C1: Electrical resistivity of different soil and rock types

325 C1.1 Basic physics of electric current flow

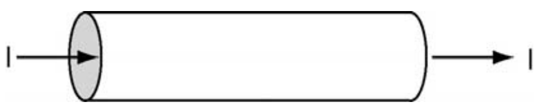
(a) Simple resistor in circuit

Ohm's Law states that for a resistor, the resistance (in ohms), R is defined as $R = \frac{V}{I}$

V = voltage (volts); I = current flow (amps)

(b) Electric current flow in a finite volume

Ohm's Law as written above describes a resistor, which has no dimensions. In considering the flow of electric current in the Earth, we must consider the flow of electric current in a finite volume. Consider a cylinder of length L and cross section A that carries a current I



J = current density = $\frac{\mathrm{I}}{A}$

R = resistance of cylinder $\propto \frac{\mathrm{L}}{A} = \frac{\rho L}{A}$

where ρ is the **electrical resistivity** of the material (ohm-m). This is the resistance per unit volume and is an inherent property of the material.

$$\rho = \frac{RA}{L}$$

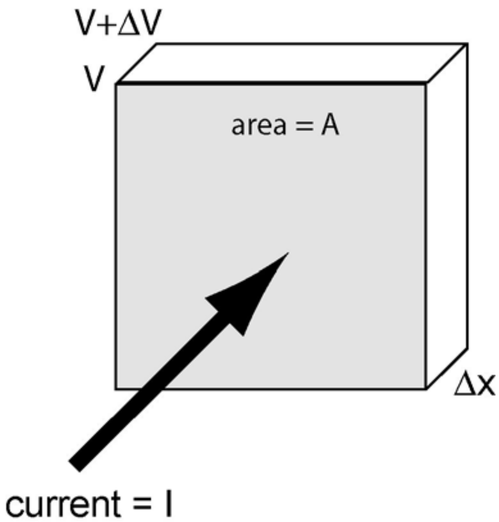
If we were to examine two cylinders made of the same material, but with different dimensions, they would have the **same** electrical resistivity, but **different** electrical resistances.

Often it is more convenient to discuss the **conductivity** (σ) which is measured in Siemens per metre.

$$\sigma = 1/\rho$$

(c) Electric current flow across a slab of material

Consider an electric current (I) flowing through a slab of material with resistivity, ρ and cross-sectional area, A



Applying Ohms Law

$$R = \frac{V}{I}$$

$$\frac{\rho \Delta x}{A} = \frac{\Delta V}{I}$$

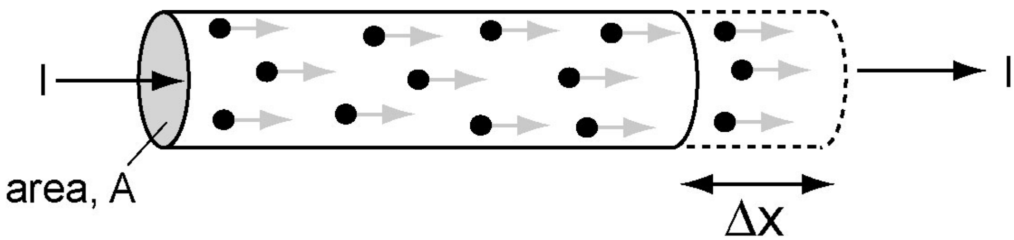
Rearranging gives $$\frac{\Delta V}{\Delta x} = \frac{I\rho}{A}$$

Taking limits $$\frac{dV}{dx} = E = \frac{I\rho}{A} = J\rho$$

Thus Ohms Law for a continuous medium can be written as $J = \sigma E$ where E is the electric field strength (Volts per m)

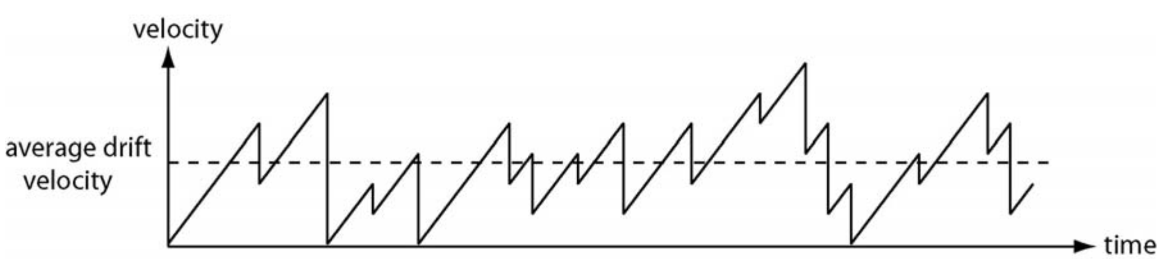
(d) Charge carriers

Electric current will flow through a medium as charge carriers move under an applied electric field (E). How is the resistivity (ρ) related to the number and type of charge carrier? Consider current flow through a cylinder of length *L* and area *A*.



