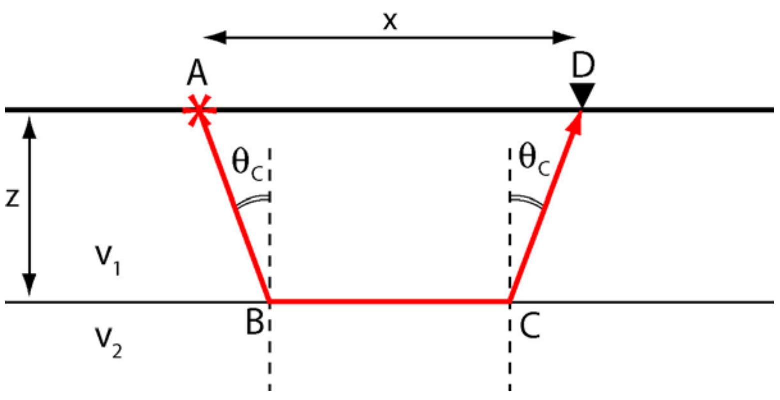
## C3.1 Seismic refraction – single horizontal interface




The P-wave is refracted at the interface between the two layers. Since $v_1 > v_2$ the wave is refracted **towards** the horizontal. As the angle of incidence is increased, the geometry results in a **head wave** travelling **horizontally** in layer 2. From Snell's Law we can write:

$$\frac{\sin\ \theta_c}{v_1} = \frac{\sin\ 90^o}{v_2}$$

Thus $\sin\theta_c = \frac{v_1}{v_2}$ and from geometry we can show that AB = CD = $z / \cos\ \theta_c$

Can also show that BC = $x - 2z \tan\ \theta_c$

Total travel time $t = t_{AB} + t_{BC} + t_{CD}$

$$t = \frac{z}{v_1 \cos\theta_c} + \frac{(x - 2z\tan\theta_c)}{v_2} + \frac{z}{v_1\cos\theta_c}$$

$$t = \frac{x}{v_2} + \frac{2z}{v_1\cos\theta_c} - \frac{2z\tan\theta_c}{v_2}$$

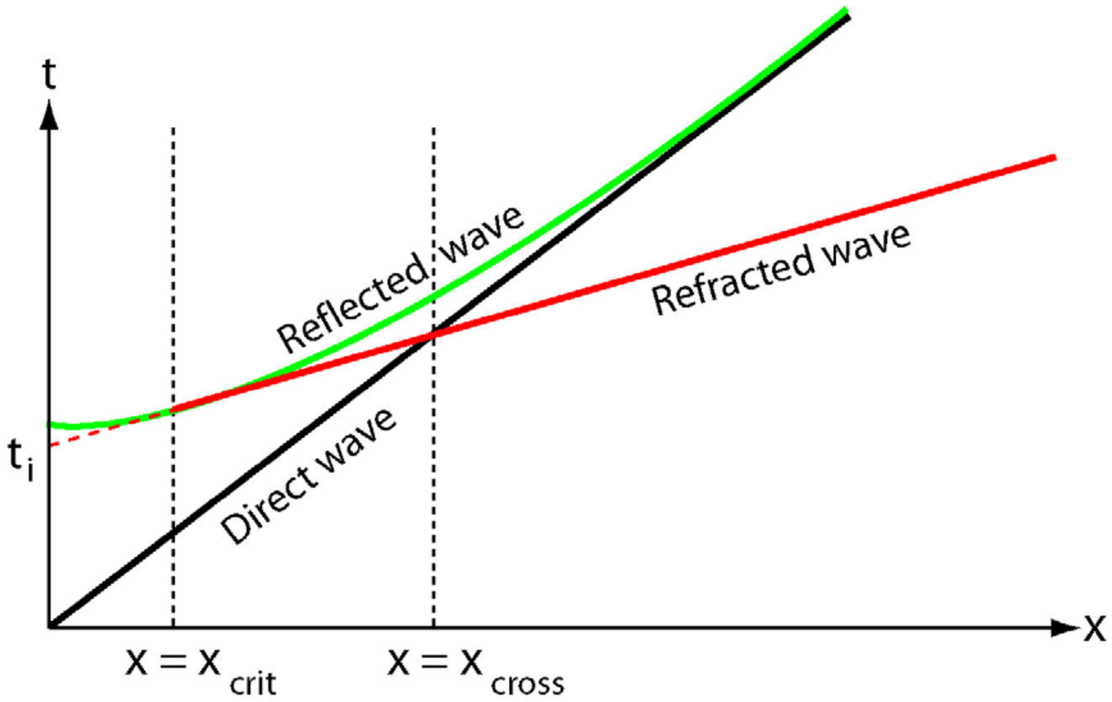
$$t = \frac{x}{v_2} + \frac{2zv_2 - 2zv_1\sin\theta_c}{v_1 v_2\cos\theta_c}$$

$$t = \frac{x}{v_2} + \frac{2z - 2z\sin^2\theta_c}{v_1 v_2\cos\theta_c}$$

$$t = \frac{x}{v_2} + \frac{2z\cos\theta_c}{v_1}$$

$$t = \frac{x}{v_2} + \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

$$t = \frac{x}{v_2} + \text{constant}$$




- The travel time curve for the refracted wave is a **straight line** with slope = $1 / v_2$

- The refracted arrival is only observed when $x > x_{crit} = 2z \tan \theta_c$

- The refracted wave is the **first arrival** at values of $x$ greater than the cross over distance ($x_{cross}$)

- When $x = x_{crit}$ the refracted and reflected waves are the same

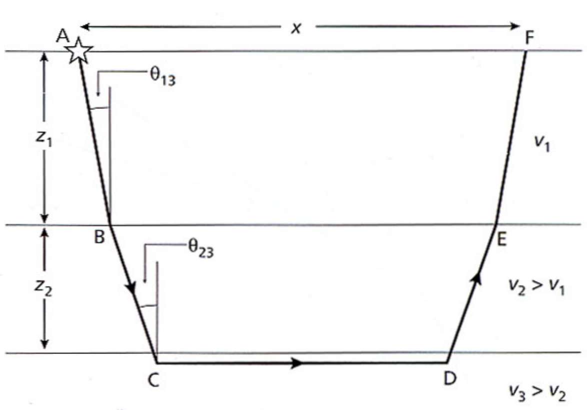
- $v_2$ can be calculated from the **slope** of the refracted wave on the $t$-$x$ plot

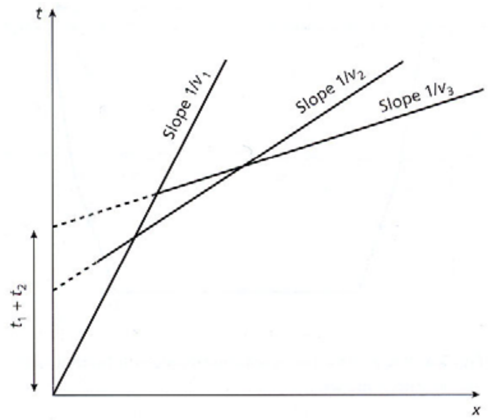
- The depth of the interface ($z$) can be found by extrapolating the travel time of the refracted wave to $x = 0$ where the travel time is

$$t_i = \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

Rearranging gives $$z = \frac{v_1 v_2 t_i}{2\sqrt{v_2^2 - v_1^2}}$$

## C3.2  Seismic refraction – multiple horizontal layers




### Direct wave

$$t = \frac{x}{v_1}$$

*Data analysis*

- compute $v_1$ from **slope** of direct arrival

### First refraction

Critical refraction occurs as the wave travels from 1>2 giving

$$sin\ \theta_c = \sin\theta_{12} = \frac{v_1}{v_2}$$

$$t = \frac{x}{v_2} + \frac{2z_1\cos\theta_{12}}{v_1} = \frac{x}{v_2} + t_1$$

*Data analysis*

- Compute $v_2$ from the **slope** of the refracted wave
- Compute $z_1$ from the **intercept time** ($t_1$) when $x = 0$ and $v_1$ and $v_2$ are already known

$$t_1 = \frac{2z_1\cos\theta_{12}}{v_1}$$

### Second refraction

Critical refraction occurs as the wave travels from 2 > 3 giving $\sin\theta_{23} = \frac{v_2}{v_3}$

Applying **Snells Law** at the interface between 1 and 2 gives $\sin\theta_{13} = \frac{v_1}{v_2}\sin\theta_{23}$

Thus $\sin\theta_{13} = \frac{v_1}{v_2}\sin\theta_{23} = \frac{v_1}{v_3}$

Total travel time for the second refraction

$$t = \frac{x}{v_3} + \frac{2z_1\cos\theta_{13}}{v_1} + \frac{2z_2\cos\theta_{23}}{v_2} = \frac{x}{v_3} + t_1 + t_2$$

*Data analysis*

- compute $v_3$ from the **slope** of the second refracted wave
- At $x = 0$, intercept time $= t_1 + t_2$
- Since $t_1$ is known we can compute $t_2$
- From $t_2 = \frac{2z_2\cos\theta_{23}}{v_2}$ we can then calculate $z_2$

