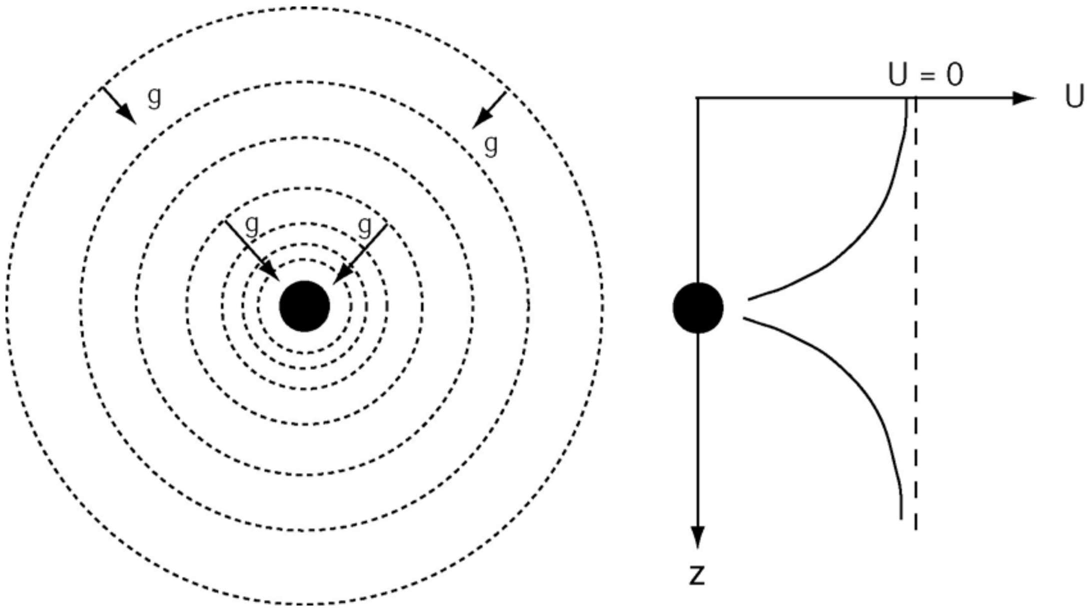
B.2 PHYSICS OF GRAVITY EXPLORATION

B.2.1 Acceleration of gravity (g) for a distributed mass

B.2.2 Gravitational potential energy and equipotential surfaces

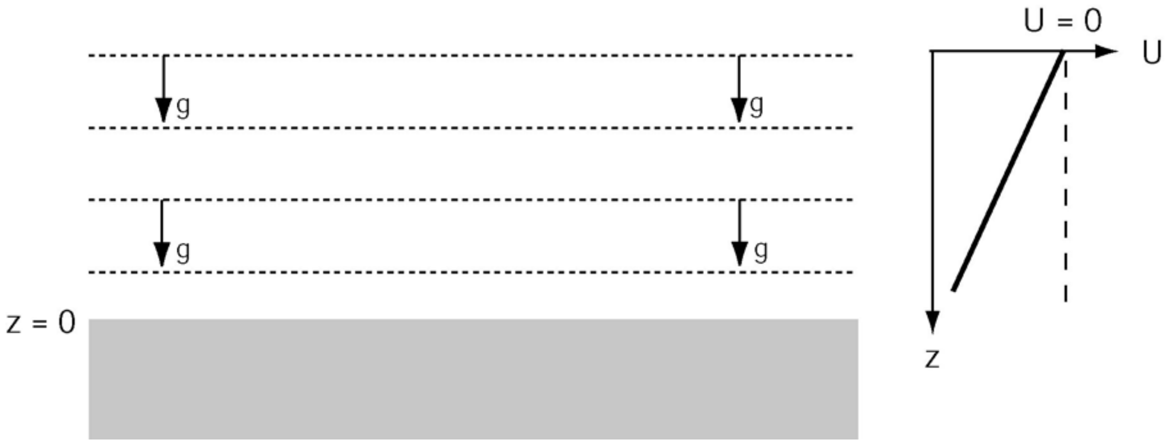
Equipotential surfaces

Example 1 : Sphere at z = 0 $\boldsymbol{g} = -\nabla U$

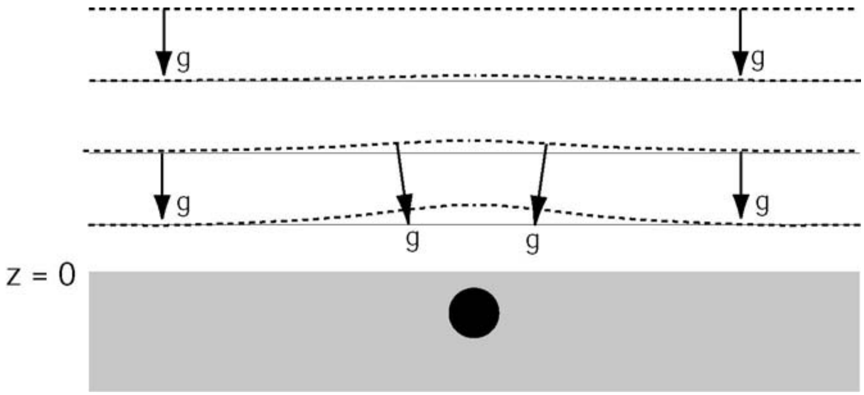


Example 2 : g constant as elevation increases. $\boldsymbol{g} = -\nabla U$

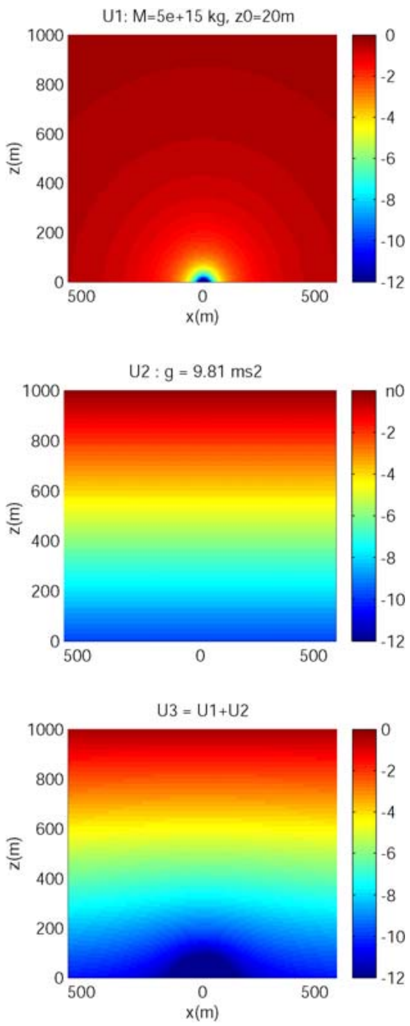
