

Novel Identification Method from Step Response

Salim Ahmed^a, Biao Huang^{a,1} and Sirish L. Shah^a

^a*Department of Chemical and Materials Engineering, 536 Chemical and Materials Engineering Building, University of Alberta, Edmonton, AB, Canada T6G 2G6*

Abstract

Methods to estimate process model parameters from both open loop and closed loop step responses are proposed. The distinctive feature of the proposed methods is that the model parameters are estimated from a single step test even if the step input is applied when the process is not at steady state. More importantly, the estimation equations are developed in terms of absolute values of variables as opposed to their deviational values. This facilitates direct use of industrial data without preprocessing. The performance of the proposed algorithms is demonstrated via simulations as well as experimental applications.

Key words: System identification, time delay, step response, deviation variable, initial condition.

1 Introduction

Step response based methods are most commonly used for system identification, especially in process industries (Gustavsson, 1973). The idea of step response was first introduced by Küpfmüller (1928) who also proposed the first method to estimate the parameters of a first order plus time delay model (FOPTD) from step response. This graphical technique, described by Oldenbourg and Sartorius (1948) and later by Rake (1980) and Unbehauen and Rao (1987), involves drawing a tangent to the inflection point of the response curve and formed the basis for a number of similar methods both for first and second order models. Strejc (1959) proposed an improvement of Küpfmüller's method in which the parameters are estimated on the basis of two points

¹ author to whom all correspondence should be addressed. E-mail: biao.huang@ualberta.ca, Tel: 1-780-492-9016, Fax:1-780-492-2881

suitably chosen on either side of the flexion point. A large number of such graphical methods are available in the literature and they have been used effectively in real life applications. For details of such methods readers are referred to (Oldenbourg and Sartorius, 1948; Rake, 1980; Seborg *et al.*, 1989; Unbehauen and Rao, 1987). The limitations of the graphical methods have been outlined in (Sundaresan *et al.*, 1978) and some other developments on the graphical method have been reported in (Huang and Clement, 1982; Huang and Huang, 1993; Rangaiah and Krishnaswamy, 1994).

A group of methods that involves estimation of the area under the response curve, has also been the subject of extensive research. Such methods, often termed as the area methods, have been reported in (Bi *et al.*, 1999; Hwang and Lai, 2004; Rake, 1980; Wang and Zhang, 2001). The method by Wang and Zhang (2001) estimates the model parameters and delay simultaneously; it, however, is not applicable for a non-zero initial state. The method by Hwang and Lai (2004) is based on the pulse response; however, it uses data corresponding to one step of the pulse at a time and is applicable for an unsteady initial state. Both of these methods may give multiple estimate of the delay. The method of moments has also emerged as an efficient technique for parameter estimation. Use of different order moments for parameter estimation has been reported in (Ba Hli, 1954). The characteristic area method (Nishikawa *et al.*, 1990) is indeed a variant of the method of moments. An improved method of moment has been proposed in (Ingimundarson and Hagglund, 2000) which is also reported in (Ingimundarson, 2003). The method of moments has been detailed in (Åström and Hägglund, 1995). Identification from step response has also been considered using the Laguerre network in (Wang and Cluett, 1995) and using state variable filter method in (Wang *et al.*, 2004).

For estimation of the open loop process parameters from a closed loop test applying a step change in the set-point, Yuwana and Seborg (1982) proposed a method for FOPTD model under proportional only controller where Padé approximation is used for the delay term. Jutan and RodriguezII (1984) proposed some modification of the method including the approximation of the delay term by a functional form. Refinement of the method has also been proposed in (Lee, 1989; Chen, 1989) and a comparison of performance of these methods was reported in (Taiwo, 1993). The method was extended for SOPTD systems by (Lee *et al.*, 1990). Closed-loop identification using the method of moments has been reported in (Nishikawa *et al.*, 1990). Also Viswanathan and Rangaiah (2000) proposed an optimization technique and Coelho and Barros (2003) proposed an integral equation approach. The method by Yuwana and Seborg (1982) was extended for unstable processes by Kavdia and Chidambaram (1996) but only for P controllers. For unstable processes with a PID controller a method is proposed by Ananth and Chidambaram (1999) that uses the coordinates of the peaks of the underdamped closed loop response curve to estimate the parameters. Identification of two input two output models from

closed loop step responses has been considered in (Li *et al.*, 2005).

A major concern of this article is about two specific issues related to the form of data. First, generally identification methods are developed to deal with variables in deviation form while data from industries are not available in that form. To obtain data in deviation form, the initial steady state values are subtracted from the raw values. However, the initial steady state values are often not known correctly due to two reasons: (i) presence of noise and (ii) often the input is introduced before the system is at the desired steady state. The second issue is whether a method can estimate the parameters in the presence of initial conditions. To the best of knowledge of the authors there is no step response based method available in the literature that can handle non-zero initial conditions. In addition if the input is applied before the system reaches the desired steady state, it is not possible to get the data in deviation form.

