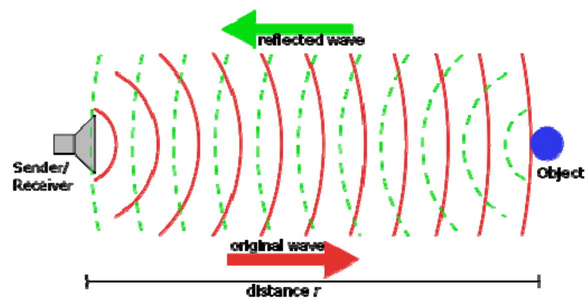
E : Ground-penetrating radar (GPR)

- The EM methods in section D use low frequency signals that travel in the Earth by diffusion. These methods can image resistivity of the Earth on various scales. Because they are diffusive, they will not be reflected from sharp interfaces between layers with different resistivities.

- Seismic exploration is the most widely used exploration method in geophysics. It uses the propagation of **elastic waves** in the Earth and can give detailed images of the subsurface. Can show that the best vertical resolution that can be obtained in one quarter of a wavelength.

- However it can be difficult to apply seismics to the study of shallow structure on the scale of 10's of metres as high frequency signals are needed. In addition, seismic sources can be awkward to operate repeatedly.

- RADAR (Radio detection and ranging) was developed during WW2 to measure the distance and velocity of an aircraft.



From http://en.wikipedia.org/wiki/Radar

- In the 1960's and 1970's radar was applied to imaging subsurface structure. Frequency must be chosen carefully to ensure that (a) signals travel as waves and (b) that the depth of penetration is not limited by the skin depth phenomena.

- Ground penetrating radar (GPR) is now a widely used technique for imaging near surface structure. The radio wave pulses can be transmitted much more easily than in seismic exploration. Most seismic data processing techniques can be easily applied to GPR data.

Calculation

What will be the round-trip travel time for a reflection from a surface 5 m from the TX-RX? Assume speed of light is 3×10^8 m/s.

Compare this with sound waves travelling at 300 m/s

E1 : Wave propagation

- In section D we considered how to determine if an EM signal will travel in the Earth as a wave or by diffusion.

- To determine if wave propagation or diffusion will dominate, can consider the ratio defined as $r = \dfrac{\sigma}{2\pi f \varepsilon}$, where f is the frequency, ε is the permittivity of the subsurface and σ is the electrical conductivity.

- Assume that the permittivity has the free space value $\varepsilon = \varepsilon_0 = 8.85\ 10^{-12}$ F/m

- If r is large, then EM diffusion will occur.

- If r is small then wave propagation occurs

E1.1 : Velocity of propagation

- In free space radio waves travel at the speed of light, c, where $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and μ_0 is the **permeability** of free space and ε_0 is the **permittivity** of free space

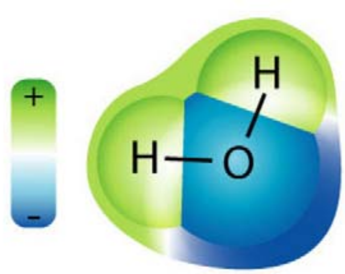
- $\mu_0 = 4\pi \times 10^{-7}$ H/m and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m giving $c = 3 \times 10^8$ m/s

- In the Earth the permeability (μ) and permittivity (ε) do not have their free space values. This is because the atoms/molecules behave as magnetic and/or electric dipoles.

- The magnetic behaviour of the atoms/molecules can be expressed in terms of **relative permeability** (μ_r) as $\mu = \mu_r \mu_0$ and section C describes this topic. The relative permeability is usually close to 1, except in deposits of magnetic minerals.

- The relative permittivity (ε_r) is defined as $\varepsilon = \varepsilon_r \varepsilon_0$. This quantity is also called the **dielectric constant** (K).

- The permittivity will be greater than 1 if the molecules can act as electric dipoles. Water is one of the most polar molecules since the hydrogen atoms develop a positive charge and the oxygen develops a negative charge.