**<u>Example of field seismogram</u>**

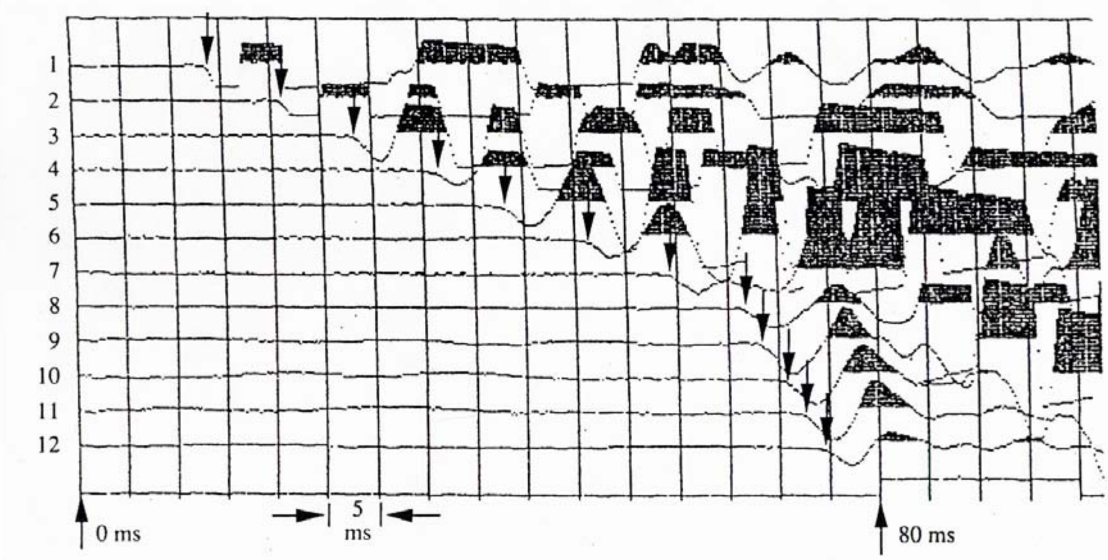
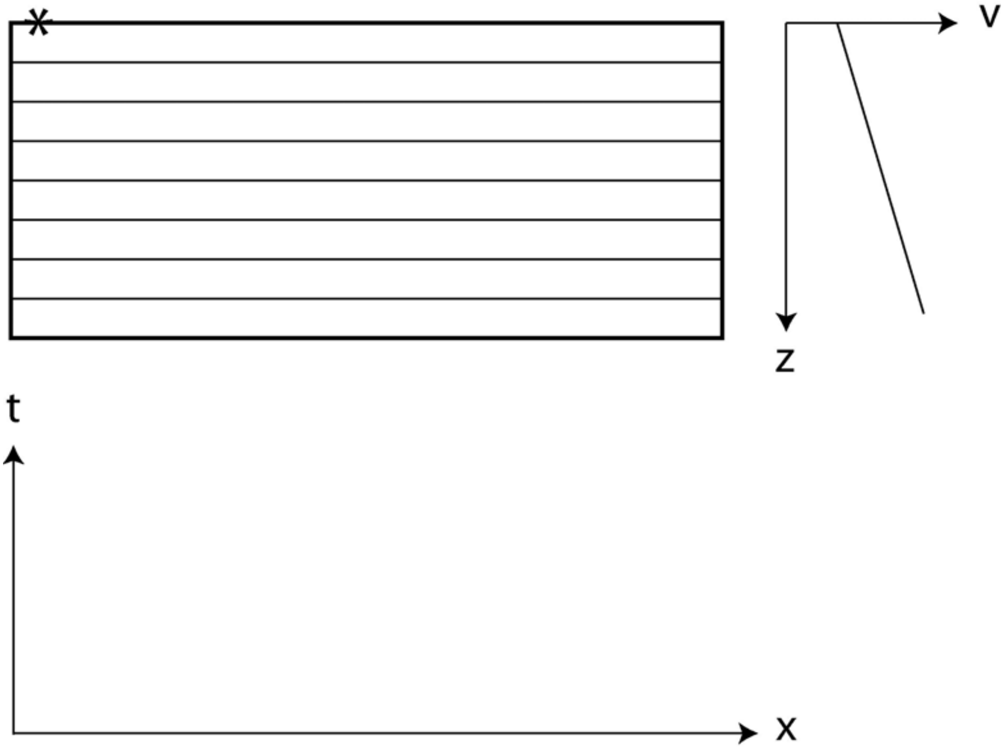



**Figure 3-6** Field seismogram from the Connecticut Valley, Massachusetts. Geophone traces are labeled 1–12. The first geophone is located 5 m from the energy source. The geophone interval is 3 m. First breaks for each trace are indicated by a downward directed arrow. Timing lines are at 5-ms intervals. The record encompasses 100 ms. This seismogram exhibits a classic two-layer pattern.

**Multiple layers with velocity increasing with depth**

The results for two layers can be generalized for an *N*-layer model and written as a series. See the text book for details. Consider the case where there is a **uniform increase** in velocity with depth. What will be the form of the ray paths and travel time curves?




What could cause velocity to increase uniformly with depth?

For each ray, the quantity called the **ray parameter**, *p*, is constant. $p = \frac{\sin\theta}{v}$

**Low-velocity layers**

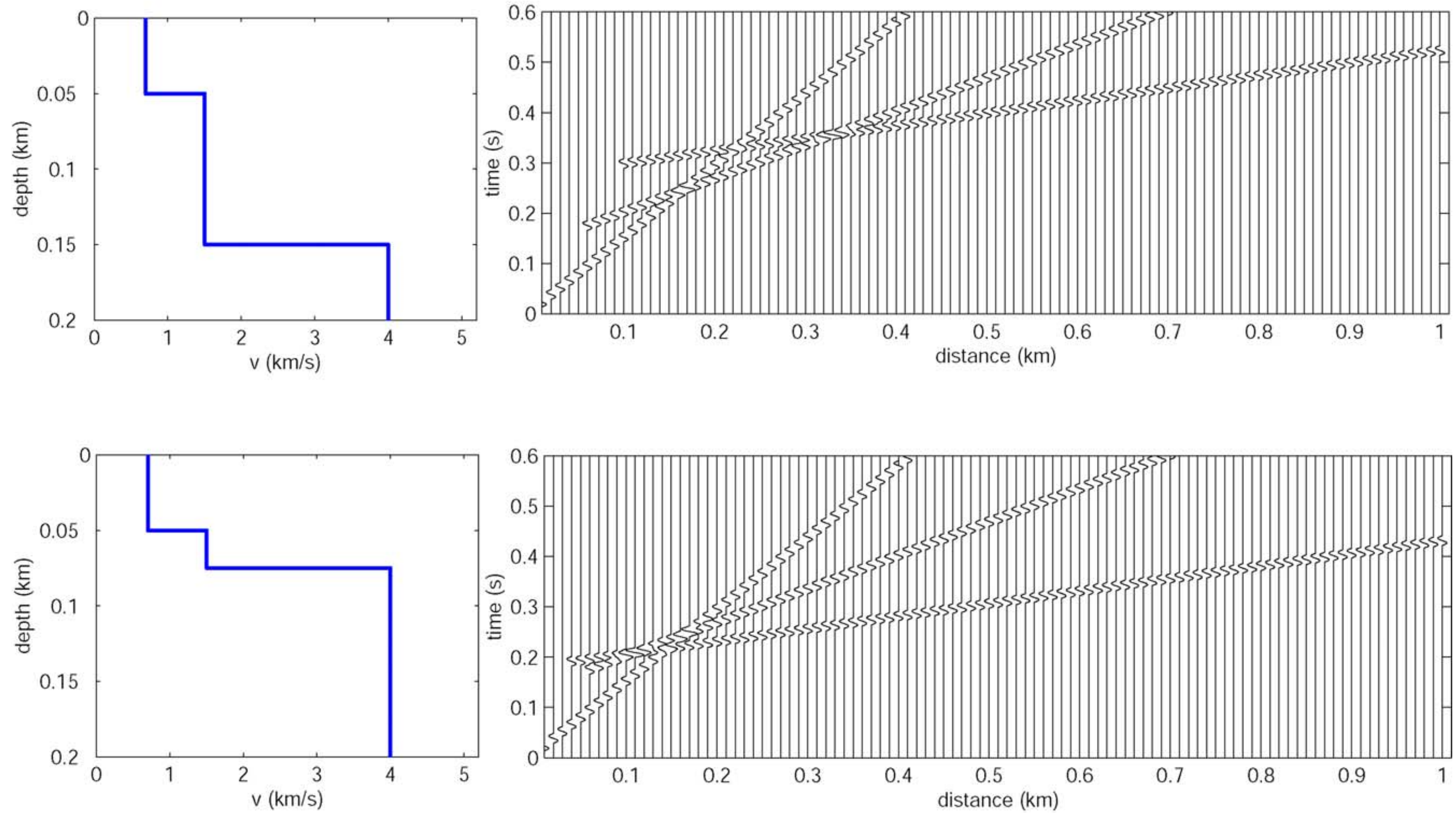
● for a head wave to propagate, an **increase in velocity** from one layer to the next is needed. If a decrease in velocity occurs, there will be no head wave and refraction will fail to detect the layer.

*Example* : If a 3-layer model has a LVZ in the second layer, then interpretation in terms of 2-layers will give a wrong answer *e.g.* soil-peat-bedrock. The peat is a low velocity layer.

**Hidden layers**

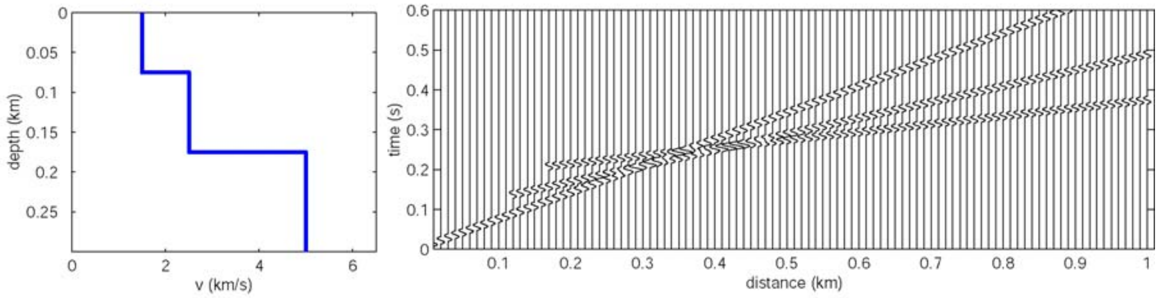
Even if a layer has an increase in velocity, then it possible for the head wave on the upper surface to **never be the first arrival**. Again if a 3-layer velocity model has a hidden second layer, then interpretation in terms of 2-layers will give a wrong answer

## C3.2 Hidden layers in seismic refraction

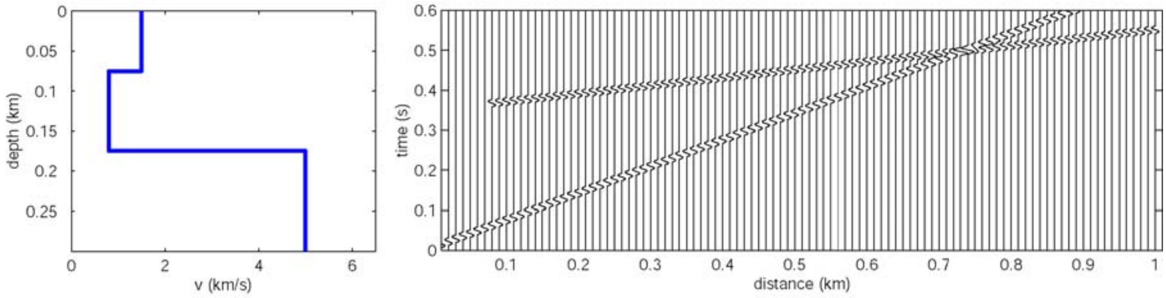


## C3.2 Seismic refraction - with a low velocity zone (LVZ)

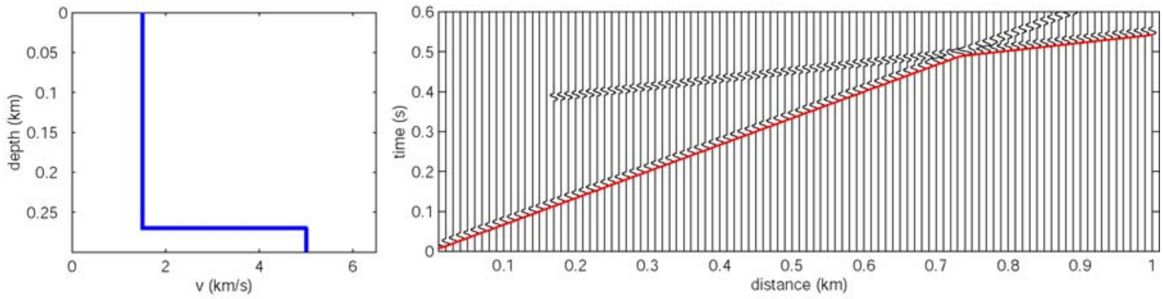
(a) 3-layer model with no LVZ




(b) 3-layer model with LVZ




(c) Interpretation of first arrivals from (b)




First arrivals shown in red. Data is fit by a 2-layer model. Which parameters are estimated correctly?

