## Geophysics 223 B1 Resistivity of rocks and minerals

## B1.1 Basic physics of electrical current flow

### B1.1.1 Simple resistor in circuit

Ohm's Law states that for a resistor, the resistance (in ohms), R is defined as

$R = \dfrac{V}{I}$  where $V$ = voltage (volts); $I$ = current flow (amps)

### B1.1.2 Resistivity and resistance

Ohm's Law as written above describes a resistor, which has no dimensions. In considering the flow of electric current in the Earth, we must consider the flow of electric current in a finite volume. Consider a cylinder of length $L$ and cross section $A$ that carries a current $I$



Current density = $J = \dfrac{I}{A}$

Resistance of cylinder, $R \propto \dfrac{L}{A} = \dfrac{\rho L}{A}$

where $\rho$ is the **electrical resistivity** of the material (ohm-m). This is the resistance per unit volume and is an inherent property of the material.

$\rho = \dfrac{RA}{L}$

If we were to examine two cylinders made of the same material, but with different dimensions, they would have the **same** electrical resistivity, but **different** electrical resistances.

Often it is more convenient to discuss the **conductivity** ($\sigma$) which is measured in Siemens per metre.

$\sigma = 1/\rho$

**B1.1.3  Electric current flow across a slab of material (optional)**

Consider an electric current (*I*) flowing through a slab of material with resistivity, $\rho$ and cross-sectional area, *A*
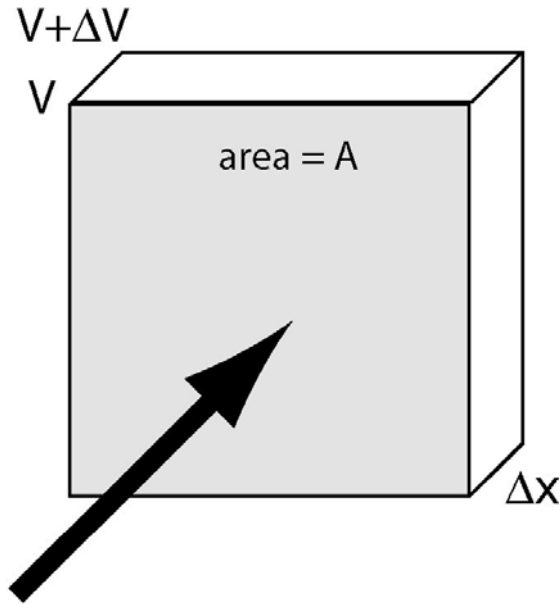


Applying Ohms Law

$$R = \frac{V}{I}$$

$$\frac{\rho \Delta x}{A} = \frac{\Delta V}{I}$$

Rearranging gives     $\dfrac{\Delta V}{\Delta x} = \dfrac{I\rho}{A}$

Taking limits     $\dfrac{dV}{dx} = E = \dfrac{I\rho}{A} = J\rho$

Thus Ohms Law for a continuous medium can be written as

$$J = \sigma E$$

where *E* is the electric field strength (Volts per m)
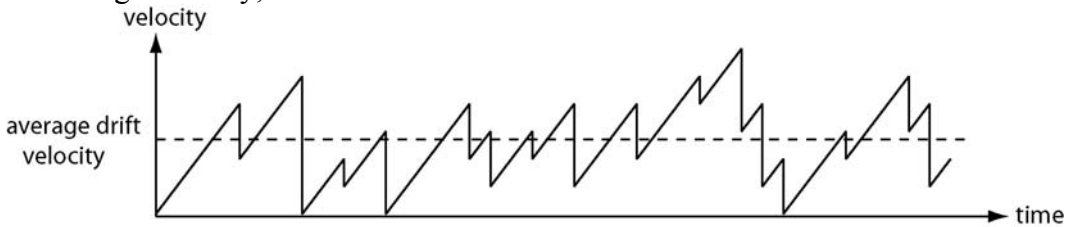
**B1.1.4 Charge carriers**
Electric current will flow through a medium as **charge carriers** move under an applied electric field (E). How is the resistivity (ρ) related to the number and type of charge carrier? Consider current flow through a cylinder of length $L$ and area $A$.



area, A
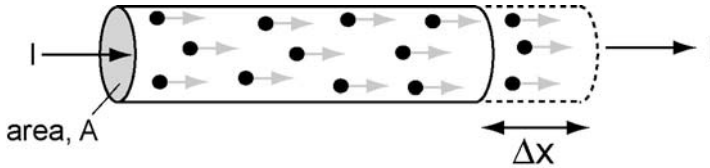
$n$ = number of charge carriers per unit volume   $q$ = the charge on each carrier

Consider one of the charge carriers. It will accelerate under the applied electric field until it strikes an atom or another charge carrier. Thus it will move through the material with an average velocity, v



The ease with which the charge carrier can move is described by the mobility, $\mu$, which is defined as the drift velocity per unit electric field = v/E

In a time $\Delta t$, the electric charges will move a distance $\Delta x$   = $v\Delta t$.