- Can write the velocity of a radar wave in the Earth (v) in terms of the relative permittivity and permeability as

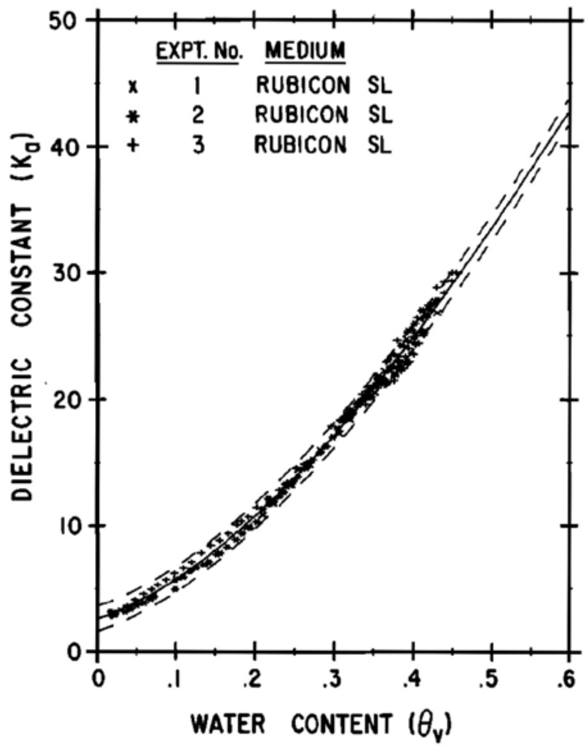
$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

- Some typical values are listed below (from Davis and Annan, 1989)

Material	ε_r	σ(mS/m)	v(m/ns)	δ (f = 100 MHz)
Air	1	0	0.3	∞
Distilled water	80	10^{-2}	0.033	15.8 m
Fresh water	80	0.5	0.033	2.24 m
Seawater	80	3000	0.01	0.03 m
Dry sand	3-5	10^{-2}	0.15	50 m
Saturated sand	20-30	0.1-1	0.06	1.58 m
Limestone	4-8	0.5-2	0.12	1.18 m
Clay	5-40	2-1000	0.06	0.05 m
Granite	4-6	0.01-1	0.13	1.58 m
Dry salt	5-6	0.01-1	0.13	1.58 m
Ice	3-4	0.01	0.16	15.8 m

- Most materials are mixtures so need to consider a combination of properties.

- Topp et al., 1980 derived an empirical relationship between the dielectric constant (K_a) and the water content (θ_v). Figure 5 from this paper is illustrated below.



$$K_a = 3.03 + 9.3\theta_v + 146\theta_v^{\ 2} - 76.7\theta_v^{\ 3}$$
$$\theta_v = -5.3x10^{-2} + 2.92x10^{-2} K_a - 5.5x10^{-4} K_a^{\ 2} + 4.3x10^{-6} K_a^{\ 3}$$

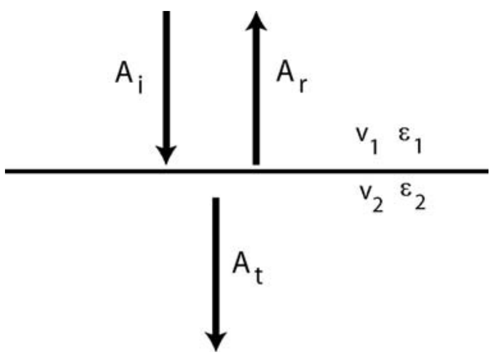
Calculation

A 200 MHz GPR signal travels in the Earth with v = 0.06 m/ns.

What is the wavelength?

E1.2 Reflection at interfaces

- When a radar pulse reaches an interface between two layers, some energy will be reflected and some will be transmitted.



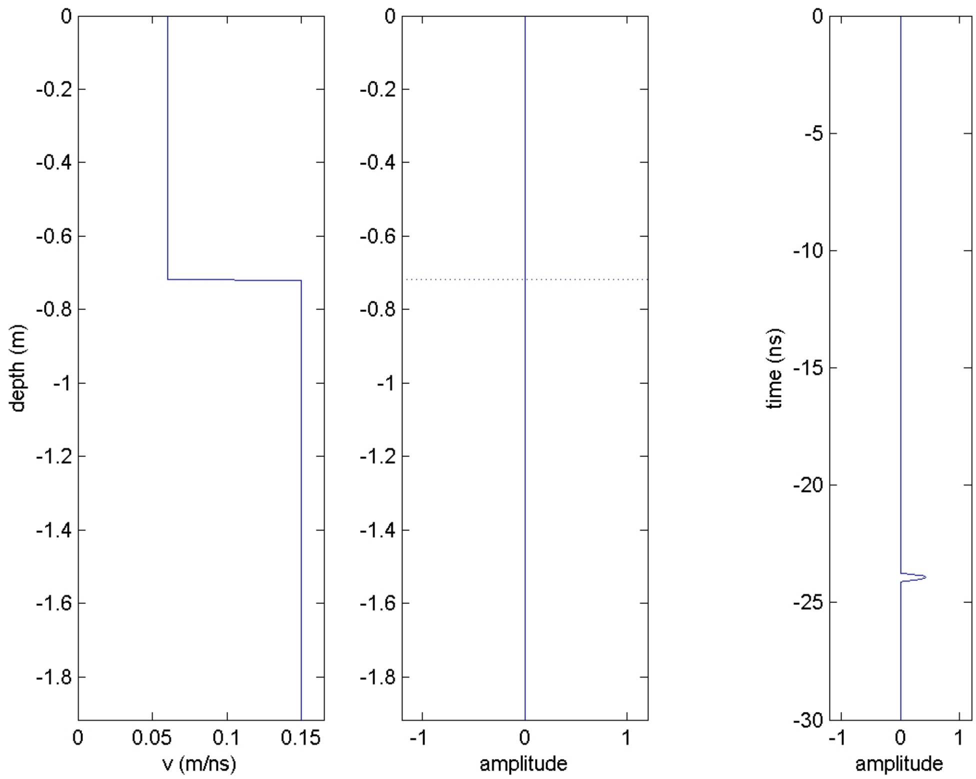
- Consider the case when the signal is **normally incident** on a horizontal interface
- In the upper layer the velocity is v_1 and in the lower layer it is v_2
- If the incident wave has an amplitude A =1, then can show that the amplitude of the reflected and transmitted wave is given by the coefficients R and T.

