

Geophysics 210 - 2008 Final exam - formula sheet

B : Gravity

Newtonian gravitation

Acceleration of gravity (g) due to a point mass (m) a distance (r) $g = \frac{Gm}{r^2}$

Gravity anomalies of sphere and cylinder

a = radius of sphere or cylinder $\Delta\rho$ = density contrast
 z = depth of the sphere or cylinder x = horizontal distance from sphere/cylinder
 $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ 1 milligal = 10^{-5} ms^{-2}
 M_S = excess mass $x_{1/2}$ = half width

Cylinder $g_z = \frac{2G\pi a^2 z \Delta\rho}{(x^2 + z^2)}$ $x_{1/2} = z$

Sphere $g_z = \frac{GM_S z}{(x^2 + z^2)^{3/2}}$ $x_{1/2} = 0.766z$

Free air correction

Change in g when elevation changes by Δh (m) $\Delta g_z = 0.3086 \Delta h$ (milligals)

Gravitational attraction of a layer / Bouguer correction

Gravitational attraction of slab with thickness Δz (m) and density contrast $\Delta\rho$ (kg m^{-3})

$$\Delta g_z = 2\pi G \Delta z \Delta\rho \quad (\text{ms}^{-2})$$

Solar system data

Polar radius of the Earth	6356.7 km
Equatorial radius of the Earth	6378.1 km
Gravitational constant, G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Mass of Earth, M_E	$5.97 \times 10^{24} \text{ kg}$
Mass of Moon, M_M	$7.36 \times 10^{22} \text{ kg}$
Mass of Sun, M_S	$1.99 \times 10^{30} \text{ kg}$
Earth-Moon distance	$3.84 \times 10^5 \text{ km}$
Earth-Sun distance	$1.50 \times 10^8 \text{ km}$

Isostasy

Airy hypothesis	$r = h \rho_c / (\rho_m - \rho_c)$
ρ_c = crustal density	ρ_m = mantle density
r = root depth	h = mountain height

C: Seismology

v = velocity of seismic wave (m/s)
 x = source-receiver offset (m)
 f = frequency of seismic wave (Hz)
 $v = x / t$
 Seismic impedance, $Z = \rho v$

t = travel time (s)
 ρ = density (kg m^{-3})
 λ = wavelength (m)
 Seismic velocity, $v = f\lambda$

Reflection and refraction

Seismic wave travelling from layer 1 to layer 2, at normal incidence

$$\text{Amplitude reflection coefficient, } R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\text{Amplitude transmission coefficient, } T = \frac{2Z_1}{Z_2 + Z_1}$$

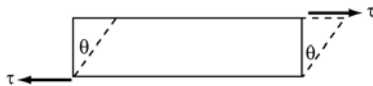
Snells Law

$$\frac{\sin \theta_i}{v_1} = \frac{\sin \theta_t}{v_2} = \frac{\sin \theta_r}{v_1}$$

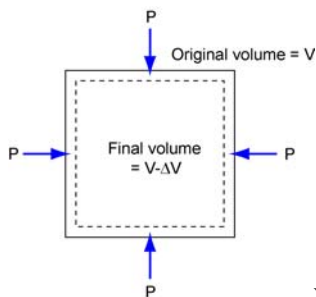
Seismic wave in layer 1 is incident on interface at angle θ_i
 Reflected at angle θ_r in layer 1; Transmitted at angle θ_t in layer 2

Stress, strain

K = bulk modulus; μ = shear modulus; ρ = density



τ = shear stress; $\tan \theta$ = shear strain; $\mu = \tau / \tan \theta$



volume stress = P ; volume strain = $(\Delta V/V)$; $K = P / (\Delta V/V)$

Seismic wave velocities

v_p = P-wave velocity v_s = S-wave velocity

$$v_p = \left[\frac{K + \frac{4}{3}\mu}{\rho} \right]^{\frac{1}{2}} \quad v_s = \left[\frac{\mu}{\rho} \right]^{\frac{1}{2}}$$

Earthquake magnitude

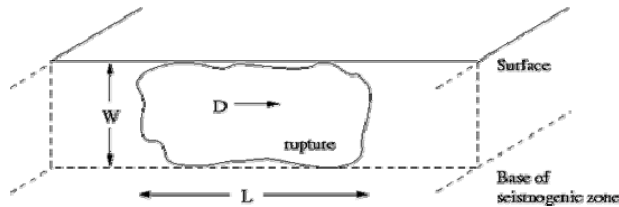
(1) **Richter scale**
$$M_S = \log_{10} \left(\frac{A_S}{T} \right) + 1.66 \log_{10} \Delta + 3.3$$

A_S = amplitude of **surface wave** motion in microns

T = period of surface wave (around 20 seconds)

Δ = distance to earthquake epicentre (degrees)

(2) Moment magnitude



Seismic moment = $M_0 = \mu DA = \mu DLW$ where μ = shear modulus

Moment magnitude as $M_W = \frac{2}{3} \log_{10} M_0 - 6$

D: Geomagnetism

Dipolar magnetic field

M = dipole moment

$$B_r = \frac{2M\mu_0 \sin \theta}{4\pi r^3} \text{ and } B_\theta = \frac{M\mu_0 \cos \theta}{4\pi r^3}$$

$$\tan I = 2 \tan \theta$$

μ_0 = magnetic permeability = $4\pi \cdot 10^{-7}$ H/m

$$F(r, \theta) = \sqrt{B_r^2 + B_\theta^2} = \frac{M\mu_0 \sqrt{3 \sin^2 \theta + 1}}{4\pi r^3}$$

