223 B3 Depth-sounding with DC resistivity

- In the previous section we assumed that the resistivity of the Earth did not vary with depth (halfspace). In general, this is not the case.

- The next scenario we will consider is a location where resistivity varies with depth, but not with horizontal position. This is referred to as a one-dimensional (1-D) Earth model.

B3.1 The effect of an interface

- To compute the apparent resistivity of a multi-layer Earth, we must consider what will happen when electric current crosses the interface between layers with different resistivity.

- Consider two layers with resistivity values ρ_1 and ρ_2 above and below the interface. It can be shown that the following **boundary conditions** will relate the electric field above and below the interface.

(1) **Electric field parallel to the interface is continuous**

$$E_1^{par} = E_2^{par}$$

From the definition of electric current density (J = σE) this requires that

$$J_1^{par}\rho_1 = J_2^{par}\rho_2$$

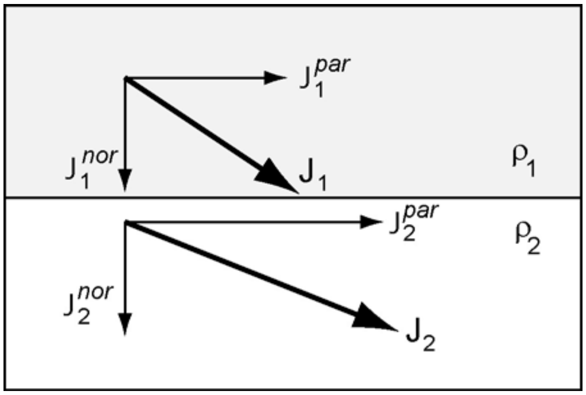
Thus if resistivity increases, the current decreases

(2) **Electric current normal to the interface is continuous**

$$J_1^{nor} = J_2^{nor}$$

This is a consequence of the conservation of electric charge.

Example 1: Resistivity decreases with depth (ρ_1 = 100 Ωm and ρ_2 = 10 Ωm)

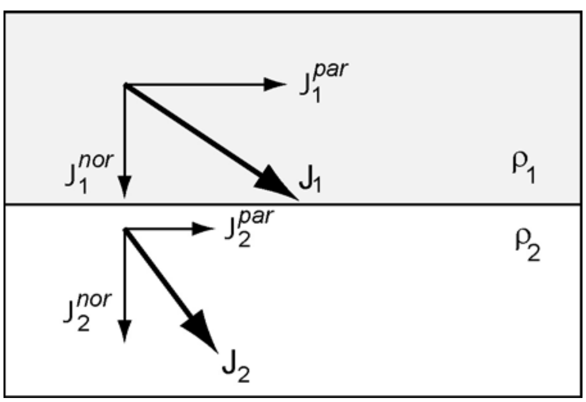


- Electric current (J) approaches the interface at an oblique angle.
- The current, J, can be resolved into two components, normal and parallel to the interface.
- Using the results from above, we can show that:

$J_1^{nor} = J_2^{nor}$ and $J_1^{par} < J_2^{par}$

Overall result: Electric current is deflected **towards** the horizontal

Example 2: Resistivity increases with depth (ρ_1 = 100 Ωm and ρ_2 = 1000 Ωm)



- Using the same arguments as above, we can show that in this case:

$J_{nor}^1 = J_{nor}^2$ and $J_{par}^1 > J_{par}^2$

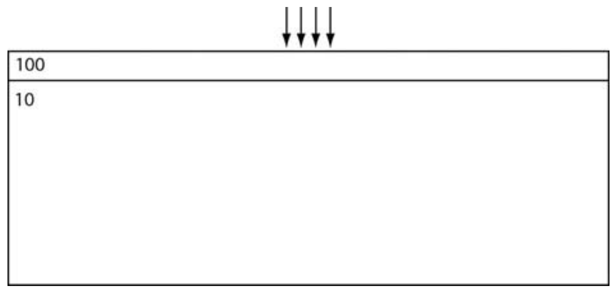
Overall result: Electric current is deflected **away from** the horizontal

B3.2 Qualitative solution for a 2-layer Earth (Wenner array)

- Consider the expanding Wenner array shown in the figures below.
- Electric current flows between the outer (current) electrodes, and the voltage is measured between the inner (potential) electrodes.
- The surface layer is 10 m thick. In each case, sketch the current flow lines.

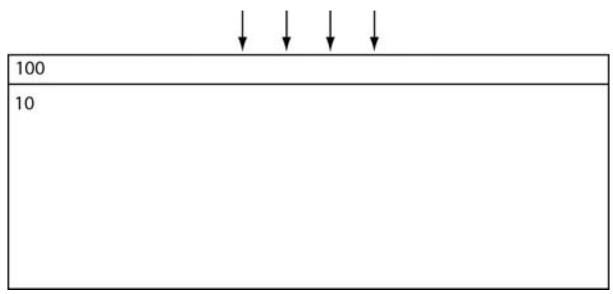
a=1 m: Electric current is confined to the upper layer.

The apparent resistivity reflects this fact and $\rho_a = \rho_1 = 100\Omega m$



a=10 m : Electric current is now flowing through both layers. The current changes direction at the interface, as described in B3.1.

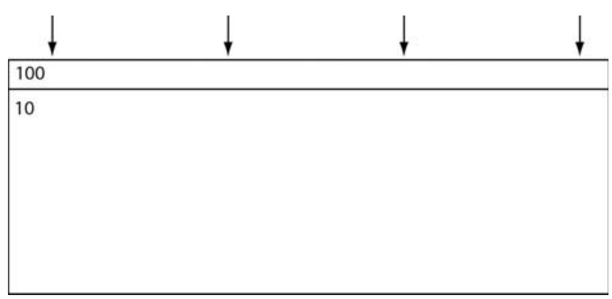
$$\rho_2 < \rho_a < \rho_1$$

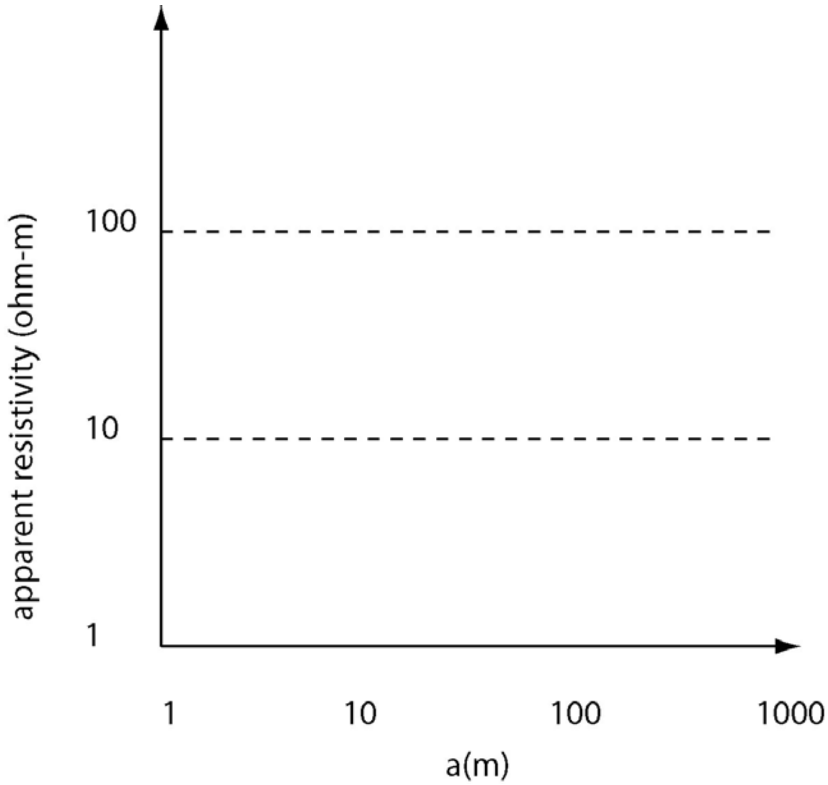


a=100 m: Most of the electric current flows in the lower layer because of geometric arguments. This is enhanced by the fact that the lower layer is a route of low resistivity, essentially a short circuit, and electric current flows preferentially in this layer.

$$\rho_2 \sim \rho_a < \rho_1$$

As the electrode spacing continues to increase, $\rho_a \rightarrow \rho_2$





- The apparent resistivity can be plotted as a function of the a-spacing.

