LAB 3: One-dimensional analysis of magnetotelluric data

- Lab report is **due by 5 p.m. on Wednesday October 16, 2024**
- Email submission to TA as a single PDF. **Include a digital copy of all MATLAB scripts.**
- Write-up should include required figures and answers to all questions.
- **All MATLAB scripts should be your own original work. Do not copy someone else's code.**
- **Do not use MATLAB scripts from other classes or from assignments in this course, unless told to do so.**
- All lab material can be downloaded from the class webpage at: https://sites.ualberta.ca/~unsworth/UA-classes/424/labs424-2024.html

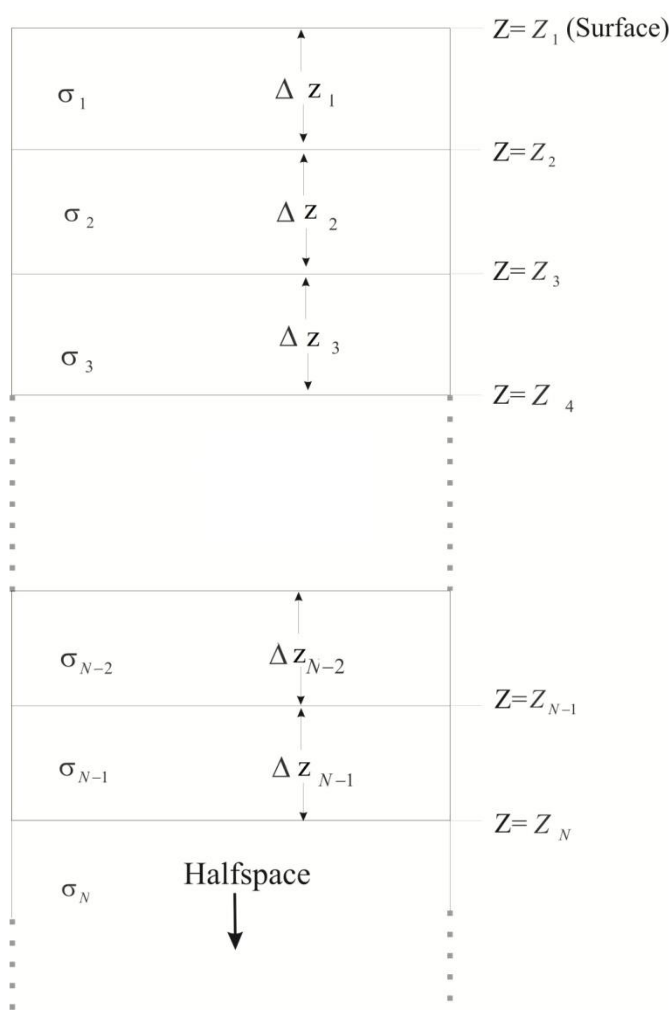

Figure 1: Illustration of Wait's recursion method.

PART 1: Introduction

In Assignment 1, you used a MATLAB code to compute the MT response of a 1-D resistivity model with 2 layers. Wait (1954) extended this analysis to consider a 1-D resistivity model with N layers (where N = any number), using a **recursion relation**. The recursion uses a **transfer function C** (which is a function of frequency) that is related to the **impedance** at the surface. The impedance at the surface of the earth can be used to calculate both apparent resistivity and phase (see class notes).

Let us consider the transfer function C_n at the top of the n^{th} layer and C_{n+1} at the top of the layer below. It can be shown that *C* is **continuous** across boundaries between layers.

Wait derived a **recursion relation** to relate C_n and C_{n+1}.

The recursion begins with the halfspace (the deepest layer, *N*), at the base of the model. On the upper surface of the halfspace (*n* = *N*) we can show that

$$C_N(\omega) = \frac{1}{\sqrt{i\omega\mu_0\sigma_N}} \tag{1}$$

We then recursively calculate the transfer function at the top of each layer above the halfspace:

$$C_n(\omega) = \frac{1}{k_n}\frac{[C_{n+1}k_n + tanh(k_n\Delta z_n)]}{[C_{n+1}k_n tanh(k_n\Delta z_n) + 1]} \tag{2}$$

where $k_n(\omega) = \sqrt{i\omega\mu_0\sigma_n}$ is the propagation constant in the n^{th} layer,

ω = angular frequency,

μ_0 = magnetic permeability of free space,

$\Delta z_n =$ the thickness of the n^{th} layer**,**

σ_n = electrical conductivity of the n^{th} layer, and

C_{n+1} = the transfer function calculated at the top of the layer below layer n.

Note that *n* ranges from 1 (the surface layer) to *N* (the halfspace).

