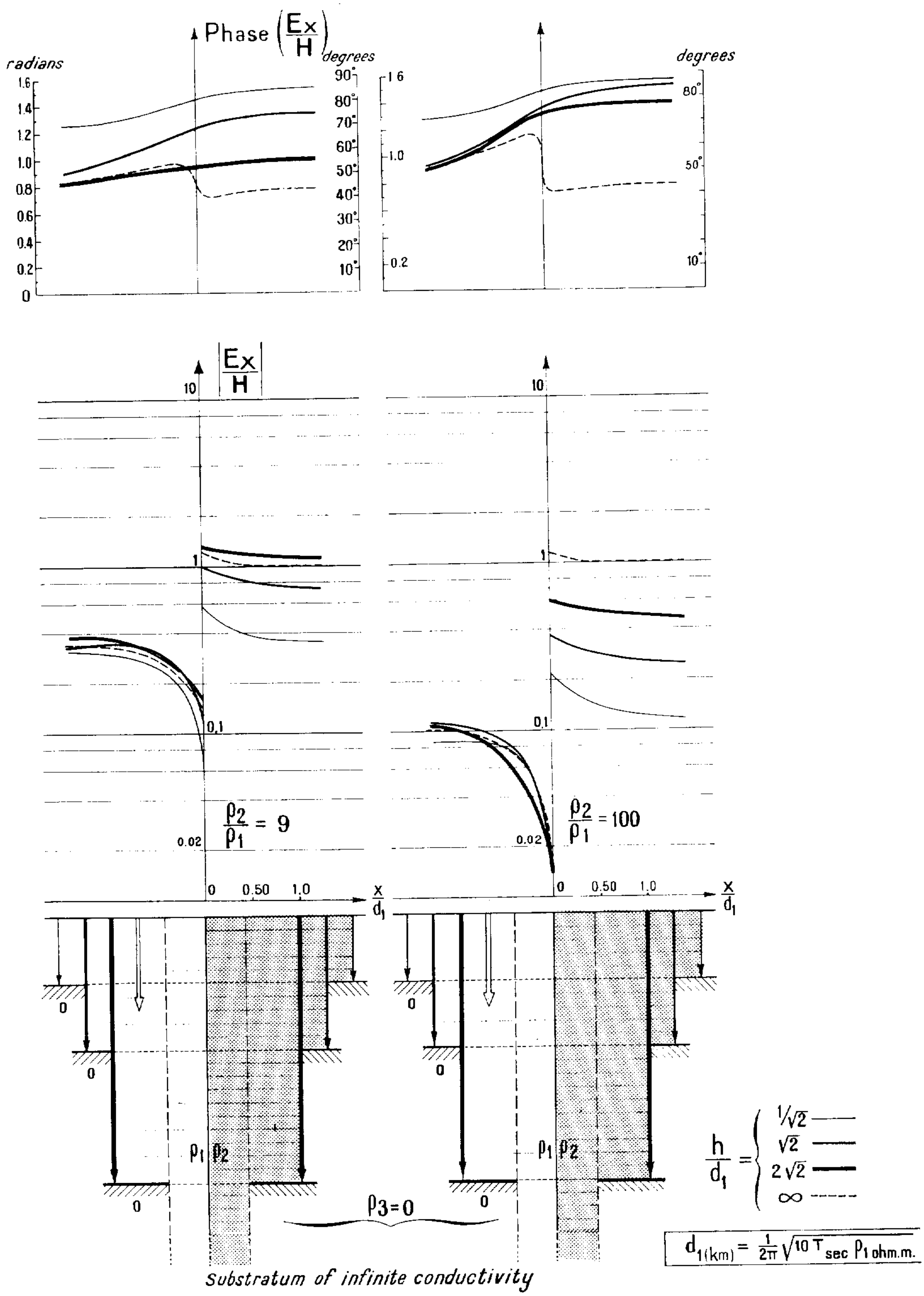

FIG. 4. Variation of the field for various depths of the basement.