To illustrate the problem some industrial data are presented in Fig. 1. The

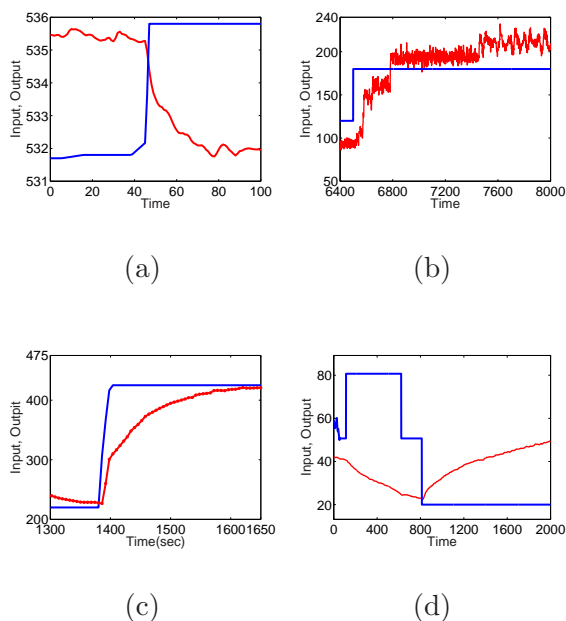


Fig. 1. *Step response of different industrial processes.*

data presented in Fig. 1(a) and 1(b) show that indeed the outputs were at some steady state values before the step inputs were applied. However, due to the presence of noise, it is not easy to determine the steady state value exactly. Figs. 1(c) and 1(d) show other situations when the step inputs were applied before the outputs had reached steady state. For such cases, the initial steady state values are unknown and consequently we cannot convert the data into deviation form.

In this work we present a new approach for identification from step response that (i) uses raw industrial data (ii) is applicable for non-zero initial conditions and estimates the initial conditions and (iii) estimates the parameters and delay simultaneously. So the method allows the use of industrial data without the required preprocessing. Also, as it can estimate the initial conditions along with the process parameters, it is not necessary to bring the process to a steady state before the input is applied.

In some cases, such as for unstable processes, it may not be possible to perform open loop tests. For such situations, we consider an identification method that formulates the estimation equation in terms of the open loop model parameters using closed loop step response and set-point data. Solution of the estimation equation directly gives the open loop model parameters. The remainder of the paper is organized as follows: section 2 describes the mathematical formulation of the identification schemes. Simulation studies are presented in section 3 followed by experimental evaluation in section 4. Conclusions are drawn in section 5.

2 Identification using raw data

2.1 Deviation vs. raw form

First, let us see how data in deviation form are obtained from the raw data. Here the subscript (\bullet_r) denotes the corresponding variable in raw form and variables without the subscript are in deviation form. These two quantities are related to as follows

$$y(t) = y_r(t) - y_{ss} \quad (1)$$

where, y_{ss} is the steady state value of the output corresponding to the steady state value of the input before the step is applied. Fig. 2 describes the real and deviation form graphically. To get the variable in deviation form we need to know the value y_{ss} which, as mentioned earlier, is sometimes difficult to measure or may be unknown. However, if we consider y_{ss} as unknown we can simply write

$$y(t) = y_r(t) - q \quad (2)$$

where, q is the initial unknown steady state value of the output. Taking Laplace transform on both sides, we get

$$Y(s) = Y_r(s) - \frac{q}{s} \quad (3)$$

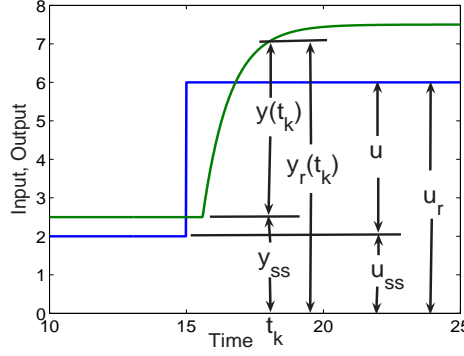


Fig. 2. Variables in deviation and real form.

2.2 Open loop identification

A new identification method is proposed that uses the data in raw form. The basic idea is to consider the initial steady state as another unknown parameter in the estimation equation. Also the method estimates the initial conditions along with the model parameters. The necessary equations are first derived in terms of the deviation variables. Later using eqn(3) the estimation equation will be presented in terms of the raw form of the variables. To describe the necessary mathematical formulation, let us consider a linear single input single output (SISO) system with time delay described by

$$\mathbf{a}_n \mathbf{y}^{(n)}(t) = \mathbf{b}_m \mathbf{u}^{(m)}(t - \delta) + e(t) \quad (4)$$

where,

$$\mathbf{a}_n = [a_n \ a_{n-1} \ \dots \ a_0] \in \mathbb{R}^{1 \times (n+1)} \quad (5)$$

$$\mathbf{b}_m = [b_m \ b_{m-1} \ \dots \ b_0] \in \mathbb{R}^{1 \times (m+1)} \quad (6)$$

$$\mathbf{y}^{(n)}(t) = [y^{(n)}(t) \ y^{(n-1)}(t) \ \dots \ y^{(0)}(t)]^T \in \mathbb{R}^{(n+1) \times 1} \quad (7)$$

$$\mathbf{u}^{(m)}(t - \delta) = [u^{(m)}(t - \delta) \ \dots \ u^{(0)}(t - \delta)]^T \in \mathbb{R}^{(m+1) \times 1} \quad (8)$$

$y^{(i)}$ and $u^{(i)}$ are i -th order time derivatives of y and u , respectively and $e(t)$ is the error term. Taking Laplace transformation on both sides of eqn(4), we can write

$$\mathbf{a}_n \mathbf{s}^n Y(s) = \mathbf{b}_m \mathbf{s}^m U(s) e^{-\delta s} + \mathbf{c}_{n-1} \mathbf{s}^{n-1} + E(s) \quad (9)$$

$Y(s)$, $U(s)$ and $E(s)$ are the Laplace transforms of $y(t)$, $u(t)$ and $e(t)$, respectively, and

$$\mathbf{s}^n = [s^n \ s^{n-1} \ \dots \ s^0]^T \in \mathbb{R}^{(n+1) \times 1} \quad (10)$$