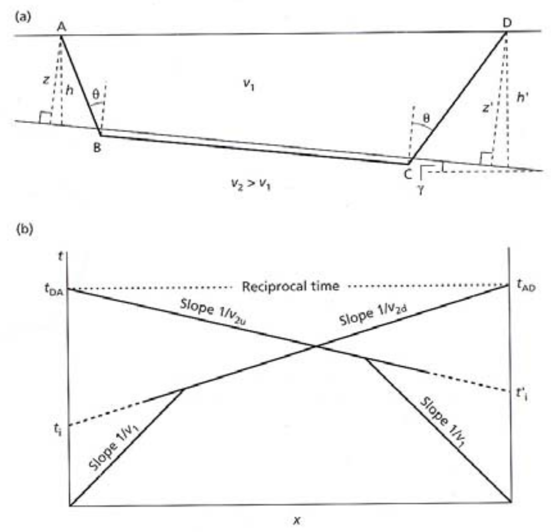
## C3.3 Seismic refraction – dipping interface

### Direct wave

$$t = \frac{x}{v_1}$$

- same as in horizontal case considered in C3.2
- compute $v_1$ from **slope** of direct arrival

### Refraction from the dipping interface




If ray is travelling down dip, then the upward leg to the geophone will **increase** in length as offset ($x$) increases. Thus the first refraction will arrive progressively later, compared to the flat interface in C3.1. This if effectively the same as the signal travelling more slowly and will increase the **slope** of the refracted wave on the travel time curve ($x$-$t$ plot)

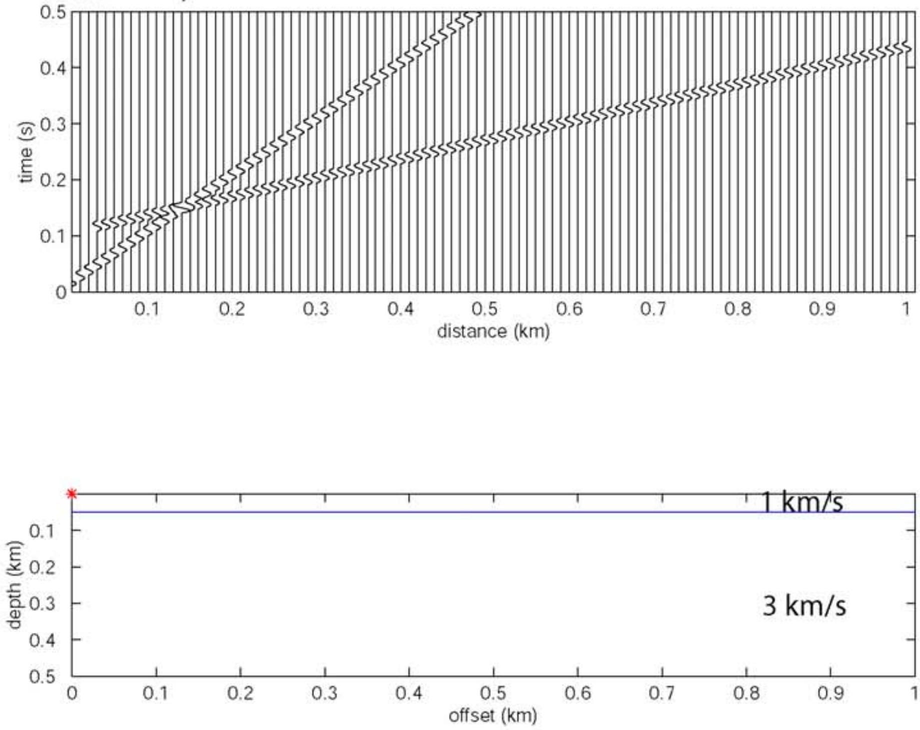
- Can show that the travel time, $t_{for} = \frac{x\sin(\theta_{12}+\gamma_1)}{v_1} + \frac{2z_{perp}\cos\theta_{12}}{v_1}$
- Can you confirm that this result is correct when $\gamma_1 = 0$?
- Apparent velocity **down dip** $= v_{2d} = \frac{v_1}{\sin(\theta_{12}+\gamma_1)} < v_2$
- Note that the apparent velocity down dip is **slower** than $v_2$ which results in a **steeper** slope on the travel time curve.

- Example of layer with $\gamma_1 = 5^\circ$ $v_1 = 1000$ m/s and $v_2 = 3000$ m/s is shown on page 9.

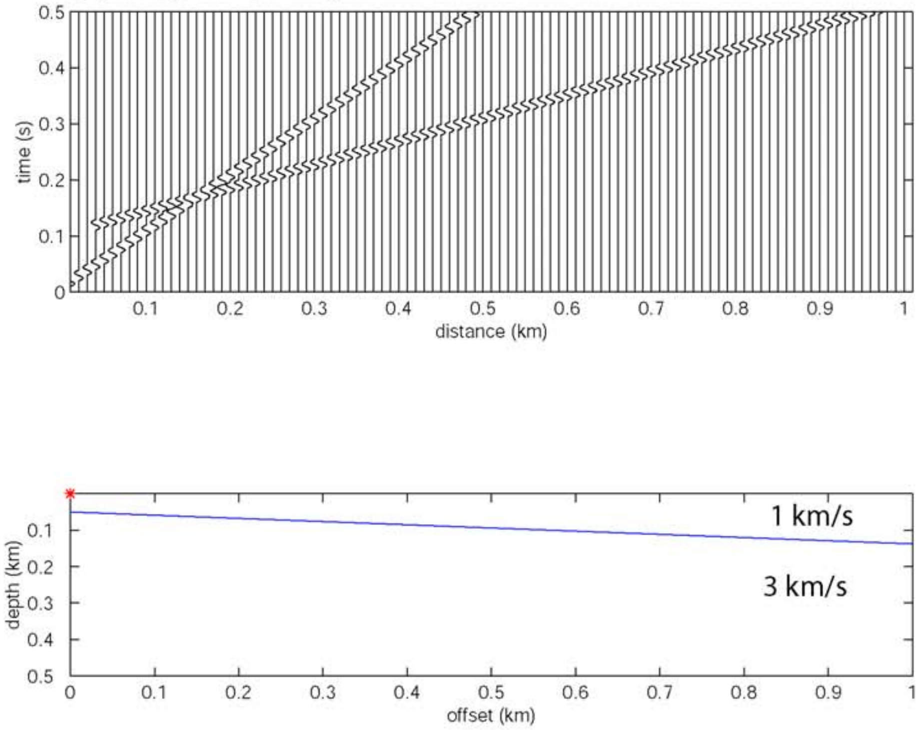
  What value of $v_{2d}$ is predicted for these values?

## C3.3 Seismic refraction - dipping layers

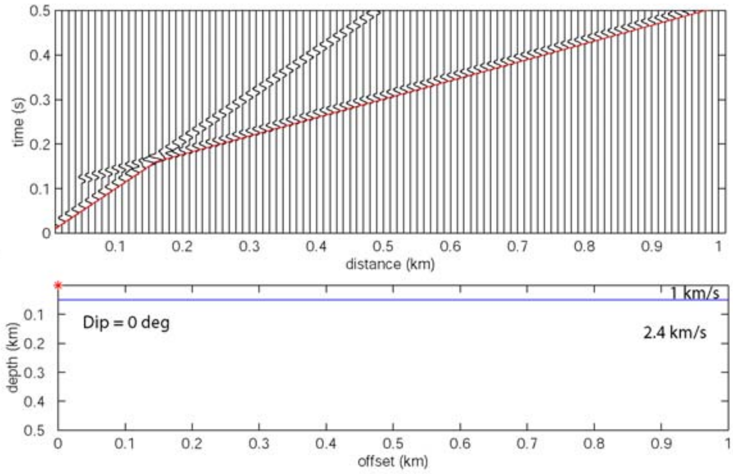
(a) No dip



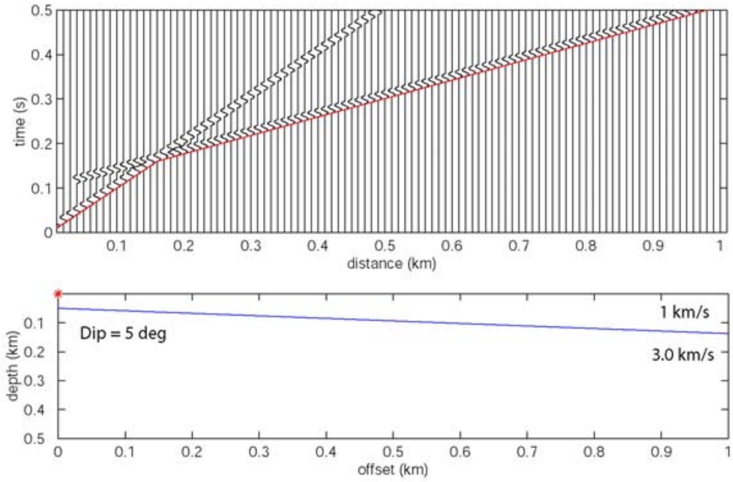

(b) 5 degree dip to right



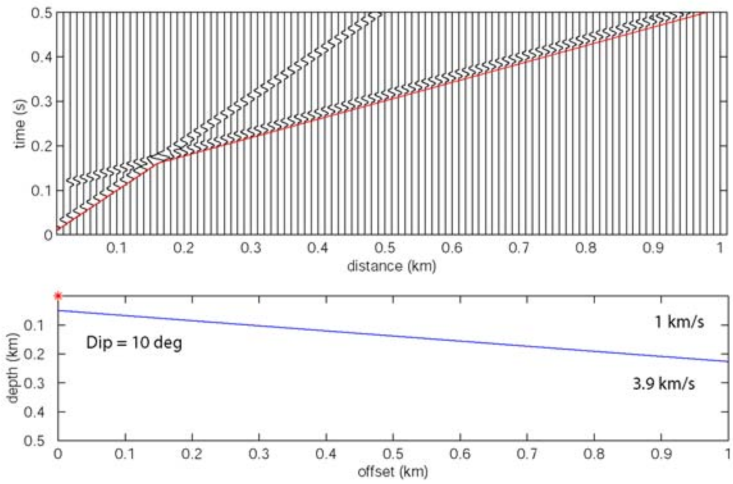
## C3.3 Seismic refraction with dipping layers