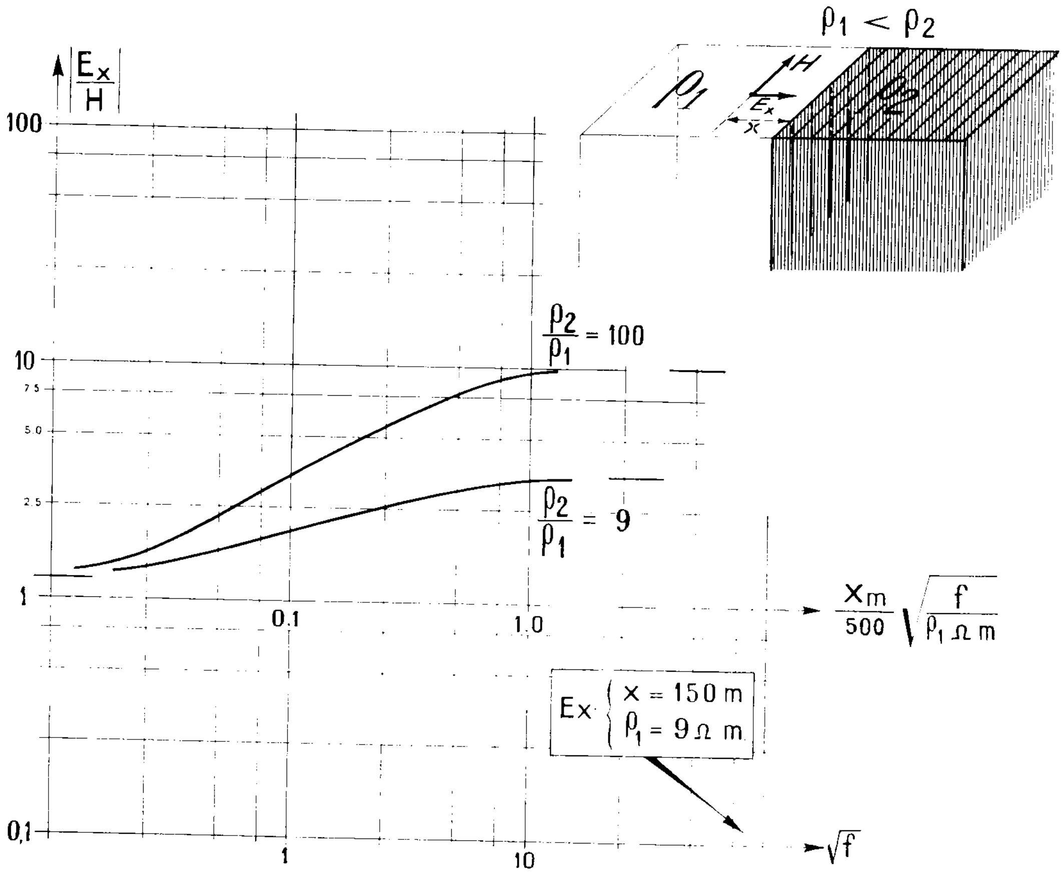

Fig. 5.

$E = -j\omega H$ and in view of the fact that, according to the foregoing,

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \quad \text{and} \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

are zero, it follows that

$$H_z = H_x = 0$$

and the only component of the magnetic field is H_y, which will be deisgnated as H, a scalar defining this field. Moreover, considering that i_z is zero at the surface, from the relation curl $\boldsymbol{H} = 4\pi \boldsymbol{i}$ it may be deduced that $\partial H_y/dx = 0$, i.e., the magnetic field at the surface is constant. Maxwell's equations make it possible to write

$$\nabla^2 H = \frac{4\pi j\omega}{\rho} H.$$

At an infinite distance from the fault, H is independent of x and the equation reduces to

$$\frac{\partial^2 H}{\partial z^2} = \frac{4\pi j\omega}{\rho} H.$$

which gives the general solution in the first medium:

$$H_1{}^0 = A_1 e^{\sqrt{(4\pi j\omega/\rho_1)}\,z} + B_1 e^{-\sqrt{(4\pi j\omega/\rho_1)}\,z}. \qquad (1)$$

If H_0 is the surface field,

$$A_1 + B_1 = H_0.$$

To carry this calculation further, a hypothesis relating to the substratum must be chosen.