The elements of \mathbf{c}_{n-1} capture the initial conditions and are defined as

$$\mathbf{c}_{n-1} = [c_{n-1} \ c_{n-2} \ \cdots \ c_0] \in \mathbb{R}^{1 \times n} \quad (11)$$

$$c_{n-i} = \mathbf{h}_i \mathbf{y}^{(n-1)}(0), \quad i = 1 \cdots n \quad (12)$$

$$\mathbf{h}_i = [\mathbf{0}^{1 \times (n-i)} \ a_n \cdots a_{n-(i-1)}] \in \mathbb{R}^{1 \times n} \quad (13)$$

$$\mathbf{y}^{(n-1)}(0) = [y^{(n-1)}(0) \ y^{(n-2)}(0) \ \cdots \ y(0)]^T \quad (14)$$

Next, we will devise a linear filter method for the estimation of the parameters. Different filter structures have been proposed in the literature for parameter estimation of continuous-time model using the linear filter approach. We adopt here a filter structure proposed in (Ahmed *et al.*, 2006) that uses a filter having integral dynamics along with a $n - th$ order lag terms. The purpose of using such a structure is to finally formulate an iterative procedure to simultaneously estimate the parameters and the delay. In (Ahmed *et al.*, 2006) the filter has a transfer function $\frac{\beta^n}{s(s+\lambda)^n}$ where the parameters λ and β are to be specified by the user. Here, we use a filter having a transfer function

$$\frac{1}{sA(s)} \quad (15)$$

where, $A(s) = \mathbf{a}_n \mathbf{s}^n$ is the denominator of the process transfer function. \mathbf{a}_n and \mathbf{s}^n have been defined by eqn(5) and (10), respectively. The purpose of including the integral term in the filter is to decouple the delay from the other parameters as shown next, while the role of the part $1/A(s)$ is the same as other linear filters which is to avoid direct derivatives of the noisy signals. Now, if we denote $P(s) = \frac{1}{sA(s)}$ and apply the filtering operation on both sides of eqn(9) we end up with the formulation

$$\mathbf{a}_n \mathbf{s}^n P(s) Y(s) = \mathbf{b}_m \mathbf{s}^m P(s) U(s) e^{-\delta s} + \mathbf{c}_{n-1} \mathbf{s}^{n-1} P(s) + P(s) E(s) \quad (16)$$

Using partial fraction expansion, the transfer function of the filter, $1/sA(s)$, can be expressed as

$$\frac{1}{sA(s)} = \frac{C(s)}{A(s)} + \frac{1}{s} \quad (17)$$

where, $C(s) = -(a_n s^{n-1} + a_{n-1} s^{n-2} + \cdots + a_1)$. Using the notations $\underline{Y}(s) = \frac{Y(s)}{A(s)}$, $\underline{Y}^I(s) = \frac{Y(s)}{s}$ and similar notations for $U(s)$ and then rearranging the estimation equation to give a standard least-square form we get the expression

$$\begin{aligned} \underline{Y}^I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} \underline{Y}(s) + \bar{\mathbf{b}}_m \mathbf{s}^{m-1} \underline{U}(s) e^{-\delta s} \\ & + b_0 [C(s) \underline{U}(s) + U^I(s)] e^{-\delta s} + \mathbf{c}_{n-1} \mathbf{s}^{n-1} P(s) + \xi(s) \end{aligned} \quad (18)$$

where,

$\bar{\mathbf{a}}_n : \mathbf{a}_n$ with its last column removed, $\bar{\mathbf{a}}_n \in \mathbb{R}^{1 \times n}$

$\bar{\mathbf{b}}_m : \mathbf{b}_m$ with its last column removed, $\bar{\mathbf{b}}_m \in \mathbb{R}^{1 \times m}$

Now using eqn(3) we can write the above equation in terms of the raw form of the output y as

$$\begin{aligned} \underline{Y}_r^I(s) - qP^I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} \underline{Y}_r(s) + \bar{\mathbf{a}}_n \mathbf{s}^{n-1} qP(s) + \bar{\mathbf{b}}_m \mathbf{s}^{m-1} \underline{U}(s) e^{-\delta s} \\ & + b_0 [C(s) \underline{U}(s) + U^I(s)] e^{-\delta s} + \mathbf{c}_{n-1} \mathbf{s}^{n-1} P(s) + \xi(s) \end{aligned} \quad (19)$$

For a step input, if the step size is denoted as h , i.e. $u(t) = u_r(t) - u_{ss} = h$, we have

$$U(s) = \frac{h}{s} \quad (20)$$

$$\underline{U}(s) = \frac{U(s)}{A(s)} = \frac{h}{sA(s)} = hP(s) \quad (21)$$

Using eqn(20) and (21) and rearranging eqn(19), we get an estimation equation in the Laplace domain as

$$\begin{aligned} \underline{Y}_r^I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} \underline{Y}_r(s) + h \bar{\mathbf{b}}_m \mathbf{s}^{m-1} P(s) e^{-\delta s} + b_0 \left[hC(s)P(s) + \frac{h}{s^2} \right] e^{-\delta s} \\ & + [\mathbf{c}_{n-1} + \bar{\mathbf{a}}_n q] \mathbf{s}^{n-1} P(s) + qP^I(s) + \xi(s) \end{aligned} \quad (22)$$

Inverse Laplace transform gives the equation in time domain as

$$\begin{aligned} \underline{y}_r^I(t) = & -\bar{\mathbf{a}}_n \underline{\mathbf{y}}_r^{(n-1)}(t) + h \bar{\mathbf{b}}_m \mathbf{P}^{m-1}(t - \delta) + b_0 [hP_c(t - \delta) + h[t - \delta]] \\ & + [\mathbf{c}_{n-1} + \bar{\mathbf{a}}_n q] \mathbf{P}^{n-1}(t) + qP^I(t) + \zeta(t) \end{aligned} \quad (23)$$

