Detection and Isolation of Model-Plant-Mismatch for Multivariate Dynamic Systems

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Outline

- Motivation & Literature Review
- Problem Formulation
- MPM Detection and Isolation
- Numerical Example
- Conclusion
- Acknowledgement

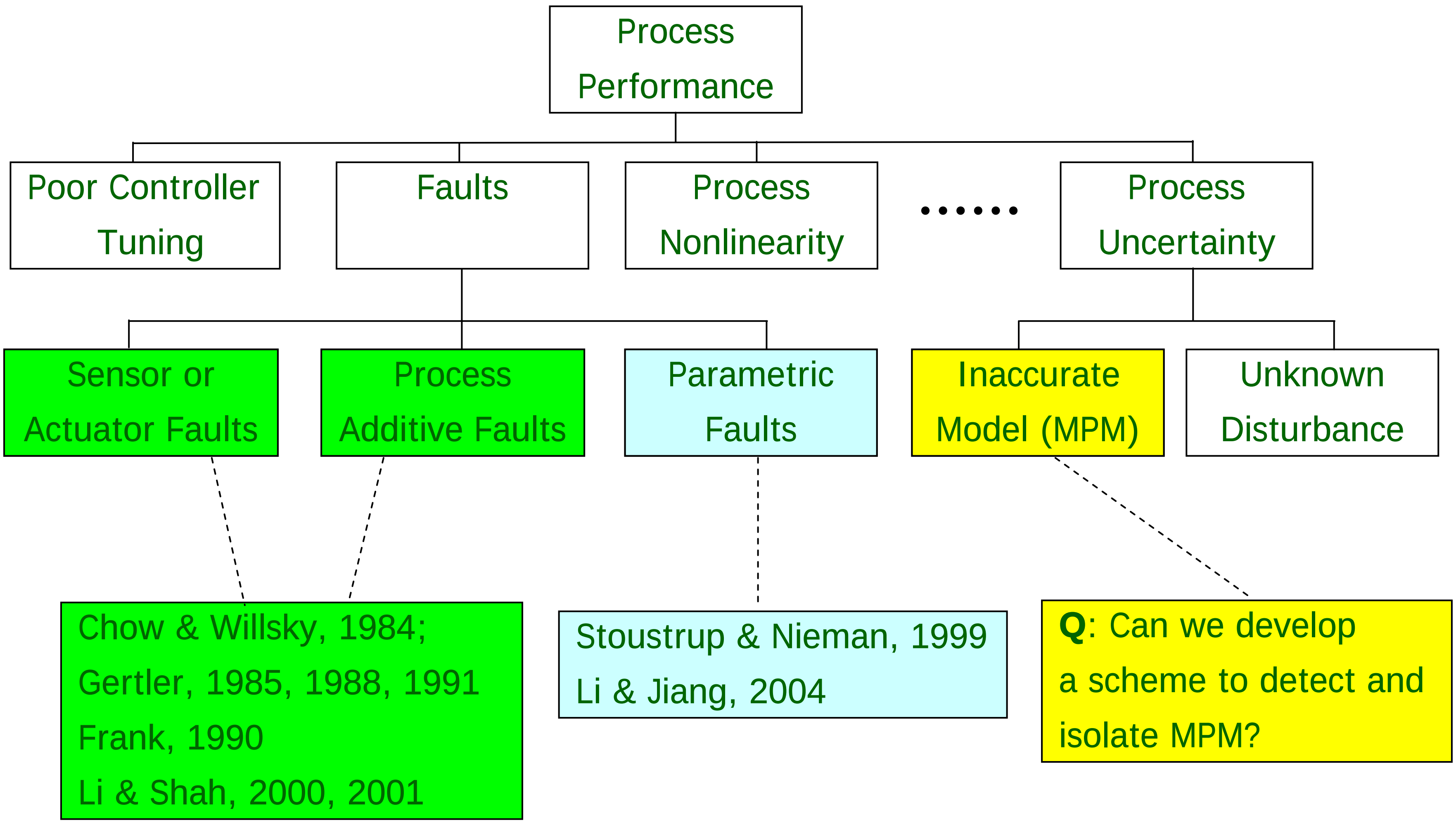
Motivation
&
Literature Review

Motivation

- In industry, model-plant-mismatch (MPM) is a common and challenging problem for existing model-based control, e.g. MPC.

- In academia, MPM is still a difficult problem in the research area of fault detection and isolation (FDI).