Let it be assumed that the substratum is of infinite resistivity. In this case, it will be further assumed that the magnetic field at infinite depth is zero. Since the magnetic field is constant in this substratum, it is also zero at its surface.

Stating

$$\sqrt{\frac{4\pi\omega}{\rho_1}}\, h = h_1,$$

$$A_1 e^{\sqrt{j}\, h_1} + B_1 e^{-\sqrt{j}\, h_1} = 0$$

is obtained.

This gives immediately:

$$A_1 = \frac{-H_0 e^{-\sqrt{j}\, h_1}}{2sh(\sqrt{j}\, h_1)}$$

and

$$B_1 = \frac{H_0 e^{\sqrt{j}\, h_1}}{2sh(\sqrt{j}\, h_1)}\,. \qquad (2)$$

The second formation can be dealt with similarly, substituting ρ_1 for ρ_2.

The presence of the fault will bring about in both media a disturbance H, which becomes zero at infinity.

The first medium (x negative) will be examined first. Obviously the disturbance P_1 is zero for $z=0$ and $z=h$. It can be developed, with respect to z, into a sine series of argument $n\pi z/h$. Each term of the series will therefore be expressed by $f_n(x)\cdot \sin n\pi z/h$.

Thus, the equation to be satisfied, namely,

$$\frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 P_1}{\partial z^2} = j\,\frac{h_1^2}{h^2}\,P_1,$$

gives

$$f''(x) = \frac{n^2\pi^2}{h^2} f(x) = j\,\frac{h_1^2}{h^2} f(x).$$

Taking into account that $f(x)$ becomes zero for $x=-\infty$, the solution will be

$$a_{1,n}\cdot e^{\sqrt{n^2\pi^2 + jh_1^2}\cdot x/h},$$

the real part of the radical being positive. A similar expression is obtained for the second medium, provided that h_1 is replaced by h_2, and that the sign of the exponent is changed, in view of the fact that this term becomes zero for x equal to $+\infty$. Hence,