This corresponds to a volume of charge carriers   = $Av\Delta t$

The total charge leaving the cylinder is thus  $\Delta q$   = $nqAv\Delta t$

By definition, the current   $I = \dfrac{\Delta q}{\Delta t} = \dfrac{nqAv\Delta t}{\Delta t} = nqAv$

Thus current density,   $J = \dfrac{I}{A} = nqv = nq\mu E$

By comparison with Ohms Law, we see that   $\rho = \dfrac{1}{nq\mu} = \dfrac{1}{\sigma}$

**Summary** : A mineral will have a low electrical resistivity (high conductivity) if it has many, highly mobile, charge carriers.

If several types of charge carriers are present, then the contribution from each charge carrier must be summed.

## B1.2 Electrical resistivity of pure minerals

Several conduction mechanisms are possible in typical Earth materials.

### B1.2.1 Metals

- Charge carriers are electrons that are not firmly attached to atoms in the lattice.

- Both the number of charge carriers (*n*) and mobility (*μ*) are high.

- This gives a very low resistivity (e.g. copper $\rho < 10^{-8}$ ohm-m)

### B1.2.2 Semiconductors

- Semi conduction occurs in minerals such as sulphides and the charge carriers are electrons or ions.

- Compared to metals, the mobility (*μ*) and number of charge carriers (*n*) are lower, and thus the resistivity is higher (typically $10^{-3}$ to $10^{-5}$ ohm-m).

- This type of conduction occurs in igneous rocks and usually shows a temperature dependence of the form (thermally activated)

$$\rho \propto e^{\frac{E}{kT}}$$

  where *T* is the temperature in *K, E* is an activation energy and *k* is the Boltzmann constant.

- When a mineral is **molten**, ions can freely move and the resistivity decreases. See B1.6 below for more details.

### B1.2.3 Insulators

- In minerals such as diamond, there are very few charge carriers. To produce a charge carrier, a carbon atom would need to be removed from the crystal lattice. This requires a lot of effort and thus the mobility would be very low. As a consequence, the resistivity of pure diamond is very high ($\rho > 10^{10}$ Ωm)

- Carbon occurs in two forms, graphite and diamond. While diamond has a high resistivity (no charge carriers), graphite has a structure that allows electrons to easily move parallel to sheets of carbon atoms. This gives a very low resistivity ($\rho = 8 \times 10^{-6}$ Ωm)
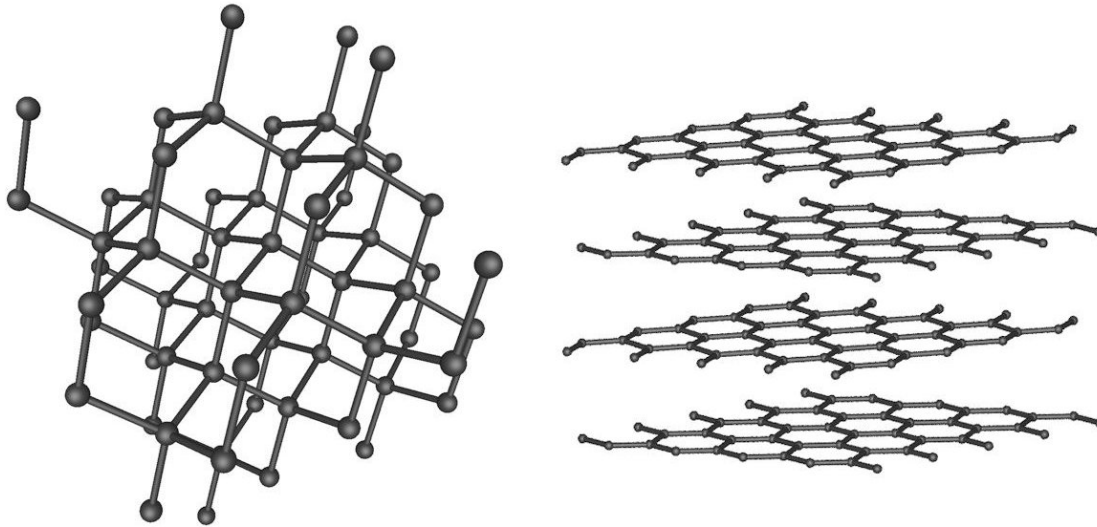
## B1.3 Electrical resistivity of mixtures

- Pure materials are rarely found in the Earth. Most rocks are mixtures of materials.

- Thus to calculate the overall resistivity, we must consider the resistivity of each component.

