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VARIATION OF ELECTRICAL CONDUCTIVITY WITH DEPTH BY THE MAGNETO-TELLURIC METHOD*

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ABSTRACT

Apparatus has been installed at the Dominion Observatory Research Station at Meanook, Alberta, for the continuous recording of earth potentials. The theory due to Cagniard (1953) and others, in which relative amplitudes of horizontal components of electric and magnetic fields are used to interpret the sub-surface structure, is applied in a modified form, to data from the Meanook records. Values of electrical conductivity between depths of 10 km and 100 km are estimated, and found to vary roughly between 10^{-13} and 10^{-14} e.m.u.

INTRODUCTION

The Magneto-Telluric method of determining the electrical conductivity of the sub-surface strata is based on the fact that natural electromagnetic variations will penetrate into the earth to a depth which depends on their frequency and on the conductivity of the rock. By applying Maxwell's equations within the framework of certain rather broad assumptions it is possible to estimate conductivity as a function of depth from measurements of earth potential and magnetic field made at the surface.

The equations governing electromagnetic variations inside a solid conducting medium are:

$$\operatorname{curl} \boldsymbol{H} = 4\pi j, \tag{1}$$

$$\operatorname{curl} \boldsymbol{E} = -\frac{\partial \boldsymbol{H}}{\partial t}, \tag{2}$$

$$j = \sigma \boldsymbol{E}. \tag{3}$$

Here H is the magnetic field vector, E the electric vector, j the current density, and σ the electrical conductivity. All quantities are expressed in e.m.u. and the magnetic permeability is taken to be unity.

The induction problem inside the earth has been investigated by Kato and Kikuchi (1950), Rikitake (1951), Cagniard (1953), Tikhonov and Lipskaya (1952) and others. Most authors have assumed that horizontal gradients of the field vectors are negligible compared to the vertical gradients, and that time variations are periodic; i.e.

$$\partial/\partial x = \partial/\partial y = 0 \quad \text{and} \quad \partial/\partial t = -\frac{2\pi i}{T}, \tag{4}$$

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where the variables are expressed in terms of cartesian co-ordinates (x, y, z) and T is the period. The positive directions of the co-ordinate axes are true north, true east, and vertically downward respectively. Physically, these assumptions require the electrical conductivity to be a function of depth only, and the sources of field variation to be such that the horizontal extent of the phenomena at the surface is much greater than their penetration depth. It is clear that the horizontal gradients will be negligible only if

$$j_z \ll (j_x^2 + j_y^2)^{1/2} \quad \text{and} \quad \dot{H}_z \ll (\dot{H}_x^2 + \dot{H}_y^2)^{1/2}. \tag{5}$$

If (4) and (5) are valid, and if we take $\bar{\sigma}$ to represent the average or effective conductivity to a penetration depth Z, equations (1), (2), and (3) lead to the approximations

$$-\frac{H_y}{Z} \approx 4\pi\bar{\sigma}E_x, \qquad \frac{H_x}{Z} \approx 4\pi\bar{\sigma}E_y \tag{6}$$

$$-\frac{E_y}{Z} \approx \frac{2\pi i H_x}{T}, \qquad \frac{E_x}{Z} \approx \frac{2\pi i H_y}{T}, \tag{7}$$

in which the operator $\partial/\partial z$ has been replaced by $1/Z$ and E_x, E_y, H_x, H_y refer to field variations of period T. From (6) and (7)

$$\bar{\sigma} \approx \frac{1}{2\left|\dfrac{E}{H}\right|^2 T}, \tag{8}$$

$$Z \approx \frac{1}{2\pi}\left|\frac{E}{H}\right| T, \tag{9}$$

where $|E/H|$ stands for the amplitude ratio $|E_x/H_y|$ or $|E_y/H_x|$.

If amplitude ratios can be measured for a series of natural periods it is possible to determine the variation of effective conductivity with depth from equations (8) and (9). Equation (8) is identical to the relation for $\bar{\sigma}$ derived more rigorously by other authors. The penetration depth defined by (9) implies an attenuation factor of $\frac{1}{2}$ instead of the usual $1/e$.

