Fault Detection of Gearbox from Vibration Signals using Time-Frequency Domain Averaging

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***Abstract*— The vibration signal of a gearbox carries the signature of the fault in the gears, and early fault detection of the gearbox is possible by analyzing the vibration signal using different signal processing techniques. Time domain average can extract the periodic waveforms of a noisy vibration signal, whereas Wavelet transformation is able to characterize the local features of the signal in different scales. This paper proposes a new technique, Time-Frequency Domain Average, which combines the time domain average and wavelet transformation together to extract the periodic waveforms at different scales from noisy vibration signals. The technique efficiently cleans up noise and detects both local and distributed faults simultaneously. A pilot plant case study has been presented to demonstrate the efficacy of the proposed technique.**

I. Introduction

Analysis of vibration signal is widely used to detect early faults in rotating machineries, especially gearboxes. There are many techniques that have been developed to detect progressing faults in gearboxes. Most of them use time domain as the base of their analysis. Among these techniques, time domain averaging has established itself as a strong vibration signal processing technique that can extract periodic waveforms from noisy signals. The time domain average of the vibration of a gear represents the vibration produced by the gear over one complete revolution, and thus indicates the variation of vibration produced by each individual tooth of the gear [4]. But even though time domain average of the signal may contain all the information about the smoothly operating gearbox, it is often hard to detect clear symptoms of any defect in the gear from only the time domain average, especially if the defect is at an early stage [9]. The technique may also fail to detect and differentiate between faults, if multiple faults are present simultaneously in multiple gears within the gearbox. A wide variety of different techniques have been explored over the years to further process the time domain average to make it more sensitive to early fault detection [5].

Recently a lot of work has been done on the analysis of vibration signals in the time-frequency domain, with a view to combine the advantages of both time and frequency domain representation of signal [3]. Undoubtedly, wavelet analysis has proved itself as the best time-frequency domain technique that keeps a good balance between the time and frequency resolution. The most important feature of Wavelet transform is its ability to characterize the local features of the signal at different scales. This is highly advantageous in examining vibration signal from a rotating machine with faults, where either large scale or small scale change in the vibration may occur, corresponding to distributed damage or local damage respectively [8]. This paper proposes a new technique that combines time-domain average with wavelet to produce a time-frequency domain representation of a signal. Time-frequency domain average captures the vibration generated by a gear at different frequencies or scales over one complete revolution. It keeps the different scale representation of the wavelet analysis intact and at the same time extracts the periodic components from the noisy signal. Therefore it is best suited to detect faults at any scale from a periodic noisy signal. Time-frequency domain average was applied on real data from a test rig introducing both small and large faults. The results indicate that the time-frequency domain averages of the data sets efficiently identified both large and small scale faults in the gearbox, even when multiple faults are present simultaneously.

II. Wavelet Analysis

Wavelet analysis has become a common tool for analyzing vibration signals to detect localized faults in rotating machineries. By decomposing a time series into time-frequency space, one is able to determine the existing frequencies in the signal as well as the duration of each individual frequency in time. If $f(t)$ is time series then Continuous Wavelet Transform of the time series can be defined in time and frequency domain as,

$$W(s,u) = \langle f, \psi_{s,u} \rangle = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\psi^*(\frac{t-u}{s})dt$$
$$= f * \overline{\psi_s}(u) \quad (1)$$

where, ψ represents the wavelet function, s and u are respectively the scale and time at which the wavelet coefficient is calculated [2]. The scaling parameter s is called the dilation factor and the parameter u that shifts the function in time is called the translation factor. Both these factors give rise to a family of wavelets from the mother wavelet by the relation,

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$$\psi_{(s,u)}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-u}{s}) \quad (2)$$

where, $\psi_{(s,u)}$ is the daughter wavelet and $\psi\frac{(t-u)}{s}$ represents the mother wavelet [1]. Different possible families of wavelets are available in wavelet application, but the Morlet wavelet has been used most commonly in the literature for analysis of vibration signal from rotating machineries. This is due to the fact that the Morlet wavelet is able to pick up impulses generated by the rotating elements.