(a) Model 1 - dip too shallow




(b) Model 2 - true model




(c) Model 3 - dip too steep

● Note that several velocity models can be found that all fit the data. $v_1$ is uniquely determined, but there is a trade-off between $\gamma_1$ and $v_2$ (non-uniqueness again)

| | |
|---|---|
| *Model 1* | $\gamma_1 = 0^\circ$ $v_1 = 1000$ m/s $v_2 = 2400$ m/s |
| *Model 2* | $\gamma_1 = 5^\circ$ $v_1 = 1000$ m/s $v_2 = 3000$ m/s |
| *Model 3* | $\gamma_1 = 10^\circ$ $v_1 = 1000$ m/s $v_2 = 3900$ m/s |

● On the basis of a single shot recorded in one direction, we cannot determine which model is the correct one.

● Suppose we fire a second shot at the far end of the geophone array, then these waves will travel **up dip**. As the refraction travels to larger offsets ($x$) the final leg in the upper layer will become shorter, and the refraction will arrive earlier. This effectively **increases the apparent velocity** and **reduces the slope** of the travel time curve.

$$t_{rev} = \frac{x\sin(\theta_{12} - \gamma_1)}{v_1} + \frac{2z'_{perp}\cos\theta_{12}}{v_1}$$

● Apparent velocity **up dip** $= v_{2u} = \dfrac{v_1}{\sin(\theta_{12} - \gamma_1)}$

For with $\gamma_1 = 5^\circ$ $v_1 = 1000$ m/s and $v_2 = 3000$ m/s, what value of $v_{2u}$ is predicted?

● Note that the travel times $t_{AD}$ (forward direction) and $t_{DA}$ (reverse direction) are the same. This is a phenomena known as **reciprocity**. Why is this true?

● When forward and reverse profiles are recorded, the correct model (#2) can be seen to be the only one that fits the travel time data (attached handout)

**Computing $v_2$ and dip of the interface**

By rearranging expressions for $v_{2u}$ and $v_{2d}$ can show that

$$\theta_{12} = \frac{1}{2}[\sin^{-1}(\frac{v_1}{v_{2d}}) + \sin^{-1}(\frac{v_1}{v_{2u}})]$$

and

$$\gamma_1 = \frac{1}{2}[\sin^{-1}(\frac{v_1}{v_{2d}}) - \sin^{-1}(\frac{v_1}{v_{2u}})]$$

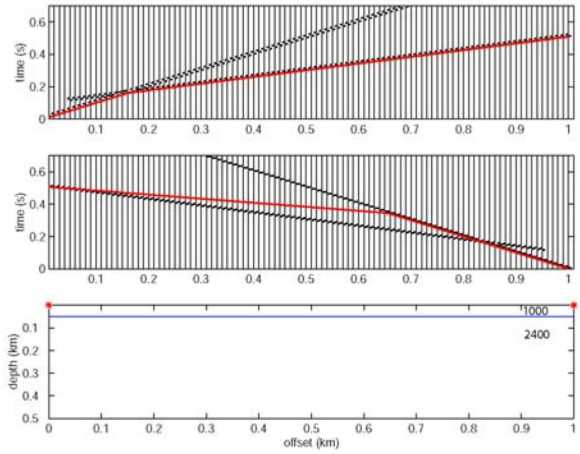
## C3.3 Seismic refraction with dipping layers

Red = synthetic data

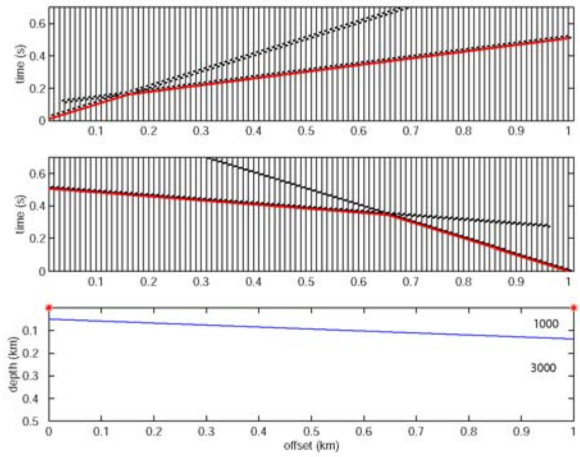
All three velocity models fit the data from the forward profile

The reverse profile shows that models (a) and (c) are not consistent with the data

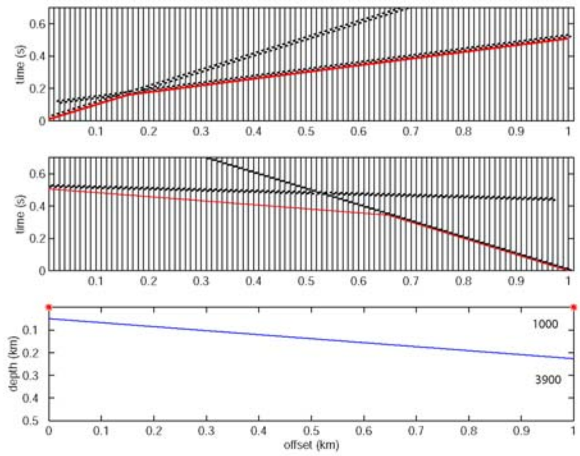
(a) Dip = 0 degrees. Synthetic seismogram fits only forward data



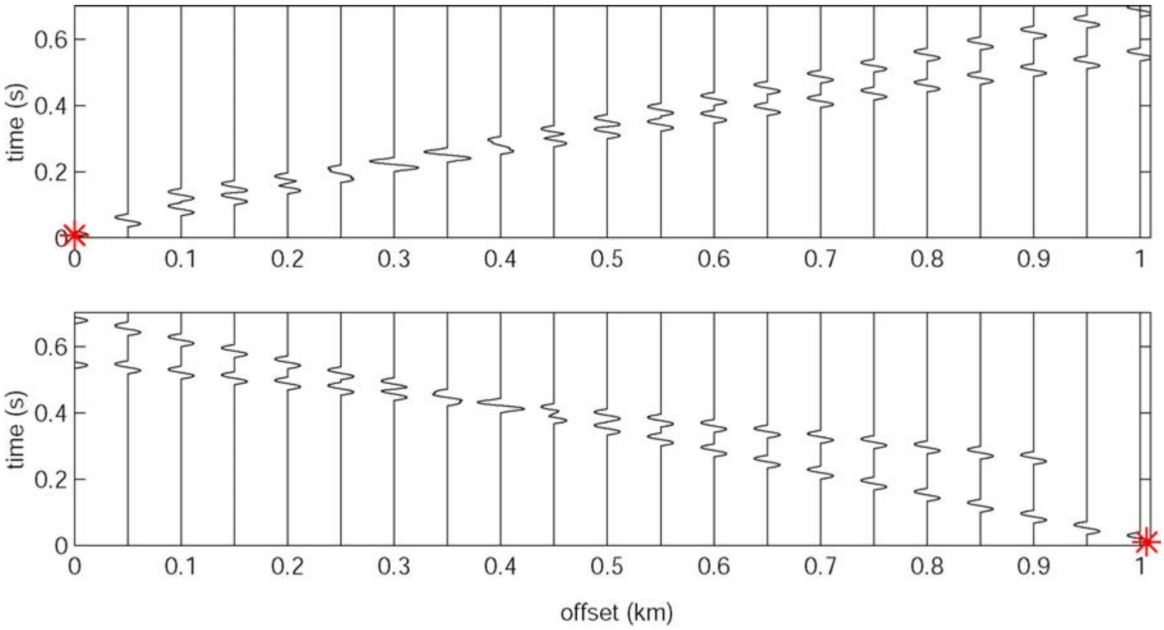

(b) Dip = 5 degrees. Synthetic seismogram fits both forward and reverse data




(c) Dip = 10 degrees. Synthetic seismogram fits only forward data



## C3.3 Seismic refraction – dipping interface - Example




Look at the travel time curve. Which way does the interface dip?

You will need this information to decide which direction is up and which is down!

**Forward profile**

| x(m) | t_dir(ms) | t_ref(ms) |
|---|---|---|
| 0.000 | 0.000 | |
| 50.000 | 33.000 | |
| 100.000 | 67.000 | 110.410 |
| 150.000 | 100.000 | 133.927 |
| 200.000 | 133.000 | 157.444 |
| 250.000 | 167.000 | 180.960 |
| 300.000 | 200.000 | 204.477 |
| 350.000 | 233.000 | 227.994 |
| 400.000 | 267.000 | 251.510 |
| 450.000 | 300.000 | 275.027 |
| 500.000 | 333.000 | 298.544 |
| 550.000 | 367.000 | 322.060 |
| 600.000 | 400.000 | 345.577 |
| 650.000 | 433.000 | 369.094 |
| 700.000 | 467.000 | 392.610 |
| 750.000 | 500.000 | 416.127 |
| 800.000 | 533.000 | 439.643 |
| 850.000 | 567.000 | 463.160 |
| 900.000 | 600.000 | 486.677 |
| 950.000 | 633.000 | 510.193 |
| 1000.000 | 667.000 | 533.710 |

**Reverse profile**

| x(m) | t_dir(ms) | t_ref(ms) |
|---|---|---|
| 0.000 | 667.000 | 533.710 |
| 50.000 | 633.000 | 517.616 |
| 100.000 | 600.000 | 501.522 |
| 150.000 | 567.000 | 485.428 |
| 200.000 | 533.000 | 469.334 |
| 250.000 | 500.000 | 453.240 |
| 300.000 | 467.000 | 437.146 |
| 350.000 | 433.000 | 421.051 |
| 400.000 | 400.000 | 404.957 |
| 450.000 | 367.000 | 388.863 |
| 500.000 | 333.000 | 372.769 |
| 550.000 | 300.000 | 356.675 |
| 600.000 | 267.000 | 340.581 |
| 650.000 | 233.000 | 324.487 |
| 700.000 | 200.000 | 308.393 |
| 750.000 | 167.000 | 292.299 |
| 800.000 | 133.000 | 276.205 |
| 850.000 | 100.000 | 260.111 |
| 900.000 | 67.000 | 244.017 |
| 950.000 | 33.000 | |
| 1000.000 | 0.000 | |

From the travel times compute the following.