Earth surface at z=0



Example 3 : Point mass located just below the Earth's surface



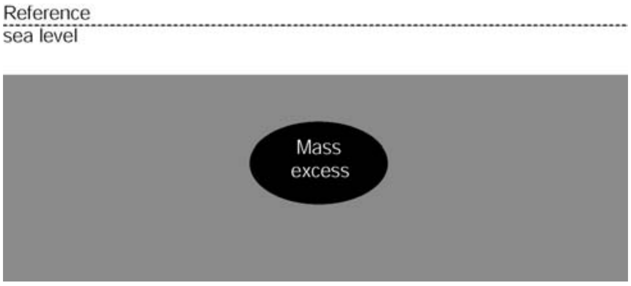
These equipotentials can be simply evaluated and plotted in a MATLAB script **Uplot.m**

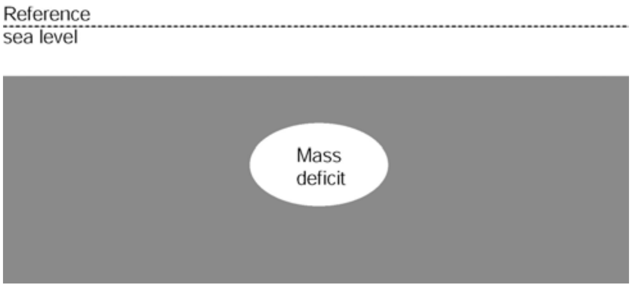


<u>The Geoid</u>

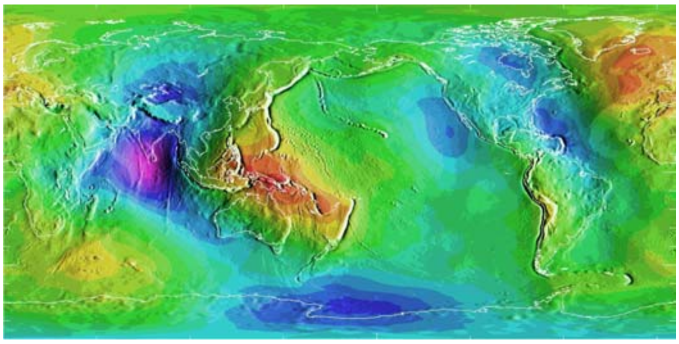
How will the sea surface respond to changes in subsurface density?

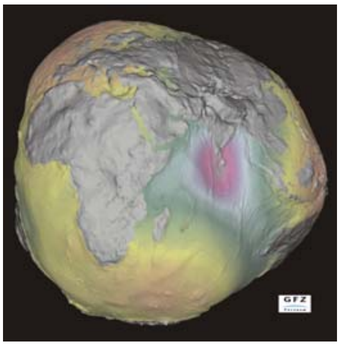
How is the sea surface related to an equipotential surface?