Literature Review on FDI



Problem Formulation

Problem Formulation

- Consider a LTI Discrete Time (DT) system:

$$\begin{cases} x(k+1) &= \mathbf{A}_o x(k) + \mathbf{B}_o u(k) + p(k) \\ y(k) &= \mathbf{C}_o x(k) + o(k) \end{cases}$$

- MPM effect:

Can we detect the mismatch?

Can we isolate the matrices that have mismatch?

$$\mathbf{A} = \mathbf{A}_o + \boxed{\Delta\mathbf{A}} \in R^{n\times n}$$
$$\mathbf{B} = \mathbf{B}_o + \boxed{\Delta\mathbf{B}} \in R^{n\times l}$$
$$\mathbf{C} = \mathbf{C}_o + \boxed{\Delta\mathbf{C}} \in R^{m\times n}$$

- A DT system with MPM:

$$\begin{cases} x(k+1) &= (\mathbf{A}_0 + \Delta\mathbf{A})x(k) + (\mathbf{B}_0 + \Delta\mathbf{B})u(k) + p(k) \\ y(k) &= (\mathbf{C}_0 + \Delta\mathbf{C})x(k) + o(k) \end{cases}$$

Problem Formulation (con't)

- Rewrite the MPM effect:

$$a(k) = \Delta\mathbf{A}x(k),\ b(k) = \Delta\mathbf{B}u(k),\ c(k) = \Delta\mathbf{C}x(k)$$

- Rewrite the system with MPM:

$$\begin{cases} x(k+1) = \mathbf{A}_0 x(k) + \mathbf{B}_0 u(k) + a(k) + b(k) + p(k) \\ y(k) = \mathbf{C}_0 x(k) + c(k) + o(k) \end{cases}$$

$$a(k) \neq 0 \Rightarrow \Delta\mathbf{A} \neq 0$$
$$b(k) \neq 0 \Rightarrow \Delta\mathbf{B} \neq 0$$
$$c(k) \neq 0 \Rightarrow \Delta\mathbf{C} \neq 0$$

Can we indicate $a(k), b(k)$ *or* $c(k)$ are non-zero?

MPM Detection and Isolation

MPM Detection

- By stacking the equation for the system with MPM

$$y_s(k) - \mathbf{H}_s^o u_s(k) = \Gamma_s^o x(k-s) + c_s(k) + o_s(k) + \mathbf{G}_s^o [a_s(k) + b_s(k) + p_s(k)]$$

$$y_s(k) = \begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix} \in R^{m_s}$$

$$m_s = m(s+1)$$

$$\Gamma_s^o = \begin{bmatrix} \mathbf{C}_o \\ \mathbf{C}_o \mathbf{A}_o \\ \vdots \\ \mathbf{C}_o \mathbf{A}_o^s \end{bmatrix} \in R^{m_s \times n}$$

$$\mathbf{H}_s^o = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_o \mathbf{B}_o & \mathbf{0} & \cdots & \vdots & \vdots \\ \mathbf{C}_o \mathbf{A}_o \mathbf{B}_o & \mathbf{C}_o \mathbf{B}_o & \ddots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_o \mathbf{A}_o^{s-1} \mathbf{B}_o & \mathbf{C}_o \mathbf{A}_o^{s-2} \mathbf{B}_o & \cdots & \mathbf{C}_o \mathbf{B}_o & \mathbf{0} \end{bmatrix} \quad \mathbf{G}_s^o = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_o & \mathbf{0} & \cdots & \vdots & \vdots \\ \mathbf{C}_o \mathbf{A}_o & \mathbf{C}_o & \ddots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_o \mathbf{A}_o^{s-1} & \mathbf{C}_o \mathbf{A}_o^{s-2} & \cdots & \mathbf{C}_o & \mathbf{0} \end{bmatrix}$$

MPM Detection (con't)

- By stacking the equation for the system with MPM

$$y_s(k) - \mathbf{H}_s^o u_s(k) = \Gamma_s^o x(k-s) + c_s(k) + o_s(k)$$
$$+\mathbf{G}_s^o[a_s(k) + b_s(k) + p_s(k)]$$

$$\mathbf{W}_o \Gamma_s^o = 0$$

$\Gamma_s^o \in R^{m_s \times n} \Rightarrow \mathbf{W}_o \in R^{(m_s - n) \times m_s}$

$$e_s(k) \equiv \mathbf{W}_o[y_s(k) - \mathbf{H}_s^o u_s(k)]$$
$$= \mathbf{W}_o[o_s(k) + \mathbf{G}_s^o p_s(k)] + \mathbf{W}_o\{c_s(k) + \mathbf{G}_s^o[a_s(k) + b_s(k)]\}$$