- A very common situation in the Earth is a rock consisting of grains (white) with pore space saturated with a liquid (green).
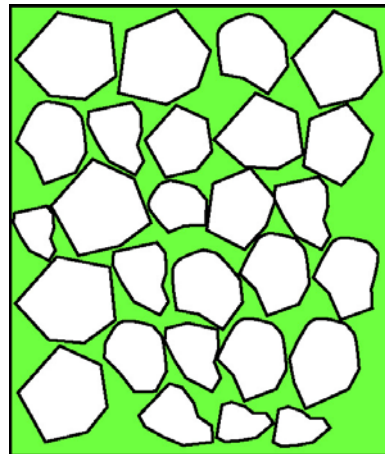


### B1.3.1  Ionic conduction in liquids

- Close to the surface, the fluid in the pore space is often water. If the water contains dissolved ions, then the resistivity of the water will be low because the ions can easily move.

- As the salinity of the brine increases, the resistivity decreases as more charge carriers become available and the resistivity decreases.

- Empirical studies (Block, 2001) show that the resistivity of brines ($\rho_w$) in the Alberta Basin varies as:

$\rho_{w} = 4.5\ TDS^{-0.85}$ where TDS is the amount of **total dissolved solids** in g/litre.

- For reference, seawater has TDS = 30 g/litre (as written as 30,000 ppm chloride)
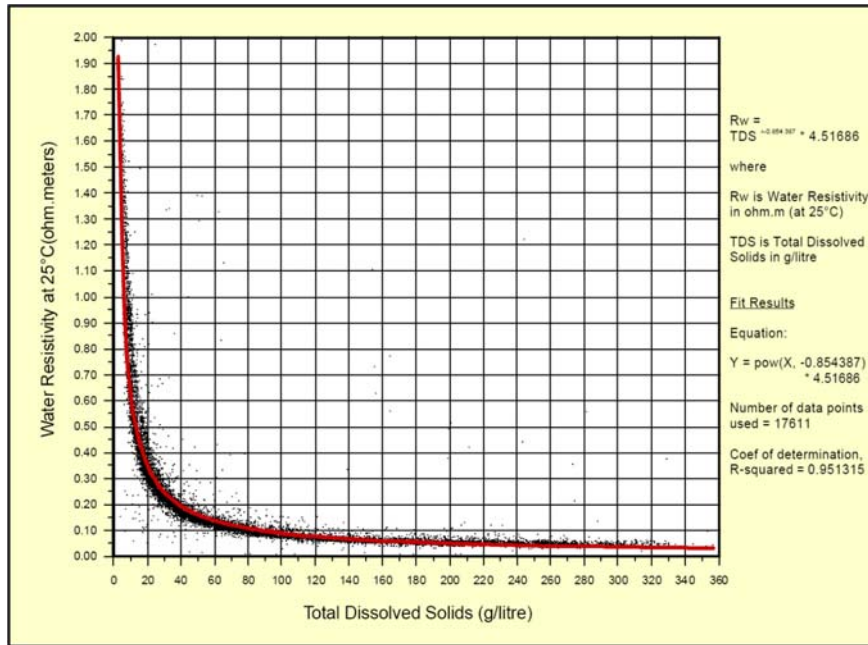


- Note also that the resistivity of the brine varies with temperature, as shown by the figure from Ussher et al (2000).

- What two processes control this behaviour?

- This temperature variation is important in deep hydrocarbon reservoirs and geothermal exploration.
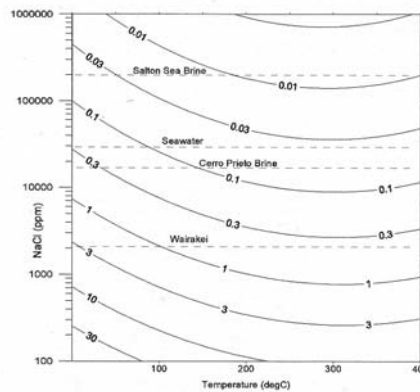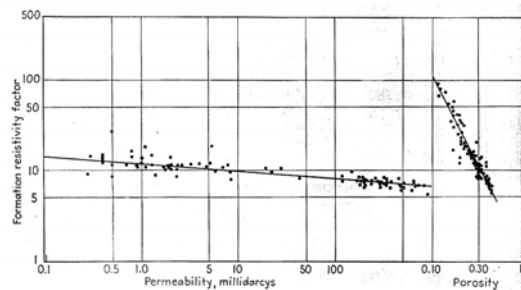
### B1.3.2 Archie's Law

- The fraction of the rock occupied by the fluid is called the porosity ($\Phi$). Assume that the resistivity of the fluid ($\rho_w$) in the pores is much lower than that of the rock grains.

- Archie (1942) discovered an **empirical relationship** for the resistivity of a completely saturated whole rock ($\rho_o$) is given by

$$\frac{\rho_o}{\rho_w} = F = \phi^{-m}$$

- where F is called the **formation factor**.