If we consider electrical conductivity to be a continuous and finite function of depth between the surface and the greatest penetration depth encountered we can write

$$\sigma = f(z), \tag{10}$$

and

$$\bar{\sigma} = \frac{1}{Z}\int_0^Z f(z)\,dz = \frac{1}{Z}\,g(Z) \tag{11}$$

represents the effective conductivity to the depth Z. Then

$$\sigma = \frac{dg(Z)}{dZ} = Z\frac{d\bar{\sigma}}{dZ} + \bar{\sigma}. \tag{12}$$

The value of conductivity at various depths can be estimated from (12) provided $\bar{\sigma}$ can be established as a known function of Z.

MEASUREMENT OF ELECTRIC AND MAGNETIC FIELDS

At the Meanook station earth potentials are measured in north-south and east-west directions. The electrodes are lead sheets 8 ft×4 ft×¼ inch laid horizontally at a depth of 8 ft in soil which is a mixture of sand and clay. The separation of the east-west electrodes is 1.60 km, while that of the north-south pair is 1.47 km. The north and east components of the horizontal geomagnetic field are measured by means of a fluxgate magnetometer. E_x and H_y are recorded together on a dual channel Brown self-balancing potentiometer, while E_y and H_x are recorded together on a similar instrument. The sensitivity can be varied from 20 mv to 2,000 mv full scale for earth potentials and from 50 gammas to 2,000 gammas full scale for the magnetics. The earth potential records are calibrated by means of a potentiometer; the magnetic records by comparison with standard photographic traces from La Cour variometers.

The resistance through the ground between a pair of electrodes varies with the moisture content of the soil but has never been found to be much in excess of 100 ohms. The input impedances of the earth potential recording circuits are greater than 20,000 ohms. Thus, very little current is drawn by the recording apparatus and changes in soil resistivity and contact resistances at the probes should not affect the recorded potential differences. When data were being collected for this analysis the chart speeds of the Brown recorders were either 6 inch per hour or 20 inch per hour. An effort was made to keep the sensitivities adjusted so that the magnetic and earth potential traces remained on scale and were subject to variations of roughly the same size.

THE VARIATION OF CONDUCTIVITY WITH DEPTH AT MEANOOK

Continuous recordings of E_x, E_y, H_x, and H_y were made throughout 1958 and from these estimates of the amplitude ratios $|E_x/H_y|$ and $|E_y/H_x|$[1] were obtained for periods ranging from about 40 sec to 1,000 sec. No attempt at harmonic analysis was made, but instances were selected on the charts when both E and H traces exhibited a fairly good sinusoidal wave form. The ratio of amplitudes was measured in each case and converted to e.m.u. Values of E_x/H_y were grouped together according to their periods and averaged. The first group contained values with periods less than one minute, the next values with periods be-

[1] The vertical bars about E/H will be omitted henceforth. It is to be understood that the ratios refer to modulus only. The effects of phase differences are not considered in this paper.