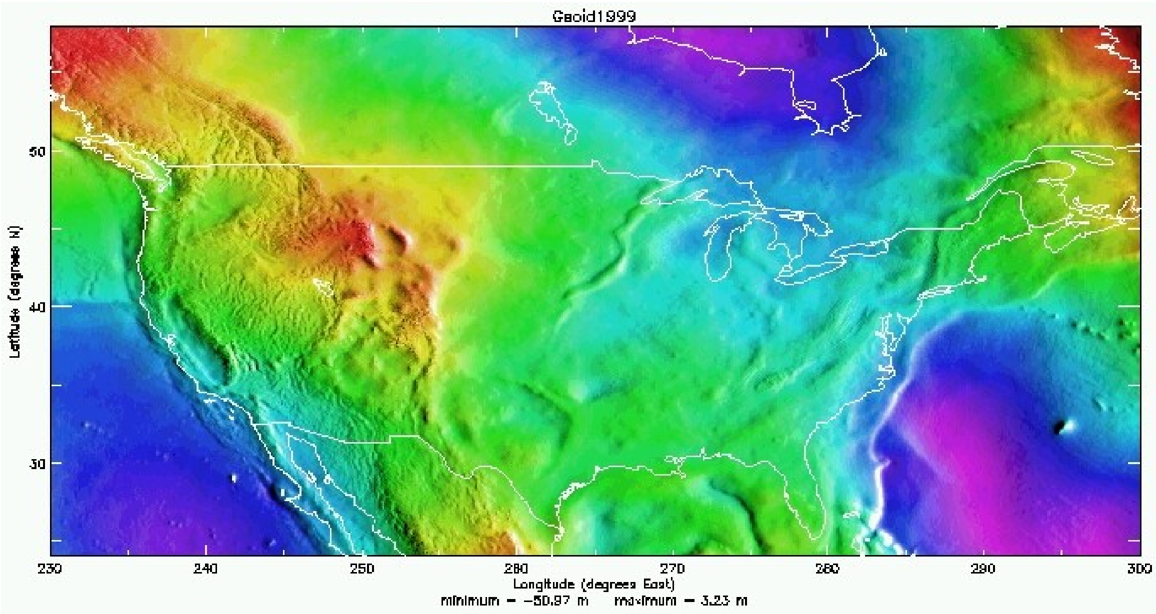


The geoid is defined as the **equipotential** surface of the Earth gravity field that **most closely approximates the mean sea surface**. At every point the geoid surface is perpendicular to the local plumb line. It is therefore a natural *reference for heights* - measured along the plumb line. At the same time, the geoid is the most graphical representation of the Earth gravity field.





Purple : -107 m Orange : + 85 m
Data from http://www.ngs.noaa.gov/GEOID/
More information can be found at http://solid_earth.ou.edu/notes/geoid/earths_geoid.htm



B.2.3 Applications of Gauss's theorem

$$\int_V \nabla.\mathbf{g}\ dV = \int_S \mathbf{g.dS}$$

(a) Consider a sphere containing a point mass at the centre

On the surface of the sphere, **g** is uniform and has magnitude,

$$g = -\frac{Gm}{r^2}$$

Thus we can write

$$\int_S \mathbf{g.dS} = g \text{ x (area of sphere)} = \frac{Gm}{r^2} \text{ x } (4\pi r^2) = 4\pi Gm$$

Now using Gauss's theorem

$$\int_S \mathbf{g.dS} = 4\pi Gm = \int_V \nabla.\mathbf{g}\ dV$$

For a very small sphere, $\int_V \nabla.\mathbf{g}\ dV = (\nabla.\mathbf{g}) \text{ x (volume of sphere)} = 4\pi Gm$

Dividing each side by the volume of the sphere gives

$$\nabla.\mathbf{g} = 4\pi G\rho$$

where ρ is the average density of the sphere.

(b) Expression for total mass

$$\int_S \mathbf{g}.d\mathbf{S} = \int_V \nabla.\mathbf{g}\ dV$$

Now substituting $\nabla.\mathbf{g} = 4\pi G\rho$ gives

$$\int_S \mathbf{g}.d\mathbf{S} = \int_V 4\pi G\rho\ dV$$

$$\int_S \mathbf{g}.d\mathbf{S} = 4\pi G \int_V \rho\ dV = 4\pi G\, x \text{ (total mass within the surface } S)$$

Thus gravity data can (in principle) tell **us how much mass** is within a volume. However the **distribution of this mass cannot be determined uniquely**.

Example : consider two spherical shells of radius *a*. At the centre of one is a **point mass**, m_1. At the centre of the other is a sphere of radius r_2, also mass m_1.

Thus $4\pi G \int_V \rho\ dV$ is the same for each shell, and therefore $\int_S \mathbf{g}.d\mathbf{S}$ must also be the same.

Since **g** will be uniform across the whole surface of the spherical shell (from symmetry)

$$\int_S \mathbf{g}.d\mathbf{S} = g_1 \times A_1 = g_2 \times A_2$$

where g_1 and g_2 are the gravitational accelerations on each shell, and A_1 and A_2 their surface areas. Since $A_1 = A_2$, it is obvious that $g_1 = g_2$.

In other words the gravitational effects of the two mass distributions are identical.

We can tell **how much mass** is in there, but not **where it is** within the sphere.