III. Time Domain Averaging

Time domain averaging is a strong signal processing technique to extract periodic waveforms from noisy signals. It can be used to represent the vibration produced by the meshing of teeth over one complete revolution of the gear in time domain. Therefore, variation in the vibration generated by the individual gear tooth can be identified by visual inspection of the average. The time domain average of the vibration of a single tooth can be obtained by the elimination of all except the fundamental and harmonics of the tooth meshing frequency from the time domain average of the gear vibration. The residual signal, generated by subtracting this signal from the original time domain average, gives the departure of the vibration from the average. Digital technique has been developed that processes the time domain average more to remove the pattern of vibration produced by normal teeth meshing. As a result, small variations between the individual tooth become apparent and earlier detection of faults become possible [4]. But most of the time when the vibration signal is noisy, it is hard to detect faults at an early stage using only this technique.

IV. Time-Frequency Domain Averaging - the Newly Proposed Approach

Vibration signal of a gearbox is periodic and it is usually corrupted with a lot of noise. Faults in the gear would produce intermittent frequency within every period of the signal. Time domain average of this signal best represents the dynamics of the gears for one single revolution. It can successfully store the deterministic part of the signal while cleaning the stochastic part from a noisy vibration signal. But the spectral characteristics are lost once this technique has been used. It is necessary to keep both the time and frequency information of the signal to detect intermittent frequency caused by the presence of localized faults. Wavelet transformation cannot be applied on the averaged signal since it requires a long time series with enough samples to capture both the time and frequency information. The dynamics of the moving gears of the gearbox can be best visualized by looking at the frequency region close to the meshing frequency of the gearbox. Therefore, it is necessary to get time domain averages at every frequency of the signal close to the meshing frequency of the gearbox. In short, the detection of faults in the gears of the gearbox requires the detection of any intermittent frequency component close to the meshing frequency of the gearbox in one period of the vibration signal. In this paper a new technique, time-frequency domain averaging has been proposed which meshes the concept of both time domain averaging and wavelet analysis to detect the intermittent frequencies present within one period of a periodic signal.

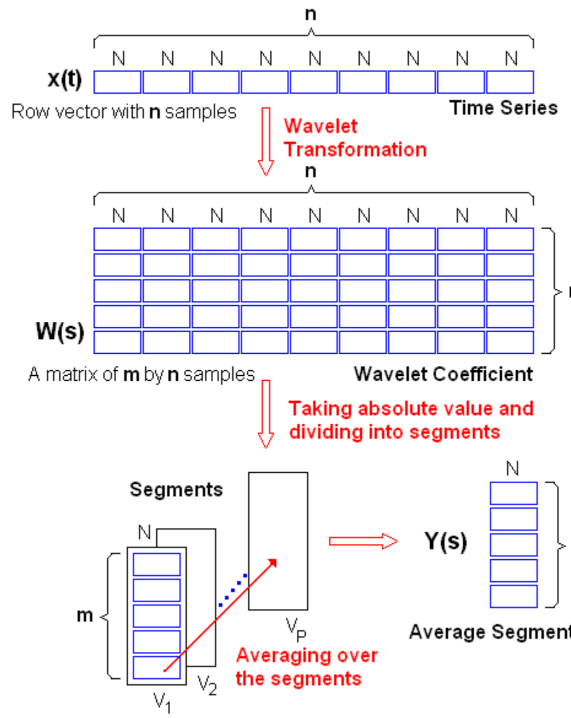


Fig. 1. Plot showing the transformation of a time series to time-frequency domain average

A. Theory

Figure 1 shows in short the procedure to calculate the time-frequency domain average from the time series. Assume, $x(t)$ is a time series of a periodic vibration signal with n samples and a period of N. Wavelet transformation of $x(t)$ would generate wavelet coefficient $W(s)$, which is a matrix of $m \times n$ dimension where m is the number of scales. The matrix $W(s)$ may be a complex matrix depending on the wavelet used. By taking the absolute value of each of the element of the matrix $W(s)$, the matrix $V(s)$ is produced. All elements of $V(s)$ are real. If $V_1, V_2, V_3 \dots V_p$ are sub-matrices of the matrix $V(s)$, so that each of them corresponds to one period of the time series signal, then these sub-matrices would have the dimension $m \times N$, where $n = N \times p$. Now, if $Y(s)$ is the time-frequency domain average matrix of the signal then $Y(s)$ would be the average of the matrices $V_1, V_2, V_3 \dots V_p$. If a geometric average is performed then each element of the matrix $Y(s)$ can be calculated from $V(s)$ as,

$$Y(k,l) = \left(\prod_{i=1}^{p} V_i(k,l)\right)^{\frac{1}{p}} \quad (3)$$

where, $k = 1, 2, \dots m$, and $l = 1, 2, \dots N$. Here m is the number of scales. By performing wavelet transformation of