$v_1$ = __________ m/s

$v_{2d}$ = __________ m/s

$v_{2u}$ = __________ m/s

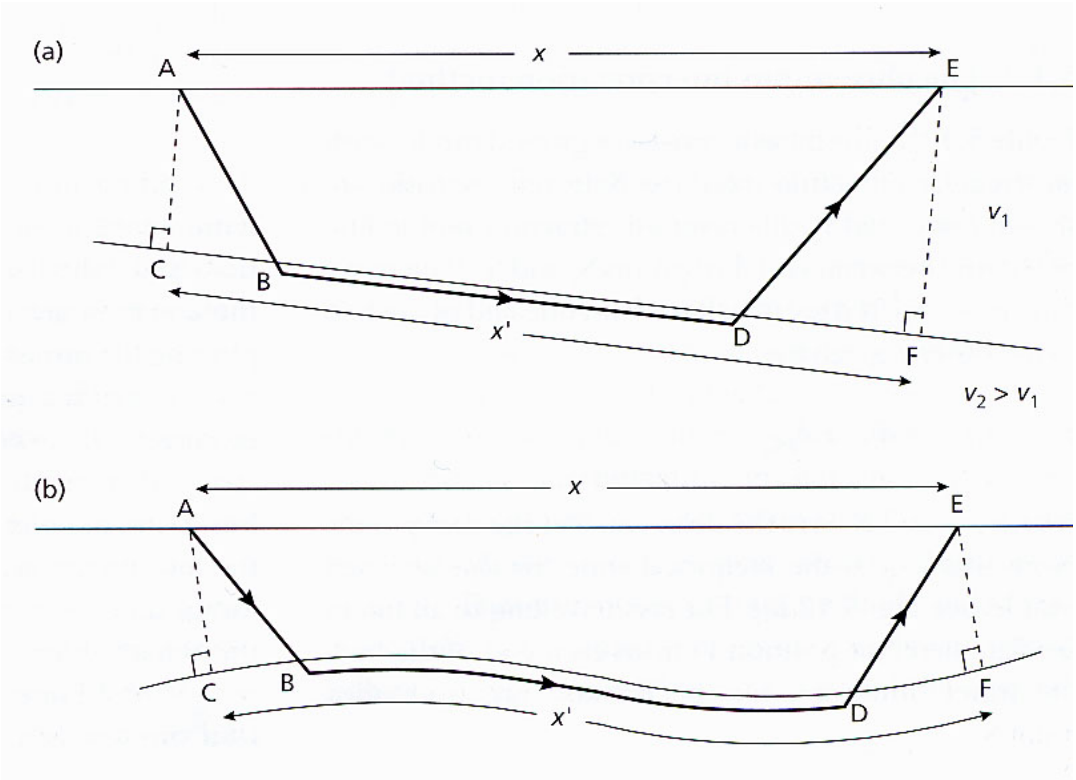
$\theta_{12}$ = __________ degrees

$v_2 = v_1 / \sin \theta_{12}$ = __________ m/s

$\gamma_1$ = __________degrees

Use the modelling program **refract_v3.m** to confirm the answers.

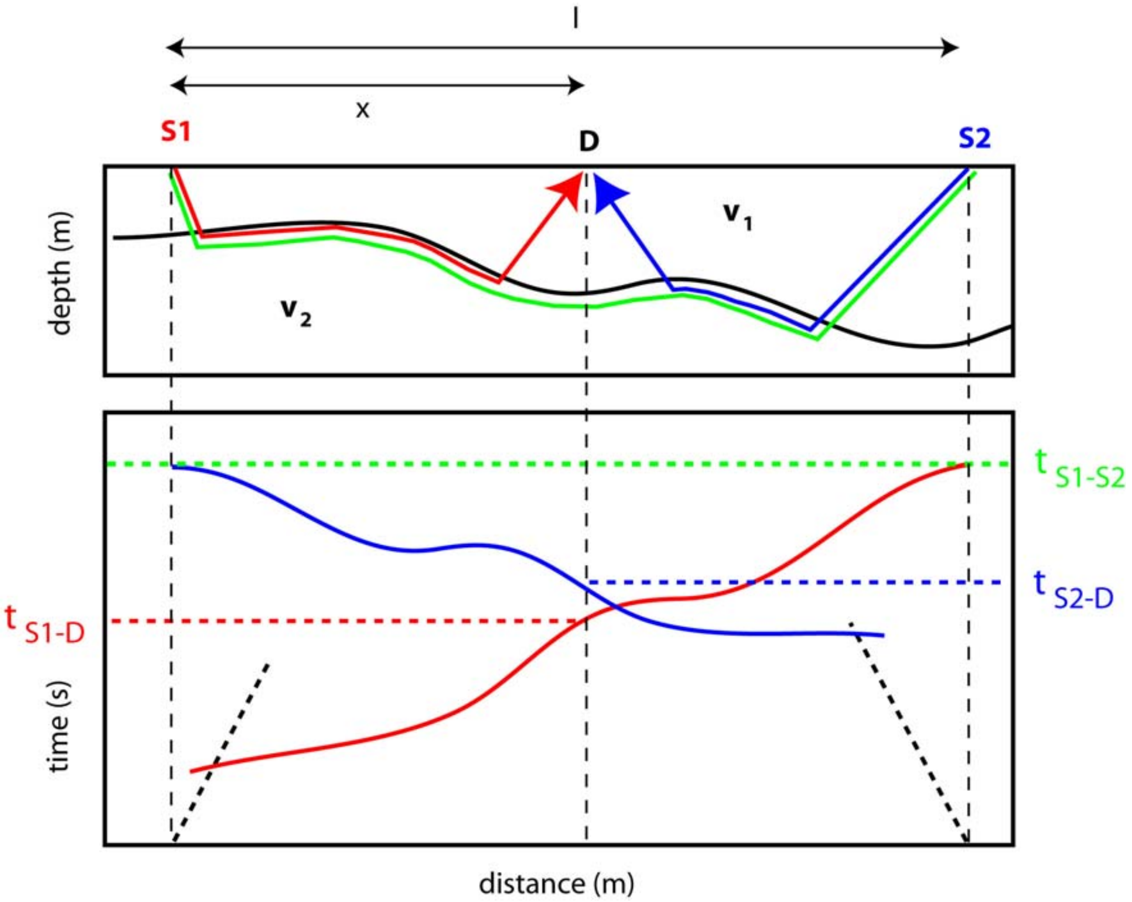
## C3.4 Seismic refraction – non planar interfaces




### C3.4.1 Basics and concept of delay time

- the **delay time** at the shot is the **extra time** needed for the wave to travel AB, compared to the time to travel CB.
- delay time at detector is the **extra time** needed for wave to travel DE, compared to DF
- can see this from the travel time curve $t = x/v_2 + \delta_{ts} + \delta_{td}$
- for horizontal interface, $\delta_{ts} = \delta_{td} = z \cos \theta / v_1$
- with just a single shot, we cannot separately determine $\delta_{ts}$ and $\delta_{td}$
- with a reversed profile, we can individually determine $\delta_{ts}$ and $\delta_{td}$
- for a non horizontal interface, with many geophones $\delta_{td}$ contains information about how the depth of the interface (z) varies with position (x')
- two common methods of finding $\delta_{td}$ are the **plus-minus method** and the **generalized reciprocal method**. More details shortly.

## C3.4.2 Plus-minus interpretation method




Consider the model with two layers and an undulating interface. The refraction profile is reversed with two shots ($S_1$ and $S_2$) fired into each detector (D). Consider the following three travel times:

(a) The reciprocal time is the time from $S_1$ to $S_2$

$$t_{S_1S_2} = \frac{l}{v_2} + \delta_{S_1} + \delta_{S_2} = t_{S_2S_1}$$

(b) Forward shot into the detector

$$t_{S_1D} = \frac{x}{v_2} + \delta_{S_1} + \delta_D$$

(c) Reverse shot into the detector

$$t_{S_2D} = \frac{(l-x)}{v_2} + \delta_{S_2} + \delta_D$$

Our goal is to find $v_2$ and the delay time at the detector, $\delta_D$. From the delay time, $\delta_D$, we can find the depth of the interface.