- On a log-log plot of $\rho_o$ as a function of $\Phi$, a straight line should result with slope – m. This exponent *m* termed the **cementation factor**. Typical values include: 1.8-2.0 for consolidated sandstones to 1.3 for unconsolidated sands.
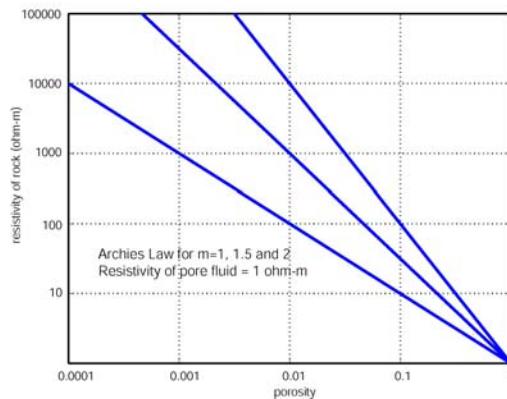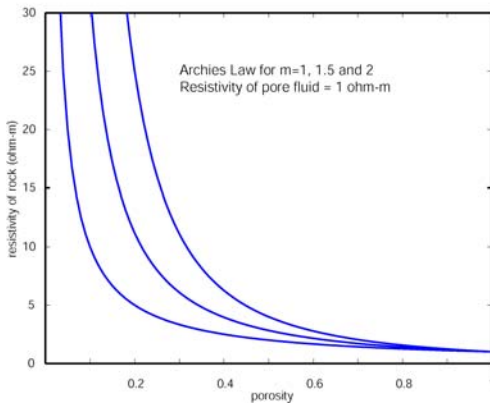
- The graph on the right is taken from Archie (1942) for Nacatoch sand from Lousiana. What is the value of *m* for this set of samples?
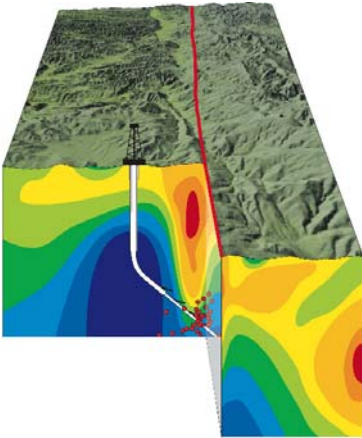


- What is the difference between permeability and porosity?

- Why are they correlated?

- The following plots show theoretical results when $\rho_w = 1$ $\Omega$m.

**Example of using Archie's law**



- Magnetotelluric exploration of the San Andreas fault at Parkfield has revealed that the fault is characterized by a low-resistivity wedge. This has been interpreted a zone of breccia, often termed the damage zone (Unsworth et al, 1997).

- In this area the groundwater is very saline with $\rho_w = 0.26$ Ωm.

- What porosity is required to explain a bulk resistivity of 3 Ωm (orange zone)? Consider the possible values of m, and assume the rock is saturated.

**Physical interpretation of the cementation factor, m**
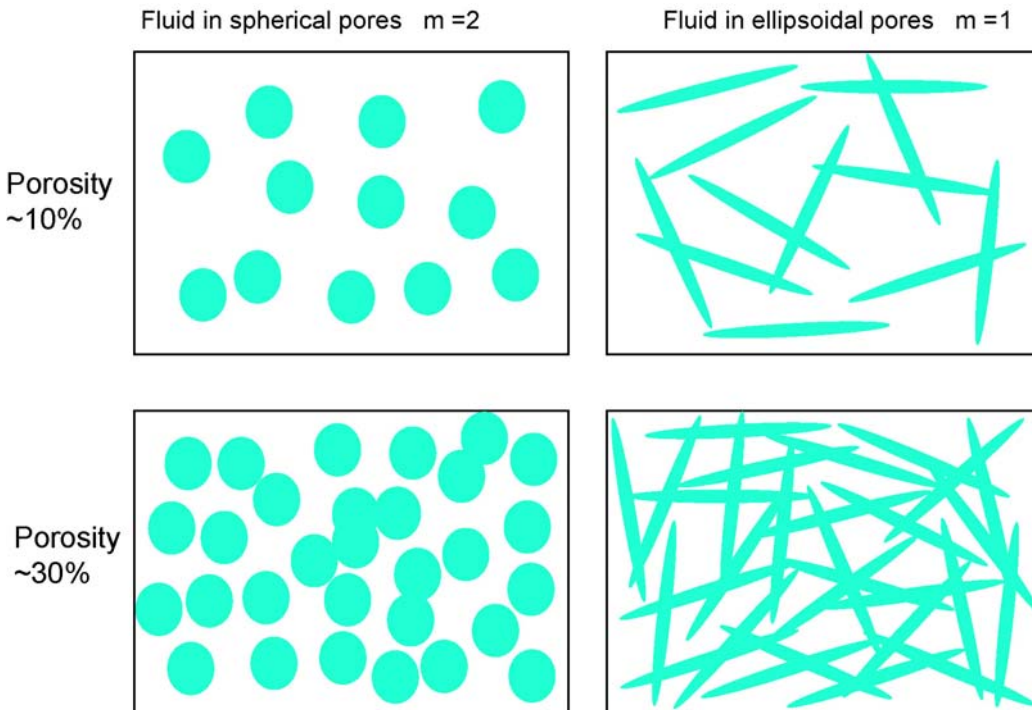


- Note that the elongated pores will connect to form an interconnected electrical network at a lower porosity than the spherical pores.

