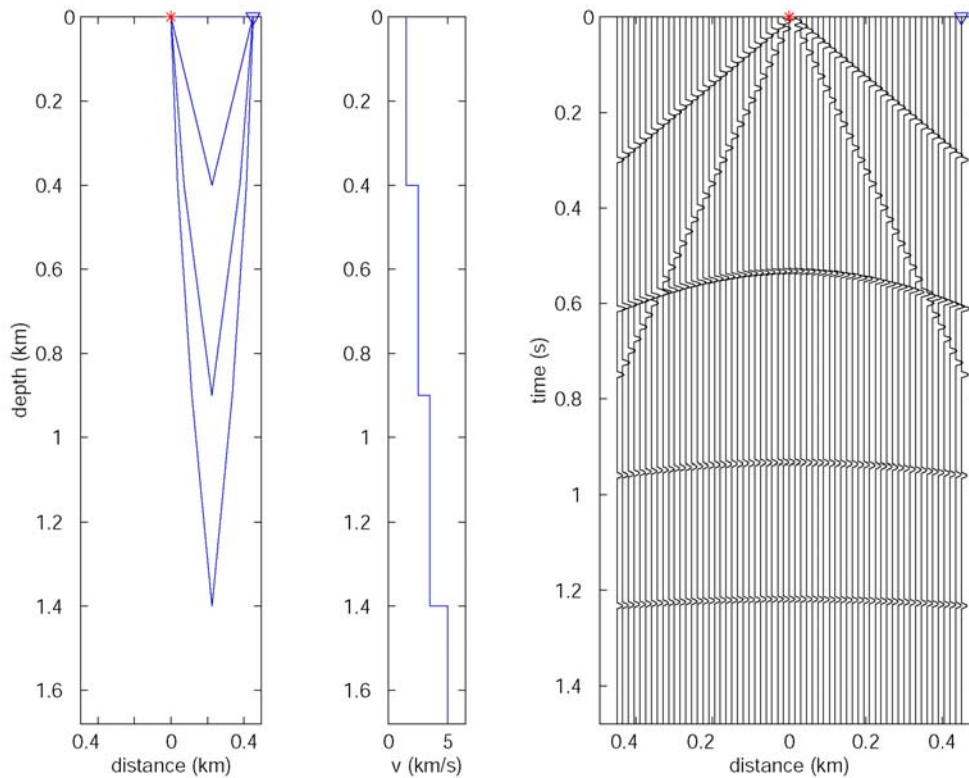


C2.2 Travel time curves for multiple reflectors

- The ray paths and travel times for multiple layers can be computed using **ray-tracing**, as demonstrated in Lab 3.
- MATLAB script [reflect_layers_v1.m](#) performs simple ray tracing.

