Detection and Isolation of Model-Plant-Mismatch for Multivariate Dynamic Systems

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Outline

- Motivation & Literature Review
- Problem Formulation
- MPM Detection and Isolation
- Numerical Example
- Conclusion
- Acknowledgement
Motivation
&
Literature Review
Motivation

- In industry, model-plant-mismatch (MPM) is a common and challenging problem for existing model-based control, e.g. MPC.

- In academia, MPM is still a difficult problem in the research area of fault detection and isolation (FDI).
Literature Review on FDI

Process Performance

- Poor Controller Tuning
  - Sensor or Actuator Faults

- Faults
  - Process Additive Faults
    - Stoustrup & Nieman, 1999
  - Parametric Faults
    - Li & Jiang, 2004

- Process Nonlinearity

- Process Uncertainty
  - Inaccurate Model (MPM)
  - Unknown Disturbance

Q: Can we develop a scheme to detect and isolate MPM?
Problem Formulation
Problem Formulation

- Consider a LTI Discrete Time (DT) system:

  \[
  \begin{align*}
  x(k + 1) &= A_0 x(k) + B_0 u(k) + p(k) \\
  y(k) &= C_0 x(k) + o(k)
  \end{align*}
  \]

- MPM effect:
  
  Can we detect the mismatch?  
  Can we isolate the matrices that have mismatch?  

  \[
  \begin{align*}
  A &= A_0 + \Delta A \in R^{n \times n} \\
  B &= B_0 + \Delta B \in R^{n \times l} \\
  C &= C_0 + \Delta C \in R^{m \times n}
  \end{align*}
  \]

- A DT system with MPM:

  \[
  \begin{align*}
  x(k + 1) &= (A_0 + \Delta A) x(k) + (B_0 + \Delta B) u(k) + p(k) \\
  y(k) &= (C_0 + \Delta C) x(k) + o(k)
  \end{align*}
  \]
Problem Formulation (con’t)

- Rewrite the MPM effect:

\[ a(k) = \Delta A x(k), \quad b(k) = \Delta B u(k), \quad c(k) = \Delta C x(k) \]

- Rewrite the system with MPM:

\[
\begin{align*}
    x(k + 1) &= A_0 x(k) + B_0 u(k) + a(k) + b(k) + p(k) \\
    y(k) &= C_0 x(k) + c(k) + o(k)
\end{align*}
\]

- Can we indicate \( a(k) \neq 0 \Rightarrow \Delta A \neq 0 \), \( b(k) \neq 0 \Rightarrow \Delta B \neq 0 \), \( c(k) \neq 0 \Rightarrow \Delta C \neq 0 \)?

Can we indicate \( a(k), b(k) \) or \( c(k) \) are non-zero?
MPM Detection and Isolation
By stacking the equation for the system with MPM

\[ y_s(k) - H_s^o u_s(k) = \Gamma_s^o x(k - s) + c_s(k) + o_s(k) + G_s^o a_s(k) + b_s(k) + p_s(k) \]

\[ y_s(k) = \begin{bmatrix} y(k - s) \\
                        y(k - s + 1) \\
                        \vdots \\
                        y(k) \end{bmatrix} \in \mathbb{R}^{m_s} \]

\[ \Gamma_s^o = \begin{bmatrix} C_0 \\
                                C_0A_0 \\
                                 \vdots \\
                                C_0A_0^{s-1} \end{bmatrix} \in \mathbb{R}^{m_s \times n} \]

\[ m_s = m(s + 1) \]

\[ H_s^o = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\
                           C_0B_0 & \cdots & \cdots & \cdots & 0 \\
                           C_0A_0B_0 & C_0B_0 & \cdots & 0 & 0 \\
                           \vdots & \vdots & \ddots & \vdots & \vdots \\
                           C_0A_0^{s-1}B_0 & C_0A_0^{s-2}B_0 & \cdots & C_0B_0 \end{bmatrix} \]

\[ G_s^o = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\
                             C_0 & 0 & \cdots & \cdots & \cdots \\
                             C_0A_0 & C_0 & \cdots & 0 & 0 \\
                             \vdots & \vdots & \ddots & \vdots & \vdots \\
                             C_0A_0^{s-1} & C_0A_0^{s-2} & \cdots & C_0 \end{bmatrix} \]
MPM Detection (con’t)

- By stacking the equation for the system with MPM

\[ y_s(k) - H_s^o u_s(k) = \Gamma_s^o \xi(k-s) + c_s(k) + o_s(k) + G_s^o [a_s(k) + b_s(k) + p_s(k)] \]

\[ W_o \Gamma_s^o = 0 \]

\[ \Gamma_s^o \in \mathbb{R}^{m_s \times n} \Rightarrow W_o \in \mathbb{R}^{(m_s-n) \times m_s} \]

\[ e_s(k) \equiv W_o [y_s(k) - H_s^o u_s(k)] \]

\[ = W_o [o_s(k) + G_s^o p_s(k)] + W_o [c_s(k) + G_s^o [a_s(k) + b_s(k)]] \]