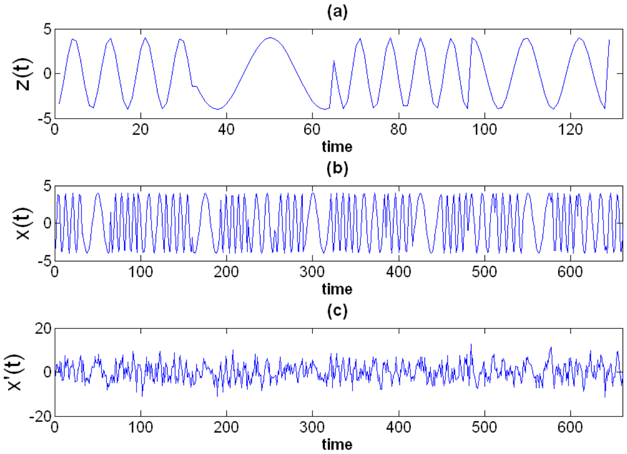
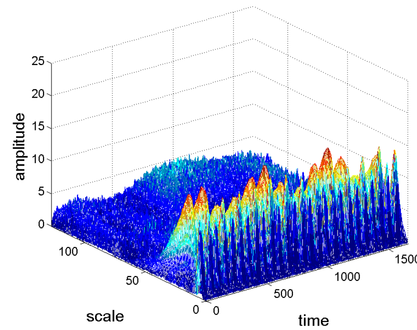


Fig. 2. Plot showing (a) one period of the simulated time series (b) simulated time series before adding noise (c) simulated time series after adding noise with a SNR of 1 (first 660 samples have been shown)

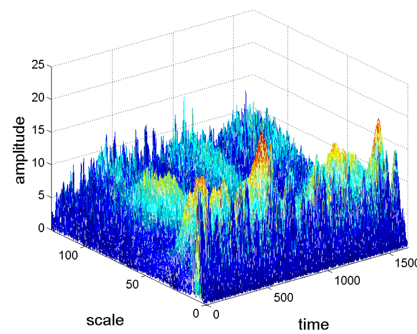
the time series, the information on the frequency response is incorporated in the matrix. The absolute value of the matrix is then divided into segments where each segment holds the frequency and time domain information of one period. Averaging these segments gives the time-frequency domain average segment and it is a representation of the dynamics of one revolution of the gear. Geometric average ensures that the signal of periodic impulses due to faults in the gear multiply with each other and the magnitude of their average increases in a geometric rate. Periodic peaks would therefore produce significant peak in the average segment though their individual magnitude may be low. White noise would have no periodicity and therefore would fail to produce any significant peak in the average segment.

B. Illustrative Example

An example has been presented here to illustrate the use of time-frequency domain average technique along with its effectiveness. The example demonstrates the ability of the technique to detect the deterministic part of multiple intermittent frequencies existing in a periodic noisy signal with a good time resolution. A time series signal $z(t)$ has been generated with the 4 frequencies 0.12, 0.04, 0.14 and 0.08 Hz (corresponding to 6.5, 20, 5.7 and 10 units in scale) one after the other for 32 samples each and at the same amplitude. Figure 2(a) shows the signal $z(t)$. This short time series of 128 samples has been repeated 50 times to generate the time series $x(t)$ so that $z(t)$ represents a period of the periodic signal $x(t)$. Figure 2(b) shows a portion of the signal $x(t)$, clearly depicting the periodic nature of the time series. Wavelet transform of the signal $x(t)$ gives the wavelet coefficients $W(s)$, shown in Figure 3(a). White noise ε, such that the signal to noise ratio with $x(t)$ is 1, has been added to $x(t)$ to generate the noise corrupted signal $x'(t)$ shown in Figure 2(c) . Wavelet coefficient $W'(s)$ of the signal $x'(t)$ has been plotted in Figure 3(b). Both Figure 2(c) and Figure 3(b) show a signal highly corrupted by noise.