$e_s(k)$: Primary residual vector (PRV)

$e_s^*(k)$: Zero-mean, Gaussian distributed random noise vector

$e_s^f(k)$: Non-zero vector, if there is MPM

PRV Properties

- If we assume the covariance of $p(k)$ & $o(k)$ as

$$\mathbf{R}_p \in R^{n \times n}, \qquad \mathbf{R}_o \in R^{m \times m}$$

- Then the covariance of $p_s(k)$ & $o_s(k)$ is:

$$\mathbf{R}_{s,p} = \mathbf{I}_{s+1} \otimes \mathbf{R}_p \in R^{n_s \times n_s}$$

$$\mathbf{R}_{s,o} = \mathbf{I}_{s+1} \otimes \mathbf{R}_o \in R^{m_s \times m_s}$$

- Then the covariance of $e_s^*(k) = \mathbf{W}_o[o_s(k) + \mathbf{G}_s^o p_s(k)]$ is:

$$\mathbf{R}_{s,e} = \mathbf{W}_o(\mathbf{G}_s^o \mathbf{R}_{s,p} \mathbf{G}_s^{oT} + \mathbf{R}_{s,o})\mathbf{W}_o^T \in R^{(m_s - n) \times (m_s - n)}$$

PRV Properties (con't)

- The PRV follows multivariate Gaussian distribution

No MPM $$\mathbf{e}_s(k) = \mathbf{e}_s^*(k) \sim \aleph(0, \mathbf{R}_{s,e})$$

MPM $$\mathbf{e}_s(k) = \mathbf{e}_s^*(k) + \mathbf{e}_s^f(k) \sim \aleph(\mathbf{e}_s^f(k), \mathbf{R}_{s,e})$$

- The PRV can be further transformed into a square weighted residual which is sensitive to the mismatch in $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$.

$$\eta_{ABC}(k) = (e_s(k))^T \times \mathbf{R}_{s,e}^{-1} \times e_s(k)$$

MPM Detection Index (1)

- It follows chi-square distribution if there is no MPM:

$$\eta_{ABC}(k) \sim \chi^2_{m_s-n}(\alpha)$$

- Given a confidence limit, e.g. $\alpha = 1\%$.

$$\eta_{ABC}(k) > \chi^2_{m_s-n}(\alpha) \quad \Rightarrow$$

This is mismatch in **A** or **B** or **C** .

$$\eta_{ABC}(k) < \chi^2_{m_s-n}(\alpha) \quad \Rightarrow$$

This is no mismatch in **A**, **B** and **C** .

MPM Detection Indices (2) & (3)

- Other two MDIs are proposed in the similar way:
  - $\eta_{AC}(k)$ is only sensitive to the mismatch in $\{\mathbf{A}, \mathbf{C}\}$;
  - $\eta_{C}(k)$ is only sensitive to the mismatch in $\{\mathbf{C}\}$.

Isolation Logic

$\eta_{ABC}(k)$	$\eta_{AC}(k)$	$\eta_{C}(k)$	Fault Matrix
1	0	0	**B**
1	1	0	**A** or **AB**
1	1	1	**C** or **AC** or **BC** or **ABC**

'1': the according MDI does indicate MPM

'0': the according MDI doesn't indicate MPM

Isolation Logic (con't)

- Usually, matrix **C** is the sensor gain matrix, which is unlikely to deviate.
- Therefore, if we assume there is no mismatch in matrix **C**, the isolation logic becomes:

$\eta_{ABC}(k)$	$\eta_{AC}(k)$	Fault Matrix
1	0	**B**
1	1	**A** or **AB**

'1': the according MDI does indicate MPM

'0': the according MDI doesn't indicate MPM

Numerical Example

The simulated process

- The simulated process is a second order dynamic system.
- The system matrices in discrete time domain are:

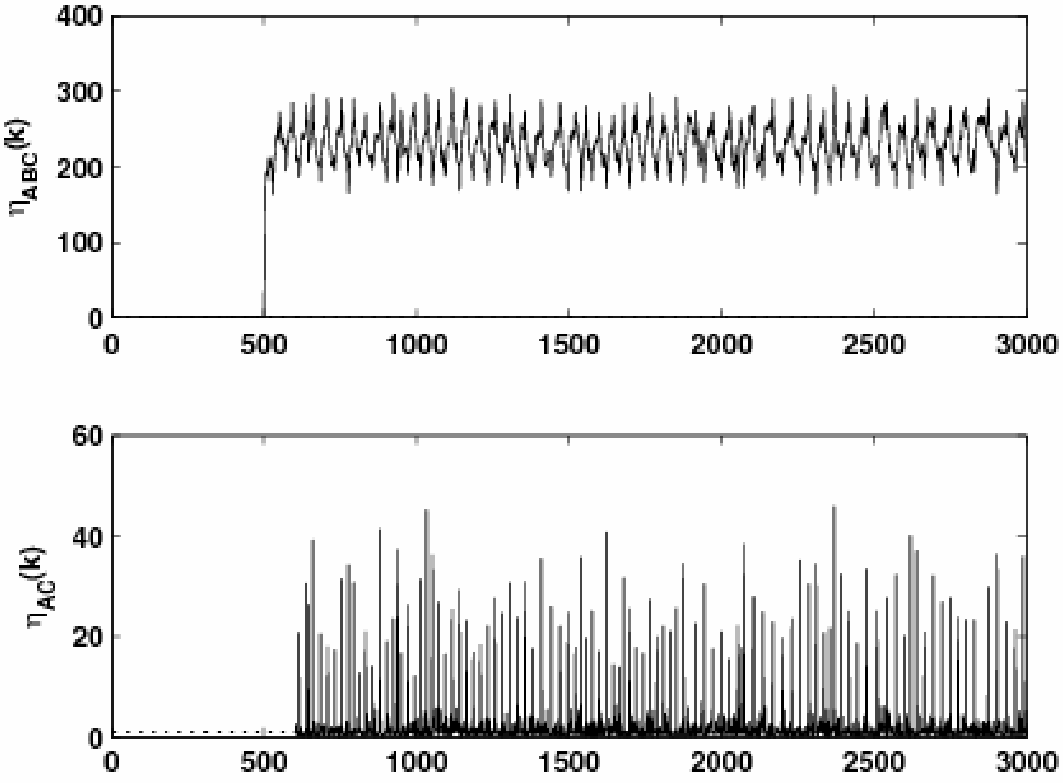
$$\mathbf{A}_0 = \begin{bmatrix} 0.6082 & -0.0100 \\ 2.2668 & 0.0364 \end{bmatrix} \quad \mathbf{B}_0 = \begin{bmatrix} 0.5602 & 0.0008 \\ 1.3760 & 0.3026 \end{bmatrix}$$

$$\mathbf{C}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Matrix **C** is the sensor gain matrix and we assume there is no mismatch in this matrix.

Case(1): mismatch in **A** matrix

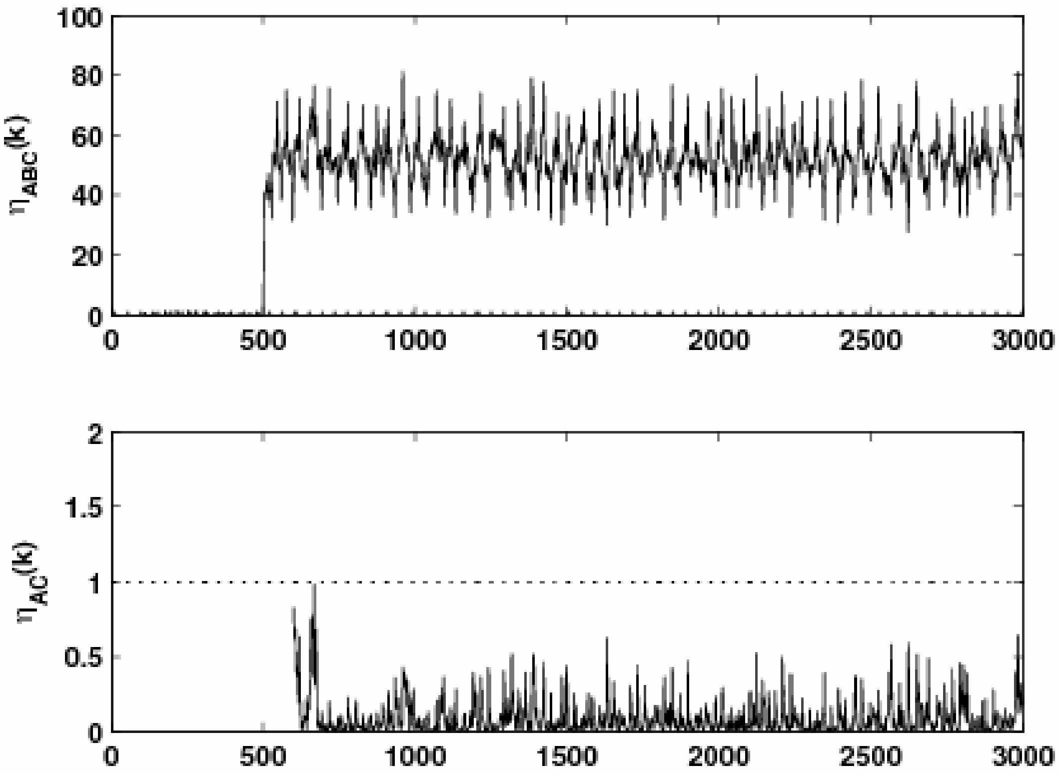
- 10% mismatch in **A** matrix;
- 3000 samples of input and output data;
- Mismatch is introduced at 500th sample;
- Noise-to-signal ration (NSR) for the two outputs are 11.19% and 2.73%.