- Since the electric current goes deeper as the *a*-spacing increases, this curve gives an impression of how electrical resistivity varies with depth.

- There is not an exact correlation between the *a*-spacing and depth being sampled by the electric currents.

- Note that a log-log plot is used for this curve.

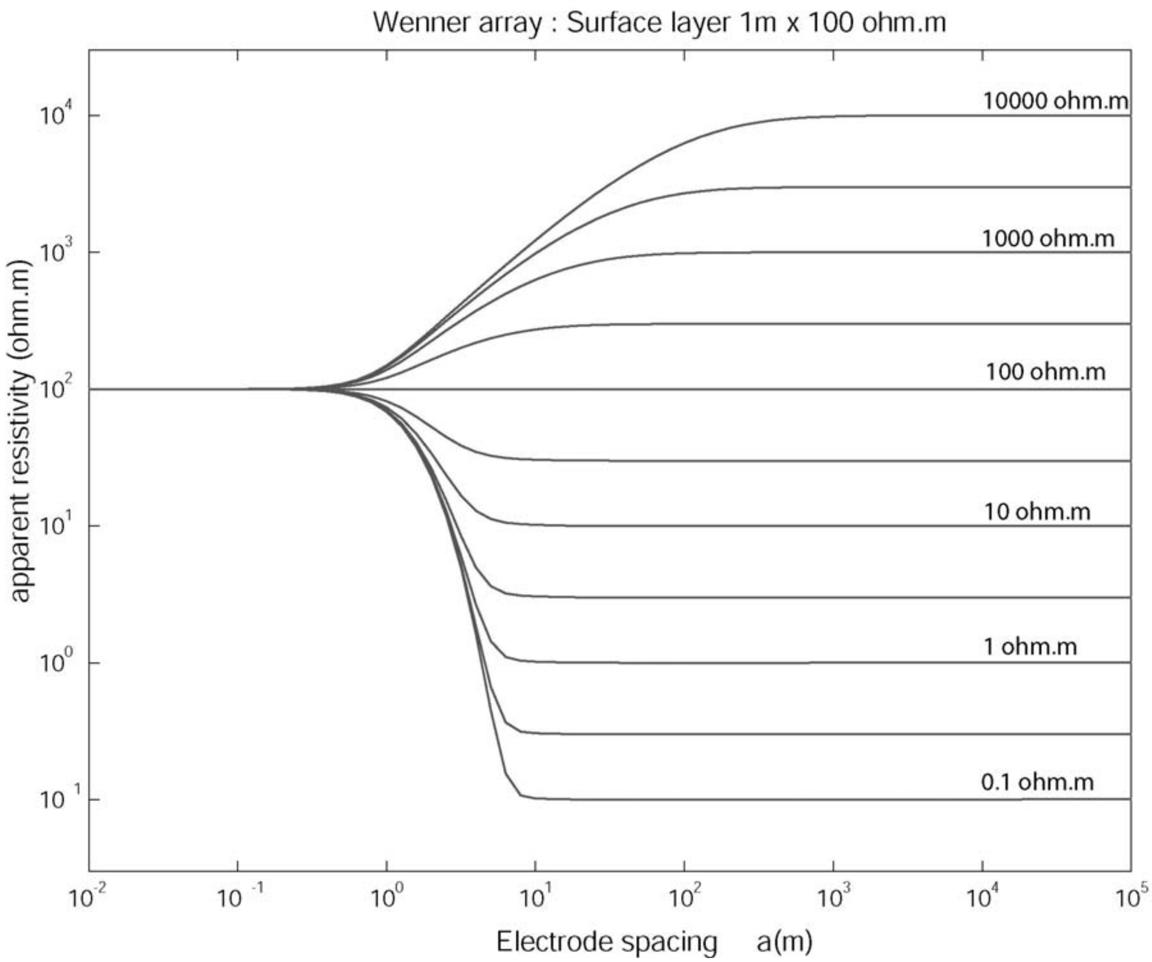
- Modeling and interpretation is needed to convert the *a*-spacing into a true depth.

B3.3 Quantitative solution for 2-layer Earth (Wenner array)

- A quantitative solution can be derived through the method of images, or other more complex calculations. Details can be found in the textbook.

- A simple MATLAB program was used to compute the apparent resistivity as a function of *a*-spacing. This was based on Mooney et al., *Geophysics*, (1966).

Example 1: In all models, the surface layer has a resistivity of 100 Ωm and is 1 m thick. The second layer has a range of resistivities, as shown in the figure.



Note that:

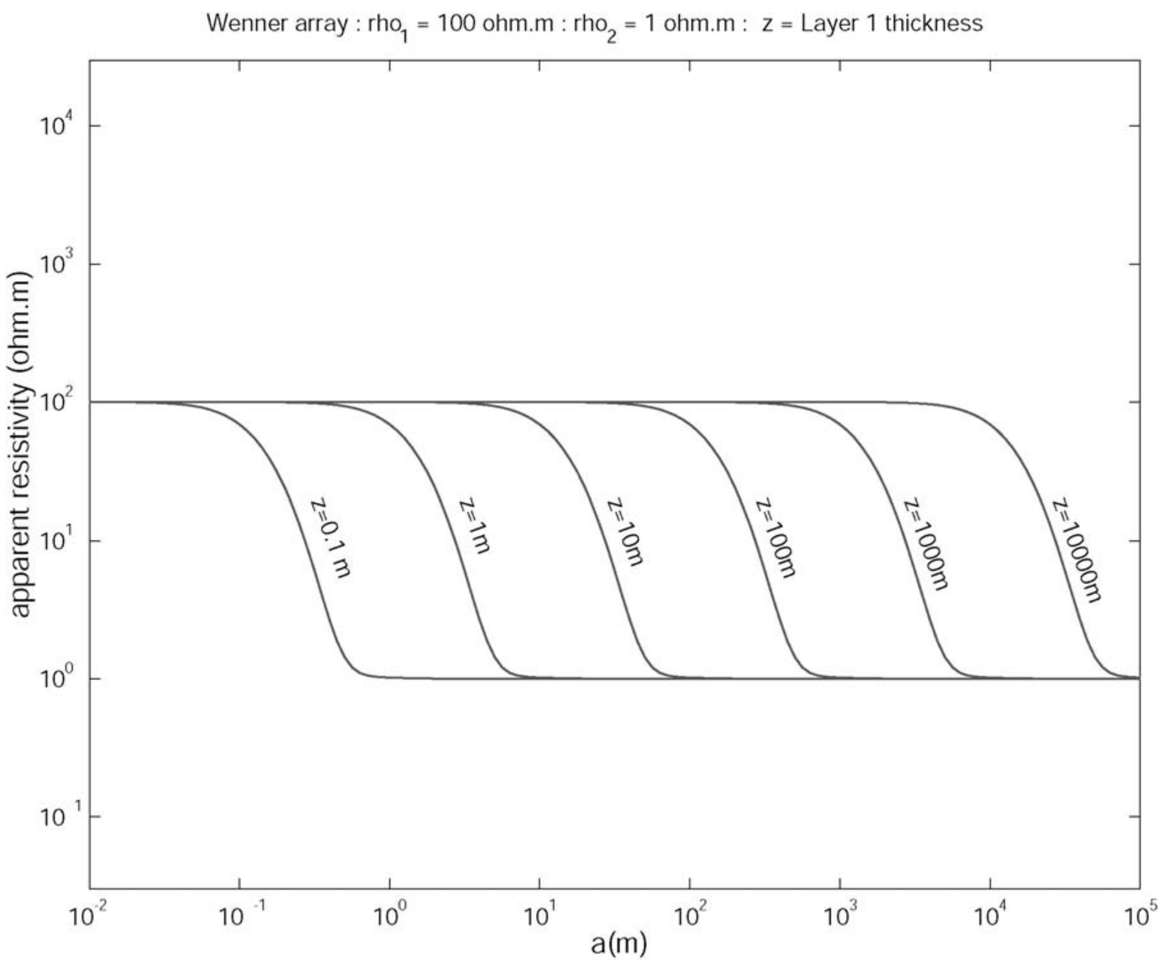
(1) The effect of the lower layer is only observed when the *a*-spacing is greater than the layer thickness.