n = number of charge carriers per unit volume
q = the charge on each carrier

Consider one of the charge carriers. It will accelerate under the applied electric field until it strikes an atom or another charge carrier. Thus it will move through the material with an average velocity, v



The ease with which the charge carrier can move is described by the mobility, μ, which is defined as the drift velocity per unit electric field = v/E

In a time Δt, the electric charges will move a distance $\Delta x = v\Delta t$.

This corresponds to a volume of charge carriers $= Av\Delta t$

The total charge leaving the cylinder is thus $\Delta q = nqAv\Delta t$

By definition, the current, $I = \frac{\Delta q}{\Delta t} = \frac{nqAv\Delta t}{\Delta t} = nqAv$

Thus current density, J $= \frac{I}{A} = nqv = nq\mu E$

By comparison with Ohms Law, we see that $\rho = \frac{1}{nq\mu}$

Thus a material will have a low electrical resistivity if it has many, highly mobile, charge carriers.

If several types of charge carriers are present, then the contribution from each type must be summed.

<u>325 C1.2 Electrical resistivity of pure elements and compounds</u>

Several conduction mechanisms are possible in typical Earth materials. A list of some minerals is given on Telford, page 285.

- electronic conduction occurs in pure metals. Here the charge carriers are electrons and their high mobility gives a very low resistivity ($<10^{-8}$ ohm-m)

- semi conduction occurs in minerals such as sulphides. Here the charge carriers are electrons, ions or holes. Compared to metals, the mobility and number of charge carriers are lower, and thus the resistivity is higher (typically 10^{-3} to 10^{-5} ohm-m).

  This type of conduction occurs in igneous rocks and usually shows a temperature dependence of the form (thermally activated)

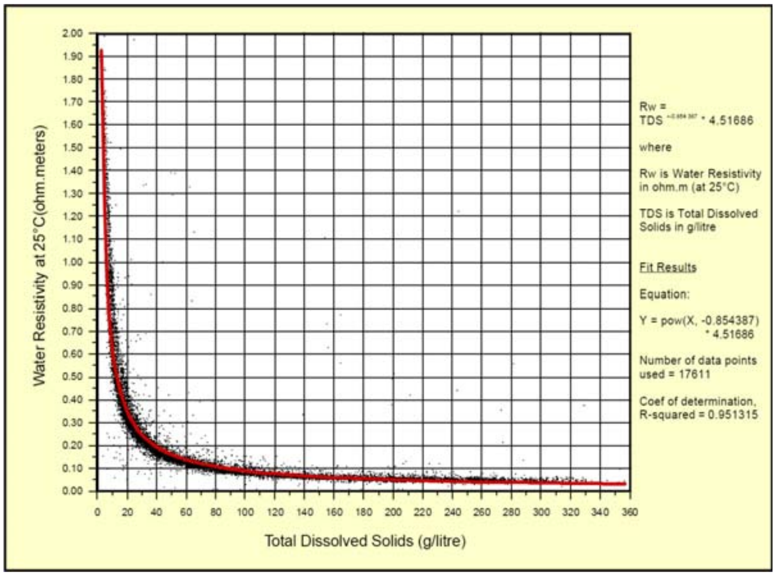
  $$\rho \propto e^{\frac{E}{kT}}$$

  where T is the temperature in K, E is an activation energy and k is the Boltzmann constant.

- Ionic conduction occurs in aqueous fluids or molten rocks. In this case the charge carriers are ions that can move through the fluid. The figure below shows that resistivity in brines decreases as the total dissolved solids (TDS) increases.

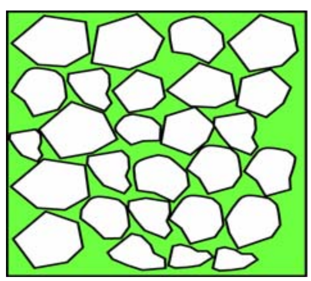
  $\rho = 4.5\ \mathrm{TDS}^{-0.85}$ (ohm-m)

  Can you explain the shape of the curve?



325 C1.3 Electrical resistivity of multiphase materials

Pure materials are rarely found in the Earth and most rocks are a mixture of two or more phases (solid, liquid or gas). Thus to calculate the overall electrical resistivity of a rock, we must consider the individual resistivities and then compute the overall electrical resistivity. Consider a sandstone saturated with salt water. The grains are quartzite and have a high resistivity (> 1000 ohm-m).
In contrast, the pore fluid is conductive (~1 ohm-m).