Example 1

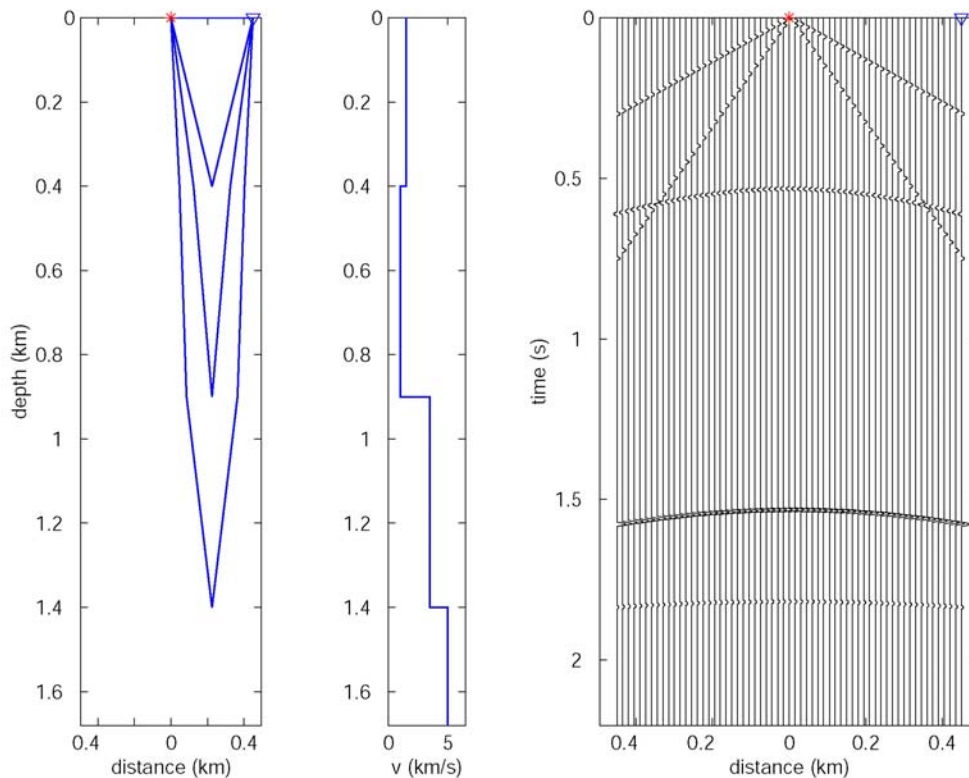


Reflected amplitudes modelled assuming normal incidence
 Attenuation, geometric spreading or AVO effects not included
 Direct wave and ground roll included

Numerical data in file *travel-time-3layer-ex1*

x (m)	ref1(ms)	ref2(ms)	ref3(ms)
0.000	533.333	933.333	1219.048
30.000	533.707	933.455	1219.110
60.000	534.829	933.819	1219.298
90.000	536.695	934.426	1219.610
120.000	539.296	935.276	1220.047
150.000	542.623	936.367	1220.608
180.000	546.659	937.698	1221.294
210.000	551.394	939.269	1222.103
240.000	556.807	941.074	1223.037
270.000	562.881	943.119	1224.094
300.000	569.589	945.395	1225.274
330.000	576.918	947.904	1226.575
360.000	584.833	950.642	1228.000
390.000	593.322	953.609	1229.545

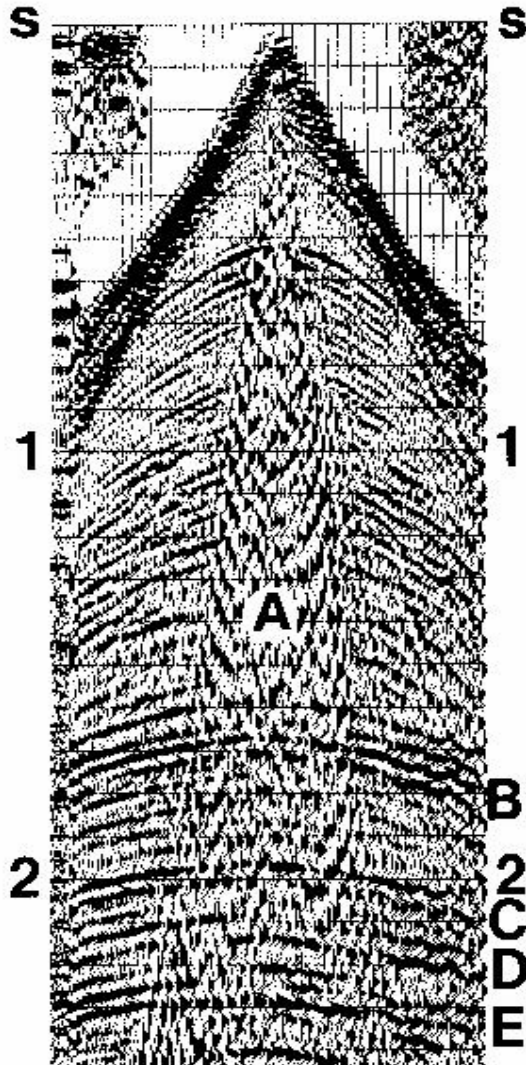
Example 2



Reflected amplitudes modelled assuming normal incidence
Attenuation, geometric spreading or AVO effects not included
Direct wave and ground roll included