- Is the permeability of the two cases different?

**B1.3.3 Partial saturation**

The analysis above assumes that all the pore space is filled with the low resistivity water. Archie's study was motivated by the application to hydrocarbon reservoirs, and thus partial saturation must be considered.

Typically oil and gas in the pore spaces will be resistive.

Additional experiments by Archie showed that the bulk resistivity ($\rho$) is given by

$$\frac{\rho}{\rho_o} = S^{-n}$$

where S is the saturation, defined as the fraction of pore space filled with fluid and *n* is (another) empirical constant.
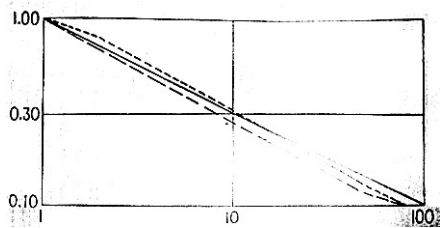


What value of *n* is implied in the figure on the left that shows S (vertical axis) as a function of $\rho/\rho_o$ (horizontal axis)?

Figure 3, Archie (1942)

Combining the above equations gives the general form of Archie's Law

$$\rho = \rho_w S^{-n} \phi^{-m}$$

Archie's Law can also be extended to include the case where some conduction occurs through the solid phase (Ussher et al, 2000; Glover et al, 2000)

**B1.3.4 Influence of fluid distribution on bulk resistivity**

To emphasize how fluid distribution controls the bulk resistivity, consider two rock samples that both have $\Phi = 0.1$
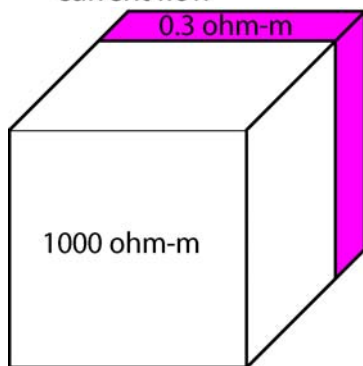
In the first example the rock (1000 $\Omega$m) and fluid (0.3 $\Omega$m) form a series electrical circuit.

In the second case, the rock and fluid form a parallel circuit.

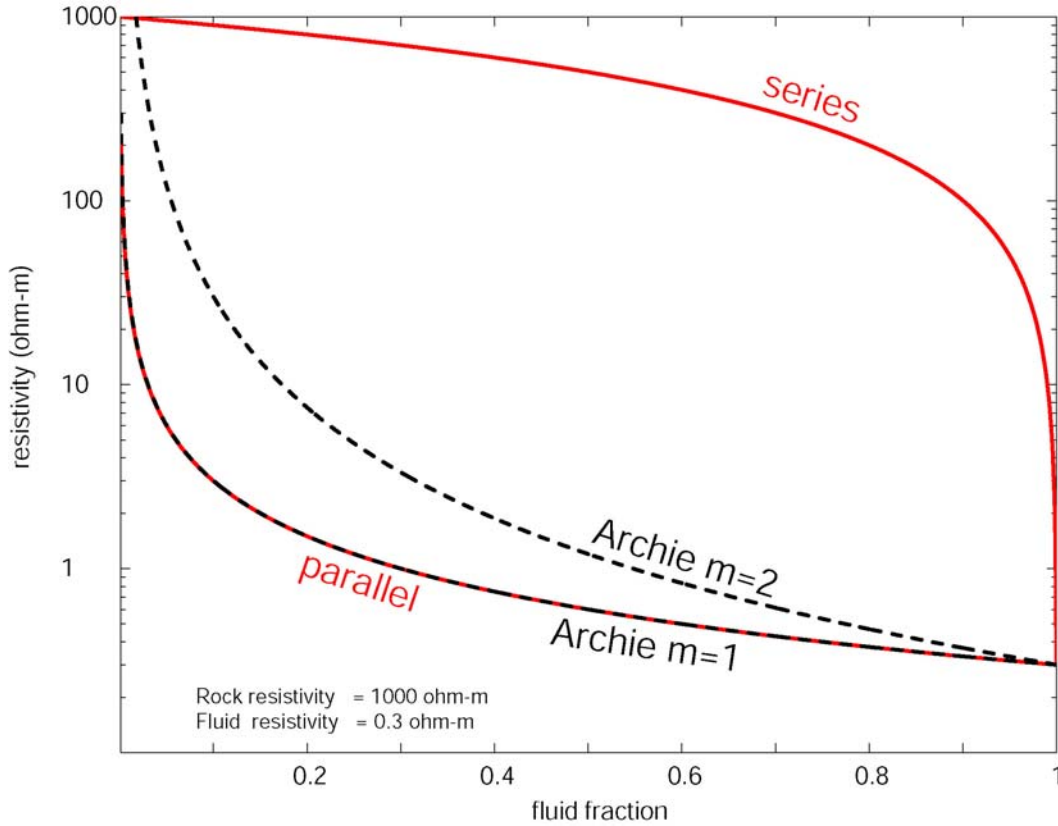What is the overall electrical resistivity of each case?
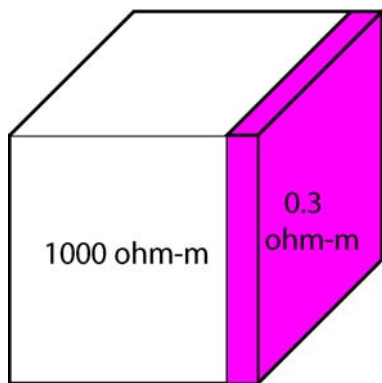