The term $\mathbf{P}_n(t)$ contains the impulse response of the filter and is defined as

$$\mathbf{P}^n(t) = [P_n(t) \cdots P_0(t)]^T \in \mathbb{R}^{(n+1) \times 1} \quad (24)$$

$$P_i(t) = L^{-1} [s^i P(s)] \quad (25)$$

$$P^I(t) = L^{-1} \left[\frac{P(s)}{s} \right] \quad (26)$$

$$P_c(t) = L^{-1} [P(s)C(s)] \quad (27)$$

Now for the step input

$$h[t - \delta] = ht - h\delta \quad (28)$$

Applying eqn(28) in eqn(23) and rearranging it we get an estimation equation in least-squares form as

$$\begin{aligned} \underline{y}_r^I(t) = & -\bar{\mathbf{a}}_n \underline{\mathbf{y}}_r^{n-1}(t) + h \bar{\mathbf{b}}_m \mathbf{P}_+^{m-1}(t - \delta) \\ & - b_0 \delta h + [\mathbf{c}_{n-1} + \bar{\mathbf{a}}_n q] \mathbf{P}^{n-1}(t) + qP^I(t) + \zeta(t) \end{aligned} \quad (29)$$

where,

$$\mathbf{P}_+^{m-1}(t - \delta) = \begin{bmatrix} \mathbf{P}^{m-1}(t - \delta) \\ P_c(t - \delta) + t \end{bmatrix} \quad (30)$$

Or equivalently

$$\gamma(t) = \phi(t)\theta + \zeta(t) \quad (31)$$

where,

$$\gamma(t) = \underline{y}_r^I(t) \quad (32)$$

$$\phi(t) = \begin{bmatrix} -\underline{\mathbf{y}}_r^{(n-1)}(t) \\ h\mathbf{P}_+^{m-1}(t - \delta) \\ -h \\ \mathbf{P}^{n-1}(t) \\ P^I(t) \end{bmatrix} \quad (33)$$

$$\theta = [\bar{\mathbf{a}}_n \quad \mathbf{b}_m \quad b_0\delta \quad \mathbf{c}_{n-1} + \bar{\mathbf{a}}_n q \quad q] \quad (34)$$

Eqn(31) can be written for $t = t_k, k = 1, 2 \dots N$, where N is the total number of available data points, and combined to give the estimation equation as

$$\mathbf{\Gamma} = \mathbf{\Phi}\theta + \zeta \quad (35)$$

2.3 Identification under closed-loop conditions

Due to safety or economic reasons it may not be always possible to open control loops for identification. Also for unstable and marginally stable processes open loop test is not a practical option. In this section a closed loop identification method based on a step change in the set-point is introduced. The method directly estimates the parameters of the open loop transfer function model along with the time delay. We assume here that the controller is completely known.

For a process model described by eqn(4) and for a known controller, $K(s)$, the fundamental relation between the output and set-point for an initial steady state condition of the set-point can be described by

$$Y(s) = \frac{\mathbf{b}_m \mathbf{s}^m K(s) e^{-\delta s}}{\mathbf{a}_n \mathbf{s}^n + \mathbf{b}_m \mathbf{s}^m K(s) e^{-\delta s}} R(s) + \frac{\mathbf{c}_{n-1} \mathbf{s}^{n-1} + \mathbf{d}_{m-1} \mathbf{s}^{m-1} e^{-\delta s}}{\mathbf{a}_n \mathbf{s}^n + \mathbf{b}_m \mathbf{s}^m K(s) e^{-\delta s}} + W(s) \quad (36)$$

where, $R(s)$ is the Laplace transform of the set-point, $r(t)$, and $W(s)$ is the error term. Now, in equation error form the closed loop expression relating

the output to the set-point can be expressed as

$$\mathbf{a}_n \mathbf{s}^n Y(s) = \mathbf{b}_m \mathbf{s}^m K(s) e^{-\delta s} [R(s) - Y(s)] + \mathbf{c}_{n-1} \mathbf{s}^{n-1} + \mathbf{d}_{m-1} \mathbf{s}^{m-1} e^{-\delta s} + V(s) \quad (37)$$

where,

$$\mathbf{d}_{m-1} = [d_{m-1} \ d_{m-2} \ \cdots \ d_0] \in \mathbb{R}^{1 \times m} \quad (38)$$

$$d_{m-i} = \mathbf{g}_i \mathbf{y}^{(m-1)}(0), \quad i = 1 \cdots m \quad (39)$$

$$\mathbf{g}_i = [\mathbf{0}^{1 \times (m-i)} \ b_m \cdots b_{m-(i-1)}] \in \mathbb{R}^{1 \times m} \quad (40)$$

$$\mathbf{y}^{(m-1)}(0) = [y^{(m-1)}(0) \ y^{(m-2)}(0) \ \cdots \ y(0)]^T \quad (41)$$

Other terms of the equation have been defined previously. Now applying the filtering operation on both the output and the set-point with the filter $P(s) = \frac{1}{sA(s)}$ and rearranging the equation we get

$$\begin{aligned} \underline{Y}^I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} \underline{Y}(s) + \bar{\mathbf{b}}_m \mathbf{s}^{m-1} K(s) [\underline{R}(s) - \underline{Y}(s)] e^{-\delta s} \\ & + b_0 K(s) [\underline{R}^I(s) - \underline{Y}^I(s)] e^{-\delta s} + \mathbf{c}_{n-1} \mathbf{s}^{n-1} P(s) \\ & + \mathbf{d}_{m-1} \mathbf{s}^{m-1} P(s) e^{-\delta s} + \epsilon(s) \end{aligned} \quad (42)$$