(a) before adding noise



(b) after adding noise with a SNR of 1

Fig. 3. Plots of wavelet coefficients of the simulated time series.

The noise overshadows the original signal so much that the original signal cannot be visualized in Figure 2(c) or Figure 3(b), and it is even impossible to have any idea about the periodicity of the signal.

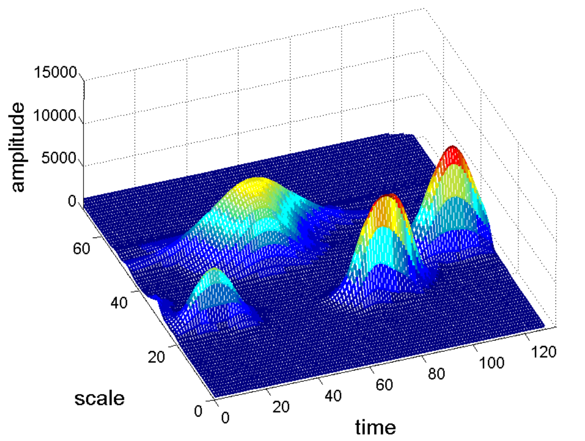
To apply time-frequency domain averaging on the generated signal $x'(t)$, the following steps have been followed:

1) Wavelet transformation $W'(s)$ of the generated signal $x'(t)$ is calculated.
2) The absolute value of each of the elements of the matrix $W'(s)$ is taken to form the matrix $V'(s)$.
3) Equation (3) is used to calculate the time-frequency domain average matrix $Y(s)$ of the matrix $V'(s)$ with a period of 128.

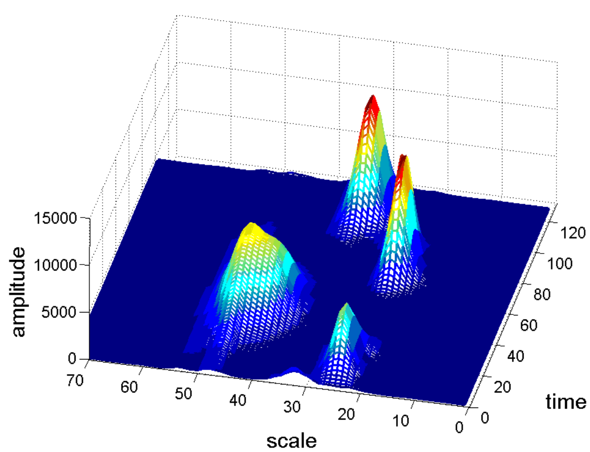
Figure 4 shows the plot of the time-frequency domain average matrix $Y(s)$. The four frequencies can be seen clearly with a good time resolution. Power spectrum may have successfully detected the presence of the frequencies but it could not give any information on the time of the existence of the frequencies in one period. Figure 4(a) clearly shows the time localization of the frequencies while Figure 4(b) shows the presence of the four frequencies.

V. Gear Fault Detection

The major faults of a gear in a gear box can be classified into 3 major categories - surface wear spalling, cracked tooth, and chipped or broken tooth. Extreme case of a cracked tooth is a missing tooth. Local faults in a gearbox are usually identified by detecting the presence of higher impulses in the



(a) view of TFA from time axis



(b) view of TFA from scale axis

Fig. 4. Plots showing the time-frequency domain average of the simulated time series.

vibration signal. Even if there is no fault, tooth contact may occur prematurely at the tip of the drive gear due to tooth deflection and the velocity mismatch between a pinion and a gear. This causes an impact at the time of contact which gives rise to the larger vibration at the gear meshing frequency [6]. For this reason, a close look at the vibration signal close to the gear meshing frequency reveals the presence of existing faults in the gears of the gearbox.