(2) At large values of *a*-spacing, the apparent resistivity asymptotically approaches the true resistivity of the lower layer.

(3) When the lower layer is more resistive, the apparent resistivity rises slowly as the a-spacing increases. This is because the electric current preferentially flows in the lower resistivity (upper) layer, and apparent resistivity is the average resistivity of the region in which current is actually flowing.

(4) When the lower layer has the lowest resistivity, the apparent resistivity falls quickly, as the *a*-spacing increases. Explanation is the converse of that in (3).

Example 2: In all models, the surface layer has a resistivity of 100 Ωm and the thickness is variable ranging from 0.1 to 10000 m. The second layer has a range of resistivity of 1 Ωm.



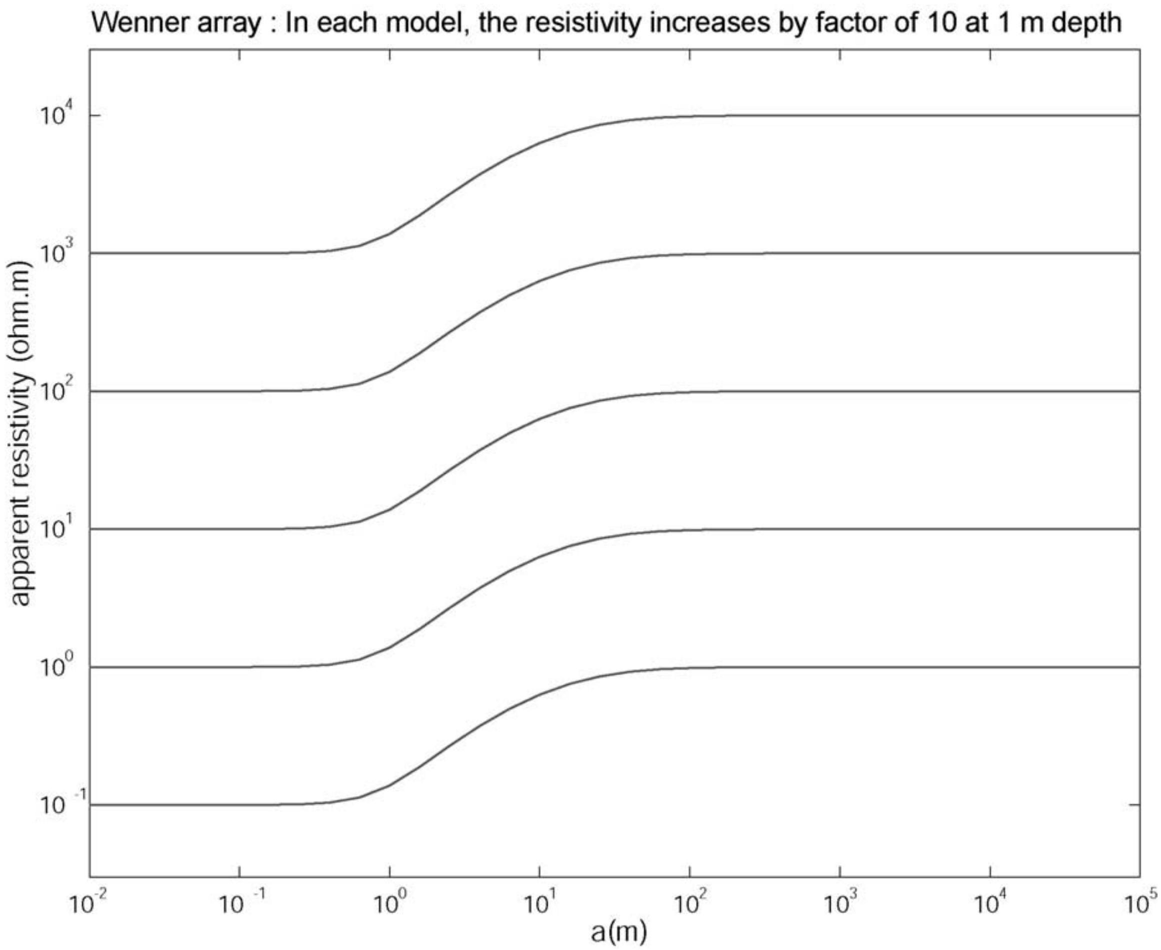
Note that:

(1) The effect of the lower layer is only observed when the *a*-spacing is greater than the layer thickness.

(2) The shape of the curve is the same in each case.

(3) If the horizontal axis was (*a/z*) then all the curves would be identical. This is the physical basis of the **master curves** discussed later on in the notes.

Example 3: Two layer models. Resistivity increases by a factor of 10 at depth of 1m in all the models.

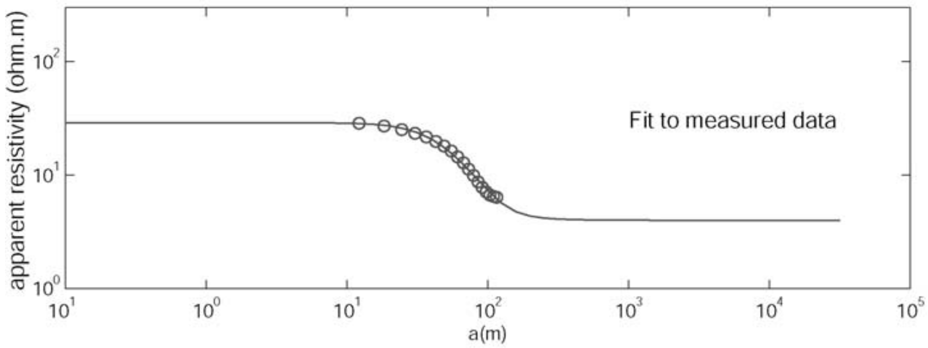


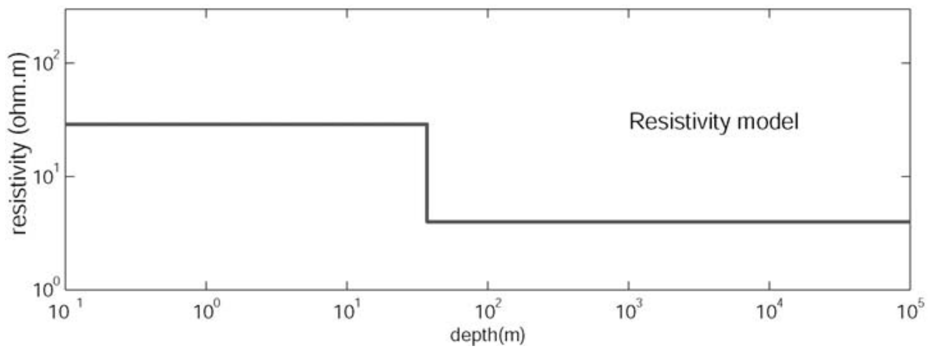
Note that:

(1) The increase in apparent resistivity occurs when the *a*-spacing of the Wenner array is approximately equal to the layer thickness.