- Remark: The MDIs have confirmed that there is mismatch in matrix **A**.

Case(2): mismatch in **B** matrix

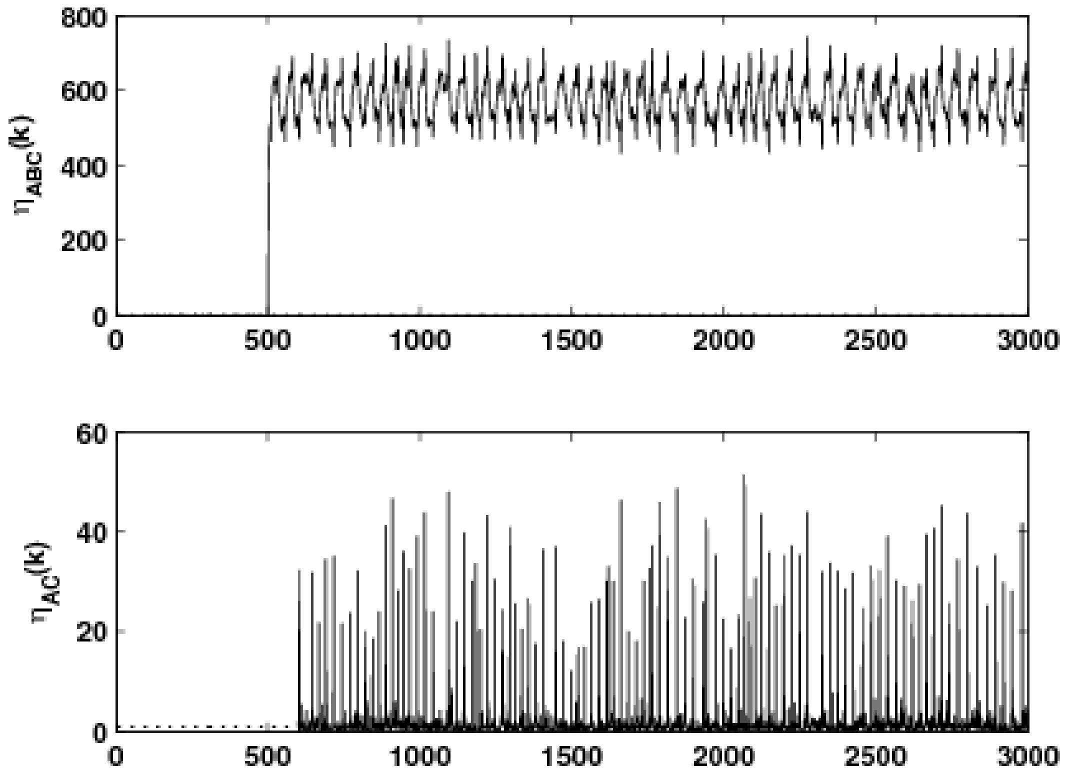
- 10% mismatch in **B** matrix;
- 3000 samples of input and output data;
- Mismatch is introduced at 500th sample;
- Noise-to-signal ration (NSR) for the two outputs are 14.81% and 4.12%.



- Remark: The MDIs have confirmed that the mismatch is only in matrix **B**.