A. Detection of Chipped Tooth

Chipped tooth produces backlash as the clearance between two consecutive teeth in the gear increases. Excessive backlash causes a difference in the angular distance traversed by the input and the output shaft at the time of contact of the chipped tooth. The impact initiated by the chipped tooth produces impulses in the vibration signal at higher or lower frequency than the gear meshing frequency. Usually nonlinearity arises within the system by the generation of a new frequency and also the resonance frequency of the gear may appear in the signal. If the time-frequency domain average of the signal is taken, then for each chipped tooth a peak appears at the meshing frequency, and at the same time another peak appears in a frequency close to the meshing frequency.

B. Detection of Missing Tooth

Missing tooth in a gear produces larger impacts than any other fault present in the gearbox. When the space of the missing tooth comes in contact with the pinion gear, the tooth of the pinion gear slips in the empty space and fails to make a proper contact. As a result the next two teeth of the gears hit each other with higher impact. This can be detected as higher impulse in the vibration signal if the time-frequency domain average is taken. For each missing tooth a high peak should appear at the meshing frequency in the time-frequency domain averaged signal.

C. Multiple Shafts

Time-frequency domain average of the vibration signal captures the dynamics of one total rotation of the output shaft of the gearbox. The period used to calculate the time-frequency domain average is equal to the period of rotation of the output shaft of the gearbox. But the gearbox under investigation may have more than two shafts and the fault in the gear may be present in any one of the shafts. In that case, the period for the calculation of the time-frequency domain average should be chosen such that the dynamics of each of the shafts present in the gearbox is properly captured. To do this, the Least Common Multiple (LCM) of the rotation periods of the shafts has been calculated and is used as the period for the calculation of the time-frequency domain average. This way the time-frequency domain average would represent the total dynamic of each of the shaft at least for one period. For shafts rotating at high frequency, the period would be small and the time-frequency domain average would represent the dynamics of the shaft for multiple integer periods. For example, if there are 2 shafts rotating at frequencies of 5 Hz (period of 0.2 seconds) and 10 Hz (period of 0.1 seconds) then the period for the calculation of time-frequency domain average should be taken as 0.2 seconds (LCM of 0.2 and 0.1) and in this case the average would capture the dynamics of the second shaft for 2 periods.

VI. Pilot Plant Case Study

A pilot plant case study was performed to assess the effectiveness of the proposed technique in early detection of gear faults. Data was generated using a test rig that could simulate single and multiple faults. The rig is located in the Reliability Lab in the Mechanical Engineering Building at the University of Alberta, Canada [7]. The configuration of the test rig is shown in Figure 5. The gearbox had 3 shafts with a total of 4 gears a, b, c and d. Brake was used to create the desired load of operation. Normal gear a could be replaced by the damaged gear a', which had a chipped tooth. Similarly, normal gear d could be replaced by the damaged gear d', which had a missing tooth. Both damaged gears could be used at the same time to simulate multiple

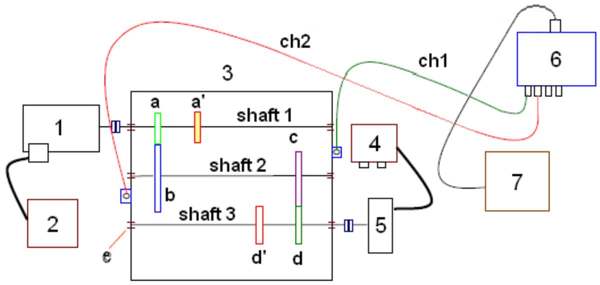


Fig. 5. Configuration of the test rig used to generate data for the case study. 1-Motor, 2-Variable Speed Motor Controller, 3-Gearbox, 4-Brake Controller, 5-Brake, 6-Siglab Vibration Analyzer, 7-Computer.

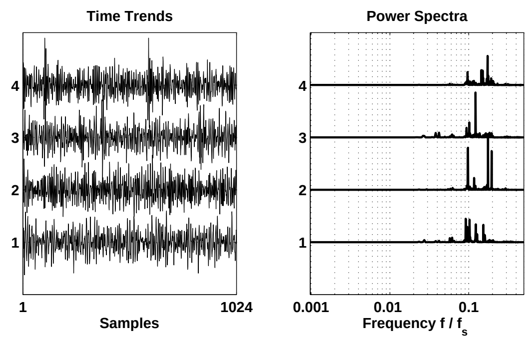


Fig. 6. Time Trend and Power Spectrum plots of the data sets generated from the test rig for case study (only first 1024 samples have been shown).

faults. Shafts 1, 2 and 3 were rotating at 10, 3.3 and 5 Hz respectively during data collection. The gear meshing frequency was 160 Hz, or 0.125 Hz in the normalized frequency (corresponding to 6.4 units in scale).