To compute the overall electrical resistivity, we must consider current flow through each phase. However, given the much higher resistivity of the grains, most current will flow through the water, with ions as the charge carriers.

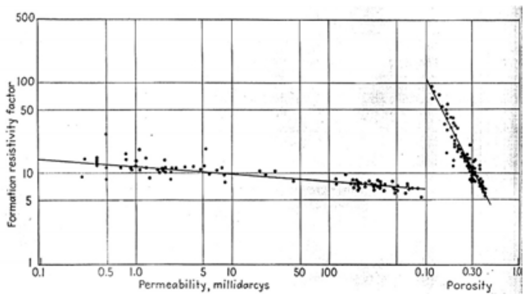
An empirical formula was developed for this scenario by Gus Archie in 1942. Archie's Law states that the resistivity of a completely saturated whole rock (ρ_o) is given by

$$\frac{\rho_o}{\rho_w} = F = \phi^{-m}$$

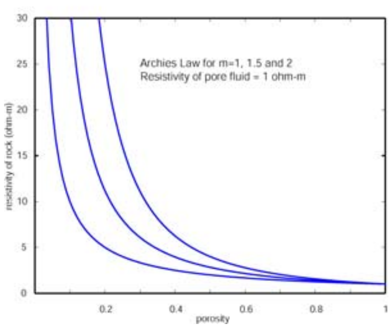
where F is called the **formation factor**, ρ_w is the resistivity of the pore fluid (water) and Φ is the porosity. On a log-log plot of ρ_o as a function of Φ, a straight line should result with slope $-m$. This exponent m termed the **cementation factor**. Typical values include:

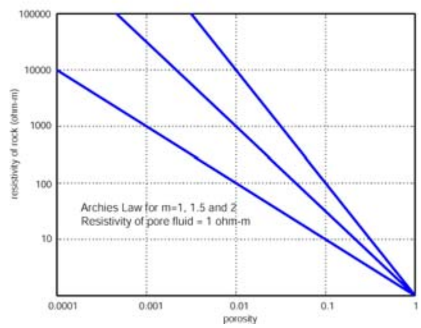
1.8-2.0 for consolidated sandstones to 1.3 for unconsolidated sands. The graph on the right is taken from Archie (1942) from Nacatoch sand from Lousiana. What is the value of *m* for this set of samples?

What is the difference between permeability and porosity? Are they correlated?

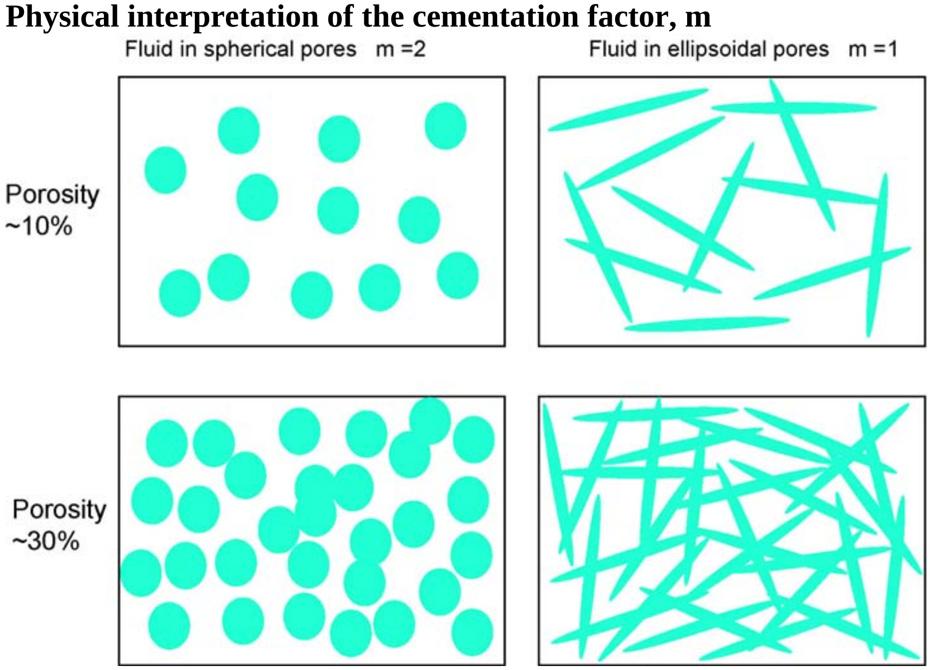


The exponent *m* is a constant termed the **cementation factor**. Typical values include: 1.8-2.0 for consolidated sandstones to 1.3 for unconsolidated sands. The following plots show **theoretical** results when ρ_w = 1 ohm-m.