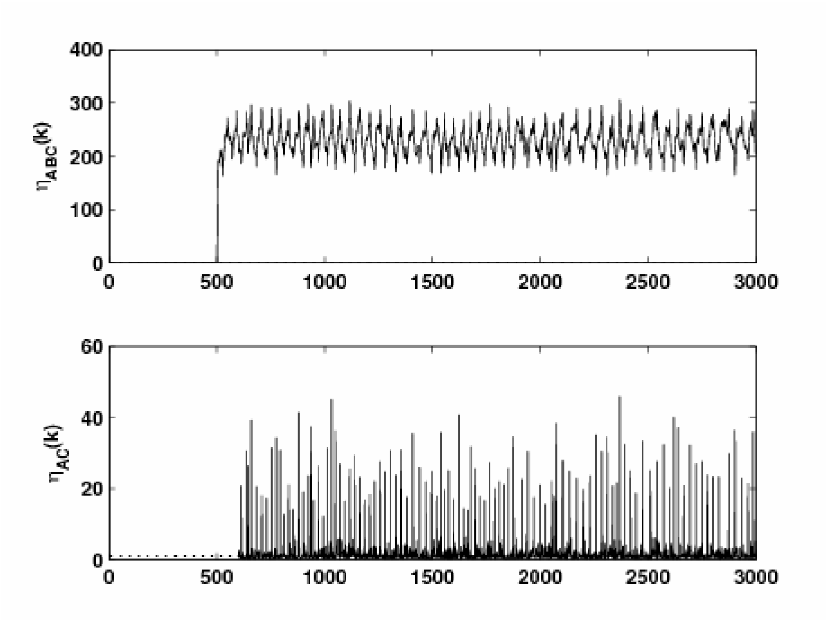
Case(3): mismatch in **A** & **B** matrices

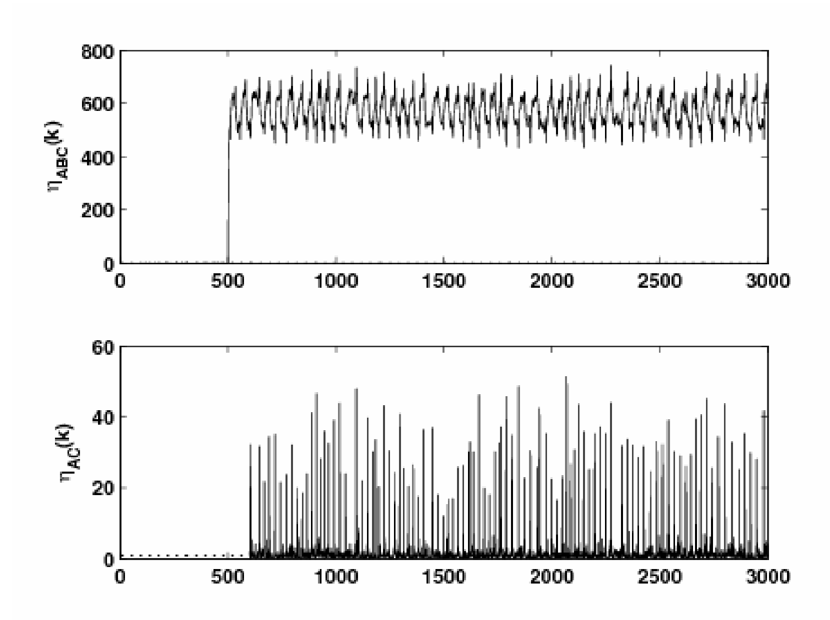
- 10% mismatch in **A** & **B** matrices;
- 3000 samples of input and output data;
- Mismatch is introduced at 500th sample;
- Noise-to-signal ration (NSR) for the two outputs are 6.94% and 1.74%.



- Remark: The MDIs have confirmed that there is mismatch in matrix **A**.

Comparison of case (1) & case (3)





- Case (1): 10% mismatch in **A**
- Case (3): 10% mismatch in **A** & **B**

- $\eta_{ABC}(k)$ is sensitive to both mismatch in **A** & **B**
- $\eta_{AC}(k)$ is only sensitive to mismatch in **A**

Conclusion

Conclusion

- The types of faults that can affect process performance haven been reviewed.
- Three different MDIs have been proposed to detect MPM.
- An isolation logic framework has been proposed to isolate the system matrices that have MPM.
- A numerical example has been presented to demonstrate the efficacy of the new scheme for detection and isolation of MPM.

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Acknowledgement

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