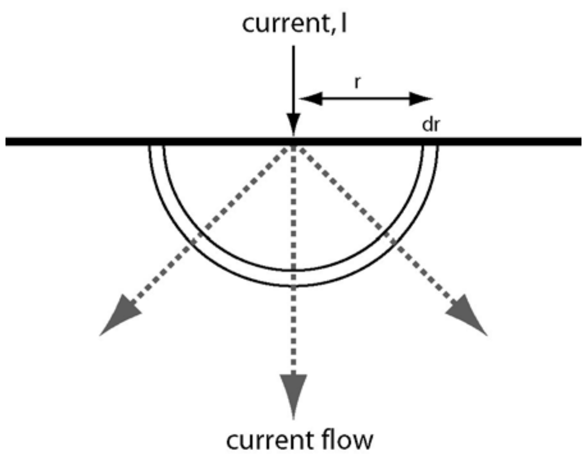
325 C2 Electric current flow in a half-space

C2.1 Potential of a single current electrode

In the lab, the electrical resistivity of a rock sample can be measured by placing flat electrode plates on each side of a rectangular sample. In this geometry the electric current flow is parallel and the simple equation derived in C1.1 can be used to compute the resistivity, ρ from the measured resistance (R).

However this approach is not practical for the measuring the Earth, since we cannot inject current from large plates. Thus we must consider the electric current flow from a simple electrode (metal spike).



Consider an electric current, I, flowing from an electrode. The air has a very high electrical resistivity, so all current flows in the Earth. From symmetry arguments, the current spreads out uniformly in all directions. Now consider a shell of rock, with radius, *r*, and thickness *dr*. The voltage (potential) drop across the shell is ΔV

The resistance of the hemispherical shell, $R = \frac{\rho L}{A} = \frac{\rho dr}{2\pi r^2} = \frac{\Delta V}{I}$

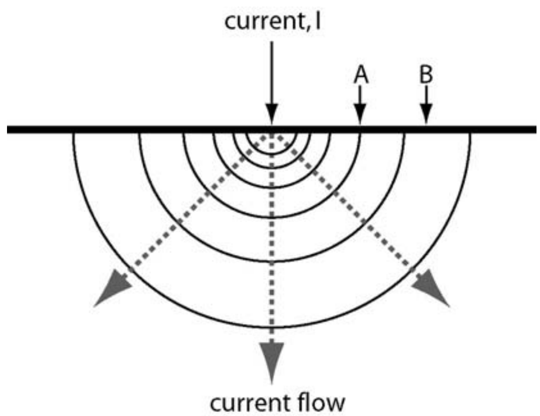
Rearranging and taking limits gives $dV = \frac{I\rho}{2\pi r^2} dr$

To compute the potential, V, apply the boundary condition that V = 0 when r = ∞ and integrate to give:

$$V = \frac{I\rho}{2\pi}\int \frac{1}{r^2}\,dr = \frac{I\rho}{2\pi}\left[-\frac{1}{r}\right]_r^\infty = -\frac{I\rho}{2\pi r}$$

Thus surfaces of equal potential (equipotentials) will be **hemispheres** centered on the electrode. Note that the electric current flow is at **right angles** to the equipotential.

Can this geometry be used to measure the resistivity of the Earth?



The voltage between the electrodes A and B is defined as $\Delta V_{AB} = V_A - V_B$

Using the above result $\Delta V_{AB} = \frac{I\rho}{2\pi}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$

where r_A and r_B are the distances from the current electrode to the potential electrodes A and B respectively. Rearranging this equation gives