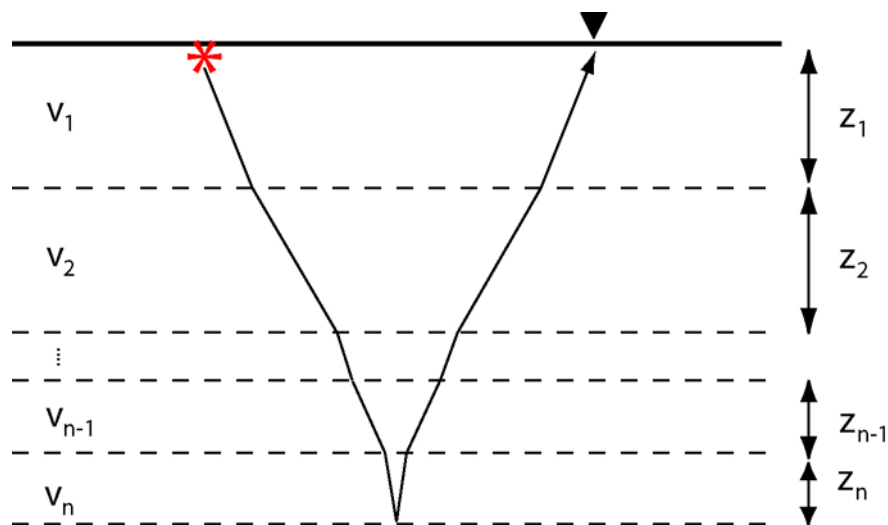
Numerical data in file *travel-time-3layer-ex2*

- The two examples both show three reflections. Note that:
 - (a) Rays are refracted at interfaces according to Snell's Law
 - (b) The polarity and amplitude of a reflection is due to the **impedance change** at the interface
 - (c) Deeper reflections are flatter (*i.e.* they exhibit **less normal moveout**)
 - (d) The high amplitude reflection from a shallow interface can weaken later reflections (from deeper structure)
 - (e) For a shot gather, the ray paths reflect (bounce) at **different points** on the reflector surface.
 - (f) This geometry, with geophones on each side of the shot is called a **split-spread**



- In the real shot gather note that:
 - (a) The vertical scale is two-way travel time in seconds.
 - (b) Positive arrivals have the trace shaded black. This helps the eye to correlate traces when noise is present. Beer sometime helps with this process, but too much has been known to have the reverse effect. Calibration is needed to optimize this task.
 - (c) Note the **direct wave**, which is the first arrival.
 - (d) The **ground roll** is long lasting and obscures reflections at small offset (A).
 - (e) Slower arrivals plot at a **steeper angle** on the seismogram (more time needed to travel the same distance)
 - (f) Reflections can be recognized by their **parabolic shape** and are labelled B,C,D, and E. Note that they are never the first arrival. This means that reflections are recorded when the ground is still moving from earlier arrivals, and reflections can be obscured by noise.

Interval velocity and average velocity



Consider the multiplayer Earth model shown above. The i^{th} layer has a thickness z_i and a velocity v_i . The ray travels through the layers 1 to n and is refracted at each interface. It is then reflected by the n^{th} interface.

The seismic wave spends a time τ_i in the i^{th} layer.