The controller transfer function $K(s)$ is different for different controller structures. For a PID controller we can write

$$K(s) = K_p + K'(s) \quad (43)$$

where, $K'(s) = \frac{K_I}{s} + K_D s$ with K_p , K_I and K_D are the proportional, integral and derivative constants, respectively. For P only controller $K'(s) = 0$ and for PI controller $K'(s) = \frac{K_I}{s}$. Following these notations and using eqn(17) we get

$$\begin{aligned} K(s) \underline{R}^I(s) = & K(s) \frac{R(s)}{sA(s)} \\ = & K_P [R^I(s) + C(s) \underline{R}(s)] + K'(s) \underline{R}^I(s) \end{aligned} \quad (44)$$

For a step change in setpoint of magnitude h , $R(s) = \frac{h}{s}$. Applying eqn(44), we can write eqn(42) for a step change in the set-point as

$$\begin{aligned} \underline{Y}^I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} \underline{Y}(s) + \bar{\mathbf{b}}_m \mathbf{s}^{m-1} K(s) [hP(s) - \underline{Y}(s)] e^{-\delta s} \\ & + b_0 \left[K_P h/s^2 + K_P hC(s)P(s) + K'(s)hP(s)/s - K(s) \underline{Y}^I(s) \right] e^{-\delta s} \\ & + \mathbf{c}_{n-1} \mathbf{s}^{n-1} P(s) + \mathbf{d}_{m-1} \mathbf{s}^{m-1} P(s) e^{-\delta s} + \epsilon(s) \end{aligned} \quad (45)$$

In closed loop operation, for a proper controller structure the overall gain of the loop is unity. So in this case it becomes straightforward to get the data in

deviation form from their raw values and we can write eqn(2) as

$$Y(s) = Y_r(s) - \frac{h_0}{s} \quad (46)$$

where, h_0 is the initial steady state value of the input. Using eqn(46) we can write eqn(45) after rearrangement as

$$\begin{aligned} \underline{Y}_r^I(s) - h_0 P^I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} [\underline{Y}_r(s) - h_0 P(s)] \\ & + \bar{\mathbf{b}}_m \mathbf{s}^{m-1} K(s) [hP(s) - (\underline{Y}_r(s) - h_0 P(s))] e^{-\delta s} \\ & + b_0 [K_P h/s^2 + K_P h C(s) P(s) + h K'(s) P^I(s) \\ & - K(s) \underline{Y}_r^I(s) + h_0 K(s) P^I(s)] e^{-\delta s} \\ & + \mathbf{c}_{n-1} \mathbf{P}^{n-1}(s) + \mathbf{d}_{m-1} \mathbf{P}^{m-1}(s) e^{-\delta s} + \epsilon(s) \end{aligned} \quad (47)$$

Taking inverse Laplace transform the above equation can be expressed in time domain at any sampling instant t as

$$\begin{aligned} \underline{y}_r^I(t) - h_0 P^I(t) = & -\bar{\mathbf{a}}_n [\underline{\mathbf{y}}_r^{(n-1)}(t) - h_0 \mathbf{P}^{n-1}(t)] \\ & + \bar{\mathbf{b}}_m [(h + h_0) \mathbf{P}_K^{m-1}(t - \delta) - \underline{\mathbf{y}}_{r_K}^{(m-1)}(t - \delta)] \\ & + b_0 [K_P h[t - \delta] + K_P h P_c(t - \delta) + h P_{K'}^I(t - \delta) \\ & + h_0 P_K^I(t - \delta) - \underline{y}_{r_K}^I(t - \delta)] \\ & + \mathbf{c}_{n-1} \mathbf{P}^{n-1}(t) + \mathbf{d}_{m-1} \mathbf{P}^{m-1}(t - \delta) + \varepsilon(t) \end{aligned} \quad (48)$$

Here, $P_K(t) = L^{-1}[K(s)P(s)]$ and other similar terms are defined in the same way. Now using the equation $h[t - \delta] = ht - h\delta$ we can get the estimation equation in a least-square form as

$$\gamma_+(t) = \phi_+^T(t) \theta_+ + \varepsilon(t) \quad (49)$$

where,

$$\gamma_+(t) = \underline{y}_r^I(t) - h_0 P^I(t) \quad (50)$$

$$\phi_+(t) = \begin{bmatrix} -\underline{\mathbf{y}}_r^{(n-1)}(t) + h_0 \mathbf{P}^{n-1}(t) \\ h \mathbf{P}_K^{m-1}(t - \delta) - \underline{\mathbf{y}}_r^{(m-1)}(t - \delta) \\ \Omega(t) - \underline{y}_{r_K}^I(t - \delta) \\ -K_P h \\ \mathbf{P}^{n-1}(t) \\ \mathbf{P}^{m-1}(t - \delta) \end{bmatrix} \quad (51)$$

$$\Omega(t) = K_P h t + K_P h P_c(t - \delta) + h P_{K'}^I(t - \delta) + h_0 P_K^I(t - \delta) \quad (52)$$

$$\theta_+ = [\bar{\mathbf{a}}_n \quad \bar{\mathbf{b}}_m \quad b_0 \quad b_0 \delta \quad \mathbf{c}_{n-1} \quad \mathbf{d}_{m-1}] \quad (53)$$