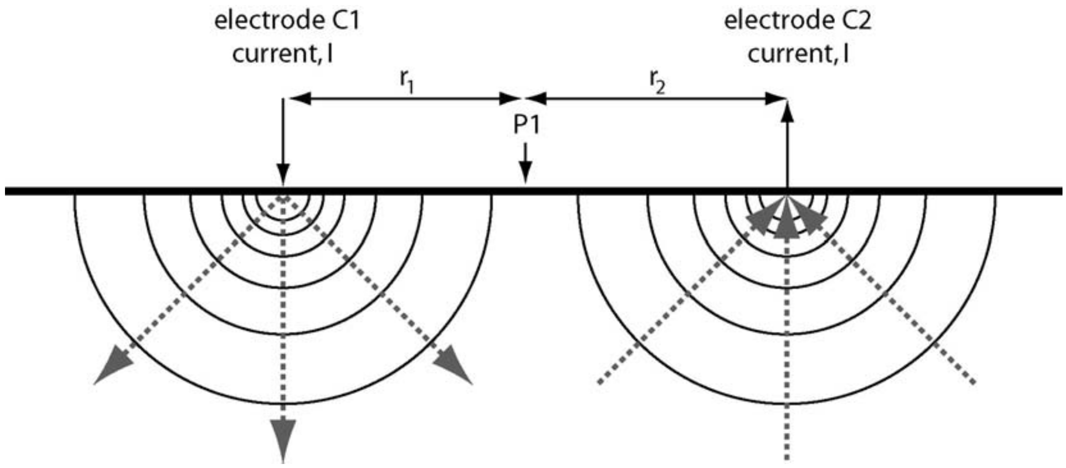
$$\rho = \frac{2\pi \Delta V_{AB}}{I(\frac{1}{r_A} - \frac{1}{r_B})}$$

Note that this is essentially Ohms Law with a geometric factor added.

Why is this not a practical way to measure the resistivity of the Earth?

C2.2 Potential of two-current electrodes, definition of apparent resistivity

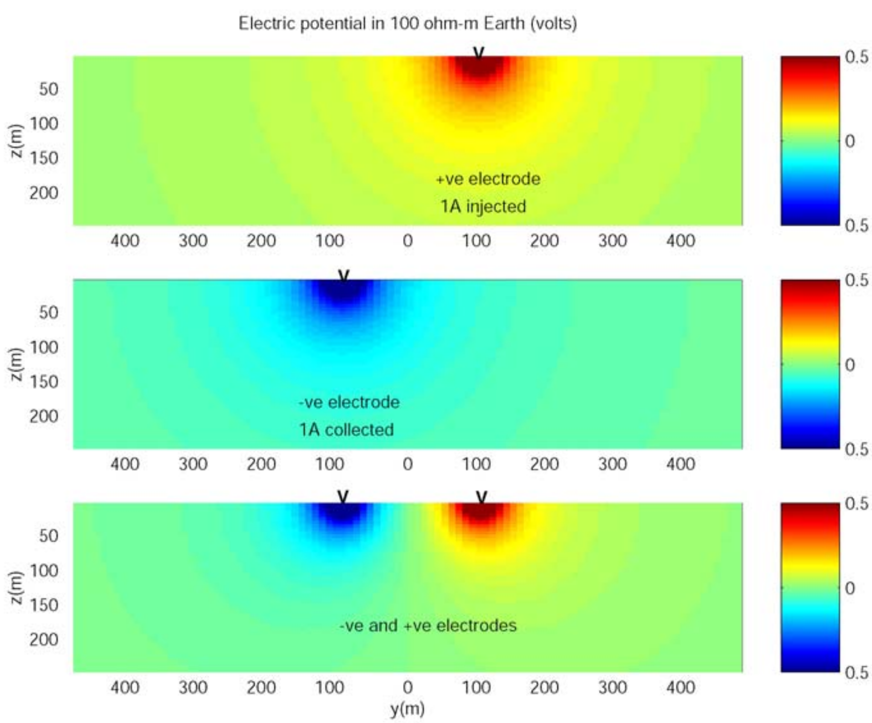
A more realistic situation uses to current electrodes. Current is injected through one electrode and withdrawn through the other.



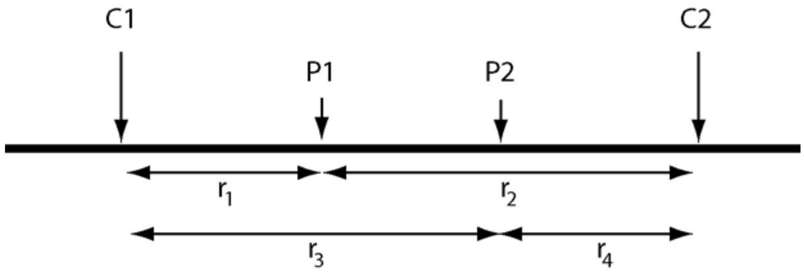
To compute the potential at electrode P1, we can simply add the potentials generated by the two current electrodes C1 and C2.

$$V_{P1} = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Note that V_{P1} will be zero as r_1 or $r_2 \rightarrow \infty$ and when $r_1 = r_2$ (on a plane equidistant between C1 and C2). This expression can be easily evaluated with a MATLAB script.



However, to measure a voltage, we need **two** potential electrodes to connect to a voltmeter. Consider the arrangement of electrodes shown below.