A. Data Description

A total of four data sets were generated from the test rig. Every time series had 8192 samples sampled at 1280 Hz. The four data sets were collected at the following conditions:

1) All normal gears used
2) One damaged gear with a Chipped tooth used - gear a was replaced by a'
3) One damaged gear with a Missing tooth used - gear d was replaced by d'
4) One damaged gear with a Chipped tooth and another damaged gear with a Missing tooth used - both gear a and d were replaced by a' and d'

The time-trend plots (with only the first 1024 data samples) and the power spectra of the four data sets are shown in Figure 6.

B. Time-Frequency Domain Average Analysis

Figure 7 shows the time-frequency domain average of the 4 data sets. The LCM of the periods of the shafts under investigation was 0.2 seconds (LCM of 0.1 and 0.2 seconds corresponding to 10 and 5 Hz respectively). Therefore each time domain average represents 2 periods of the gear a and 1 period of the gear d. Brief explanations of the plots are given below.

1) Normal Gears: Figure 7(a) shows the TFA plot for the data set generated from normal gears. No significant peaks can be observed in the plot since the gears had no fault in their teeth.

2) Gear with Chipped Tooth: Figure 7(b) shows the TFA plot for the data set representing the gear with the chipped tooth. It shows 2 peaks since the time-frequency domain average covers 2 rotation of the gear holding the chipped tooth. Since chipped tooth produces backlash, the peaks at the gear meshing frequency are accompanied by new parallel peaks that generate due to the difference in the angular distance traversed by the input and the output shaft at the time of contact of the chipped tooth. Therefore the spread of the peaks are larger than in the normal situation. If the plot is observed closely it may be noticed that the peaks of the gear meshing frequency at the scale of 6 units are quite separate from the second line of parallel peaks at the scale of 9 units.