current flow



current flow

This illustrates that the **anisotropy** of rock can be an important factor in determining the overall resistivity.

The results are sketched below for the case of $\rho_{rock} = 1000$ ohm-m and $\rho_f = 0.3$ ohm.

## C1.4 Clay minerals

If a rock contains clay minerals, then an extra conduction pathway is possible via the electrical double layer that forms at the interface of the clay mineral and the water. This effectively allows ions to move through the system with a lower effective viscosity than in the liquid phase.
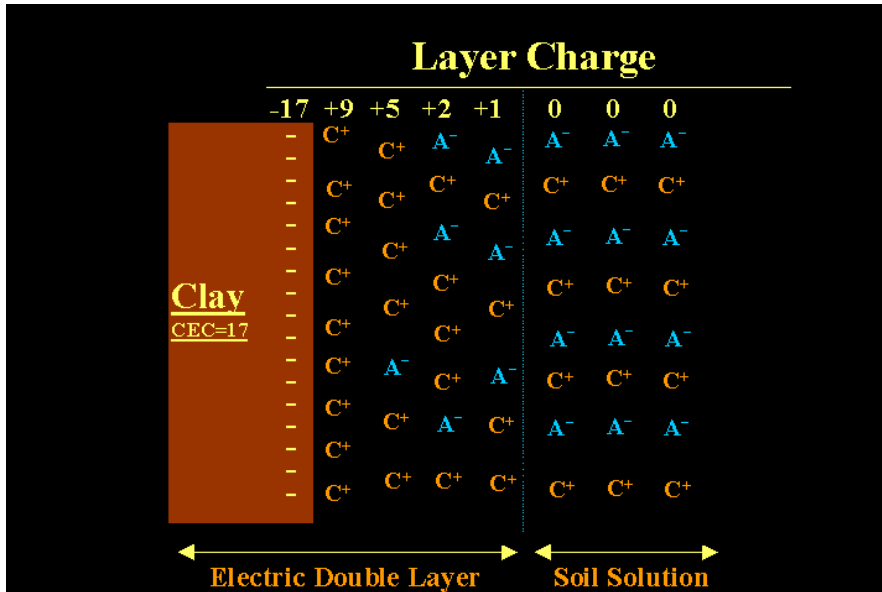
Waxman and Smits (1968) developed an equation for the resistivity due to both conduction through the both the liquid and the double layer

$$\rho = \frac{F}{(BQ_v + \dfrac{1}{\rho_w})}$$

where B is the **equivalent conductance** of the ions in solution and F is the **formation factor** and

$$Q_v = CEC(1-\phi)\frac{\rho_m}{\phi}$$

with $\rho_m$ the matrix grain density and CEC the **cation exchange capacity** of the clay. Thus the term $BQ_v$ is a measure of how much the clay contributes to conduction. Values of CEC vary from one type of clay to another. For example: smectite, CEC = 120; illite, CEC = 20. Note that smectite occurs at shallower depths (lower temperatures) than illite in geothermal fields. This is conspicuous as a low resistivity layer in many geothermal fields (Ussher et al, 2000).



http://faculty.plattsburgh.edu/robert.fuller/370%20Files/Week5Ion%20Exchange/Electricdoublelayer.htm

- Some cations are held tightly right next to the surface
- Others held more loosely further away
- Anion exclusion zone next to surface
- Soluble cations are reached only when concentrations of cations and anions are equal

More information at:
http://www.cartage.org.lb/en/themes/sciences/Chemistry/Electrochemis/Electrochemical/ElectricalDouble/ElectricalDouble.htm

http://www.zeta-meter.com/5min.pdf

## B1.5 Other causes of high electrical conductivity

In the upper 10 km, aqueous pore fluids dominate the resistivity of most rocks. However, other materials, even in small amounts, can dramatically lower the resistivity of a rock. These include:

**Graphite films:** Proposed as an explanation of why the lower crust is unusually high in conductivity (Frost et al., 2000). This requires interconnection of the films, which sometimes is difficult to maintain over geologically long time frames. See debate by Yardley and Valley (1997) and Wannamaker (2000).