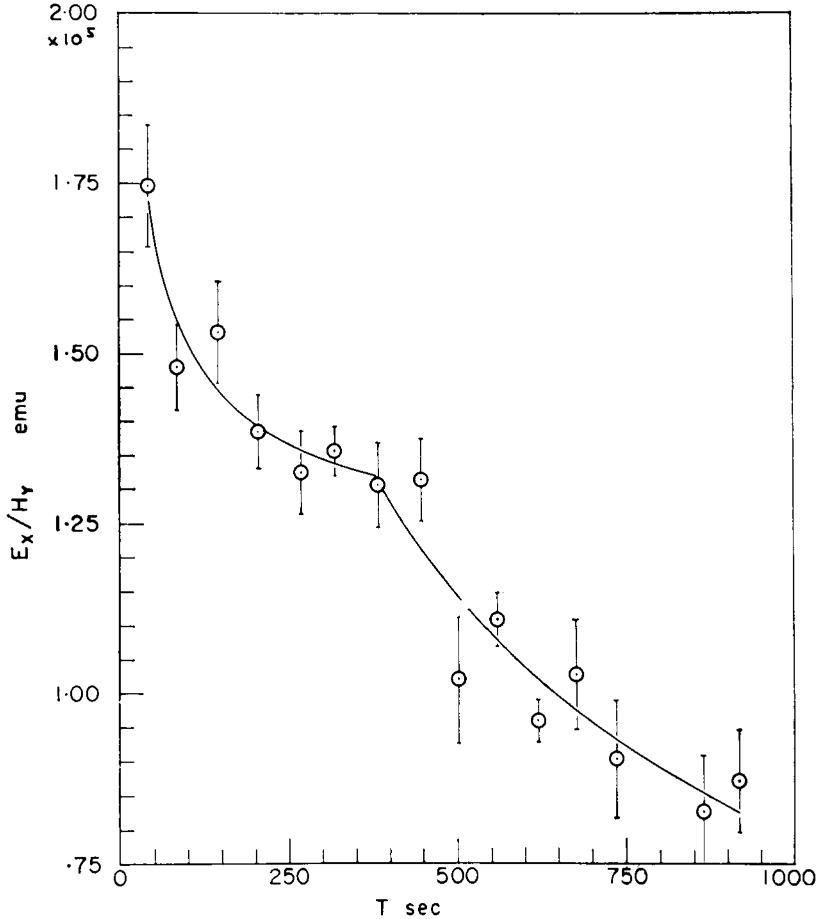

FIG. 1. Amplitude ratio E_x/H vs period T.

tween one and two minutes, and so on. The data in E_y/H_x were handled in the same manner. The results are shown in Figures 1 and 2 where average ratios are plotted against average periods. The scatter in the individual values of the ratios within a group is usually quite large. The standard errors of the means are shown; for the most part they lie between 5 percent and 10 percent. The average number of observations within a group is 42, though the number is larger than this for periods less than 400 sec and smaller for longer periods.

Equation (8) indicates that if $\bar{\sigma}=\sigma=$constant the amplitude ratio can be expressed

$$\frac{E}{H}=\frac{B}{\sqrt{T}}$$

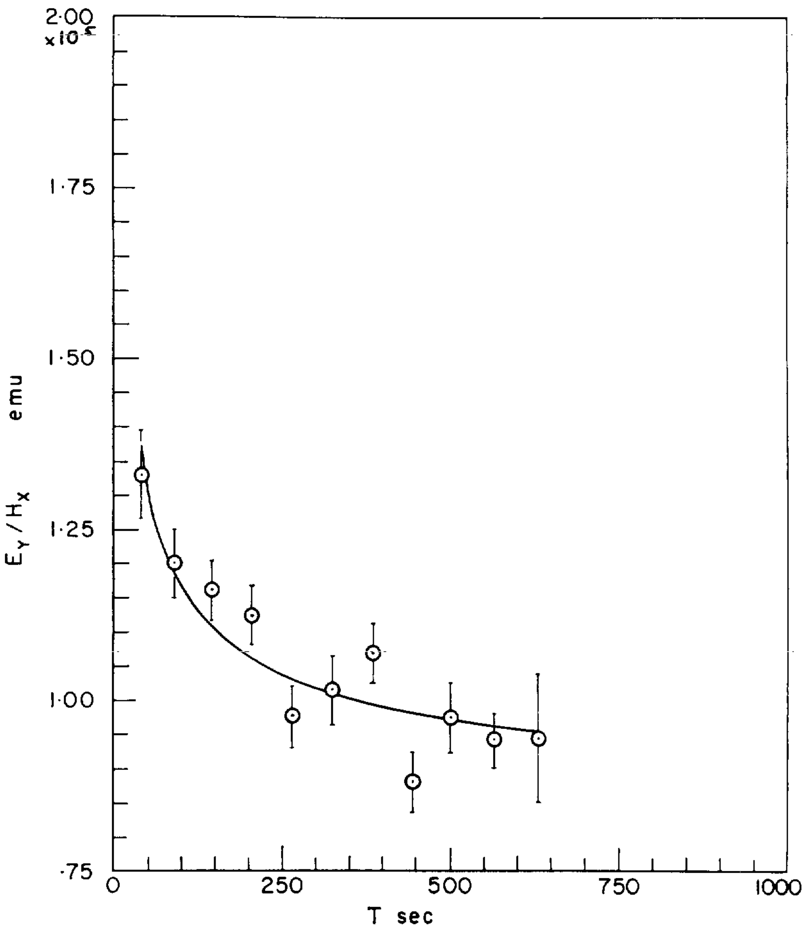

FIG. 2. Amplitude ratio E_y/H_x vs period T.

where B is a constant. In this case a plot of E/H vs $1/\sqrt{T}$ should yield a straight line through the origin with slope $B=1/\sqrt{2\sigma}$. Plots of this type are shown in Figure 3. It is clear a straight line through the origin will not provide a reasonable fit to either set of data, and that the conductivity must be subject to appreciable variation throughout the range of depths which are appropriate.