Eqn(49) can be written for $t = t_k, k = 1, 2 \dots N$, where N is the total number of available data points. To formulate the estimation equation for $t_k < \delta$, we need output data before the step input is applied. Hence we suggest recording some output data before the setpoint is changed. Combination of the N equations gives the estimation equation as

$$\mathbf{\Gamma}_+ = \mathbf{\Phi}_+ \theta_+ + \varepsilon \quad (54)$$

2.4 Parameter estimation

The parameter vector can be obtained by solving eqn(35) for open loop data or eqn(54) for closed loop setpoint and output data. However, there are two problems associated with the solution. First, for both of the cases we need to know $A(s)$ and δ , which are of course unknowns. This problem can be solved by applying an iterative procedure that adaptively adjust an initial estimate of $A(s)$ and δ until they converge. Second, the least-square solution does not give unbiased estimate in the presence of general forms of measurement noise such as colored noise. To solve the bias problem, a popular bias elimination procedure namely the instrumental variable (IV) method can be used. A bootstrap estimation of IV type where the instrumental variable is built from an auxiliary model (Young, 1970) is considered here. For the open loop method the instrumental variable is defined as

$$\psi(t) = \begin{bmatrix} -\hat{\underline{\mathbf{y}}}_r^{(n-1)}(t) \\ \phi(n : 2n + m + 2, 1) \end{bmatrix} \quad (55)$$

where, ϕ has been defined in eqn(33), $\hat{y}_r(t) = \hat{y}(t) + \hat{q}$, $\hat{y}(t) = L^{-1}[\hat{Y}(s)]$ and

$$\hat{Y}(s) = \frac{\hat{\mathbf{b}}_m \mathbf{s}^m}{\hat{\mathbf{a}}_n \mathbf{s}^n} U(s) e^{-\delta s} + \frac{\hat{\mathbf{c}}_{n-1} \mathbf{s}^{n-1}}{\hat{\mathbf{a}}_n \mathbf{s}^n} \quad (56)$$

For identification under closed loop conditions, following the above procedure the instrument matrix is obtained by replacing the $y(t)$ in $\phi_+(t)$ by $\hat{y}(t)$ i.e.,

$$\psi_+(t) = \begin{bmatrix} -\hat{\mathbf{y}}_{\mathbf{r}}^{(n-1)}(t) + h_0 \mathbf{P}^{n-1}(t) \\ h \mathbf{P}_K^{m-1}(t - \delta) - \hat{\mathbf{y}}_{\mathbf{r}_K}^{(m-1)}(t - \delta) \\ \Omega(t) - \hat{y}_{r_K}^I(t - \delta) \\ \phi_+(n + m + 2 : 2n + m + 2, 1) \end{bmatrix} \quad (57)$$

where, ϕ_+ has been defined in eqn(51), $y_r(t) = \hat{y}(t) + h_0$ and

$$\hat{Y}(s) = \frac{\hat{\mathbf{b}}_m \mathbf{s}^m K(s) e^{-\delta s}}{\hat{\mathbf{a}}_n \mathbf{s}^n + \hat{\mathbf{b}}_m \mathbf{s}^m K(s) e^{-\delta s}} R(s) + \frac{\hat{\mathbf{c}}_{n-1} \mathbf{s}^{n-1} + \hat{\mathbf{d}}_{m-1} \mathbf{s}^{m-1} e^{-\delta s}}{\hat{\mathbf{a}}_n \mathbf{s}^n + \hat{\mathbf{b}}_m \mathbf{s}^m K(s) e^{-\delta s}} \quad (58)$$

The iterative IV scheme can be embedded within the iteration steps of the proposed method and no additional step is required.

From θ or θ_+ we directly get the parameters $\bar{\mathbf{a}}_n$, \mathbf{b}_m , δ , q and \mathbf{c}_{n-1} . To retrieve $\mathbf{y}^{(n-1)}(0)$ from \mathbf{c}_{n-1} , eqn(12) can be written for $i = 1 \dots n$ to give

$$(\mathbf{c}_{n-1})^T = \mathbf{H} \mathbf{y}^{(n-1)}(0) \quad (59)$$

where, $\mathbf{H} = [(\mathbf{h}_1)^T (\mathbf{h}_2)^T \dots (\mathbf{h}_n)^T]^T \in \mathbb{R}^{n \times n}$. Finally

$$\mathbf{y}^{(n-1)}(0) = (\mathbf{H})^{-1} (\mathbf{c}_{n-1})^T \quad (60)$$

The iterative procedure for parameter estimation is summarized below as Algorithm 1.

2.5 Convergence of the iterative scheme

Extensive simulation study shows that the iterative procedure converges monotonically except for processes showing inverse response. Also the effect of the initial conditions on the response curve is similar to the inverse response for some cases. For both of these cases the iteration scheme diverges monotonically. To make the diverging scheme converge we suggest the procedure proposed in (Ahmed *et al.*, 2006) where the incremental change in δ is defined as $\Delta\delta = \delta_{i-1} - \delta_i$ and in the $(i + 1)$ -th stage of iteration the initial estimate is

Algorithm 1: Iterative procedure for parameter and delay estimation.

Step 1 - Initialization: Choose an initial estimate $A_0(s)$ and δ_0 .

Step 2 - LS step: $i=1$ Construct $\mathbf{\Gamma}$ (or $\mathbf{\Gamma}_+$) and $\mathbf{\Phi}$ (or $\mathbf{\Phi}_+$) by replacing $A(s)$ and δ by $A_0(s)$ and δ_0 and get the LS solution of θ as

$$\hat{\theta}^{LS} = (\mathbf{\Phi}^T \mathbf{\Phi})^{-1} \mathbf{\Phi}^T \mathbf{\Gamma} \quad (61)$$

Or

$$\hat{\theta}_+^{LS} = (\mathbf{\Phi}_+^T \mathbf{\Phi}_+)^{-1} \mathbf{\Phi}_+^T \mathbf{\Gamma}_+ \quad (62)$$

$\hat{\theta}^1 = \hat{\theta}^{LS}$ or $(= \hat{\theta}_+^{LS})$. Get $\hat{A}_1(s)$, the process numerator $\hat{B}_1(s)$ and $\hat{\delta}_1$ from $\hat{\theta}^1$.