**Iron oxides and metallic sulphides:** Usually localized in a discrete body, and allows easy detection of these deposits with airborne EM methods (see later in this class).

**Partial melting:** Rocks are complex mineral assemblages, so they don't melt at one temperature. In a partial melt, molten rock collects along the grain boundaries of grains that melt at higher temperature.
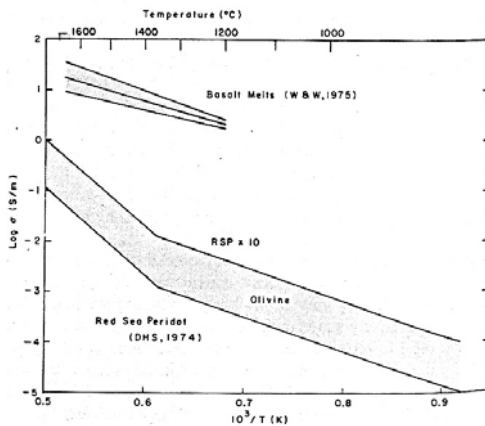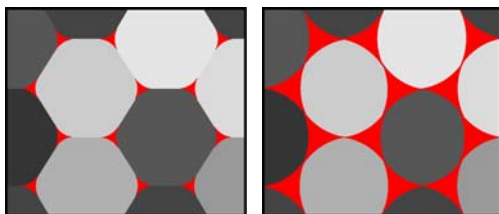


Typical resistivities of pure melt from Shankland and Waff (1977). These values are for dry rocks.

Wet rocks melt at lower temperatures and the water generally raises the conductivity.

The dihedral (wetting) angle determines if the melt will form isolated pockets or interconnected tubes along grain boundaries.
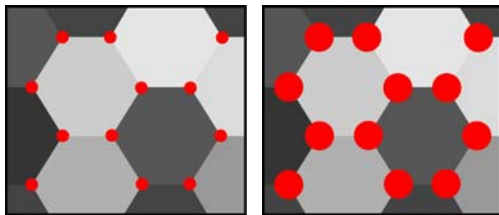


High dihedral (wetting) angle



Low dihedral (wetting) angle

1% melt          5% melt

Laboratory experiments suggest that partial melts seem to interconnect at low melt fractions, with a relatively low dihedral angle. Thus Archies Law can be used with m=1.3 (ten Grotenhuis et al., 2005).

Partial melting is believed to be widespread in the asthenosphere and beneath geothermal fields and volcanos. Partial melting may also occur in tectonically active areas (Tibet, Altiplano) where MT surveys reveal a zone of low resistivity in the crust.
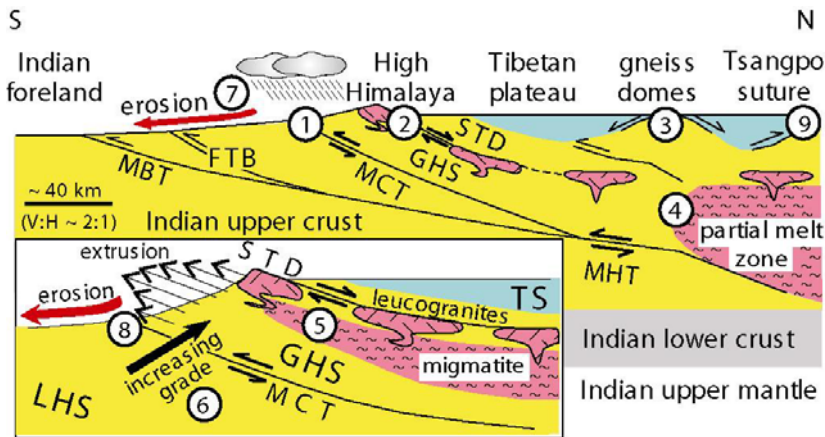