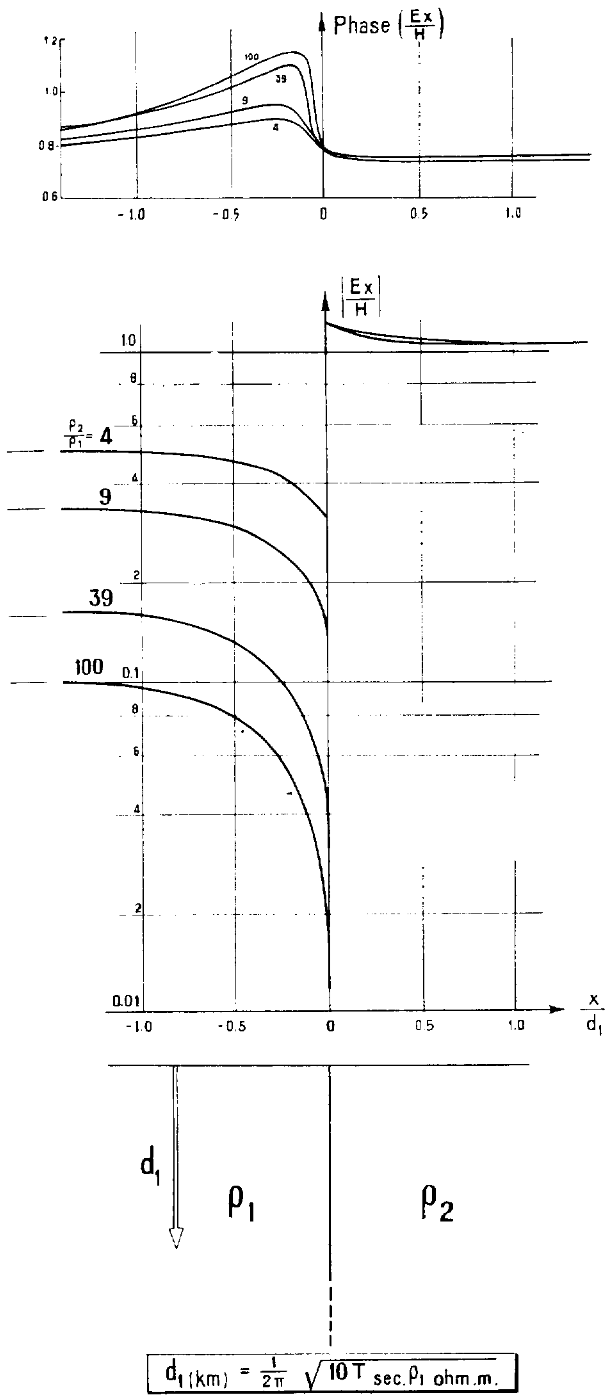

FIG. 6. Variation of the field in the case of an infinitely deep basement.

Table 2. Numerical values of the field in the case of an infinitely deep basement
$E_x = R(x) \cos \omega t + I(x) \sin \omega t = \rho_x \cos (\omega t - \theta_x)$.

$\frac{\rho_2}{\rho_1} = 100$

$\frac{x}{d_1/\sqrt{2}}$	I	R	ρ	θ
− 2	0,07815	0,06556	0,10200	0,8728
− 1	0,07512	0,04733	0,08879	1,0085
− 0.6	0,06538	0,03437	0,07387	1,0867
− 0.4	0,05592	0,02647	0,06187	1,1286
− 0.2	0,04075	0,01765	0,04441	1,1619
− 0	0,00832	0,00816	0,01656	0.795
+ 0	0,82425	0,82430	1,1615	0,785
+ 0 1	0,80202	0,81962	1,1467	0,77454
+ 0.2	0,79040	0,81529	1,1355	0,76989
+ 0.4	0,77367	0,80751	1,1183	0,76400
+ 0.6	0,76170	0,80068	1,1051	0,76044
+ 1	0,74521	0,78918	1,0854	0,75674
+ 2	0,72334	0,76852	1,0553	0,75512

$\frac{\rho_2}{\rho_1} = 39$

$\frac{x}{d_1/\sqrt{2}}$	I	R	ρ	θ
− 2	0,12356	0,10650	0,16312	0,85944
− 1	0,12001	0.08069	0,14462	0,97873
− 0.6	0,10637	0,06175	0,12301	1,0449
− 0.4	0,09278	0,04997	0,10540	1,0767
− 0.2	0,07062	0.03668	0.07959	1,0917
− 0	0,02192	0.02200	0,03103	0,783
+ 0	0.85721	0,85724	1,21234	0,785
+ 0.2	0,80705	0,84327	1,1672	0,76345
+ 0.4	0,78244	0,83129	1,1416	0,75513
+ 0.6	0,76533	0,82083	1,1222	0,75041
+ 1	0,74264	0,80339	1,0940	0,74611

$\frac{\rho_2}{\rho_1} = 9$

$\frac{x}{d_1/\sqrt{2}}$	I	R	ρ	θ
− 2	0,24935	0,22839	0,33814	0.82926
− 1	0,24735	0.19289	0.31367	0.90847
− 0,6	0,22950	0.16433	0.28226	0.94939
− 0.4	0.21014	0.14575	0.25573	0.96439
− 0,2	0.17644	0.12396	0.21563	0.95836
− 0	0.09886	0.09887	0.14361	0.785
+ 0	0.88981	0.88980	1.2545	0.785
+ 0,1	0.83849	0.87712	1.2134	0.76288
+ 0.2	0.81055	0.86556	1.1858	0.75259
+ 0.4	0.77395	0.84521	1.1460	0.74141
+ 0.6	0.75032	0.82788	1.1173	0.73629
+ 1	0.72209	0.80003	1.0777	0.73423
+ 2	0.69634	0.75594	1.0277	0.74438