**Minus term to estimate velocity ($v_2$)**

(b)-(c) will eliminate $\delta_D$

$$t_{S_1D} - t_{S_2D} = \frac{(2x - l)}{v_2} + \delta_{S_1} - \delta_{S_2}$$

$$t_{S_1D} - t_{S_2D} = \frac{2x}{v_2} + C$$

where $C$ is a constant. A plot of $t_{S_1D} - t_{S_2D}$ versus $2x$ will give a line with slope = $1/v_2$

**Plus term to estimate delay time at the detector**

(b)+(c) gives

$$t_{S_1D} + t_{S_2D} = \frac{l}{v_2} + \delta_{S_1} + \delta_{S_2} + 2\delta_D$$

Using the result (a) we get

$$t_{S_1D} + t_{S_2D} = t_{S_1S_2} + 2\delta_D$$

Re-arranging to get an equation for $\delta_D$

$$\delta_D = \frac{1}{2}(t_{S_1D} + t_{S_2D} - t_{S_1S_2})$$

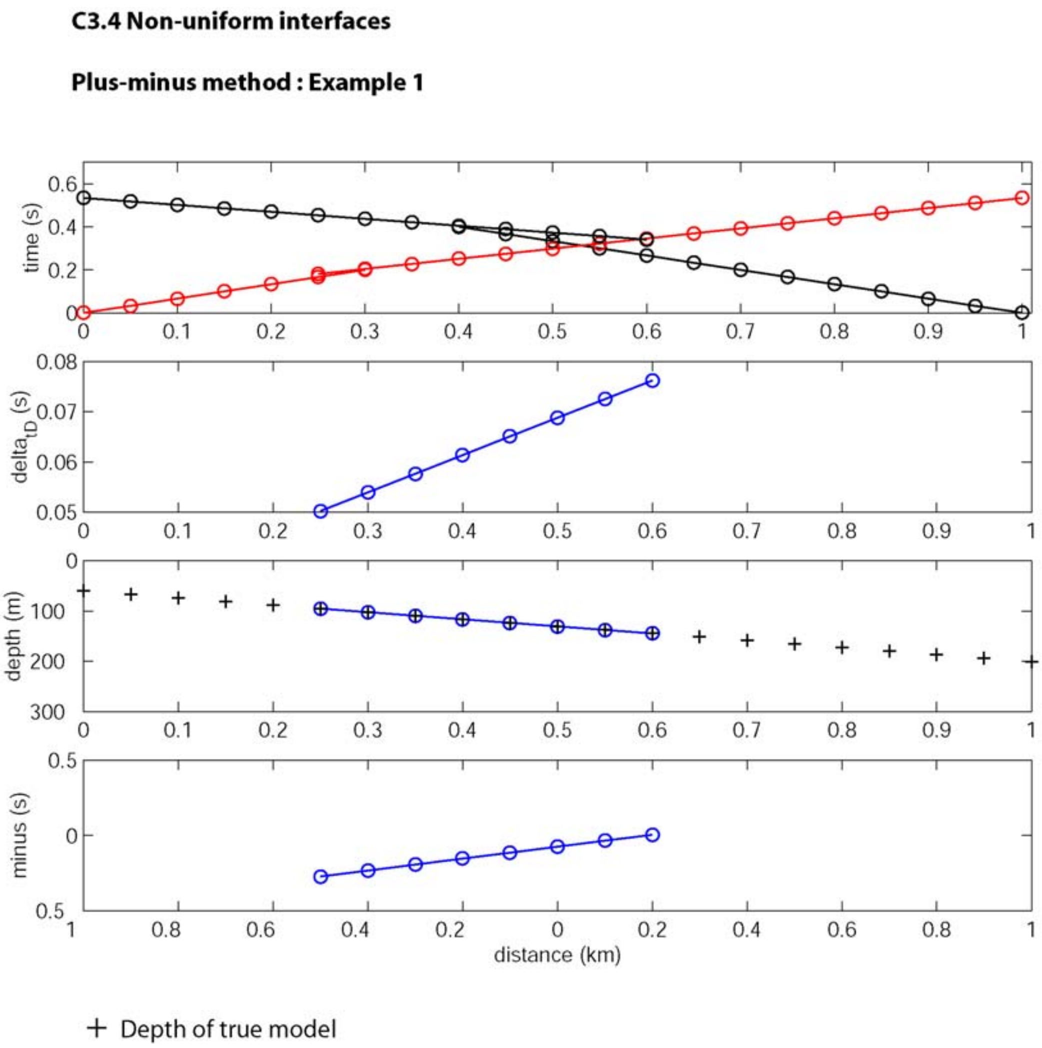
This process is then repeated for all detectors in the profile

**Example 1** : Synthetic refraction data were generated for a dipping flat interface with the MATLAB script **refract_v3_data_plus_minus.m**

The plus-minus technique was then applied using the MATLAB program **plus_minus_ex1.m**

| | | |
|---|---|---|
| *Original model* | $v_1=1500$ *m/s* | $v_2=2500$ *m/s* |
| *Interpretation with plus-minus method* | $v_1=1502$ *m/s* | $v_2=2525$ *m/s* |

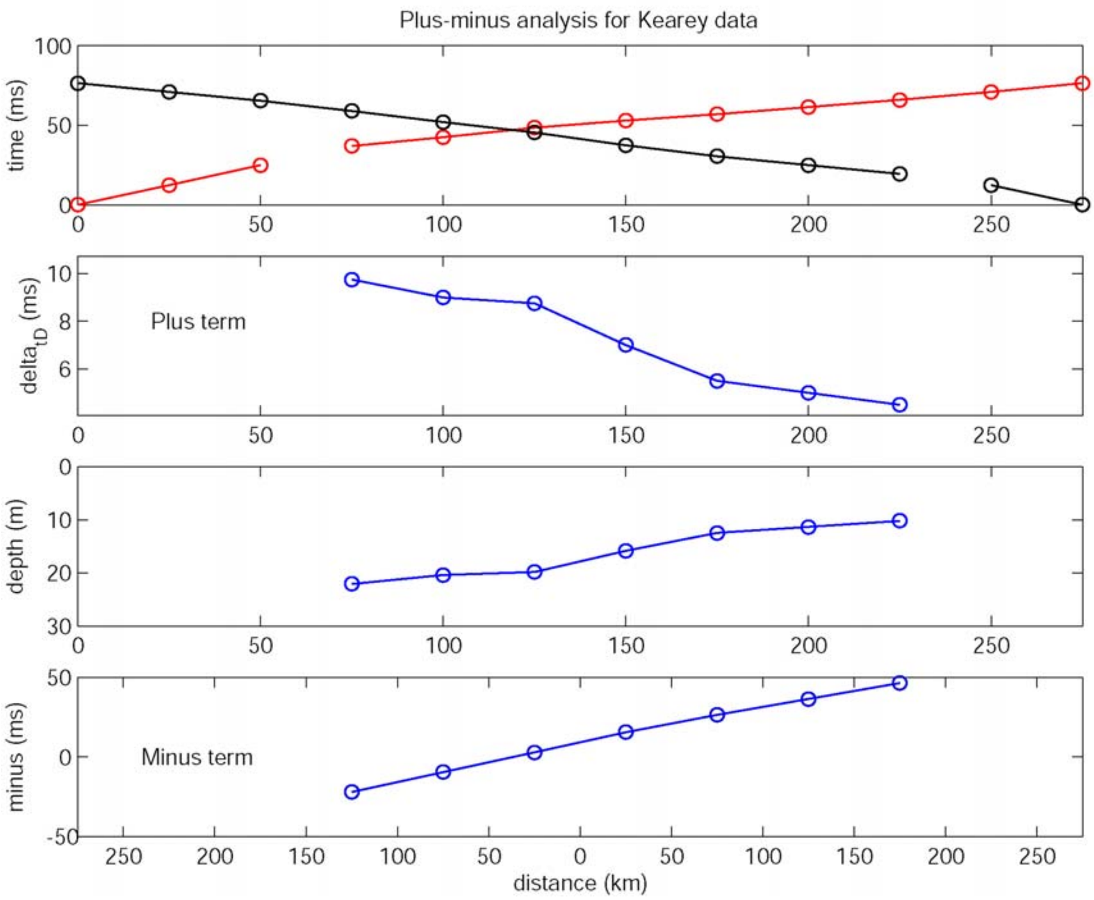
The interface dips to the right at 8˚, and is correctly imaged in this example.



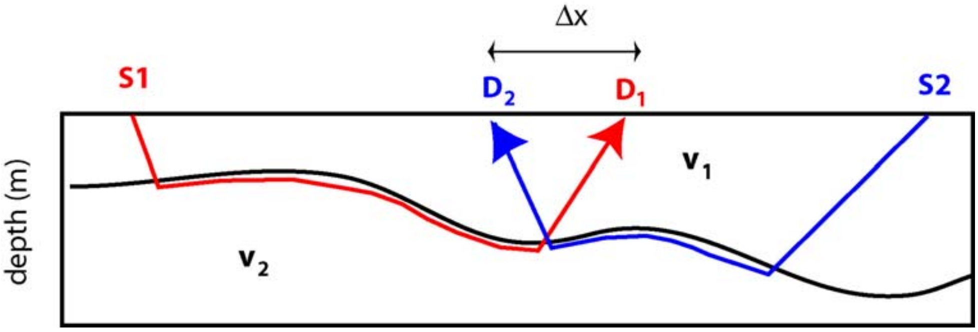
**Example 2**: Field data from Kearey and Brooks, p.123

### C3.4 Non-uniform interfaces

#### Plus-minus method : Example 2



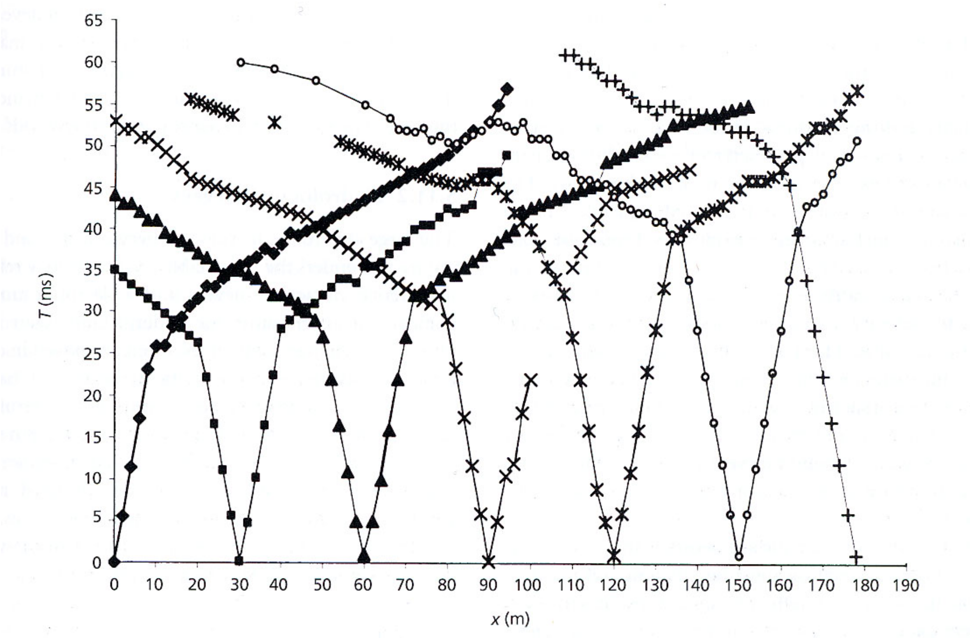
## C3.4.3 Generalized reciprocal  method




The plus-minus method assumes a linear interface between points where the ray leaves the interface. A more powerful technique if the **Generalized reciprocal** method in which pairs of rays are chosen that leave the interface at the same location. More details can be found in *Kearey page 109*.