The MT surveys image a layer around 3 Ωm. Recent laboratory studies have confirmed that these resistivities are consistent with a wet, granitic melt (Gaillard et al, 2004). Tectonic processes and erosion extrude these rocks in the Himalaya, some 10-12 million years later.
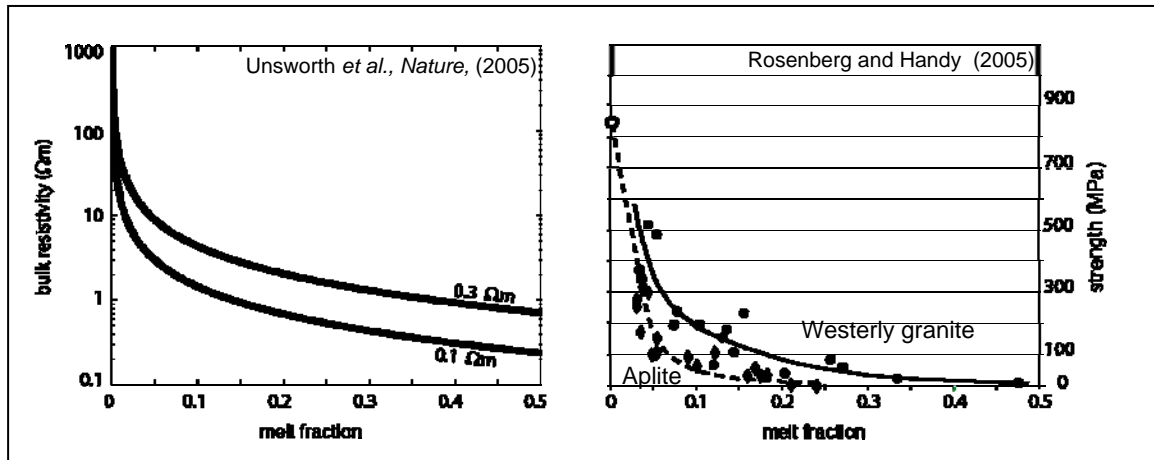
## B1.6 Summary of resistivity of rocks
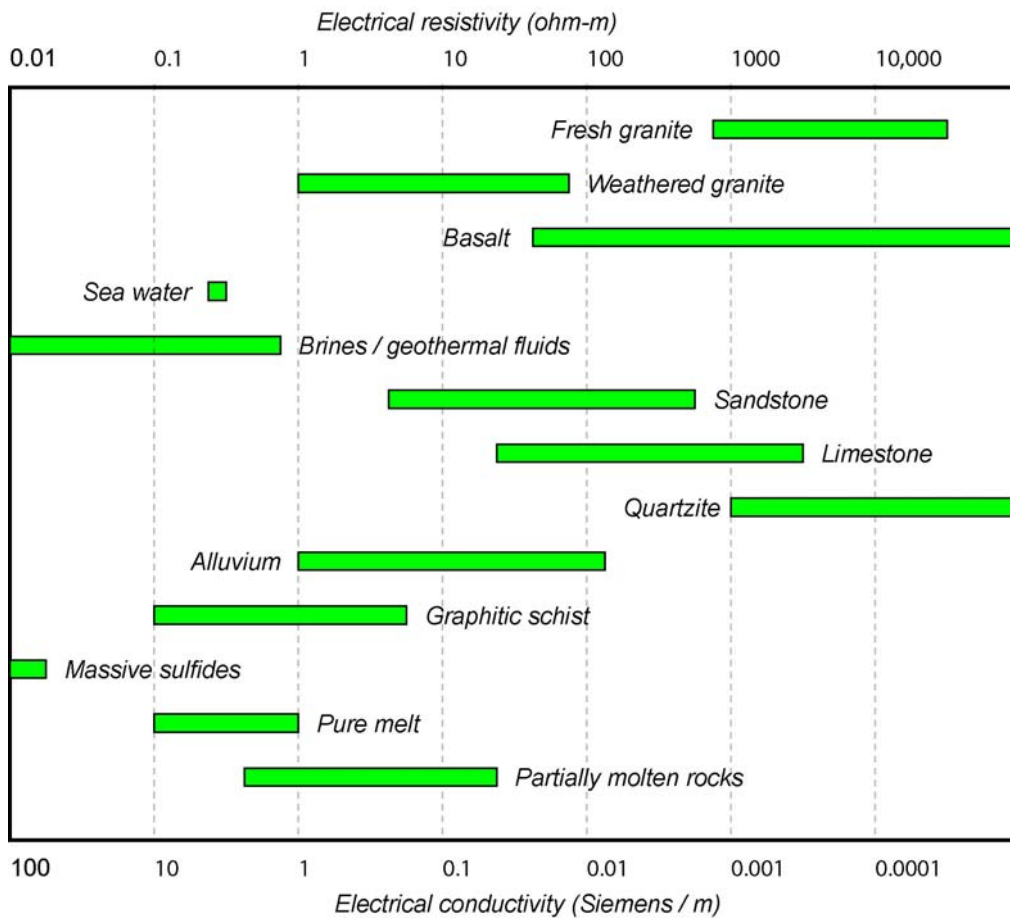


Factors that will **DECREASE** the resistivity of a rock:

(a) Add more pore fluid
(b) Increase the salinity of the pore fluid - more ions to conduct electricity
(c) Fracture rock to create extra pathways for current flow
(d) Add clay minerals
(e) Keep fluid content constant, but improve interconnection between pores

Factors that will **INCREASE** the resistivity of a rock

(a) Remove pore fluid
(b) Lower salinity of pore fluid
(c) Compaction - less pathways for electric current flow
(d) Lithification - block pores by deposition of minerals
(e) Keep fluid content constant, but decrease connection between pores

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Yoshino, T., T. Matsuzaki, S. Yamashita, T. Katsura, Hydrous olivine unable to account for conductivity anomaly at the top of the asthenosphere, *Nature*, 443, 973-976, 2006.