(2) If the vertical axis was plotted as $\frac{\rho_a}{\rho_1}$ then all the curves would be identical. This will be useful when **master curves** are considered later in this section.

B3.4 Fitting Wenner array data with a two-layer resistivity model





- Data were collected at Malagash, Nova Scotia and are listed in Telford.
- In this area, the subsurface is saturated with brine, quite close to the surface.
- These data will be interpreted in class by trial-and-error modeling with the MATLAB script **wenner2lay_fit.m**

a(ft)	rho (ohm-m)
40.	28.5
60.	27.1
80.	25.3
100.	23.5
120.	21.7
140.	19.8
160.	18.0
180.	16.3
200.	14.5
220.	12.9
240.	11.3
260.	9.9
280.	8.7
300.	7.8
320.	7.1
340.	6.7
360.	6.5
380.	6.4

B3.5 Curve matching (a history lesson)

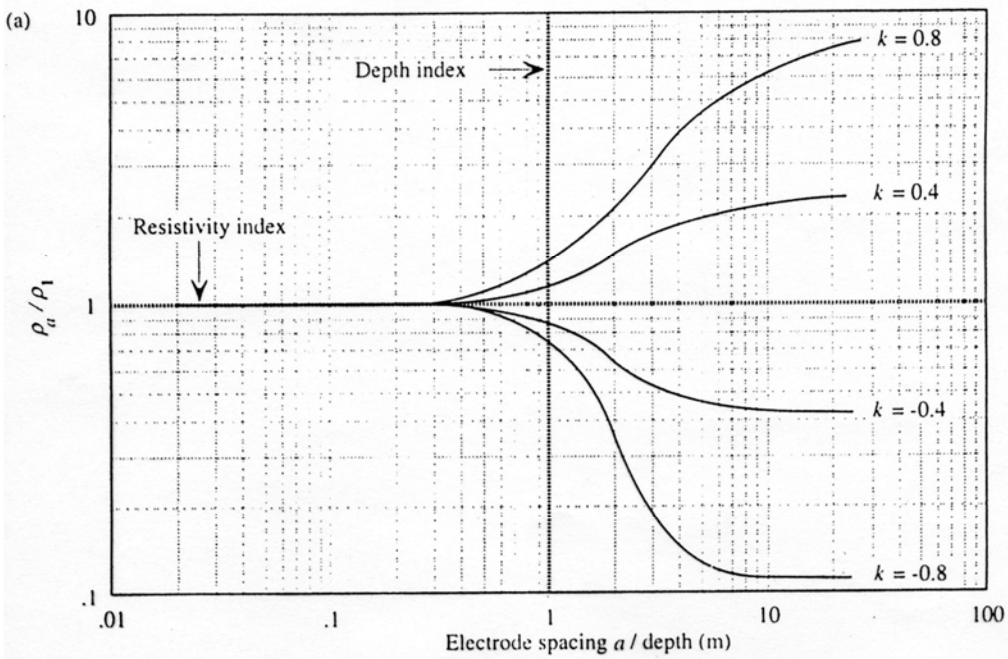


Figure 5-33 Basic procedures for curve matching. (a) Master curves for several possible *k*-values. (b) Field data plotted on the same scale as (a). (c) Answer obtained by superimposing field-data curve on master curves.

- This set of curves summarizes **all possible combinations** of upper layer resistivity (ρ_1), upper layer resistivity (ρ_2) and layer thickness (d).

- This is achieved by plotting apparent resistivity normalized by ρ_1 on the vertical axis.

- Similarly, the horizontal scale is normalized by plotting apparent resistivity divided by layer thickness (*a/d*). This reflects the fact that the lower layer will be detected when the electrode spacing is approximately equal to the layer thickness (when *a~d* or *a/d =1*).

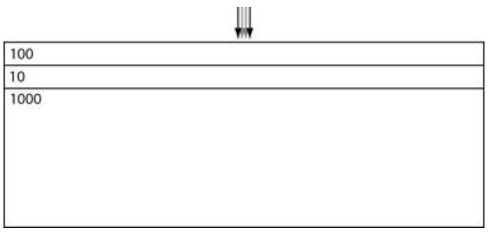
- The individual apparent resistivity curves represent a range of lower layer resistivity values (ρ_2) and $k = \dfrac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$. Positive values of *k* correspond to an increase in resistivity with depth, while negative values correspond to a decrease in resistivity with depth.

- Note that sometimes the master curves are labeled with the ratio (ρ_2 / ρ_1)

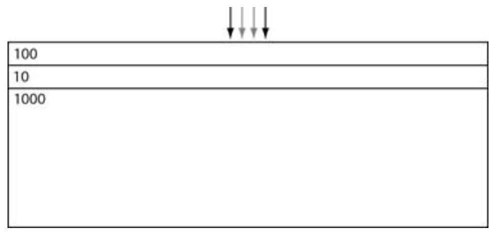
- These master curves are used by overlaying a plot of the actual data points on a set of master curves and sliding the overlay **up-down** and **left-right** until a good fit is found to one of the curves.

B3.6 Qualitative solution for apparent resistivity (Wenner array)

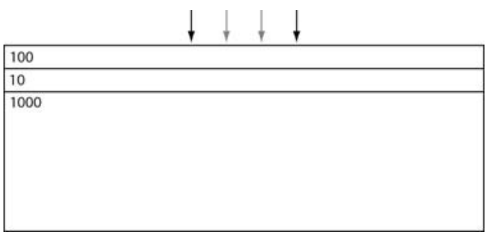
- Consider the following models with $h_1 = h_2 = 10$ m.
- Sketch the current flow patterns and estimate the apparent resistivity.
- Remember that **apparent resistivity** can be considered as the **average resistivity** over the region in which the current flows.