Physical interpretation of the cementation factor, m

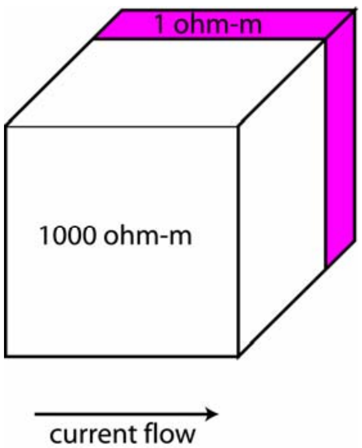


Note that the elongated pores will connect to form an interconnected electrical network at a lower porosity than the spherical pores. Is the permeability of the two cases different?

The above discussion shows that the resistivity of a fluid saturated rock depends on the **amount of fluid** and it's **distribution** (degree of interconnection).

To emphasize this point, consider the two limiting cases of the fluid distribution.

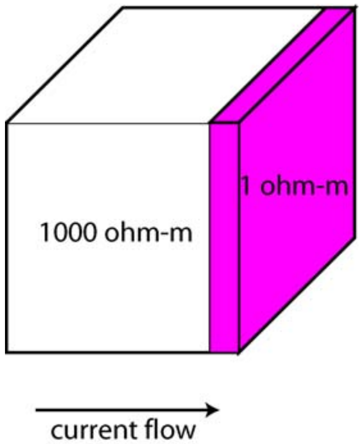
A – Fluid in cracks parallel to electric current flow



The sample has 10% porosity. This fluid geometry represents a **parallel circuit**, and electric current can effectively bypass the resistive rock grains and travel through the sample entirely in the conductive liquid.

What is the overall resistivity of the cube?

B – Fluid in cracks perpendicular to the electric current flow



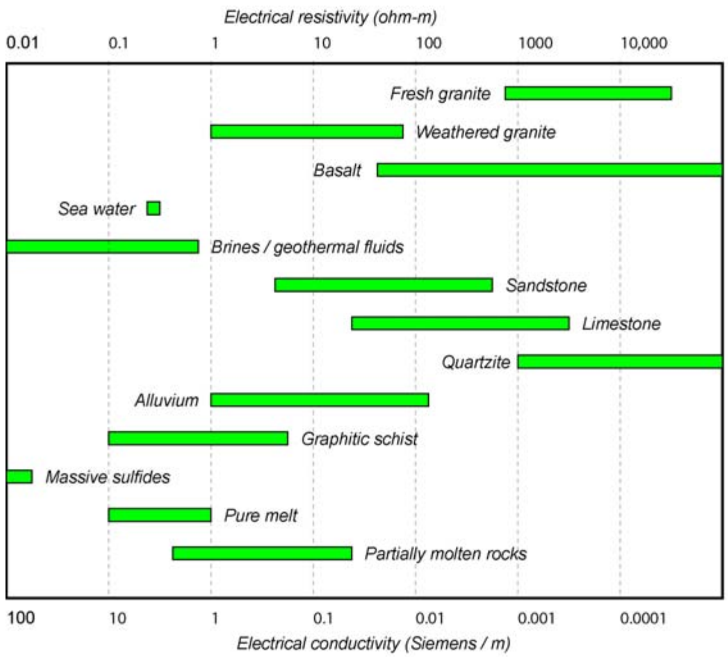
The sample has 10% porosity. This geometry represents a **series circuit**, and electric current cannot effectively bypass the resistive rock grains. It must travel through both the conductive liquid and resistive rock grains.

What is the overall resistivity of the cube?

C1.4 Clay minerals

If a rock contains clay minerals, then an extra conduction pathway is possible via the electrical double layer that forms at the interface of the clay mineral and the water. This effectively allows ions to move through the system with a lower effective viscosity (higher mobility) than in the liquid phase. More about this in Geophysics 424.

Summary
●The low resistivity phase dominates the overall resistivity.
●Overall resistivity is very sensitive to the geometry (distribution of the fluid).



Factors that will **DECREASE** the resistivity of a rock:
(a) Add more pore fluid
(b) Increase the salinity of the pore fluid - more ions to conduct electricity
(c) Fracture rock to create extra pathways for current flow
(d) Add clay minerals
(e) Keep fluid content constant, but improve interconnection between pores

Factors that will **INCREASE** the resistivity of a rock
(a) Remove pore fluid
(b) Lower salinity of pore fluid
(c) Compaction - less pathways for electric current flow
(d) Lithification - block pores by deposition of minerals
(e) Keep fluid content constant, but decrease connection between pores

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