The voltage difference measured between electrodes P1 and P2 is given by

$$\Delta V = V_{P1} - V_{P2} = \frac{I\rho}{2\pi}\left[(\frac{1}{r_1} - \frac{1}{r_2}) - (\frac{1}{r_3} - \frac{1}{r_4})\right]$$

$$= \frac{I\rho}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right]$$

Now let us make the geometry of the array simple, with the 4 electrodes separated by a distance *a*. Then we have $r_1 = r_4 = a$ and have $r_3 = r_3 = 2a$

$$\Delta V = V_{P1} - V_{P2} = \frac{I\rho}{2\pi}\left[(\frac{1}{a} - \frac{1}{2a}) - (\frac{1}{2a} - \frac{1}{a})\right]$$

$$= \frac{I\rho}{2\pi a}\left[(1 - \frac{1}{2}) - (\frac{1}{2} - 1)\right]$$

$$= \frac{I\rho}{2\pi a}$$

This represents a solution to a **forward problem** i.e. for a model of the Earth (resistivity, ρ) we can predict the value of ΔV that will be observed in a geophysical survey. Simple rearrangement gives us a solution to the corresponding **inverse problem**.

$$\rho = \frac{2\pi a \Delta V}{I}$$

This equation shows how the resistivity of the Earth can be computed from field measurements of *ΔV, I* and the electrode spacing (*a*). Again, this equation is essentially Ohms Law, with a geometric factor to account for the complex current flow pattern.

If the Earth has a uniform structure, with the resistivity equal to *ρ* at all points, then the measured resistivity value will equal the actual resistivity value of the Earth.

However, if the resistivity is variable, the resistivity computed will be an average value over the region in which the current is flowing. This average resistivity is termed the **apparent resistivity** and defined as

$$\rho_a = \frac{2\pi a \Delta V}{I}$$

The figure below shows a quantitative evaluation of the electric current flow pattern.

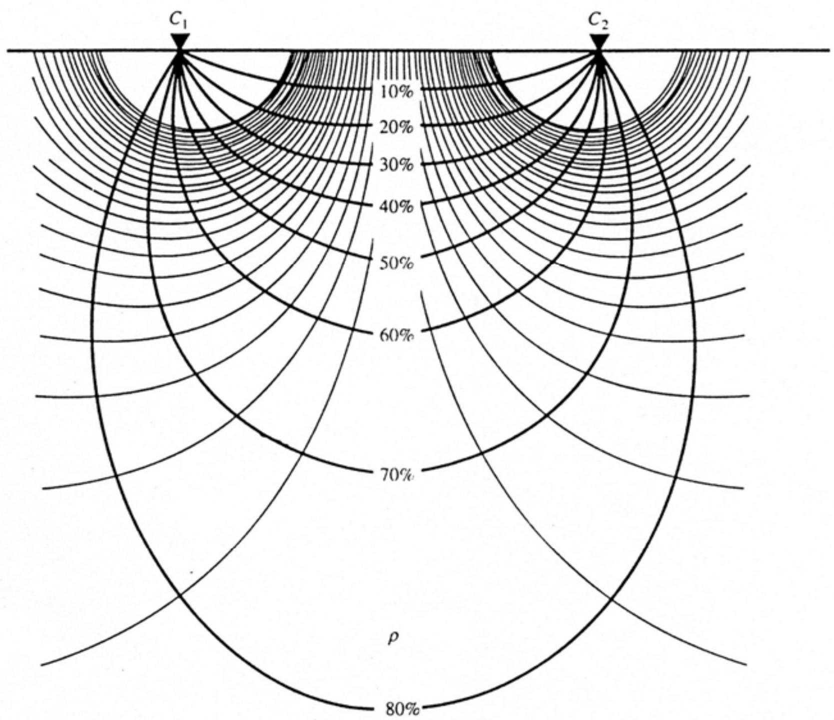


Figure 5-8 Equipotential surfaces and current lines of flow. Labels indicate percent of total current that penetrates to the depth of the line.

Note that:

- Electric current does not flow directly from one current electrode to the other in a straight line. This is because the charge carriers repel one another.
- Electric current flow is at right angles to the equipotential surfaces.
- Approximately 50% of the electric current flows within a depth *a* of the surface.
- The apparent resistivity can be considered the average resistivity over a volume that is located between the electrodes, and in the depth range from the surface to a depth equal to the *a*-spacing.