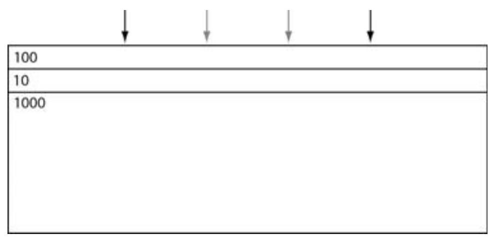
a =0.1m



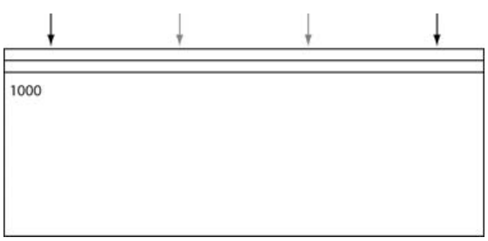
a=3 m



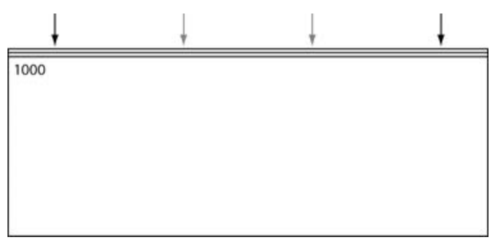
a=10 m



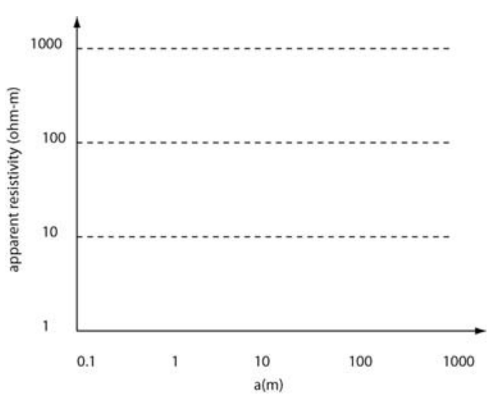
a = 30 m



a=100 m



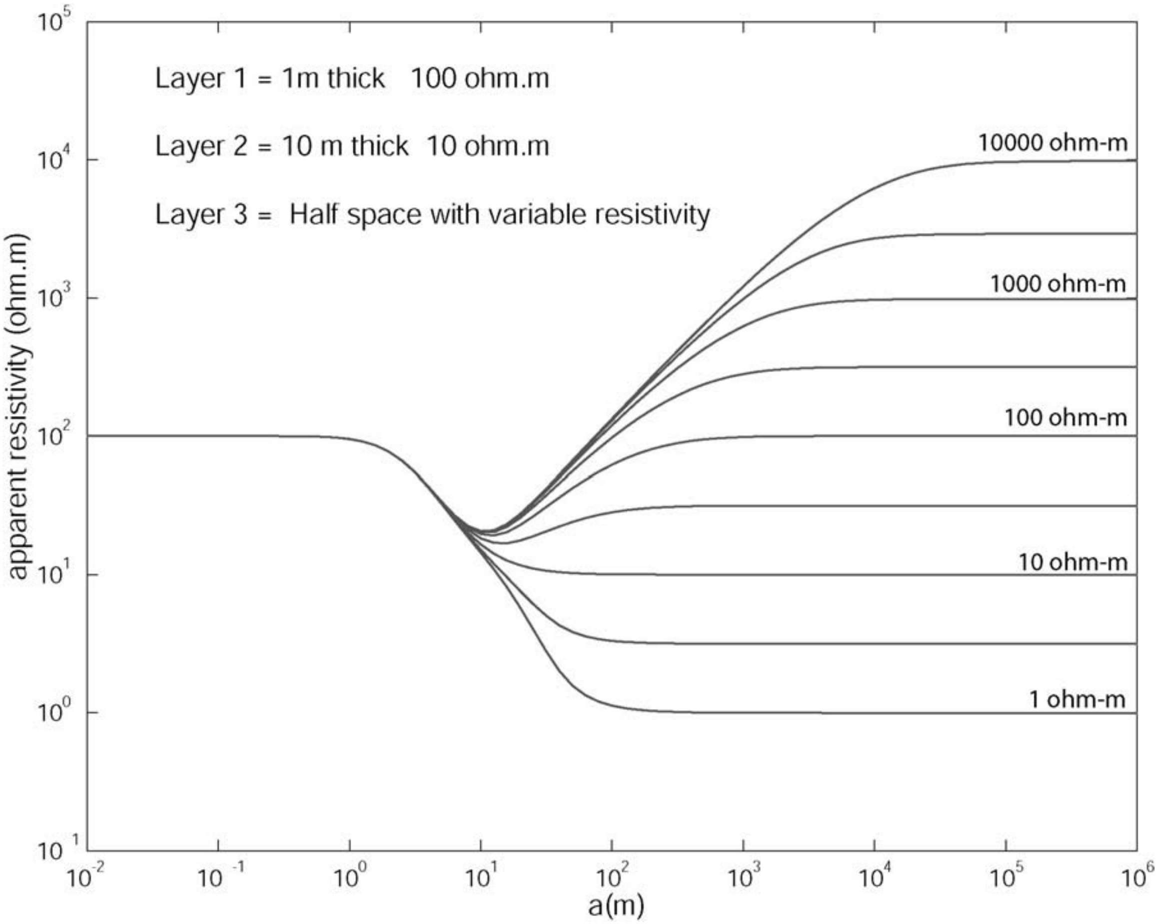
a = 1000m



B3.7 Quantitative solution for apparent resistivity (Wenner array)

- Figures below were generated using MATLAB script **wenner3lay.m**
- This uses a power series expansion to compute ρ_a as a function of *a*-spacing.

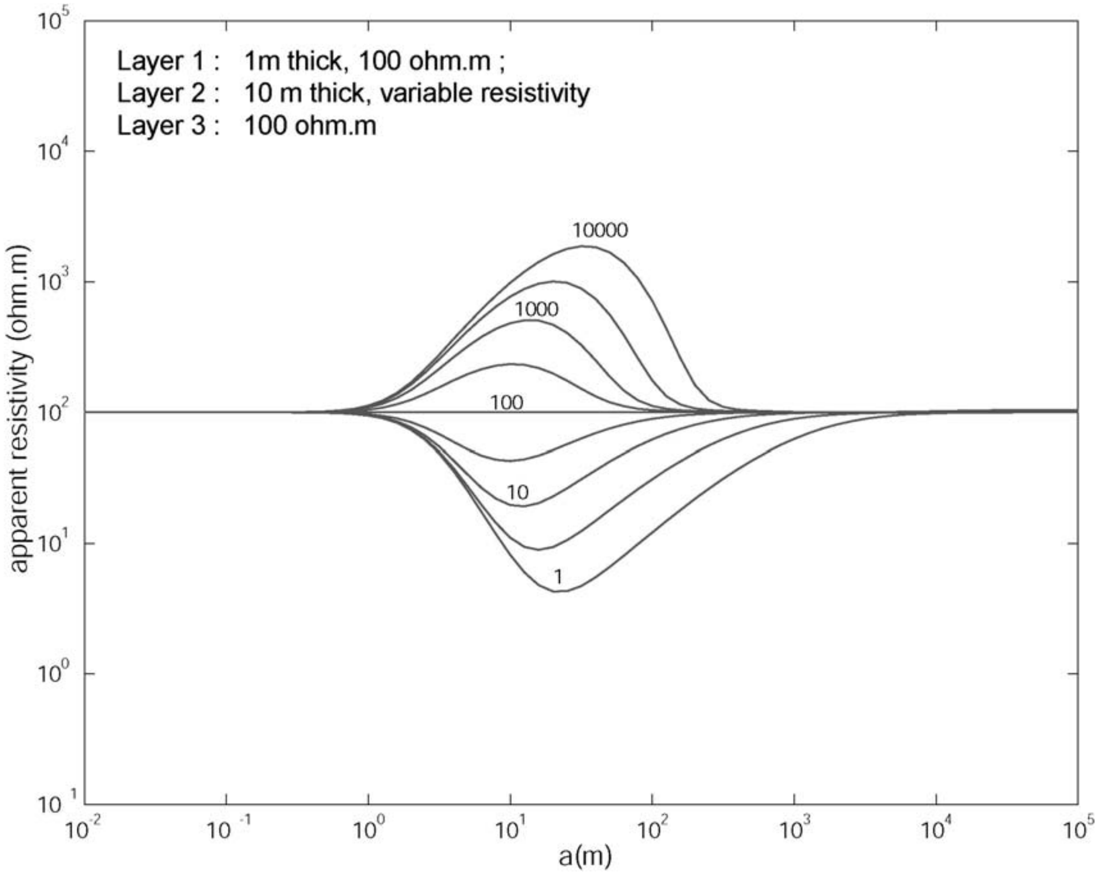
Example 1



Note

- All the apparent resistivity curves are identical until the third layer is detected with *a* ~10 m.
- At very large values of *a*-spacing, the apparent resistivity value is that of the lowest (3rd) layer. These very large values cannot be measured in practice.