Once the surface is reached (*n*=1) compute the impedance Z at the surface:

$$Z(\omega) = i\omega\mu_0 C_{n=1}(\omega) \tag{3}$$

Then use equations from the class notes to calculate the apparent resistivity and phase.

QUESTIONS:

- **1.1** Briefly describe how the magnetotelluric method works: what are typical sources of signal, what data is measured, and what kind of information is obtained about the subsurface?
- **1.2** Briefly explain how to calculate the frequency-dependent apparent resistivity and phase using Wait's recursion. This should be a summary of how your MT1D code in Part 2 works.

PART 2: Calculating Apparent Resistivity and Phase with a MATLAB function

- Write a MATLAB function using Wait's recursion to compute the apparent resistivity and phase for an N-layer Earth model.
  - Name this function **MT1D.m.** Do not plot anything in this function. Your function should simply calculate apparent resistivity and phase; the only inputs needed are a frequency array and the depths and resistivities of your model layers. Use the function format:
    [app_res,phase] = MT1D(model_depths , model_res , frequency_array).
    (Feel free to change the variable names.)
  - The model_depth array should contain depths to the top of each layer (i.e. the first depth will always be 0). We will use depths instead of layer thicknesses because the lowest layer, the halfspace, extends infinitely downward. See the files *'model-1.txt'* and *'model-2.txt'* for examples.
  - Use a logarithmically spaced frequency array (MATLAB function '*logspace*'). While testing your function, you may use a frequency range 1000 to 0.001 Hz (broadband MT).
- Then, write a MATLAB script called **plotMT1D.m** to plot the results of your **MT1D.m** function. Use the syntax above to call **MT1D.m.** This function should create a 3-panel figure using '*subplot*'. Plot the apparent resistivity using '*loglog*', and phase using '*semilogx*', with *period in seconds* along the horizontal axes. In the same figure, show the resistivity model (resistivity on horizontal axis, depth on vertical axis) using '*stairs*' or a similar approach. (Your figure should look similar to Figure 2.)
- Note: You will also need to use these codes in the next lab.

QUESTIONS:

- **2.1** Include your MATLAB function **MT1D.m** and make sure this is a stand-alone function that can be called from another function.
- **2.2** Include your plotting script **plotMT1D.m**, and make sure it calls **MT1D.m** before plotting.

PART 3: Testing your MATLAB function (quantitatively)

- Now you will test your **MT1D.m** function with some synthetic models. You will need your MATLAB code from Assignment 1, and the two text files ('*model-1.txt'* and '*model-2.txt'*). Answer all parts below.

QUESTIONS:

- **3.1** Test your **MT1D.m** function by calculating the apparent resistivity and phase for the two layer model in *'model-1.txt'*. Use a frequency range 1000 to 0.001 Hz (broadband MT). Using your **plotMT1D.m**, create a figure with the apparent resistivity, phase and resistivity depth profile. You may need to enlarge the plot window in order to show all the axis tick labels.

- **3.2** In Assignment 1 you calculated the apparent resistivity and phase for a two layer model (Question 3c). **Use the code from Assignment 1 to calculate the apparent resistivity and phase for '*model-1.txt'*** and see if you obtain the same result as Wait's recursion in question 3.1 (To plot two curves in the same figure use '*hold on*' and use different plotting styles so both curves are visible.) Include this figure, and make sure both curves are visible. Do the curves agree? If not, why? You may use a modified version of your **plotMT1D.m** to plot the comparison.

- **3.3** Display the three layer model and data from '*model-2.txt'* using your **MT1D.m** and **plotMT1D.m**. Use a frequency range 1000 to 0.001 Hz (broadband MT). Include this figure. Explain whether or not the apparent resistivity and phase curves are consistent with each other.

- **3.4** Calculate the period at which the top of the second layer (in the 3-layer model) will be detected. Use the skin depth equation and show your work. Does this agree qualitatively with your plot (from question 3.3) using **MT1D.m**?

PART 4: Forward Modelling

- Using your MATLAB functions developed in this lab, write another script called **fwdMT1D** which will:
  (1) Load and plot MT data from a text file (frequency, apparent resistivity, phase, and errors). Include error bars (given in data file). You will need to read the data from '*S3LAY_2020.dat'*.
  (2) Read in a resistivity model from a text file (with depths and resistivity). You will need to read the file *mod.txt*.
  (3) Calculate the apparent resistivity and phase for that model by calling your **MT1D.m** function. Important: Use the same frequencies from the data file to simplify the comparison.
  (4) Plot the calculated apparent resistivity, phase, and resistivity model (can modify code from **2.2**).
  (5) Calculate and show the RMS misfit between your calculated MT data and the actual data in the plot titles (can use '*sprintf* '). See Figure 2 for an example.