## C3.5 Shallow applications of seismic refraction

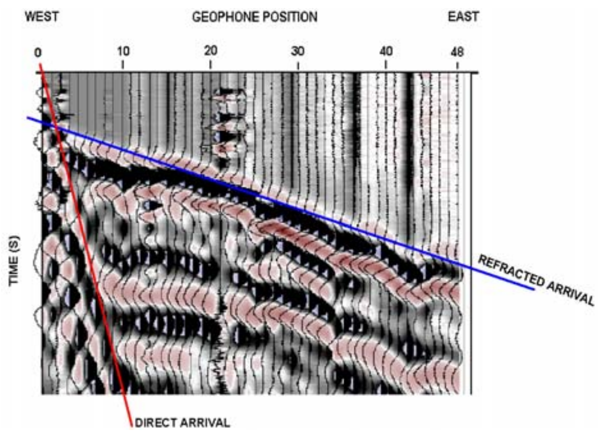
### C3.5.1 Depth of bedrock



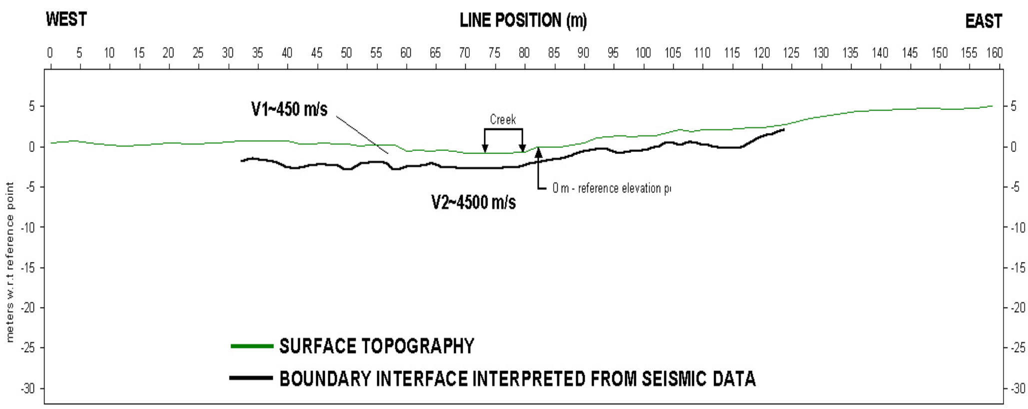

- velocity of bedrock greater than unconsolidated layer
- in this example, (Kearey 5.24) a shot point was located every 30 m
- depth to bedrock increases with *x*

**Example from Northern Alberta**

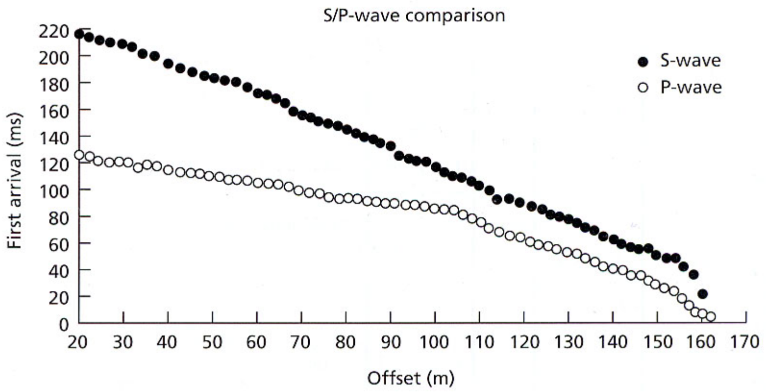
Seismic refraction was used to determine depth to bedrock at the location where a pipeline was planned to cross a creek. DC resistivity data were also collected at this location. Data courtesy of Greg Kovacs and Komex



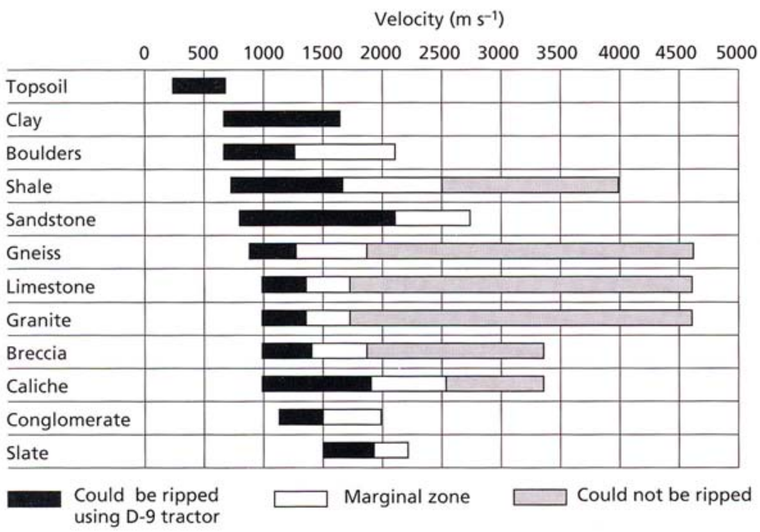

Note that the direct wave is the only the first arrival at the first 2 geophones. This is because of a very high velocity contrast between the upper and lower layers. The model below was derived from the seismic data using the general reciprocal method.



## 3.5.2 Locating water table



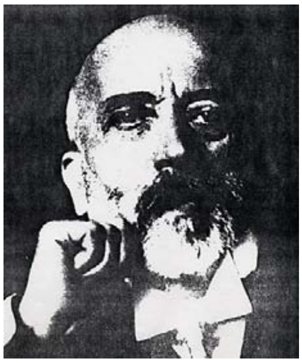

## 3.5.3 Rippability

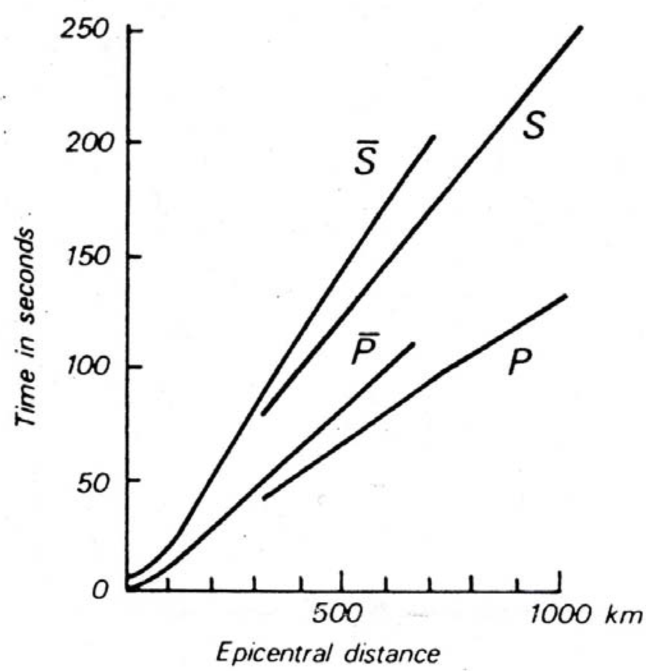


**C3.6 Crustal scale seismic refraction surveys**

**3.6.1 Discovery of the crust-mantle boundary (the Moho)**

In 1910 Andrija Mohorovicic published a paper describing the time taken for earthquake waves to travel across the Balkans. He noticed that P-waves apparently travelled at two different velocities. He inferred that this required a crustal layer, underlain by a higher velocity layer. This boundary is now called the **Mohorovicic discontinuity** or Moho.





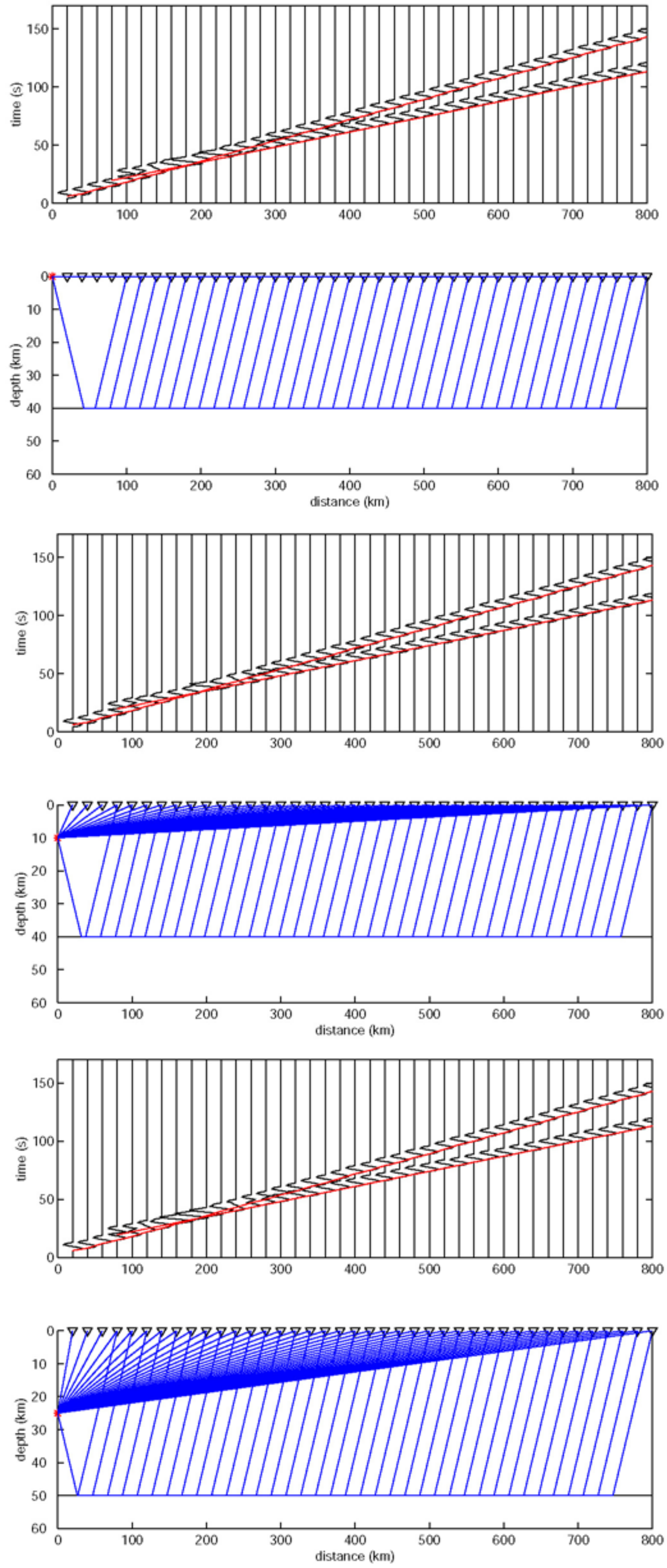

●What crustal velocity and crustal thickness is implied by the attached figure?