Example 2



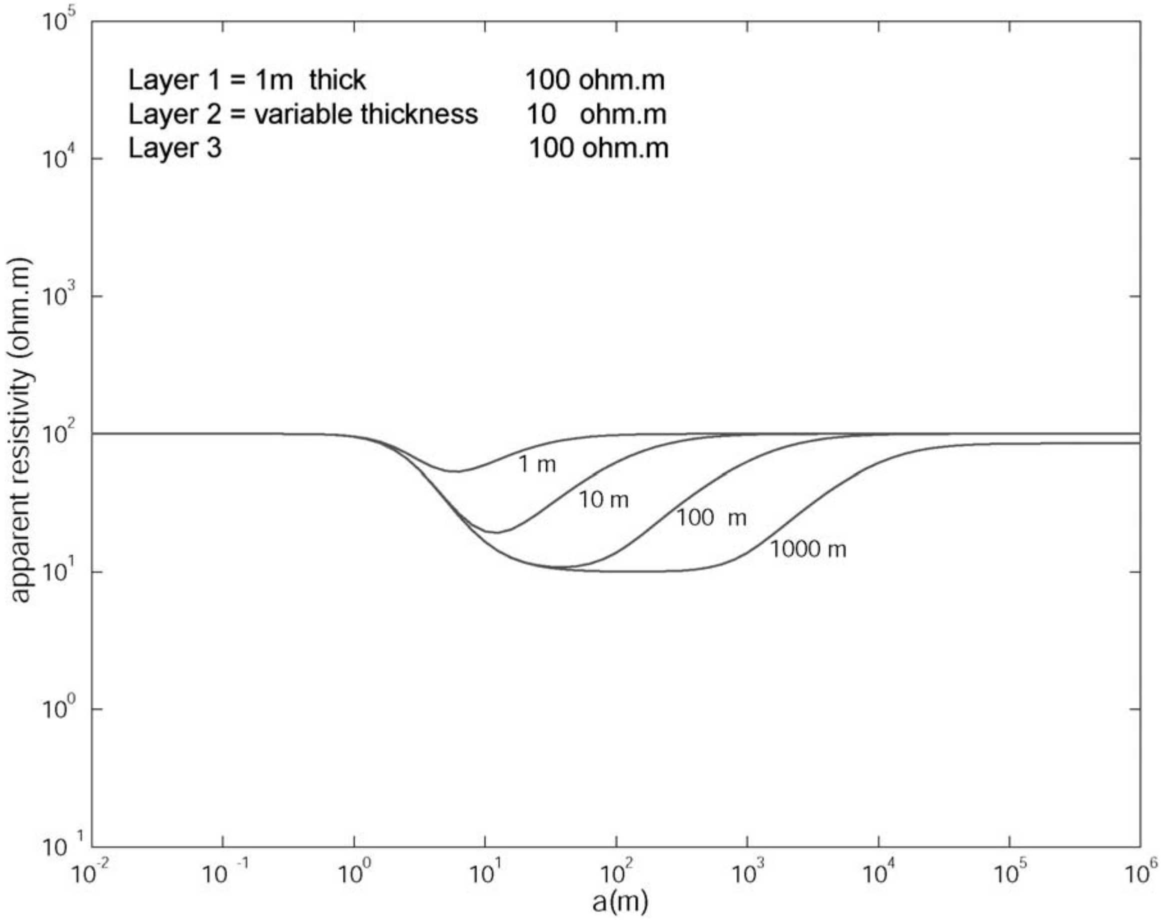
Note

- As the *a*-spacing increases and the middle layer is detected. If this layer is a conductor, then ρ_a falls quickly as *a*-spacing increases. This is because electric current flows preferentially in a conductor.

- Similarly, when the middle layer is a resistor, ρ_a increases more slowly as electric current tends to flow in the overlying conductor (Layer 1).

- Resistor and conductor are relative terms.

Example 3



Note

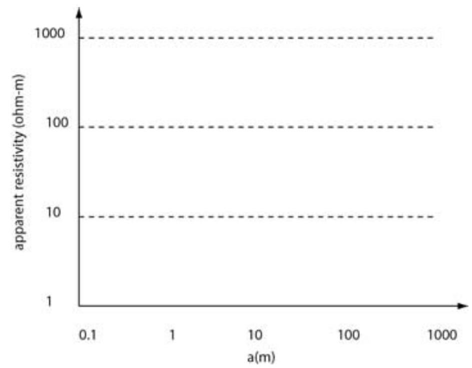
- When Layer 2 is thin, ρ_a never reaches the true value of ρ_2 since there is never a situation where all (or even a majority) of the electric current flows in Layer 2.

- When Layer 2 is very thick, ρ_a approaches the true value of $\rho_2 = 10\ \Omega m$

There are **four** possible types of 3-layer apparent resistivity curves

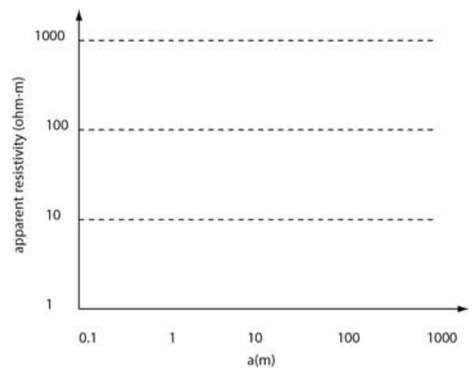
A-type $\rho_3 > \rho_2 > \rho_1$

e.g. ρ_1 =10Ωm; ρ_2 =100 Ωm; ρ_3 =1000 Ωm



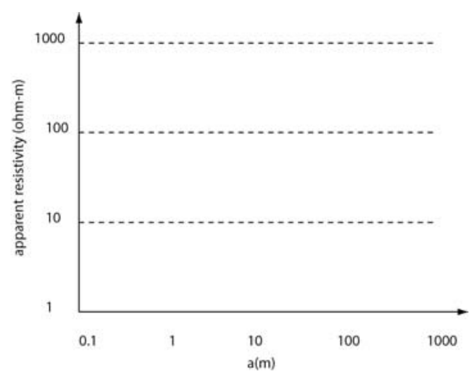
Q-type $\rho_3 < \rho_2 < \rho_1$

e.g. ρ_1 =100 Ωm; ρ_2 =10 Ωm; ρ_3 =1 Ωm



H-type $\rho_3 > \rho_2 < \rho_1$

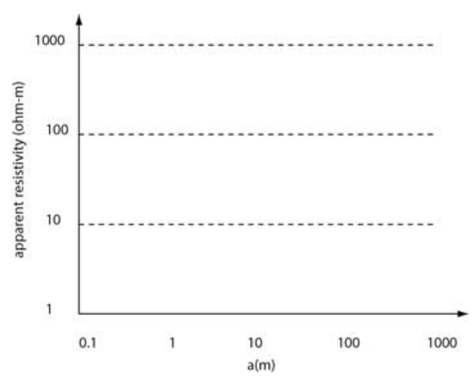
e.g. ρ_1 =100 Ωm; ρ_2 =10 Ωm; ρ_3 =1000 Ωm