The **interval velocity** is the **actual velocity** in a specific layer and is defined as $v_i = z_i / \tau_i$

For the whole ray path we can define an **average velocity** as

$$\bar{V} = \frac{\sum z_i}{\sum \tau_i} = \frac{\sum v_i \tau_i}{\sum \tau_i}$$

Another way of averaging the velocity is to use the **root-mean-square** average. This is defined as:

$$\bar{V}_{rms,n} = \left[\frac{\sum_{i=1}^n v_i^2 \tau_i}{\sum_{i=1}^n \tau_i} \right]^{\frac{1}{2}}$$

and is needed to compute interval velocities, as described below. For the case of $n = 2$

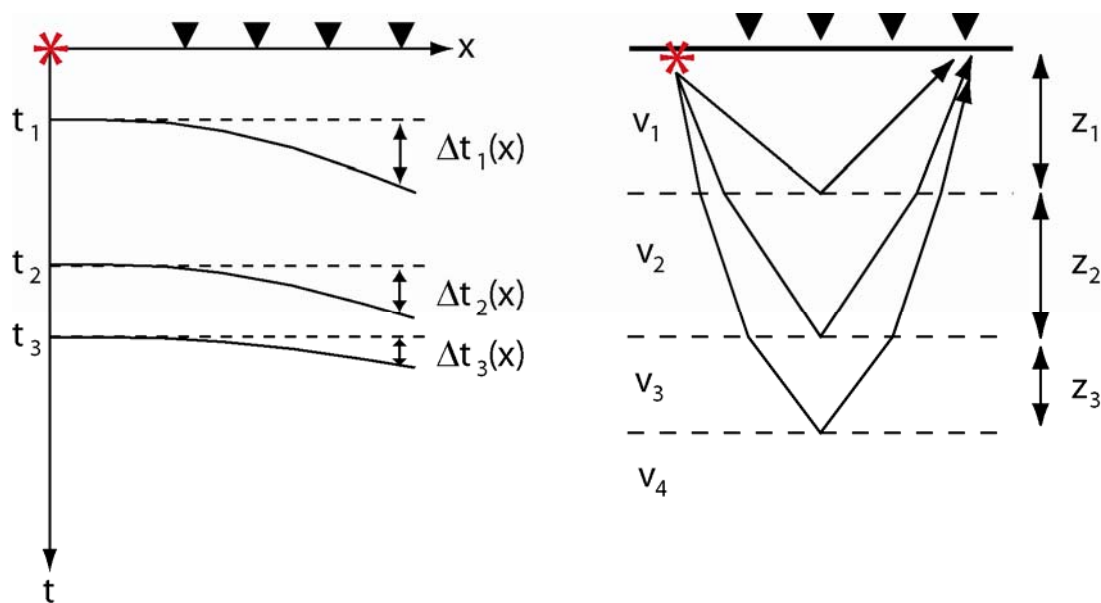
$$\bar{V}_{rms,2} = \left[\frac{v_1^2 \tau_1 + v_2^2 \tau_2}{\tau_1 + \tau_2} \right]^{\frac{1}{2}}$$

Normal move-out for multiple layers and the Dix equation

To determine the depth of the interfaces and the interval velocities, an approach similar to that used in C2.1 is needed. Note that we need more than just the travel-time at normal incidence, and should use a plot of NMO vs. offset (x). When the seismic signals travel close to the vertical direction, can show that the normal moveout for the n^{th} reflection is:

$$\Delta t = t - t_n = \frac{x^2}{2t_n \bar{V}_{rms,n}^2}$$

This is very similar to the equation in C2.1 for the normal moveout above a single interface. The only change is that the interval velocity (v_1) has been replaced by the r.m.s. velocity (i.e. the average velocity down to the n^{th} reflector). It can be shown the **root-mean-square velocity**, gives the correct average in this context.



