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ON DETERMINING ELECTRICAL CHARACTERISTICS OF THE DEEP LAYERS OF THE EARTH'S CRUST

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Magnetic and electric fields in the earth should follow certain interrelations as different characteristics of natural electromagnetic fields of the earth. Empirical comparison of a derivative of the East component of magnetic field H_y and the North component of the electric field E_x shows, that between these functions there is a rough proportionality. Between these functions there also is a phase shift.

The aim of this note is to present a theoretical consideration of the relations between magnetic and electric fields from the point of view of Maxwell equations. Although suggested theory is based on very simplified considerations, it permits establishing and refining the above relations and also allows reaching some conclusions about electrical characteristics of the earth's crust.

I use this study to formulate the problem of using natural electric fields to study the electric characteristics of the deep layers of the earth's crust, i.e. the problem of deep electrical prospecting.

To establish qualitative relations between magnetic and electric fields, I schematically consider the earth's crust as a layer $0 \leq z \leq \ell$ of finite conductivity, on a perfectly conductive basement. I consider the one-dimensional (plane-wave) problem.

The components of the magnetic field H_x, H_y, and H_z, and also the components of the electric field of the earth E_x and E_y, are known on the earth's surface $z=0$. The component E_z at $z=0$ is practically equal to zero because of the very small conductivity of the air.

At $0 \leq z \leq \ell$ these functions satisfy Maxwell equations. Disregarding the displacement currents, then each of those functions satisfies

$$\Delta u = \frac{1}{a^2}\frac{\partial u}{\partial t} \left(0 \leq z \leq \ell, \frac{1}{a^2} = \frac{4\pi\sigma\mu}{c^2}\right). \qquad (1)$$

For boundary conditions at $z=0$, E_x and E_y are directly observed at $z>0$. The magnitudes of H_y and H_x are measured at $z=0$ (in the air), however, because of continuity at $z=0$ I consider them as known on the boundary of the layer $0 \leq z \leq \ell$; H_z can also be considered as known ($\mu=1$).

As a relation between components of magnetic and electric fields, I use Maxwell equations in their explicit form. The relation

$$\frac{\mu}{c}\dot{H} = -\nabla_x E$$

leads to the equations

$$\frac{\mu}{c}\dot{H}_x = -\frac{\partial E_y}{\partial t} \quad \text{and} \quad \frac{\mu}{c}\dot{H}_y = \frac{\partial E_x}{\partial t},$$

because $E_z=0$ at $z=0$. To transform these equations to a suitable form for comparison with measurements I express $\partial E_x/\partial_z$ (and $\partial E_y/\partial_z$) as functions of E_x (and E_y) at $z=0$.

Disregarding the curvature of the fields in the horizontal direction gives the equation

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{a^2}\frac{\partial u}{\partial t},$$

and the boundary problem without initial conditions

$$u(0,t) = \mu(t), \ \mu(\ell,t) = 0.$$

The solution of this problem is very simple. Expressing the boundary value $\mu(t)$ as a superposition of harmonic functions and taking separate harmonics gives

$$\mu(t) = A\begin{smallmatrix}\cos \omega t\\ \sin \omega t\end{smallmatrix},$$

where

$$u = \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\left(A\frac{\mathrm{sh}\sqrt{\frac{i\omega}{a^2}}(\ell-z)}{\mathrm{sh}\sqrt{\frac{i\omega}{a^2}}\ell}\right)\begin{matrix}\cos \omega t\\ \sin \omega t\end{matrix},$$

and

$$\left.\frac{\partial u}{\partial z}\right|_{z=0} = \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\left(A\sqrt{\frac{i\omega}{a^2}}\coth\sqrt{\frac{i\omega}{a^2}}\ell\right)\begin{matrix}\cos \omega t\\ \sin \omega t\end{matrix}.$$

In this way I obtain for the harmonic amplitudes $H^{(\omega)}$ and $E^{(\omega)}$ of the magnetic and electric fields at frequency ω

$$i\omega\frac{\mu}{c}H_x^{(\omega)} = E_y^{(\omega)}\sqrt{\frac{i\omega}{a^2}}\coth\sqrt{\frac{i\omega}{a^2}}\ell,$$

and analogously for $H_y^{(\omega)}$ and $E_x^{(\omega)}$.

Assuming that $\alpha = \sqrt{\frac{\omega}{a^2}\ell}$ is a small quantity and then substituting *coth* $\alpha \sim 1/a$ (when $\mu = 1$) gives

$$i\omega H_x^{(\omega)} = E_y^{(\omega)} \frac{c}{\ell},$$

i.e., at low frequencies the amplitude of the derivative of the magnetic field $H_x^{(\omega)}$ is proportional to the electric field $E_y^{(\omega)}$. This corresponds to the empirically established fact that these two values are proportional. The value of the coefficient of proportionality permits determining the ℓ, the thickness of the conducting layer of the earth's crust.

The nature of the relationship at high frequencies, as shown in the above formula, should be completely different. These considerations lead to the conclusion that the value of α is small enough. However, as previously mentioned, H_x is roughly proportional to E_y. To be more accurate, consider subsequent terms in the power expansion of coth α

$$\coth \alpha \approx \frac{1}{\alpha}\left(1 + \frac{\alpha^2}{3} + \ldots\right),$$

and thereby obtain

$$i\omega H_x^{(\omega)} = E_y^{(\omega)} \frac{c}{\ell}\left(1 + \frac{i\omega}{a^2}\,\ell\right) = E_y^{(\omega)} a_0 + i\, E_y^{(\omega)} a_1 .$$

That is, in this case the amplitude of the derivative of $H_x^{(\omega)}$ is a linear combination of $E_y^{(\omega)}$ and $E_y^{(\omega)}$. Furthermore

$$\ell = \frac{c}{a_0} \text{ and } \ell\sigma = \frac{a_1}{4\pi}\, c .$$

That way, if phase shift is taken into account it should allow determination, not just of ℓ, but also of the electrical resistivity of the conducting layer of the earth's crust.

In this paper, I do not deal with other formulas which could be derived from the same circle of ideas.

Now, briefly compare the above formulas with the results of measurements. Table 1 shows the results of processing measurements to obtain daily variations (harmonic I) and the next three harmonics (II, III, IV) data for Zue[1] in 1944 and for Tucson (Rooney, 1949) in 1933, 1934, 1935 and 1936. The values of ℓ are shown in hundreds of kilometers, and the resistivities ϱ are in $\Omega \cdot$m.

Harmonics	Zue 1944		Tucson 1933		Tucson 1934		Tucson 1935		Tucson 1936	
	ℓ	ϱ	ℓ	ϱ	ℓ	ϱ	ℓ	ϱ	ℓ	ϱ
I	0.99	3.6	11.4	147	10.7	154	9.8	122	10.8	188
II	1.6	2.6	8.9	330	10.6	246	9.5	218	10.5	280
III	0.95	3.0	9.1	254	8.7	248	9.7	342	8.6	319
IV	0.60	2.7	8.2	250	7.9	244	7.7	276	7.2	404
Average	1.1	3.0	9.4	245	9.5	233	9.2	240	9.3	298

I do not discuss in detail the experimental data or the values derived.

Because the preliminary results shown indicate the possibility of exploring the earth's crust and even deeper layers in some areas, it is natural to propose using the method of comparing the magnetic and electric fields as a means of electrical exploration of depths of the earth's crust.

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[1]Additional measurements in two papers communicated by B. B. Novyshev.