Geophysics 424, Assignment 1

Electromagnetic exploration techniques

This assignment is due at **5 pm** on **Wednesday October 11th 2023**

1. Prove that for a vector **E** the following identity is true

$$\nabla \wedge (\nabla \wedge \mathbf{E}) = \nabla(\nabla . \mathbf{E}) - \nabla^2 \mathbf{E}$$

2. Consider a plane EM wave, travelling vertically downwards (z-direction) and polarized with the electric field in the x-direction.

   Starting from Maxwell's equations show that:

$$H_y = \frac{-1}{i\omega\mu} \frac{\partial E_x}{\partial z}$$

   You can assume a harmonic time dependence with angular frequency, ω.

3. In this question, you will work through the derivation of the MT response for a 2 layer Earth. This was outlined in the class notes.

   a. Solve the 4 equations derived from the boundary conditions to obtain expressions for B_1 and B_2 in terms of A_1 , k_0, k_1 and h.

      Hint : Eliminate A_2 and C

   b. Show that at *z=0*

$$Z_{xy} = \frac{E_x}{H_y} = \frac{i\omega\mu}{k_1} \frac{[(k_1 + k_2)e^{2k_1 h} + (k_1 - k_2)]}{[(k_1 + k_2)e^{2k_1 h} - (k_1 - k_2)]}$$

   c. Evaluate this expression by writing a MATLAB script. Compute the apparent resistivity (ρ_a) and phase (Φ) using the following numerical values.

ρ_1 = 100 ohm-m
ρ_2 = 1 ohm-m
h = 5000 m

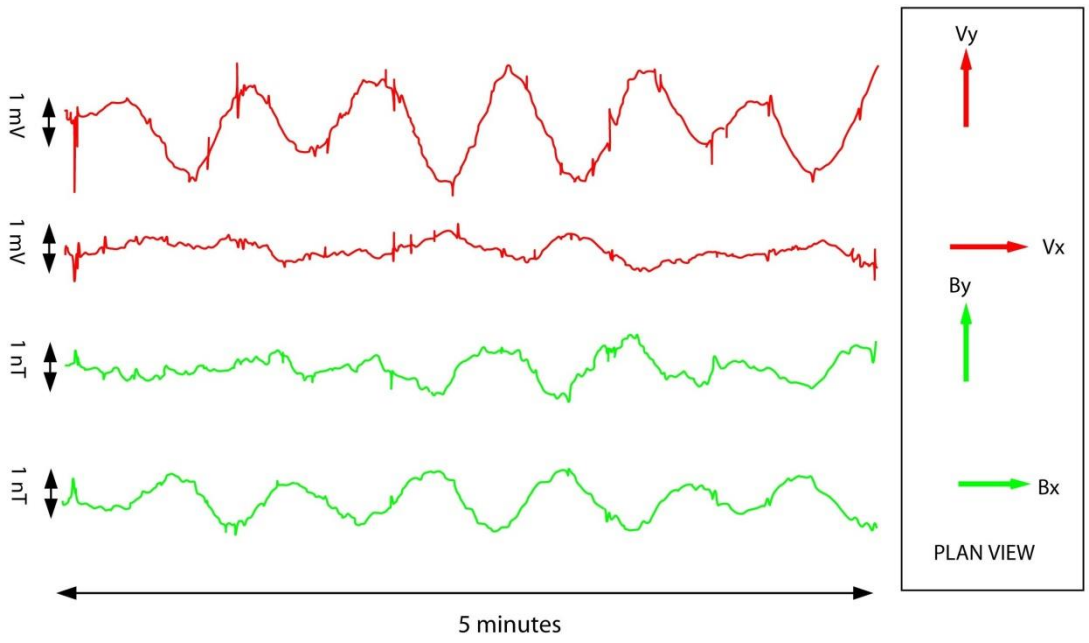
Frequency range : 1000-0.0001 Hz

Use a logarithmic scale for both apparent resistivity and frequency.

Include a copy of the MATLAB script with your answer

d. Validate your solution using the skin depth equation

4. To illustrate how apparent resistivity is calculated from an MT time series, look at the time series in the plot below.



(a) V_y is the voltage in the *y*-direction. Estimate the peak-to-peak value. This voltage is measured with a 100 m dipole. What is the value of the electric field strength, E_y?

(b) Convert E_y to **field units**. These are **millivolts per kilometre**.

(c) Measure the peak-to-peak value of the magnetic field, B_x, in nT. The field unit for magnetic field measurements is **nT**.

(d) Estimate the period, T, of the sinusoidal E_y-B_x variation in seconds.

(e) In class we derived that $\rho_{yx} = \dfrac{1}{\omega\mu}\left|\dfrac{E_y}{H_x}\right|^2$

Convert this equation to field units, noting that $B_x = \mu H_x$. Show that

$$\rho_{yx} = \frac{T}{5}\left|\frac{E_y^{field}}{B_x^{field}}\right|^2$$

(f) Calculate the value of apparent resistivity (ρ_{yx}) for this period.

(g) What is the approximate **phase difference** between E_y and B_x? Comment on your answer.