Consider the seismogram with three reflections that is shown above:

For each reflection we can measure the **zero-offset travel times** (t_1 , t_2 , t_3). If necessary a plot of t^2 versus x^2 may be needed (see C2.1)

Next the **r.m.s. velocities** are computed from the **normal moveout**. Consider the 4th geophone at distance x . The normal moveout at this value of x are Δt_1 , Δt_2 and Δt_3 . From these NMO values, the average (r.m.s.) velocities can be computed.

$$NMO = \Delta t_1(x) = t(x) - t_1 = \frac{x^2}{2t_1 \bar{V}_{rms,1}^2}$$

Re-arranging this equation gives

$$\bar{V}_{rms,1} = \sqrt{\frac{x^2}{2t_1 \Delta t_1(x)}}$$

For the other reflections it is obvious that

$$\bar{V}_{rms,2} = \sqrt{\frac{x^2}{2t_2\Delta t_2(x)}} \quad \text{and} \quad \bar{V}_{rms,3} = \sqrt{\frac{x^2}{2t_3\Delta t_3(x)}}$$

The next stage of the data analysis is to compute the interval (true) velocities (v_1, v_2, v_3) from the average (r.m.s.) velocities ($V_{rms,1}, V_{rms,2}, V_{rms,3}$)

This is achieved with the **Dix equation** that states:

$$v_n = \left[\frac{V_{rms,n}^2 t_n - V_{rms,n-1}^2 t_{n-1}}{t_n - t_{n-1}} \right]^{\frac{1}{2}}$$

While this appears complicated, it is really a simple **recursion equation**. This means that if we know the velocity in one layer, the equation tells us what the velocity will be in the layer below.

To start this process, we begin at the surface. For the first reflection, the seismic signal travels only in Layer 1. Thus $v_1 = V_{rms,1}$

Now $V_{rms,2}$ represents an average velocity of layers 1 and 2. Thus to find v_2 we can use the Dix equation with $n = 2$

$$v_2 = \left[\frac{V_{rms,2}^2 t_2 - V_{rms,1}^2 t_1}{t_2 - t_1} \right]^{\frac{1}{2}}$$

This can then be repeated as many times as necessary. For the next layer down we can write

$$v_3 = \left[\frac{V_{rms,3}^2 t_3 - V_{rms,2}^2 t_2}{t_3 - t_2} \right]^{\frac{1}{2}}$$

Example : Look at the data in the table above for the three reflection seismogram in Example 1. The travel times for the first reflection are the same as for the example in C2.1

The solution to that problem gave:

$$v_1 = 1500 \text{ m s}^{-1} ; z_1 = 400 \text{ m and } t_0 = 0.533 \text{ s}$$

Note that $V_{rms,1} = v_1 = 1500 \text{ m s}^{-1}$

Consider the second reflection

Choose a geophone, $x =$ _____

Zero offset travel time, $t_2 =$ _____

Normal moveout $\Delta t_2 =$ _____

r.m.s. velocity for Layers 1 and 2 $V_{rms,2} =$ _____

Use the Dix equation to compute the interval velocity for layer 2:

$$v_2 = \left[\frac{V_{rms,2}^2 t_2 - V_{rms,1}^2 t_1}{t_2 - t_1} \right]^{\frac{1}{2}} = \underline{\hspace{2cm}}$$

Does this agree with the true model?

Now compute the thickness of the second layer. To do this note that the reflection spends a time $t_2 - t_1$ in the second layer. In this time, it travels a distance $2z_2$. Thus we can write