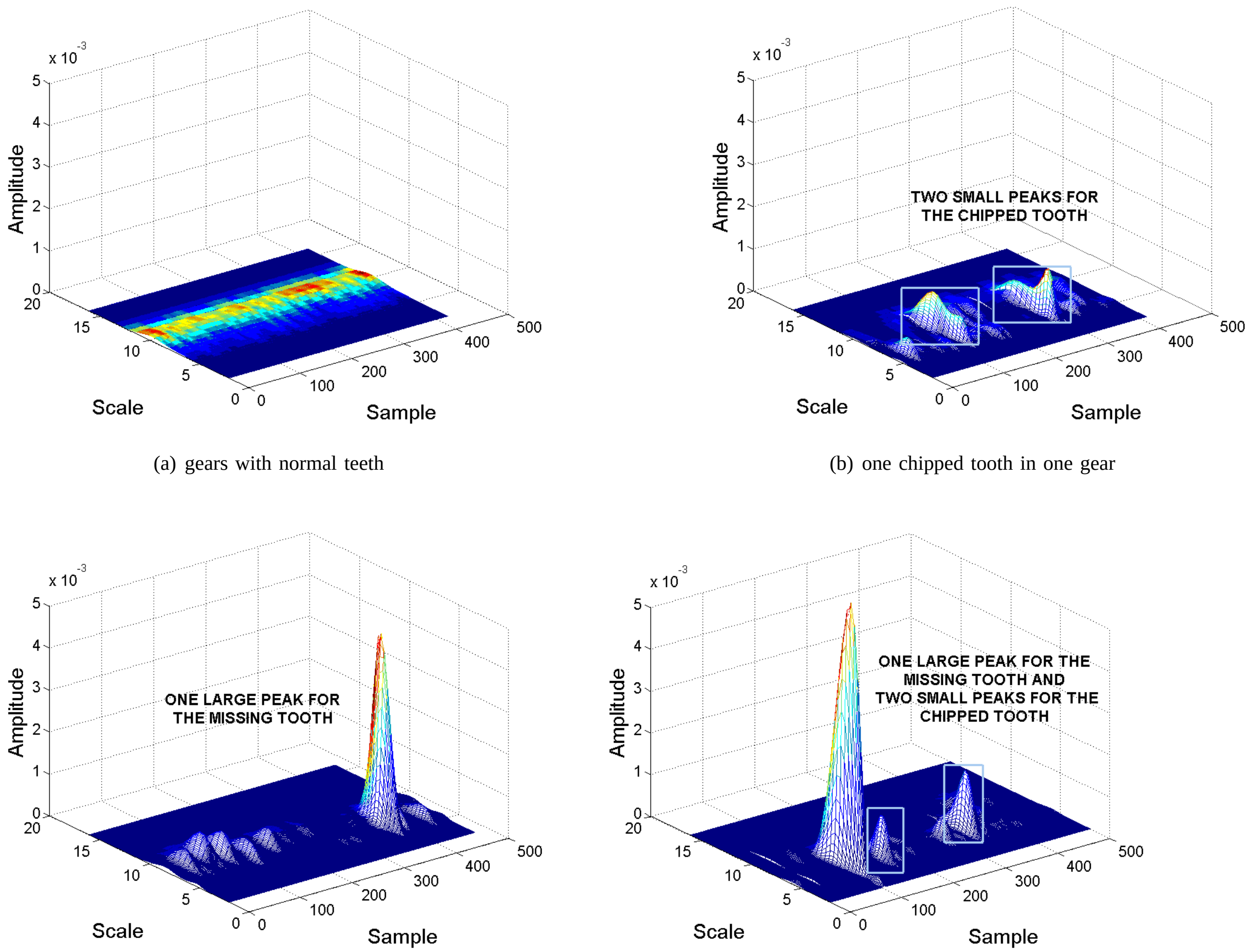
3) Gear with Missing Tooth: Figure 7(c) shows the TFA plot for the data set representing the gear with the missing tooth. The plot shows one large peak. The time-frequency domain average covers one rotation of the gear containing the missing tooth and therefore the peak corresponds to the missing tooth in the gearbox.

4) Gear with Both Chipped Tooth and Missing Tooth: Figure 7(d) shows the TFA plot for the data set representing the gear with one chipped tooth and one missing tooth. It shows one large peak for the missing tooth. Also two smaller peaks can be seen for the chipped tooth as the time-frequency domain average covers two rotations of the gear holding the chipped tooth. The two smaller peaks have a larger spread on the scale axis (from 4 to 10 units in scale) though the presence of the large peak due to the missing tooth suppresses the smaller peaks and makes them difficult to be distinguished.

The position of the peaks in the time axis depends on the position of the gear at the time the data collection is initiated. Therefore the large peak in Figure 7(c) has shifted its position in Figure 7(d). The plots of the time-frequency domain average of the data sets clearly indicate the exact faults present in the gearbox. The plots demonstrate that the technique is not only able to capture large faults like a missing tooth, but is also sensitive to even the slightest chipping in the tooth of a gear. The success of the time-frequency domain average technique on real gearbox data indicates its effectiveness and reliability.

VII. Conclusion

A new technique has been proposed that combines wavelet and time domain averaging to analyze a time series and performs averaging in the time-frequency domain. The strength of the technique lies in the way it preserves the frequency domain information while performing the average, and captures the deterministic part of the periodic signal for one period removing the stochastic part efficiently. Gearbox vibration signals are usually periodic and noisy. Time-frequency domain average technique successfully removes the noise from



(a) gears with normal teeth

(b) one chipped tooth in one gear

(c) one missing tooth in one gear

(d) one chipped tooth in one gear and one missing tooth in another

Fig. 7. Plots of the data sets generated from the test rig for case study.

the signal and captures the dynamics of one period of the signal. The presence of fault in any gear of the gearbox gives rise to a peak in the plot of the time-frequency domain average. Missing tooth produces large peak and chipped tooth produces peaks with parallel side peaks at the meshing frequency. Simultaneous multiple faults in the gearbox can be identified by looking at the peaks of the plot of time-frequency domain average. The newly proposed method was successfully evaluated on a pilot plant test rig.

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