(1) Archie's Law and resistivity of saline fluids

$$\rho = \rho_w S^{-n} \phi^{-m} \qquad \rho_w = 4.5\ TDS^{-0.85}$$

ρ = bulk resistivity of a rock
Φ = porosity
m = cementation factor ($1 < m < 2$)
TDS = salinity (g/litre).
ρ_w = fluid resistivity
S = fluid saturation.
n = saturation exponent = 1

(2) Resistivity and resistance

A sample of rock has resistivity (ρ), length (L) and cross-sectional area (A). The resistance (R) is given by

$$R = \frac{\rho L}{A}$$

(3) Maxwell's equations

$\nabla . \mathbf{E} = C/\varepsilon_0$	$\int_S \mathbf{E} . d\mathbf{S} = C/\varepsilon_0$	Coulombs Law
$\nabla . \mathbf{B} = 0$	$\int_S \mathbf{B} . d\mathbf{S} = 0$	Magnetic flux
$\nabla \wedge \mathbf{B} = \mu \mathbf{J} + \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t}$	$\oint \mathbf{B} . d\mathbf{l} = \mu \int_S \mathbf{J} . d\mathbf{s}$	Ampère's Law
$\nabla \wedge \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} . d\mathbf{l} = - \frac{\partial}{\partial t} \int_S \mathbf{B} . d\mathbf{S}$	Faraday's Law

E = electric field strength
J = electric current density
B = magnetic flux density
H = magnetic field strength
μ = magnetic permeability
ε = dielectric permittivity
C = charge density

$\mathbf{B} = \mu \mathbf{H}$ $\qquad \mu_0 = 4\pi \times 10^{-7}\ H/m$

$\mathbf{J} = \sigma \mathbf{E}$ $\qquad \varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

$Z_{xy} = \frac{E_x}{H_y}$ = impedance

$$\nabla \wedge \mathbf{A} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix} \qquad \nabla \cdot \mathbf{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z$$

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(4) Skin depth

$\delta = \frac{500}{\sqrt{\sigma f}}$ f is frequency in Hertz, σ is the conductivity in *S/m*, δ in metres

(5) Low induction number EM systems (frequency domain)

s = TX-RX separation; v = **vertical** dipoles; h = **horizontal** dipoles

$R_V(z) = \frac{1}{(4z^2+1)^{1/2}}$; $R_H(z) = (4z^2+1)^{1/2} - 2z$

Normalized depth : $z = \frac{d}{s}$

Two layer conductivity model

Conductivities σ_1 and σ_2, separated by an interface at depth = d

Apparent conductivity

$\bar{\sigma}_v = \sigma_1(1 - R_v(z)) + \sigma_2 R_v(z)$ $\bar{\sigma}_h = \sigma_1(1 - R_h(z)) + \sigma_2 R_h(z)$

Three layer conductivity model

Conductivities [σ_1 , σ_2 ,σ_3] separated by interfaces at depths = d_1 and d_2

Apparent conductivity

$$\bar{\sigma}_v = \sigma_1(1 - R_v(z_1)) + \sigma_2(R_v(z_1) - R_v(z_2)) + \sigma_3 R_v(z_2)$$

$$\bar{\sigma}_h = \sigma_1(1 - R_h(z_1)) + \sigma_2(R_h(z_1) - R_h(z_2)) + \sigma_3 R_h(z_2)$$

(6) Time domain EM

Transmitter current = I (amps); B_z = magnetic field (T)
Transmitter area = A *(m^2)* Number of turns on transmitter = N
Earth conductivity = σ*(S/m)* Time after switch-off = t *(s)*

Late time transient decay: $\frac{dB_z(t)}{dt} = \frac{\mu NIA}{20}\left(\frac{\mu\sigma}{\pi}\right)^{\frac{3}{2}} t^{-\frac{5}{2}}$

Effective penetration depth at time = t $\delta_T = \left[\frac{2t}{\sigma\mu}\right]^{\frac{1}{2}}$

(7) Ground penetrating radar

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

c = speed of light = 3 x 10^8 m/s; Moisture content= θ_v
Relative permittivity = ε_r Relative permeability = μ_r
$\varepsilon_r = 3.03 + 9.3\theta_v + 146\theta_v^2$ $\theta_v = -5.3x10^{-2} + 2.92x10^{-2}\varepsilon_r - 5.5x10^{-4}\varepsilon_r^2$