$$v_2 = \frac{2z_2}{t_2 - t_1}$$

Re-arranging the equation gives

$$z_2 = \frac{v_2(t_2 - t_1)}{2} = \underline{\hspace{2cm}}$$

Repeat this for the third reflection

Choose a geophone, $x =$ _____

Zero offset travel time, $t_3 =$ _____

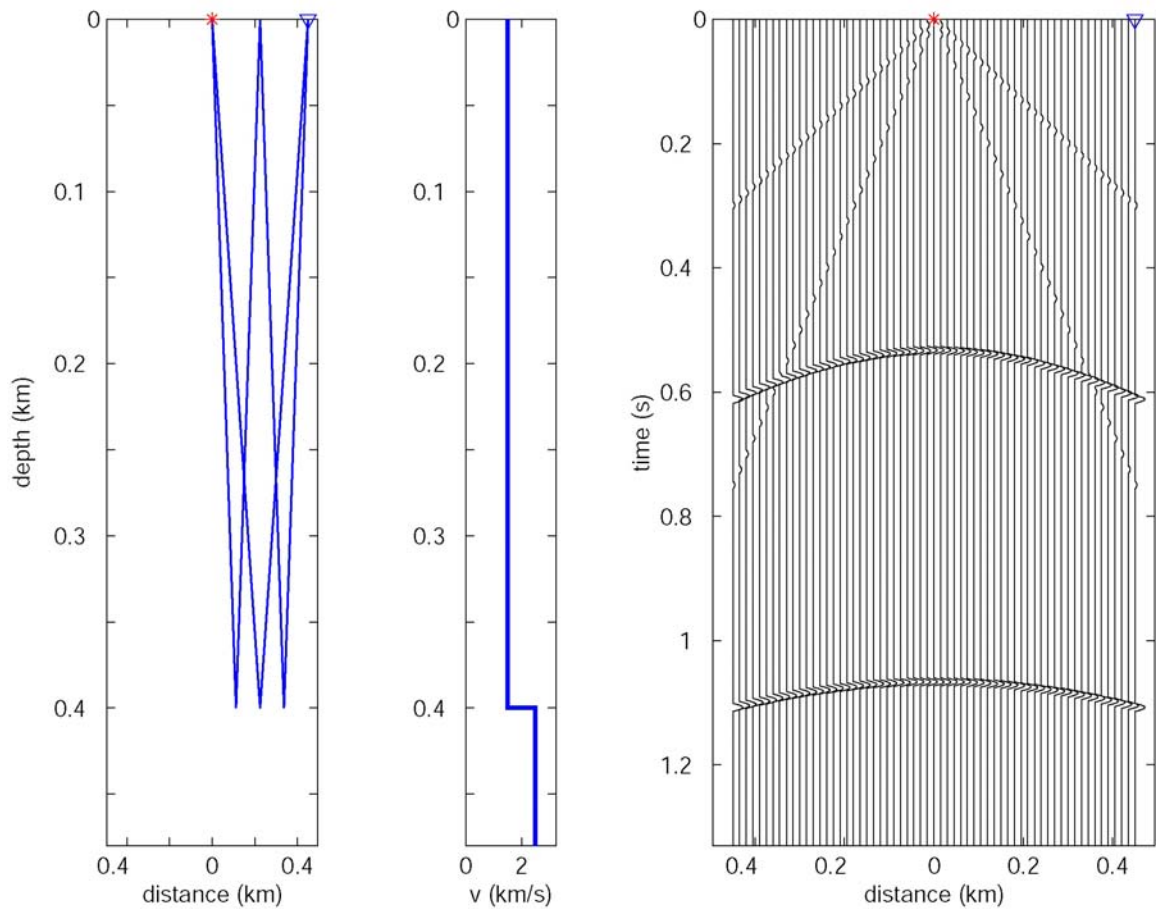
Normal moveout $\Delta t_3 =$ _____

$V_{rms,3} =$ _____

$$v_3 = \left[\frac{V_{rms,3}^2 t_3 - V_{rms,2}^2 t_2}{t_3 - t_2} \right]^{\frac{1}{2}} = \underline{\hspace{2cm}}$$

$$z_3 = \frac{v_3(t_3 - t_2)}{2} = \underline{\hspace{2cm}}$$

Multiple reflections



- (a) The second reflection is a **multiple bounce** from the interface at 0.4 km depth.
- (b) The travel time of the multiple at zero-offset is exactly **double** that of the single reflection. This can help identify a multiple.
- (c) Note that when the seismic energy bounces at the Earth-Air interface, the reflection coefficient can be large. Remember that

$$R = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1}$$

Consider the upward travelling wave as it approaches the surface. ρ_2 is now the density of the air which is essentially zero. This gives $R = -1$ and almost all energy is reflected downwards.

- (d) For an example of multiples in single channel seismic data, see Kearey Figure 4.56 and the discussion in C2.3