<u>QUESTIONS:</u>

- **4.1** Use the synthetic data found in '*S3LAY_2020.dat'* to test your function. **Find a good fit for this data** (r.m.s. misfit < 1.5), by creating a 3-layer model. You can start with the values given in the *mod.txt* file. Include the figure showing the resistivity model and the fit between the calculated and observed data (following the guidelines above). Also include what depths and resistivities you used (i.e. from your *mod.txt*).

- **4.2** Make the same plot using the data generated from your *mod.txt* and the noisy data in '*S3LAY_noisy2020.dat'*. Slightly adjust your model to obtain a better fit for the noisy data. Try to maintain a smooth model while keeping the r.m.s. misfit reasonably low. Again, include what model depths and resistivities you used (i.e. *mod.txt*) and a figure showing the data fit and resistivity model.

- **4.3** Comment on the difference between the data curves from both data files. Does your original model also fit the noisy data acceptably? Comment on why or why not? Was it harder to achieve an acceptable fit?

- **4.4** Compare the resistivity models you used to fit '*S3LAY_2020.dat'* and '*S3LAY_noisy2020.dat'*. Does a small amount of noise significantly change your resistivity model? Comment on how this would affect a geological interpretation of the resistivity model.

- **4.5** Include all codes used in these questions.

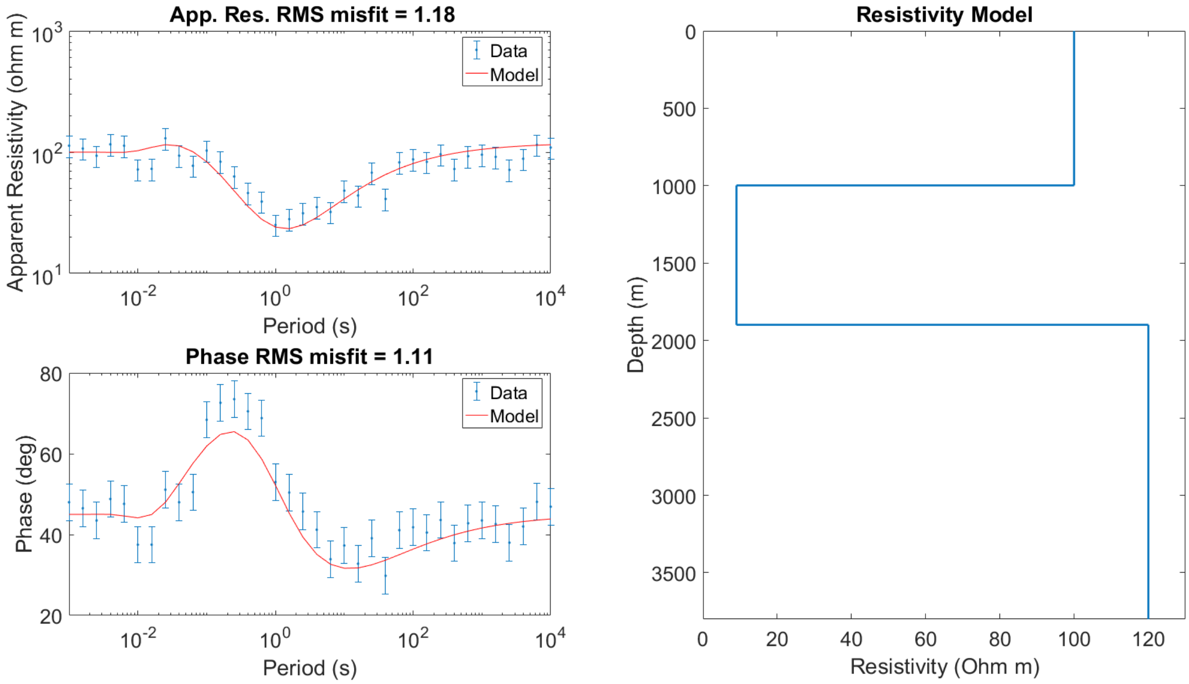

Figure 2: Example of comparing the measured data to the calculated data. The right panel shows the model used to calculate the modeled data.

REMINDER: The r.m.s misfit (root mean square) for ρ_{obs} & ρ_{calc} (observed and calculated apparent resistivity), *N* number of data points, and *error* – the amount of error in each data point:

$$r.m.s.misfit = \sqrt{\frac{\sum_{1:N}\left(\frac{\rho_{obs} - \rho_{calc}}{error}\right)^2}{N}}$$

Use the same equation to calculate the phase r.m.s. misfit:

$$r.m.s.misfit = \sqrt{\frac{\sum_{1:N}\left(\frac{\phi_{obs} - \phi_{calc}}{error}\right)^2}{N}}$$

Reference

Wait, J.R., "*On the relation between telluric currents and the earth's magnetic field*", Geophysics, **19**, 281-289, 1954.