Step 3 - IV step: $i = i+1$. Construct $\mathbf{\Gamma}$, $\mathbf{\Phi}$ and $\mathbf{\Psi}$ or their closed loop equivalents for closed loop data by replacing $A(s)$, $B(s)$ and δ by $\hat{A}_{i-1}(s)$, $\hat{B}_{i-1}(s)$ and $\hat{\delta}_{i-1}$ and get the IV solution of θ as

$$\hat{\theta}^i = (\mathbf{\Psi}^T \mathbf{\Phi})^{-1} \mathbf{\Psi}^T \mathbf{\Gamma} \quad (63)$$

or

$$\hat{\theta}_+^i = (\mathbf{\Psi}_+^T \mathbf{\Phi}_+)^{-1} \mathbf{\Psi}_+^T \mathbf{\Gamma}_+ \quad (64)$$

Obtain $\hat{A}_i(s)$, $\hat{B}_i(s)$ and $\hat{\delta}_i$ from $\hat{\theta}^i$ and repeat **step 3** until \hat{A}_i and $\hat{\delta}_i$ converge.

Step 4 - Termination: When \hat{A}_i and $\hat{\delta}_i$ converge, the corresponding $\hat{\theta}^i$ is the final estimate of parameters and includes estimates of q , δ and the initial conditions.

taken as $\delta_i + \Delta\delta$. For detail of this procedure readers are referred to (Ahmed *et al.*, 2006).

2.6 Choice of A_0 and δ_0

The initiation of the iteration procedure involves choice of A_0 and δ_0 . In theory, there is no constraint on the choice of A_0 except that the filter should not be unstable. Moreover, as the filter is updated in every step, the final estimate of the parameters is found to be not much sensitive to the initial choice. Nevertheless, to provide a good initial guess, we suggest to choose A_0 based on process information. If we have an estimate of the process cut-off frequency, λ , we suggest choosing $A_0 = \frac{1}{(s+\lambda)^n}$. Similarly, for δ_0 a choice based on process information would save computation. In case where process information is unavailable we suggest choosing a small positive value for δ_0 .

3 Simulation Study

3.1 Open loop identification

3.1.1 Example 1: Effect of initial condition

To demonstrate the effect of initial condition on the estimation of the process model, a first order process having the following transfer function is used

$$G(s) = \frac{1.25}{20s + 1} e^{-7s} \quad (65)$$

Figure 3 shows the response of the process to three successive steps. At the beginning of every steps the process is very close to steady state conditions. Data presented here are free of noise. For noisy data it is even harder to determine the steady state condition. Now to apply most of the methods available in literature, a preprocessing of the data is required. An approximated steady state value is first subtracted from the raw measurements to get the data in deviation form. To show the effect of this preprocessing, we will present results obtained using the MATLAB SYSID Toolbox. Besides other requirements such as regular sampling, SYSID Toolbox can handle data only in deviation form. If an initial steady state value is estimated from the data and data are preprocessed using that value, for the three steps, the toolbox gives three different models. Fig. 4(a) shows the step response of the three models estimated using the data from the three steps. From the figure we see that although the process reached very close to the steady state value before the steps were applied, the estimated models differ from the true model as well as among themselves. In particular the gain values are different. On the other hand, if we use the proposed method, we get three models having almost the same parameters. The step responses of the three models estimated using the propose method are shown in Fig. 4(b). Here we see that the responses coincide and overlap with the step response of the true process. It is worthwhile to mention here that in MATLAB there are some options to preprocess the data e.g., to remove mean and to choose a particular segment of data. The often used quick start option indeed detrends the data and chooses a segment of the step response to do the identification. Though we are not presenting here any results using MATLAB's preprocessing steps, the SYSID toolbox gave very different models when the data from the three steps were used and the data were preprocessed using MATLAB's quick start option. Simply removing the means also fails to produce consistent results for step response based identification.

Figure 5 shows the step responses of the same process when it is initially far away from the steady state conditions. It is readily understandable that any sort of preprocessing by subtraction of an approximated steady state value

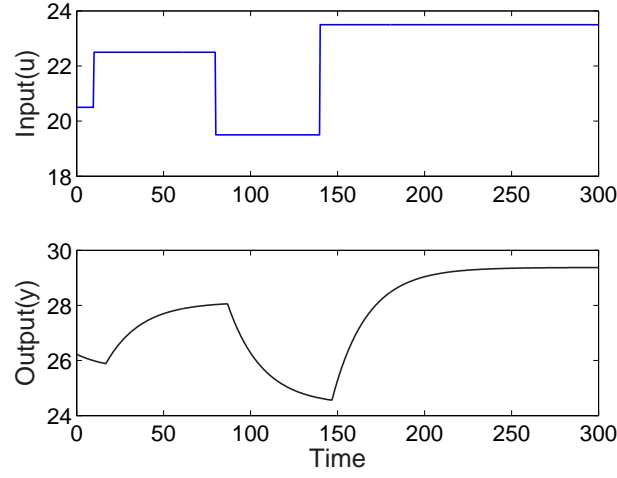


Fig. 3. Output response of the process considered in example 1 to three successive steps in the input.