$$R = \frac{A_r}{A_i} = \frac{v_2 - v_1}{v_2 + v_1} \qquad T = \frac{A_t}{A_i} = \frac{2v_1}{v_2 + v_1}$$

- If the velocity variation is solely due to changes in permittivity, then

$$R = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \qquad T = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

Example 1 : Increase in velocity with depth

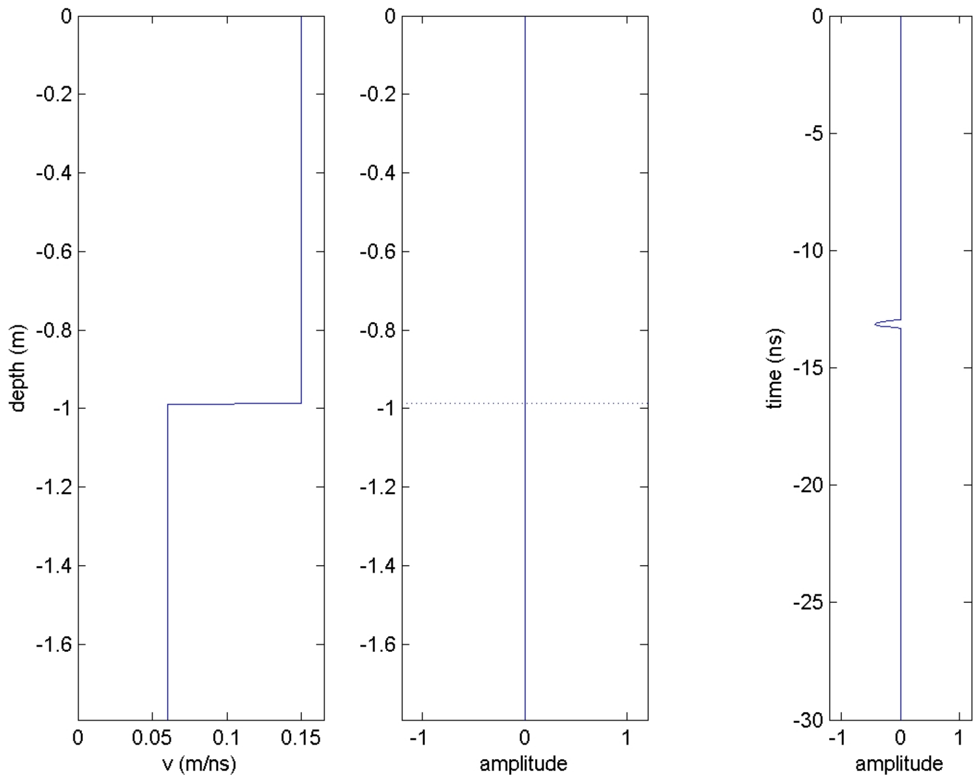


- This example has an increase in velocity from 0.06 to 0.15 m/ns which would correspond to water saturated sediments overlying granite.