Figure 3 does suggest that the E_x/H_y data might be adequately fitted by two straight lines, and the E_y/H_x data by a single straight line. The equations are of the form:

$$\frac{E_x}{H_y} = A_1 + \frac{B_1}{\sqrt{T}} \text{ in the range 0.051 to 0.150 of } T^{-1/2}$$

$$\frac{E_x}{H_y} = A_2 + \frac{B_2}{\sqrt{T}} \text{ in the range 0.033 to 0.051 of } T^{-1/2}$$

$$\frac{E_y}{H_x} = A_3 + \frac{B_3}{\sqrt{T}} \text{ in the range 0.040 to 0.158 of } T^{-1/2}$$

The constants were determined by least squares and are:

$$A_1 = 1.109 \times 10^5 \qquad B_1 = 4.11 \times 10^5$$
$$A_2 = -0.073 \times 10^5 \qquad B_2 = 27.2 \times 10^5$$
$$A_3 = 0.815 \times 10^5 \qquad B_3 = 3.50 \times 10^5.$$

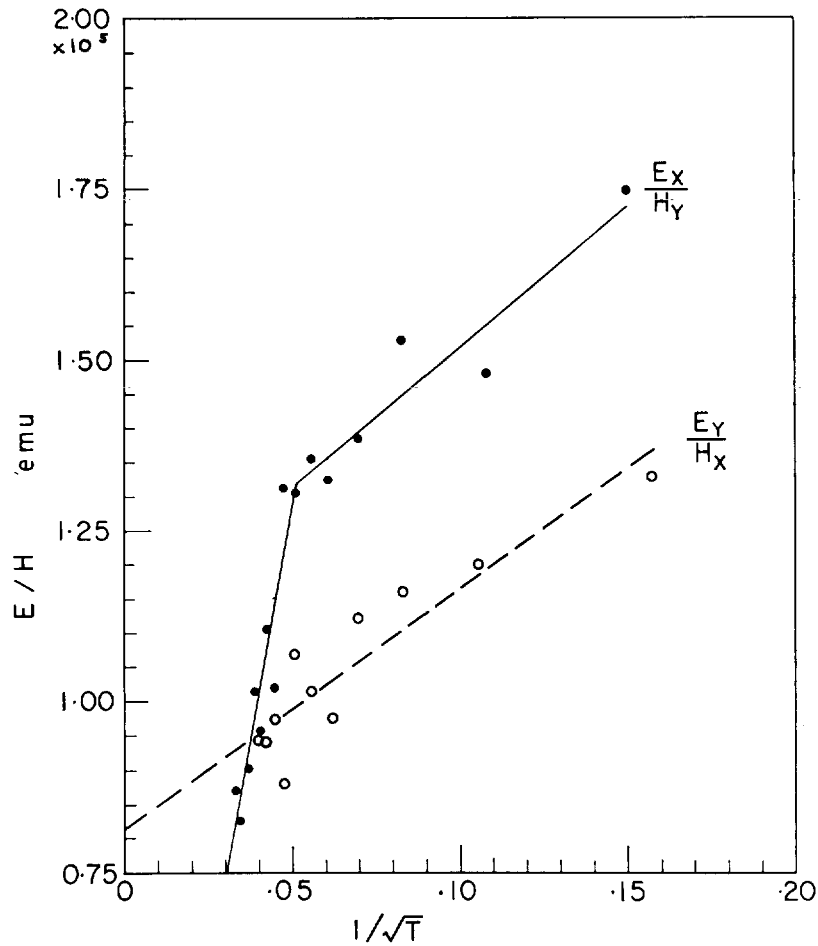

FIG. 3. Amplitude ratios vs $T^{-\frac{1}{2}}$.