K-type $\rho_3 < \rho_2 > \rho_1$

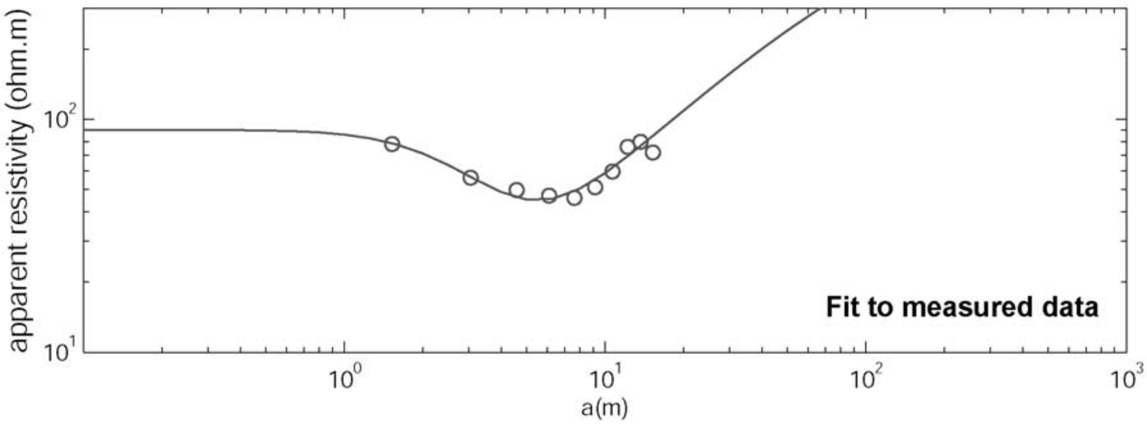
e.g. ρ_1 =100 Ωm; ρ_2 =1000 Ωm; ρ_3 =10 Ωm

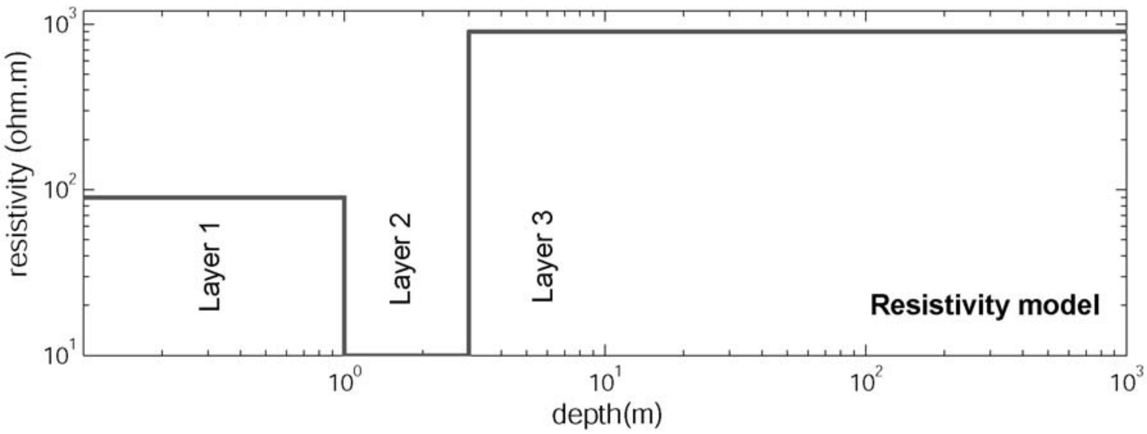
Sketch the apparent resistivity curves when h_1=3 m and h_2 = 10 m.



Question: Consider the A-type and Q-type resistivity curves. Would it always be obvious from these curves that there are 2 or 3 layers present?

B3.8 Fitting Wenner array data with a three-layer resistivity model





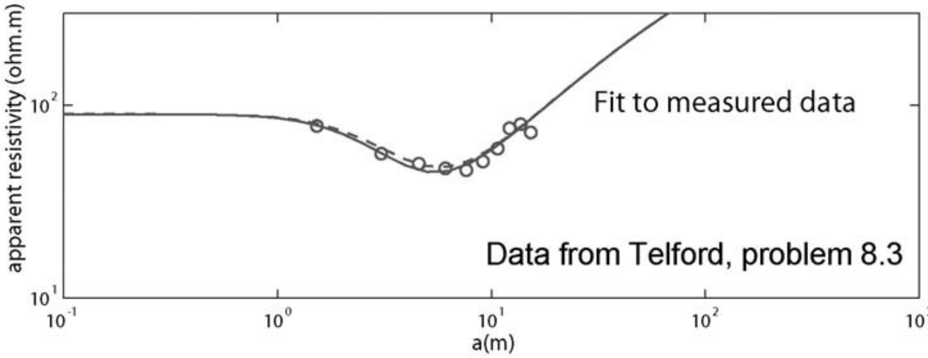
- Simple trial and error forward modeling (**wenner3lay_fit.m**) can be used to find a 3-layer model that will fit the Wenner array data plotted above. These data can be found in Telford Table 8.3.

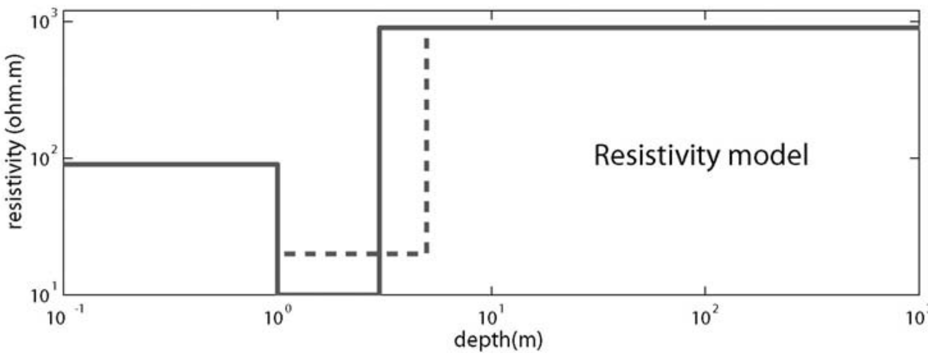
- Note that in this case, the range of *a*-spacing values are quite limited. We don't have an *a*-spacing that is small enough to sample just the upper layer (1), or a long enough spacing to sample just the lower layer (3).

- Master curves could also be used to interpret these Wenner array data. However, this is more complicated than for the two layer geometry considered in B3.4

- All possible combinations of the five model parameters [h_1, h_2, ρ_1, ρ_2, ρ_3] cannot be displayed on a single graph. Thus a book showing many master curves is needed and analysis can become time-consuming.

B3.9 Equivalence and non-uniqueness





Model 1: $h_2 = 2$ m and $\sigma_2 = 0.01$ S/m **Model 2**: $h_2 = 4$ m and $\sigma_2 = 0.02$ S/m

- Both models fit the same Wenner array data. This is an example of **non-uniqueness** in geophysical data interpretation.

- The models shown above have the same values of ρ_1, ρ_3 and h_1. However the values of ρ_2 and h_2 are different.

- Note that for both models, Layer 2 has the same ratio h_2 / ρ_2. This quantity is termed the **conductance**, C, and

  $$C = h_2 / \rho_2 = h_2 \sigma_2 = 2 \text{ x } 0.1 = 4 \text{ x } 0.05 = 0.02 \text{ Siemens (S)}$$

  where σ_2 is the electrical conductivity of the layer and h_2 is the layer thickness. DC resistivity exploration can determine the conductance of the layer reliably.

- However in this situation, DC resistivity cannot distinguish between various combinations of h_2 and σ_2 that give the same conductance. The two models that both fit the data are said to be **equivalent**.