●How do the travel time curves differ from those we have considered with a hammer or explosive source?

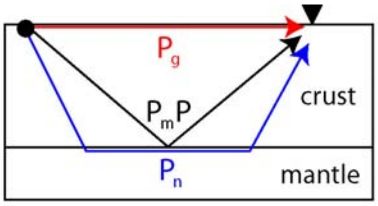
●Why can the S-waves be detected, since they are not first arrivals?

● In modern seismic terminology $P = P_n$ and $\overline{P} = P_g$

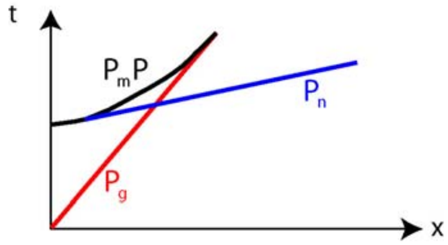
# C3.6 Depth of Moho from seismic refraction data




data in red (line)
model response (wiggles)



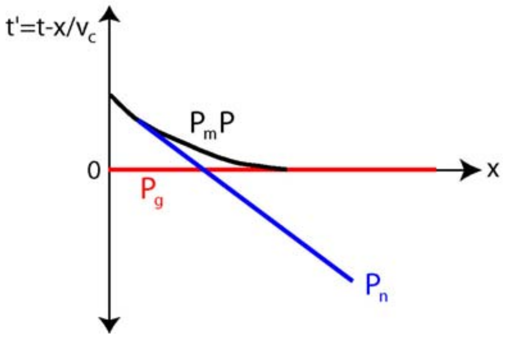

- the direct wave is referred to as $P_g$ and often travels as a parabola. Why?
- the head wave that travels in the upper mantle is called $P_n$
- reflection from the Moho is called $P_mP$




- **reduced travel time** is sometimes plotted on the vertical axis.

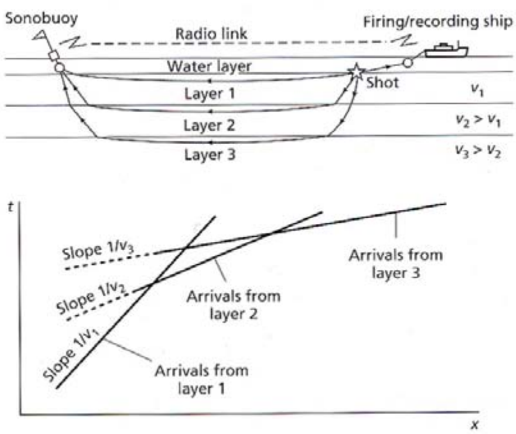
  $t' = t - x/v_{red}$

  where $v_{red}$ is the **reduction velocity**. This has the effect of making arrivals with $v=v_{red}$ plot **horizontally** on a *t-x* plot.

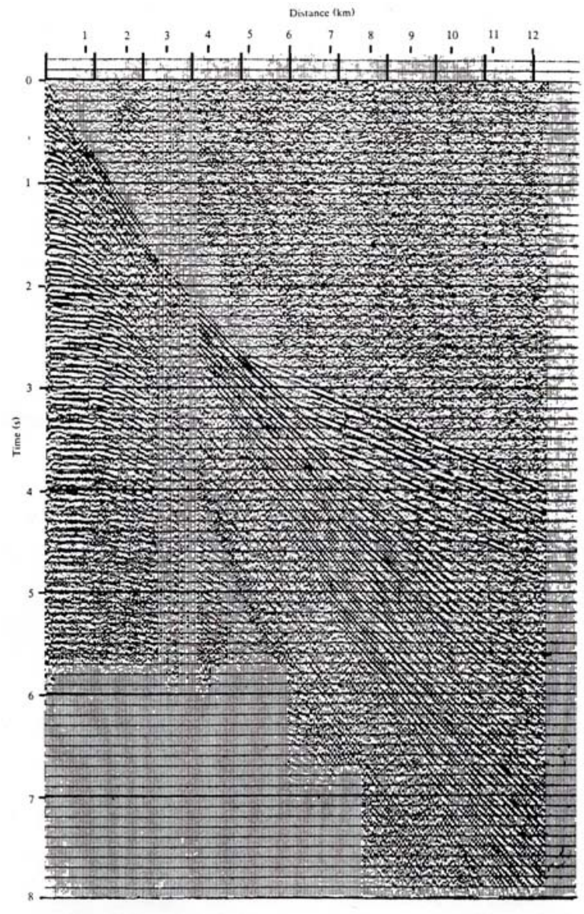



- in the figure on the left, the crustal P-wave velocity was used as the reduction velocity.

## 3.6.2 Seismic refraction at sea



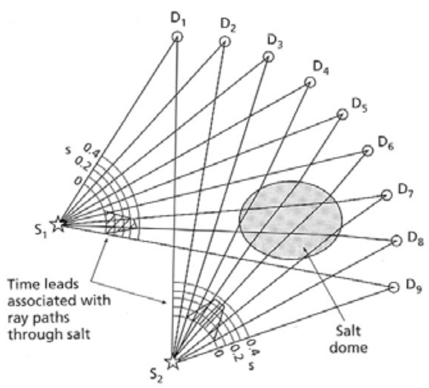

- signals can be detected either by seafloor recording, using a long streamer, two-ship survey or a sonobuoy.
- refractions (head waves) occur at seafloor and at deeper interfaces.
- sonobuoy sends data back via radio telemetry. Modern sonobouys are often disposable.




- Can you locate the direct water wave on this section? (P-wave velocity is 1500 m/s)
- Locate a refraction and measure the velocity.
- Where would you expect to see normal incidence reflections?

### 3.6.3 Fan shooting

Technique first used in the 1920's in the search for salt domes. The higher velocity of the salt causes earlier arrivals for signals that travel though the salt.



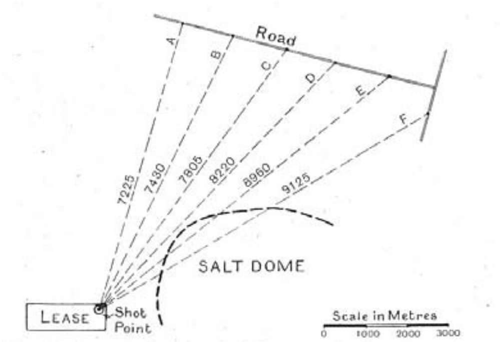



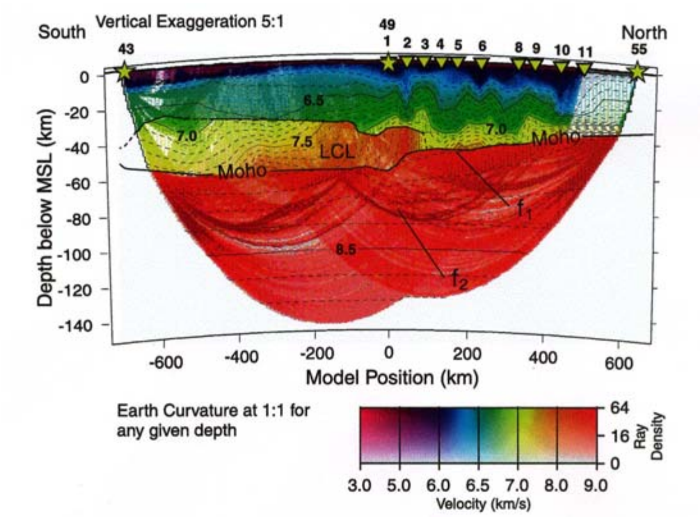

Fig. 80. Diagram showing the "shot-point" and positions of six recording seismographs.

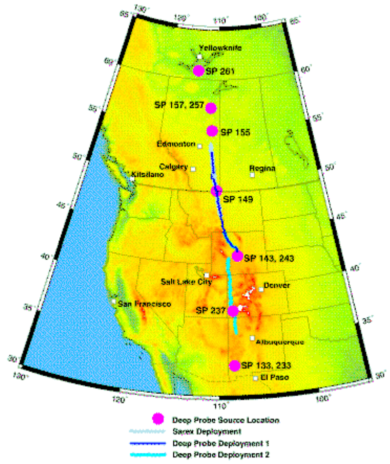
Kearey and Brooks

Eve and Keys, *Applied Geophysics*, 1928

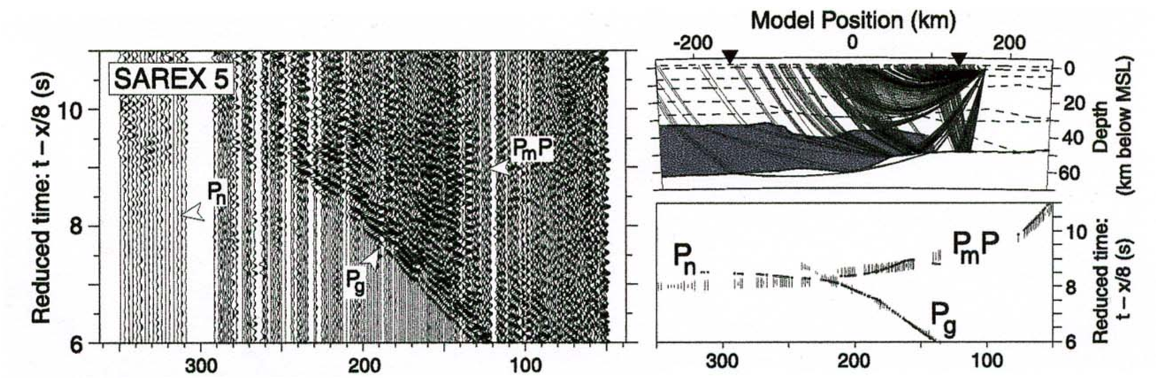
### 3.6.4 Tectonic studies of the continental lithosphere with refraction

**Deep Probe experiment**




● used explosive shots up to 2400 kg with seismic recorders deployed on a profile from 60˚N to 43˚N




● ray tracing used to model the data. Measures the variation in Moho depth and crustal structure. Figure above shows ray tracing for SAREX shot 5. Note that with a reduction velocity of 8 km/s, $P_n$ plots as a horizontal line, while the slower $P_g$ has a positive slope.

Gorman, A.R. *et al*, Deep probe: imaging the roots of western North America, *Canadian Journal of Earth Sciences*, **39**, 375-398, 2002.

Clowes, R. *et al*, Crustal velocity structure from SAREX, the Southern Alberta seismic experiment, *Canadian Journal of Earth Sciences*, **39**, 351-373, 2002.