The three computed straight lines are shown in Figure 3. Application of the "chi-square" test to the residuals indicates in each case that the linear representation is adequate for the data. The smooth curves of Figures 1 and 2 are obtained by plotting the computed values of E/H against T instead of $T^{-\frac{1}{2}}$.

Since we now express amplitude ratios in the form

$$\frac{E}{H} = A + \frac{B}{\sqrt{T}},$$

equations (8) and (9) become

$$\bar{\sigma} = \{2(A^2T + 2AB\sqrt{T} + B^2)\}^{-1}, \tag{13}$$

and

$$Z = \frac{1}{2\pi}(AT + B\sqrt{T}). \tag{14}$$

These give, on elimination of T

$$16\pi^2A^2Z^2\bar{\sigma}^2 - (8\pi AZ + 2B^2)\bar{\sigma} + 1 = 0. \tag{15}$$

Equations (13), (14), and (15) hold only in the specified ranges of $T^{-\frac{1}{2}}$.

The effective conductivity is plotted against penetration depth for both cases in Figure 4. Values derived from the observations are shown for comparison with the curves computed from equations (13) and (14). The discontinuity in the curve computed from the E_x/H_y data at $Z = 80$ km. corresponds to the intersection of the two straight lines in Figure 3.

From (12) and (15) we derive

$$\sigma = \frac{B^2\bar{\sigma}}{B^2 + 4\pi AZ(1 - 4\pi AZ\bar{\sigma})}. \tag{16}$$

The final curves showing conductivity as a function of depth are given in Figure 5.

DISCUSSION

It is important to consider the validity of the initial assumption that horizontal gradients of the field vectors are negligible compared to the vertical ones. Horizontal gradients can arise in two ways. Appreciable variation of electrical conductivity or magnetic susceptibility over horizontal planes at the depths considered would lead to field gradients of internal origin. Furthermore, overhead current systems in the ionosphere may be neither sufficiently remote nor of suitable configuration to produce transient variations at the surface which are uniform over horizontal distances up to 100 km. At Meanook the latter effect is likely to be important because the station is close enough (5 degrees in geomagnetic latitude) to the auroral zone for the magnetic activity to be subject

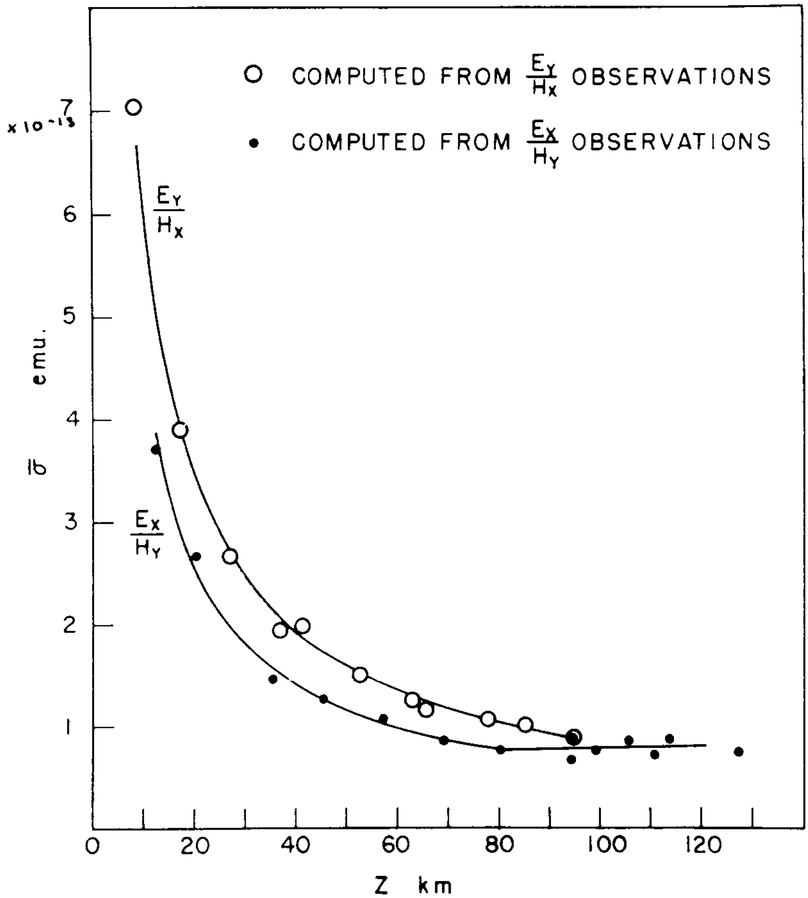

FIG. 4. Effective electrical conductivity $\bar{\sigma}$ vs depth Z.

