Gravity measurements are made on a surface profile across a buried sphere. The sphere has an excess mass $M_S$ and the centre is at a depth $z$. At a distance $x$, the vertical component of $g$ is given by

$$g_z = \frac{GM_S z}{(x^2 + z^2)^{3/2}}$$

This curve is drawn below for a sphere with:

- Radius, $a = 50$ m
- Depth, $z = 100$ m
- Density contrast, $\Delta \rho = 2000$ kg m$^{-3}$
- Excess mass, $M_S = 10^9$ kg

Note that:

- $g_z$ has its maximum value directly above the sphere at $x = 0$ m.
- The maximum value is $g_z^{\text{max}} = \frac{GM_S}{z^2}$
- The value of $x$ where $g_z = (g_z^{\text{max}})/2$ is called the half-width of the curve ($x_{1/2}$).
  
  Can show that $x_{1/2} = 0.766 z$

- Far away from the sphere, $g_z$ becomes very small
Gravity measurements are rarely made on a single profile. Usually they are made on a grid of points. This allows us to make a map of \( g_z \).

**Question:** What will the map look like for the buried sphere

### B2.2 Buried horizontal cylinder

When gravity measurements are made across a buried cylinder, it can be shown that the variation in \( g_z \) will be:

\[
g_z = \frac{2G\pi a^2 z\Delta \rho}{(x^2 + z^2)}
\]

This curve is drawn below for a cylinder with:

- radius, \( a \) = 50 m
- depth of axis, \( z \) = 100 m
- density contrast, \( \Delta \rho \) = 2000 kg m\(^{-3}\)
- horizontal location, \( x = 0 \) m

![Graph showing the variation of \( g_z \) with \( x \) for a buried horizontal cylinder](image)

Note that:

- the maximum value of \( g_z \) is located directly above the axis of the cylinder.
- From the plots, we can see that this value is larger than \( g_z^{\max} \) for a sphere? Why?
- For a cylinder, we can show that the half-width \( x_{1/2} = z \)

**Question:** Compare the profiles across the sphere and a cylinder. Would this information allow you to decide if the buried object was a sphere or a cylinder?
Question: If $g_z$ is measured on a grid of points, what will the resulting map look like? Would this be a better way to distinguish between a sphere and cylinder?
Forward and inverse problems in geophysics

B2.1 and B2.2 illustrate the gravity anomaly that we would expect to observe above a known geological target. This is called a forward problem in geophysics, and is a useful exercise in understanding if measurements would be able to detect a particular structure.

**Forward problem:**  *Density model of Earth > Predicted gravity data (anomaly)*

However, we are usually more interested in solving the opposite problem. When gravity data has been collected in a field survey, we want to find out the depth and size of the target. This is called an inverse problem in geophysics.

**Inverse problem:**  *Measured gravity data > Density model of Earth*
Example: Gravity data interpretation example

Consider some gravity data collected on a profile crossing a spherical iron ore body.

- Where is the centre of the ore body? \( x = \ldots \) metres
- What is the maximum value of \( g_z \)? \( g_z^{\text{max}} = \ldots \) mgal
- At what distance \((x_{1/2})\) has \( g_z \) fallen to half this value? \( x_{1/2} = \ldots \) metres
- The depth of the sphere can be derived using the equation \( x_{1/2} = 0.766 \, z \)
  Rearranging this gives \( z = 1.306 \, x_{1/2} \)
  \( z = \ldots \) metres
- To determine the excess mass, we use the equation \( g_z^{\text{max}} = \frac{GM_s}{z^2} \)

We know \( z \) and have measured \( g_z^{\text{max}} \) so we need to rearrange this equation to find \( M_s \)
This gives

\[ M_s = \frac{g_{z}^{\text{max}} z^2}{G} \]

Remember to convert \( g_{z}^{\text{max}} \) from milligals to ms\(^{-2}\)!

\[ M_s = \text{___________kg} \]

- Can the radius \((a)\) and density contrast \((\Delta \rho)\) of the sphere be determined?

### B2.3 Uniform layer

A layer has an infinite extent, thickness \( \Delta z \) and a density \( \rho \). The gravitational attraction of this slab at \( P \) is:

\[ g_{z} = 2\pi G \rho \Delta z \]

Note that \( g_{z} \) does **not depend** on the distance from the layer to the point \( P \). Why?

Consider the two density models shown below.

\[ \rho = 1000 \quad \Delta z = 200 \text{ m} \]

\[ \rho = 2000 \quad \Delta z = 100 \text{ m} \]

What can we say about the gravitational acceleration \((g_{z})\) of the two models?

This is an example of **non-uniqueness** in geophysics, and occurs when more than one Earth model can explain the same set of geophysical data.
**B2.4 General polygon – simple computer modelling**

Simple density models such as a sphere and a cylinder can help us get an idea of what gravity anomaly ($g_z$) is produced by simple density models. However, to represent the real Earth, more complicated model geometries are clearly needed. The development of powerful, portable computers has made this possible.

In the lectures and labs we will use a very simple MATLAB program that computes $g_z$ for a N-sided prism. This is called 2-D modelling, since the polygon is assumed to be infinite in length in the strike direction (out of the plane of the page). Fully 3-D gravity modelling is also widely used, but will not be discussed in detail in Geophysics 224.

**B2.4 Example 1 – verification**

When using a computer modelling program, it is important to be suspicious about the answer. The first time the program is used, you should try and use a simple model for which the answer is already known. If you can get the same answer by two methods, then maybe you can trust the program.

Consider using the program to compute the gravity anomaly of a cylinder. The cylinder can be approximated by an 8-sided polygon, as shown below. The dots show the exact (known) answer for a cylinder.

![Diagram](image)

**Question**: Why is $g_z$ for the polygon less than for a cylinder?
With a 16-sided polygon, the agreement is significantly better.

**B2.4 Example 2 – sedimentary basin**

An over simplified sedimentary “basin” is shown below sheet. Note that the “basin” is less dense than the basement rocks, so the pull of gravity is slightly weaker over the basin. The horizontal line shows the expected value of $g_z$ for an infinite layer with the same thickness and density as the basin.
Now consider a similar basin, that is twice as deep and with a lower density contrast.

![Graph showing depth and gravity anomaly](image)

Note the following two features:

- The minimum value of $g_z$ is the same as for the first basin. Why?
- The edge effects extend further into the basin than in the first case.

In the lab we will use this program to analyse some real gravity data collected across a sedimentary basin in Washington and an impact crater in Mexico.

* MJU 2006*