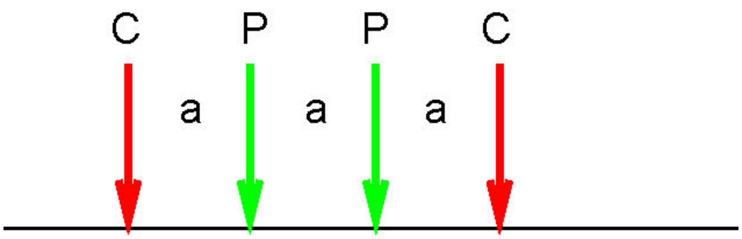
- Another problem with interpreting DC resistivity data can be that a thin layer may not resolved, especially when it's conductance is much less than adjacent layers. This phenomenon is called **suppression**.

- Also consider the case of the A and Q-type curves listed above. A model may have 3 layers, but the curve will not show this.

B3.10 Wenner and Schlumberger arrays

Two types of electrode array are routinely used in DC resistivity, as described below.

B3.10.1 Wenner array



$$\rho_a = 2\pi a \left(\frac{\Delta V}{I}\right)$$

- Vary the *a*-spacing to get resistivity as a function of depth
- *a*-spacing is increased logarithmically, with 5-7 values per decade.
- The maximum value of *a*-spacing should be at least 3-5 times the maximum depth of investigation.

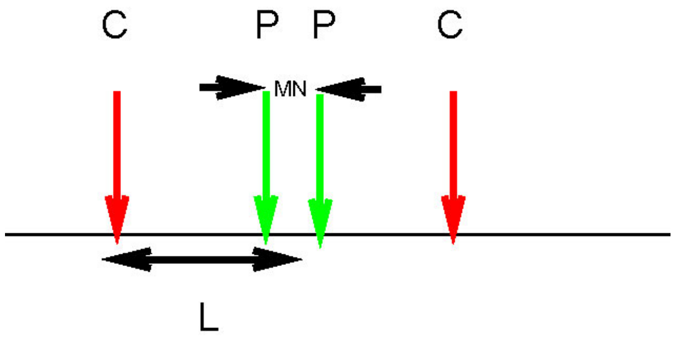
Advantages of Wenner array

- Simple to compute apparent resistivity

Disadvantages of Wenner array

- Sensitive to local variations in resistivity structure near the potential electrodes (*e.g.* rocks, wet soil)
- Need to move 4 electrodes for each measurement (slows down fieldwork)
- Long-wires for deep sounding (overcome by dipole-dipole method)

3.10.2 Schlumberger array



$$\rho_a \approx \frac{\pi L^2}{MN}\left(\frac{\Delta V}{I}\right)$$

Field procedure

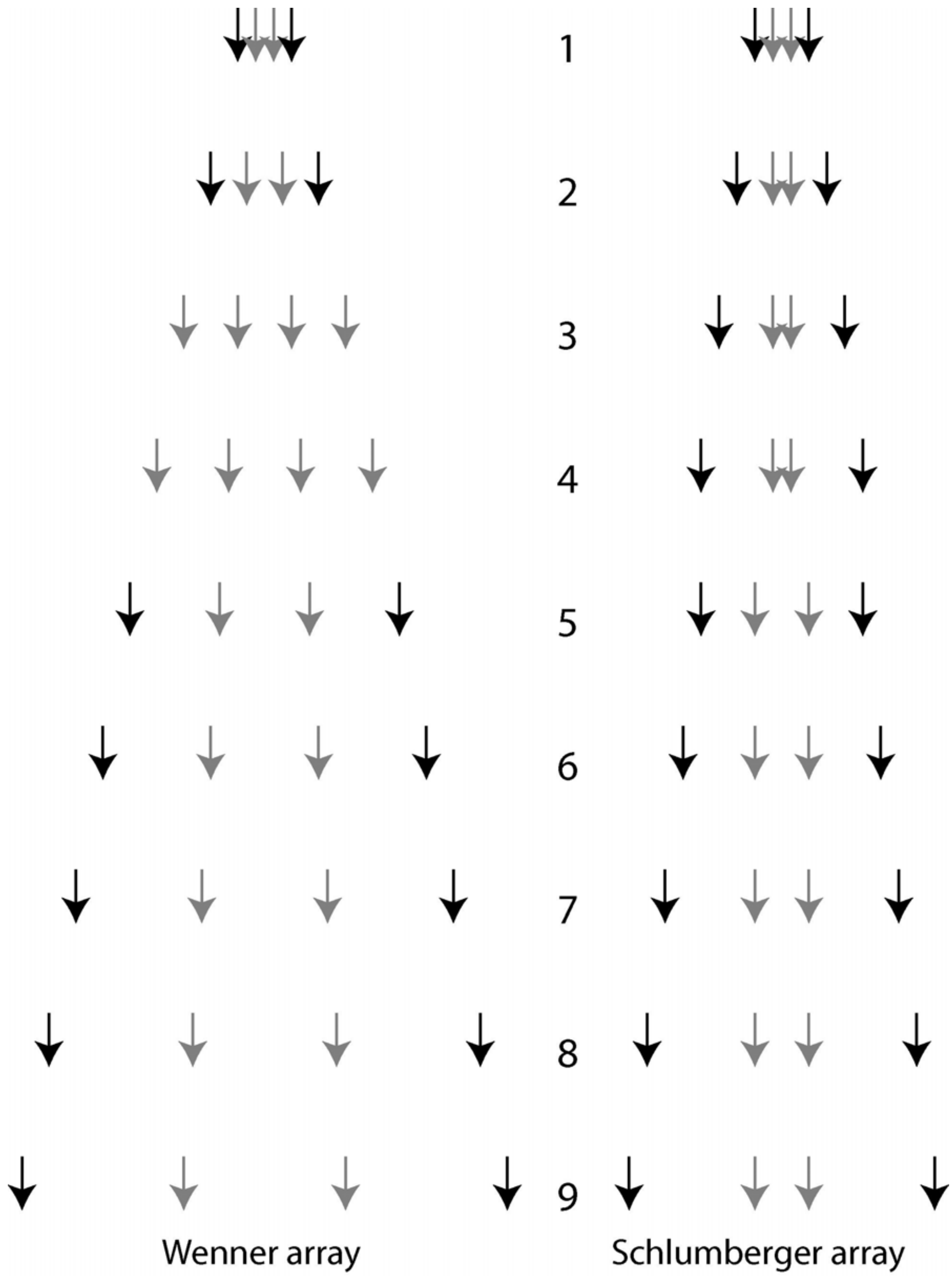
- This array keeps the potential electrodes (P) stationary while the current electrodes (C) are moved out.
- The array is symmetric with current electrodes a distance L from the centre.
- The potential electrodes are separated by a distance MN.
- As the current electrodes are moved out, ΔV becomes smaller and ultimately becomes too small to measure. At this point, the current electrodes are moved out and measurements continue.
- A more complicated formula is needed for apparent resistivity.
- *a*-spacing is increased logarithmically, with 5-7 values per decade.
- The maximum value of L in a Schlumberger array should be at least 3-5 times the maximum depth of investigation.

Advantages of Schlumberger array

- Only move **two electrodes** for each data point.
- Potential electrodes **moved less often** than in the Wenner array. This reduces the noise due to near surface heterogeneity.

Disadvantages of Schlumberger array

- Complications of computing the apparent resistivity
- Long-wires for deep sounding (overcome by dipole-dipole method)


1
2
3
4
5
6
7
8
9
Wenner array
Schlumberger array