to fairly rapid variation along the magnetic meridian. Since auroral zone currents are believed to occur at heights of the order of 150 km the transient field variations at the surface could vary considerably over a distance of 100 km particularly in the magnetic north-south direction.

From (5) it is evident that periodic field variations will have negligible horizontal gradients if

$$\frac{|H_z|}{\sqrt{|H_x|^2 + |H_y|^2}} \ll 1. \tag{17}$$

Since the only vertical force records available at Meanook are standard run magnetograms with sensitivities and chart speeds which are much less than those of the Brown recorders, a direct comparison between the amplitudes of vertical and horizontal force variations is not possible. However, hourly ranges of H_x, H_y, and H_z at the times when individual estimates of E_x/H_y and E_y/H_x were made were scaled off the standard run magnetograms. It was found that the ratio of hourly range activity in vertical force to hourly range activity in horizontal force varied from 0.1 to 2.5, the average value being 0.56. This gives a crude indication that for the longer periods the amplitude ratio of (17), while

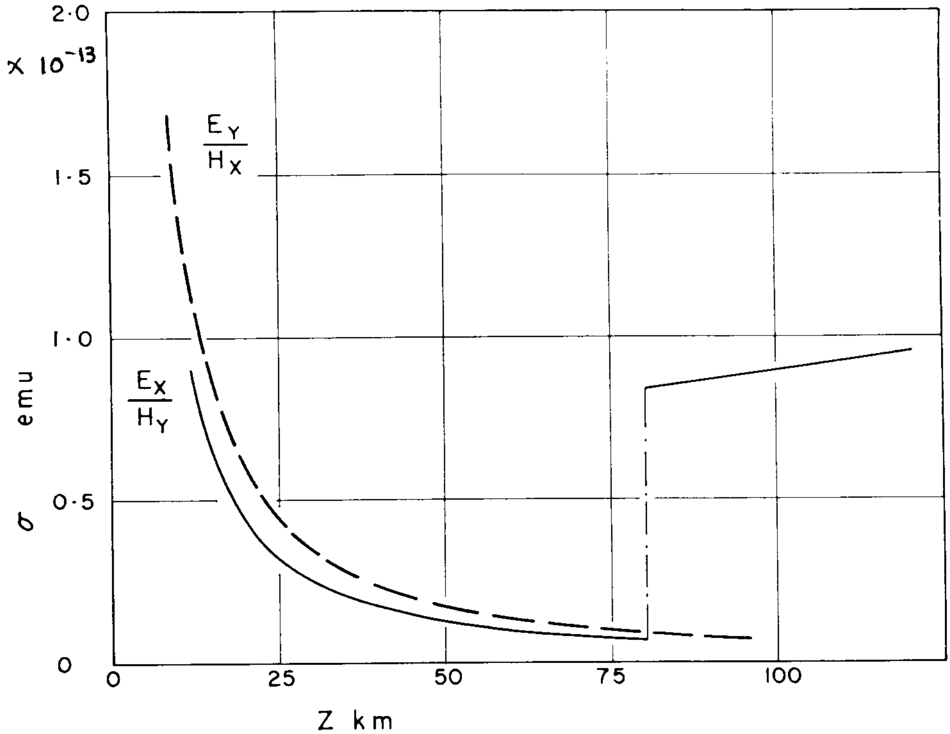

FIG. 5. Electrical conductivity σ vs depth Z.

subject to considerable variation, has an average value of the order of $\frac{1}{2}$. If this is the case, the horizontal gradients may be generally smaller than the vertical ones, but not small enough to be considered negligible.