$\frac{\rho_2}{\rho_1} = 4$

$\frac{x}{d_1/\sqrt{2}}$	I	R	ρ	θ
− 2	0,36573	0.34807	0.50483	0,81004
− 1	0,36592	0.31551	0.48314	0.85925
− 0.6	0.35028	0.28728	0.45301	0.88390
− 0.4	0.33208	0.26818	0.42683	0.89145
− 0.2	0.29903	0.24507	0.38660	0.88425
− 0	0.21744	0.21745	0.30750	0.785
+ 0	0.86971	0.86970	1.2299	0.785
+ 0.1	0.81626	0.85550	1.1820	0.76155
+ 0.2	0.78715	0.84272	1.1531	0.75131
+ 0.4	0.75161	0.82060	1.1127	0.74154
+ 0.6	0.73015	0.80222	1.0847	0.73839
+ 1	0.70713	0.77390	1.0483	0.74034

$$P_i = \sum_{n=1}^{\infty} a_{i,n} \sin \frac{n\pi z}{h} e^{\pm\sqrt{n^2\pi^2 + jh_i^2} \cdot x/h} \quad (3)$$

$(i = 1 \text{ or } 2).$

But H is continuous at $x=0$, i.e.,

$$H_{1|x=0} = H_{2|x=0}$$

and, since

$$H_1 = H_1{}^0 + P_1 \text{ and } H_2 = H_2{}^0 + P_2,$$

this gives

$$\left.\begin{aligned} H_1{}^0 + P_1 &= H_2{}^0 + P_2 \\ \text{or} \quad & \\ P_1 - P_2 &= H_2{}^0 - H_1{}^0 \end{aligned}\right\} \text{ for } x = 0. \quad (4)$$

Now $H_2{}^0 - H_1{}^0$ can be developed by standard methods into a sine series of argument $n\pi z/h$. Since $H_2{}^0 - H_1{}^0$ is zero for $z=0$ and $z=h$,

$$H_2{}^0 - H_1{}^0 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi z}{h},$$

where

$$b_n = \frac{2}{h} \int_0^{\infty} (H_2{}^0 - H_1{}^0) \frac{\sin n\pi z}{h} dz.$$

Substituting for $H_1{}^0$ the solution (1) and for $H_2{}^0$ the similar expression for the second formation, the result:

$$b_n = 2\pi j H_0(h_1^2 - h_2^2) \cdot \frac{n}{(n^2\pi^2 + jh_1^2)(n^2\pi^2 + jh_2^2)} \quad (5)$$

is obtained. Now, equating coefficients of sin $n\pi z/h$ on both sides of equation (4), where for P_1 and P_2 the expression (3) (with $x=0$) is substituted, we obtain

$$a_{1,n} - a_{2,n} = b_n. \quad (6)$$

Moreover, E_z is continuous across the fault, which entails continuity of $\rho(\partial H/\partial x)$. Hence,

$$\rho_1 \frac{\partial H_1}{\partial x} = \rho_2 \frac{\partial H_2}{\partial x} \quad \text{for } x = 0,$$

and, as H_1^0 and H_2^0 are independent of x,

$$\rho_1 \frac{\partial P_1}{\partial x} = \rho_2 \frac{\partial P_2}{\partial x} \quad \text{for } x = 0.$$

This gives, from (3),

$$a_{1,n} \frac{n^2\pi^2 + jh_1^2}{h_1} + a_{2,n} \frac{n^2\pi^2 + jh_2^2}{h_2} = 0. \quad (7)$$