- Note the **positive** reflection coefficient as predicted by $R = \dfrac{v_2 - v_1}{v_2 + v_1}$

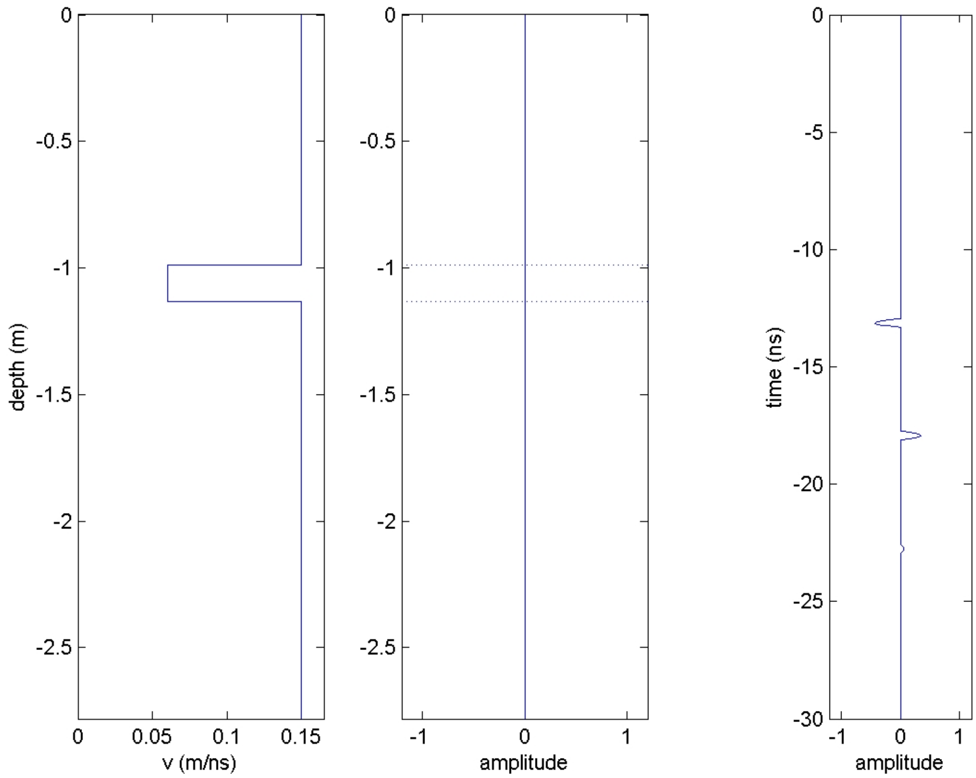
- Confirm that the round trip travel time is correctly computed.

Example 2 : Decrease in velocity with depth



- Velocity decreases from 0.15 to 0.06 m/ns corresponding to the water table
- Note the decrease in velocity that corresponds to the increase in permittivity
- Reflection has negative polarity as predicted by $R = \dfrac{v_2 - v_1}{v_2 + v_1}$

Example 3 : Low velocity layer



- Low velocity layer produced by a zone of water saturated sand.
- The 3 layers have velocities v_1, v_2 and v_3
- Note the **negative polarity reflection** from the top of the layer which is caused by a decrease in velocity according to $R = \frac{v_2 - v_1}{v_2 + v_1}$
- The second reflection comes from the base of the low velocity layer and has a positive polarity. To calculate the amplitude must consider both transmission at the top of the layer and reflection at the base.

$$\text{Amplitude} = R = \left(\frac{2v_1}{v_2 + v_1}\right)\left(\frac{v_3 - v_2}{v_3 + v_2}\right)\left(\frac{2v_2}{v_1 + v_2}\right)$$

Note that the middle term can be positive or negative. However the first and third terms (transmission on downward and upward paths) are both positive. Thus the polarity of the second reflection is determined by the velocity change where it is reflected.

- Can you confirm the relative magnitude of these two reflections for the numerical values used in this case?

- In the animation you can also see reverberation (multiple internal reflections) in the low velocity layer.

- As the layer gets thinner, the reflections arrive closer in time. Can show that the two reflections can only be distinguished if $t > \frac{\lambda}{4}$ where t is the thickness of the layer.

Calculation

A 100 MHz survey takes place in a region where dry sand has v = 0.15 m/ns. What is the thinnest layer that could be detected?

E1.3 Factors that reduce the amplitude of radar waves

The amplitude of radar waves will be reduced by two primary effects

(a) Geometric spreading. As the wave spreads out to cover a larger area, **conservation of energy** requires that the amplitude decreases. This is very similar to the way that ripples on a pond decrease in amplitude.

(b) Attenuation through the skin depth effect. This is the most important effect and you should **always** compute the skin depth from the frequency and conductivity. At high frequency in a high conductivity medium, the skin depth can be just centimetres (see table above).