- Zero-mean, Gaussian distributed random noise vector

- Non-zero vector, if there is MPM
PRV Properties

- If we assume the covariance of $p(k) \& o(k)$ as

$$R_p \in \mathbb{R}^{mxn}, \quad R_o \in \mathbb{R}^{mxm}$$

- Then the covariance of $p_s(k) \& o_s(k)$ is:

$$R_{s,p} = I_{s+1} \otimes R_p \in \mathbb{R}^{m_s \times n_s}$$

$$R_{s,o} = I_{s+1} \otimes R_o \in \mathbb{R}^{m_s \times m_s}$$

- Then the covariance of $e_s^*(k) = W_o [o_s(k) + G_s^o p_s(k)]$ is:

$$R_{s,e} = W_o (G_s^o R_{s,p} G_s^{oT} + R_{s,o}) W_o^T \in \mathbb{R}^{m_s-n \times (m_s-n)}$$
PRV Properties (con’t)

- The PRV follows multivariate Gaussian distribution

  \[
  \begin{align*}
  &\text{No MPM} \quad e_g(k) = e^*_g(k) \sim \mathcal{N}(0, R_{g,e}) \\
  &\text{MPM} \quad e_g(k) = e^*_g(k) + e^f_g(k) \sim \mathcal{N}(e^f_g(k), R_{g,e})
  \end{align*}
  \]

- The PRV can be further transformed into a square weighted residual which is sensitive to the mismatch in \(\{A, B, C\}\).

  \[
  \eta_{ABC}(k) = (e_g(k))^T \times R_{g,e}^{-1} \times e_g(k)
  \]
MPM Detection Index (1)

- It follows chi-square distribution if there is no MPM:
  \[ \eta_{ABC}(k) \sim \chi^2_{m_s-n}(\alpha) \]

- Given a confidence limit, e.g. \( \alpha = 1\% \).
  
  \[ \eta_{ABC}(k) > \chi^2_{m_s-n}(\alpha) \Rightarrow \text{This is mismatch in A or B or C.} \]
  
  \[ \eta_{ABC}(k) < \chi^2_{m_s-n}(\alpha) \Rightarrow \text{This is no mismatch in A, B and C.} \]
Other two MDIs are proposed in the similar way:

- $\eta_{AC}(k)$ is only sensitive to the mismatch in $\{A, C\}$;

- $\eta_C(k)$ is only sensitive to the mismatch in $\{C\}$.
# Isolation Logic

<table>
<thead>
<tr>
<th>$\eta_{ABC}(k)$</th>
<th>$\eta_{AC}(k)$</th>
<th>$\eta_C(k)$</th>
<th>Fault Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$B$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$A$ or $AB$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$C$ or $AC$ or $BC$ or $ABC$</td>
</tr>
</tbody>
</table>

'1': the according MDI does indicate MPM  
'0': the according MDI doesn’t indicate MPM
Isolation Logic (con’t)

- Usually, matrix $\mathbf{C}$ is the sensor gain matrix, which is unlikely to deviate.

- Therefore, if we assume there is no mismatch in matrix $\mathbf{C}$, the isolation logic becomes:

<table>
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<tr>
<th>$\eta_{ABC}(k)$</th>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$\mathbf{B}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$\mathbf{A}$ or $\mathbf{AB}$</td>
</tr>
</tbody>
</table>

'1': the according MDI does indicate MPM
'0': the according MDI doesn’t indicate MPM
Numerical Example
The simulated process

- The simulated process is a second order dynamic system.

- The system matrices in discrete time domain are:

\[
A_0 = \begin{bmatrix}
0.6082 & -0.0100 \\
2.2668 & 0.0364
\end{bmatrix} \quad B_0 = \begin{bmatrix}
0.5602 & 0.0008 \\
1.3760 & 0.3026
\end{bmatrix}
\]

\[
C_0 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

- Matrix \( C \) is the sensor gain matrix and we assume there is no mismatch in this matrix.
Case(1): mismatch in $A$ matrix

- 10% mismatch in $A$ matrix;
- 3000 samples of input and output data;
- Mismatch is introduced at 500th sample;
- Noise-to-signal ration (NSR) for the two outputs are 11.19% and 2.73%.

**Remark:** The MDIs have confirmed that there is mismatch in matrix $A$. 
Case(2): mismatch in $B$ matrix

- 10% mismatch in $B$ matrix;
- 3000 samples of input and output data;
- Mismatch is introduced at 500th sample;
- Noise-to-signal ration (NSR) for the two outputs are 14.81% and 4.12%.

**Remark:** The MDIs have confirmed that the mismatch is only in matrix $B$. 
Case(3): mismatch in \textbf{A} \& \textbf{B} matrices

- 10% mismatch in \textbf{A} \& \textbf{B} matrices;
- 3000 samples of input and output data;
- Mismatch is introduced at 500\textsuperscript{th} sample;
- Noise-to-signal ration (NSR) for the two outputs are 6.94\% and 1.74\%.

\begin{itemize}
  \item Remark: The MDIs have confirmed that there is mismatch in matrix \textbf{A}.
\end{itemize}
Comparison of case (1) & case (3)

- Case (1): 10% mismatch in $A$
- Case (3): 10% mismatch in $A$ & $B$

- $\eta_{ABC}(k)$ is sensitive to both mismatch in $A$ & $B$
- $\eta_{AC}(k)$ is only sensitive to mismatch in $A$
Conclusion
Conclusion

- The types of faults that can affect process performance have been reviewed.

- Three different MDIs have been proposed to detect MPM.

- An isolation logic framework has been proposed to isolate the system matrices that have MPM.

- A numerical example has been presented to demonstrate the efficacy of the new scheme for detection and isolation of MPM.
Acknowledgement
Acknowledgement

- NSERC, Matrikon and ASRA for financial support
- Colleagues of CPC group