The two relations (6) and (7), where, in (6) ex-expression (5) is substituted for b_n, give

$$a_{1,n} = 2\pi j \cdot H_0[h_1^2 - h_2^2] \frac{n}{[n^2\pi^2 + jh_1^2][n^2\pi^2 + jh_2^2]\left[1 + \dfrac{h_2^2}{h_1^2} \dfrac{\sqrt{n^2\pi^2 + jh_1^2}}{\sqrt{n^2\pi^2 + jh_2^2}}\right]}. \quad (8)$$

Finally, E_x can be obtained from the equation

$$E_{ix} = \frac{1}{4\pi} \rho_i \frac{\partial H_i}{\partial x},$$

which becomes in the first formation:

$$E_{1x} = \frac{1}{4\pi} \rho_1 \frac{\partial(H_1^0 + P_1)}{\partial z}.$$

Using expressions (1) and (2) on the right-hand side for H_1^0, and expressions (3) and (8) for P_1, differentiating with respect to z and putting $z=0$, the following result is obtained:

$$E_{1,x|z=0} = \frac{\omega h H_0}{h_1^2}\left[-\sqrt{j}\, h_1 \frac{1 + jth\left(\dfrac{h_1}{\sqrt{2}}\right) tg\left(\dfrac{h_1}{\sqrt{2}}\right)}{th\left(\dfrac{h_1}{\sqrt{2}}\right) + jtg\left(\dfrac{h_1}{\sqrt{2}}\right)} + 2\pi^2 j[h_1^2 - h_2^2] \times U \right],$$

with

$$U = \sum_{n=1}^{\infty} \frac{n^2 e^{-\sqrt{n^2\pi^2 + jh_1^2}\,|x|/h}}{[n^2\pi^2 + jh_1^2][n^2\pi^2 + jh_2^2]\left[1 + \dfrac{h_2^2}{h_1^2} \cdot \dfrac{\sqrt{n^2\pi^2 + jh_1^2}}{\sqrt{n^2\pi^2 + jh_2^2}}\right]}.$$

For the other medium it suffices to permute h_1 and h_2. Now the final result, the ratio of the electric and the magnetic fields at the surface E_{1x}/H_1, is equal to

$$\frac{E_{1x}}{H_0},$$

as the magnetic field is constant at the surface.

Let it now be assumed that the substratum is infinitely conductive. In this case, E_x must be zero at the depth h. This implies that

$$\left[\frac{\partial H}{\partial z}\right]_{z=h} = 0.$$

In the first medium, at infinity,

$$H_1^0 = A_1 e^{\sqrt{j}\, h_1(z/h)} + B_1 e^{-\sqrt{j}\, h_1(z/h)},$$

is found along with the relations:

$$A_1 + B_1 = H_0$$

and

$$A_1 e^{\sqrt{j}\,h_1} - B_1 e^{-\sqrt{j}\,h_1} = 0.$$

This gives

$$A_1 = \frac{H_0 e^{-\sqrt{j}\,h_1}}{2ch[\sqrt{j}\,h_1]}$$

$$E_x = \frac{\omega h H_0}{h_1^2}\left[-\sqrt{j}\,h_1 \frac{th\left(\frac{h_1}{\sqrt{2}}\right) + jtg\left(\frac{h_1}{\sqrt{2}}\right)}{1 + jth\left(\frac{h_1}{\sqrt{2}}\right)tg\left(\frac{h_1}{\sqrt{2}}\right)} + 8\pi^2 j[h_1^2 - h_2^2]u\right],$$

with

$$u = \sum_{n=0}^{n=\infty} \frac{(2n+1)^2 e^{-\sqrt{(2n+1)^2\pi^2+4jh_1^2}\cdot|x/2h|}}{[(2n+1)^2\pi^2 + 4jh_1^2][(2n+1)^2\pi^2 + 4jh_2^2]\left[1 + \frac{h_2^2}{h_1^2}\frac{\sqrt{(2n+1)^2\pi^2 + 4jh_1^2}}{\sqrt{(2n+1)^2\pi^2 + 4jh_2^2}}\right]}.$$

and

$$B_1 = \frac{H_0 e^{\sqrt{j}\,h_1}}{2ch[\sqrt{j}\,h_1]},$$

and the same applies for the second medium.