Figures 1, 2, and 3 reveal that values of E_x/H_y are, in general, quite different from E_y/H_x at corresponding values of the period T. The discrepancies are too large to be explained by errors in the measurement of amplitudes, and it is again concluded that the conditions of horizontal uniformity imposed by setting $\partial/\partial x = \partial/\partial y = 0$ do not closely approximate actual conditions in the Meanook region. None the less, each set of data gives electrical conductivities which agree to within a factor of two between depths of 10 km and 80 km. The conclusion that the curves of Figure 5 show the correct order of magnitude and the general trend of the variation in this range seems justified.

A sudden increase in σ from 10^{-14} e.m.u. to 10^{-13} e.m.u. at 80 km is indicated by the E_x/H_y data. The discontinuity was forced into the curve when it was decided to fit two straight lines to these data as in Figure 3. This is much the simplest method of obtaining an analytical expression. A smooth curve of the form

$$\frac{E_x}{H_y} = A + \frac{B}{\sqrt{T}} + \frac{C}{T} + \cdots$$

could be fitted, but a function of at least 3rd degree in $T^{-\frac{1}{2}}$ would be required. Such a procedure is scarcely worth while. The conductivity must increase by roughly an order of magnitude between 60 km and 100 km to be consistent with the E_x/H_y data at longer periods. A smooth curve would only replace the discontinuity at 80 km by a fairly rapid rise in this region. Since condition (17) is most unlikely to be fulfilled at longer periods, and in the absence of corroborative evidence from the E_y/H_x data, it is far from certain that this rise in conductivity is real. The best that can be said is that a rise at about 80 km is not inconsistent with the E_y/H_x values plotted in Figure 3. However, good estimates of this ratio were not obtained at long enough periods to establish a significant trend at depths close to 100 km.

Surface values of σ vary from about 10^{-11} e.m.u. for sea water to 10^{-16} e.m.u. and less for certain types of dry earth and rock. An estimate of the surface conductivity can be obtained by extrapolating the conductivity depth curves of Figure 5 to $Z=0$. This is equivalent to putting $Z=0$ in equation (15) from which we derive $\bar{\sigma}_0=\sigma_0=1/(2B^2)$. The extrapolated values are $\sigma_0=3\times10^{-12}$ e.m.u. for the E_x/H_y data, and $\sigma_0=4\times10^{-12}$ e.m.u. for the E_y/H_x data.

In the Athabasca region, the Precambrian basement is overlain by Devonian and Cretaceous sediments to a depth of about 1.8 km. These rocks are predominately shales, sandstones, dolomites and limestones. Their conductivity, even allowing for moisture content, is unlikely to exceed 10^{-13} e.m.u. The extrapolated surface values appear to be high by at least an order of magnitude. The most likely explanation is that the computed conductivities decrease too rapidly with depth in the neighborhood of 10–15 kms. The curves would probably be subject to modification at these depths if the range of E/H determinations could be extended to periods of 10 sec or less.

There have not been many previous estimates of electrical conductivity at depths between 10 and 100 kms. Coster (1948) measured the conductivity of several rock samples at various temperatures up to 1,000°C. His conclusion that the conductivity at 100 km below the surface would be about 10^{-13} e.m.u. agrees well with the present result. He also notes that the conductivity would be expected to decrease over the first few kms below the surface as the rocks become progressively drier. At greater depths (and higher temperatures) the conductivity should increase as a consequence of the semi-conduction process in the basic rocks of the mantle. Lahiri and Price (1939) made estimates to a depth of about 1,500 km by comparing the induced fields of magnetic daily variation and storm time variation to their inducing fields. They concluded that the mean conductivity of the earth down to 600 km is about 2×10^{-13} e.m.u., and that at some regions near the surface the value must be higher than this. However, they account for their high surface value by treating the upper layer as a uniform ocean having a depth of about 1 km. With this interpretation the conductivity of the solid earth below would not exceed 10^{-15} e.m.u. to depths of 200 or 300 km. Rikitake (1950) adopts a similar figure for this range. While a high surface value is certainly indicated in the present analysis, the oceans are much too remote to affect the result. Between 10 and 100 km the estimated values are between one and two orders of magnitude higher than the one adopted by Lahiri and Price and Rikitake.

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