As previously, $H_2^0 - H_1^0$ can be developed into a sine series, but this time, the argument being $n\pi z/2h$, the function $H_2 - H_1$ is completed by symmetry around the plane $z=h$ up to $z=2h$.

It can easily be seen that the even terms are zero. Then,

$$H_2^0 - H_1^0 = 16\pi j H_0(h_1^2 - h_2^2)\sum_{n=0}^{\infty} \frac{(2n+1)\sin\frac{(2n+1)\pi z}{2h}}{[(2n+1)^2\pi^2 + 4jh_1^2][(2n+1)^2\pi^2 + 4jh_2^2]}$$

All these terms satisfy the condition $\partial H/\partial z = 0$, for $z=h$.

Let the values of $a_{1,2n+1}$ be determined as above by the condition of continuity of H and E_z for $x=0$. It is found that

$$a_{1,2n+1} = \frac{16\pi j H_0(h_1^2 - h_2^2)(2n+1)}{[(2n+1)^2\pi^2 + 4jh_1^2][(2n+1)^2\pi^2 + 4jh_2^2]\left[1 + \frac{h_2^2}{h_1^2}\frac{\sqrt{(2n+1)^2\pi^2 + 4jh_1^2}}{\sqrt{(2n+1)^2\pi^2 + 4jh_2^2}}\right]}.$$

The disturbance is similarly developed into this sine series of the same argument. For the first medium,

$$a_{1,2n+1}\sin\frac{(2n+1)\pi z}{2h}\, e^{\sqrt{(2n+1)^2\pi^2+4jh_1^2}\cdot x/2h}$$

is found.

From this it is deduced that

It can be seen that the terms of the disturbance for an infinitely conductive substratum can be deduced from the odd terms of this disturbance for an infinitely resistive substratum, by merely replacing h_1 and h_2 by $2h_1$ and $2h_2$ and x by $x/2$.

Let it be assumed that the substratum is at infinite depth (infinite fault). This is a common limit for the two preceding cases.

Taking

$$p_1 = \sqrt{\frac{\rho_1}{4\pi\omega}} \quad \text{and} \quad p_2 = \sqrt{\frac{\rho_2}{4\pi\omega}},$$

with the conditions at infinity,

$$H_1^0 = H_0 e^{-\sqrt{j}\cdot z/p_1} \quad \text{and} \quad H_2^0 = H_0 e^{-\sqrt{j}\cdot z/p_2}.$$

The previous deductions can be repeated by substituting Fourier integrals (in sines) for Fourier series. Then,

$$\frac{E_x}{H_0} = \omega p_1^2 \left[-\sqrt{j}\frac{1}{p_1} + \frac{2j}{\pi}[p_2^2 - p_1^2] \int_0^\infty \frac{q^2 e^{-\sqrt{q^2 p_1^2 + j}\,|x/p_1|}}{[q^2p^2 + j]\,[q^2p_2^2 + j]\left[1 + \dfrac{p_1}{p_2}\dfrac{\sqrt{q^2p_1^2 + j}}{\sqrt{q^2p_2^2 + j}}\right]} dq \right]$$

is obtained.

For the other medium, it suffices to permute p_1 and p_2.

The case of a current parallel to the fault, i.e., the case in which the only component of E is E_y. Another hypothesis is necessary so that the problem be well determined. It will be assumed that at the surface, E_y is not dependent on x, which amounts to assuming that the vertical component of the magnetic field is zero at the surface. From these data, the magnetic field, H_x, can be calculated by the same method as above. This is done by estimating the fields for x equal to infinity and computing the disturbance developed into series of sines, in view of the fact that both E and H_z must be continuous across the fault.

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