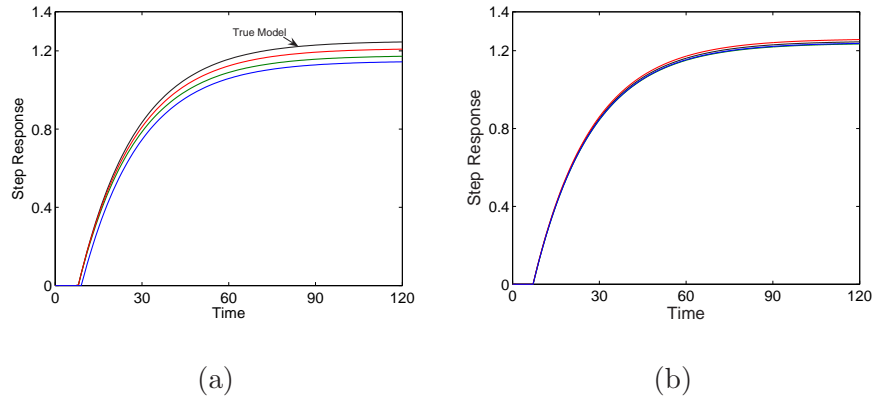


Fig. 4. Step response of the estimated models (a) MATLAB SYSID Toolbox (b) proposed method (example 1)

may produce misleading results. In some cases the estimated gain may even have a wrong sign. However, using the proposed method we get models whose step responses coincide with that of the true process as shown in Fig. 6.

3.2 Identification under closed loop condition

3.2.1 Example 2: Unstable process

We consider here a second order process with a PI controller with the process transfer function and controller as

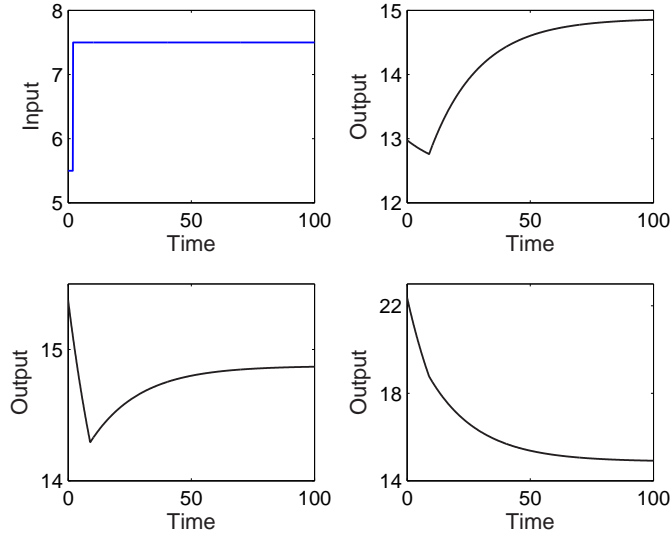


Fig. 5. Step response from initial conditions far away from steady state (example 1).

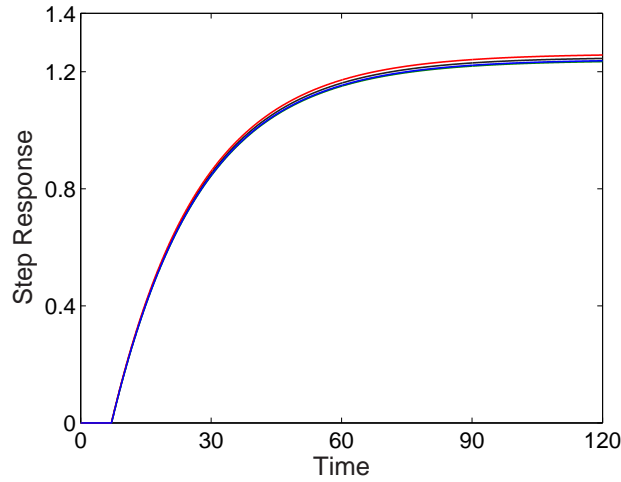


Fig. 6. Step response of the models using data when process initially at far away from steady state (example 1).

$$G(s) = \frac{s + 1}{s^2 + s - 2} e^{-0.04s} \quad (66)$$

$$K(s) = 10 + \frac{15}{s} \quad (67)$$

This open loop unstable process has been considered in (Garnier *et al.*, 2000), however, without any delay. The sampling interval was set to 1 ms. Figures 7(a) and 7(b) show the closed loop response of the unstable process and that of the estimated model for the same controller. The identification data set shows that the process was at an unsteady state when the step change was made in the set-point. The validation data shows that the method gives a good estimate of the model parameters in the presence of initial condition. To study

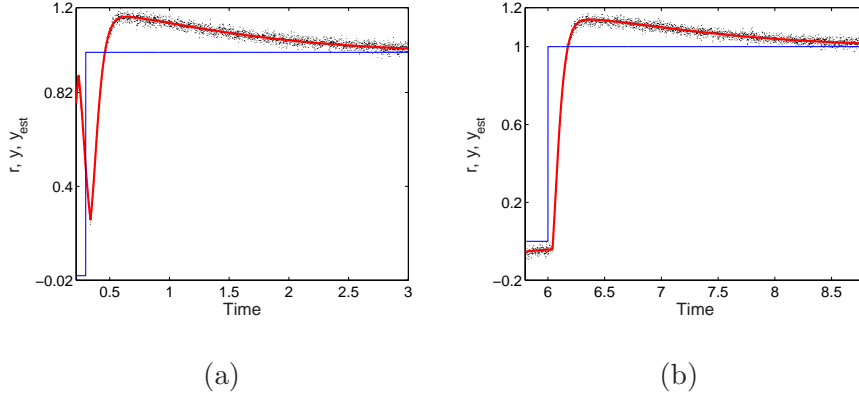


Fig. 7. Closed loop step response of the process and model. (a)Identification data (b) Validation data (example 2).

the effect of noise on the parameter estimates, 100 Monte Carlo simulations are carried out for a NSR of 10%. For this study a zero initial condition was assumed. Figures 8a and 8b show the Bode diagram of the 100 estimated

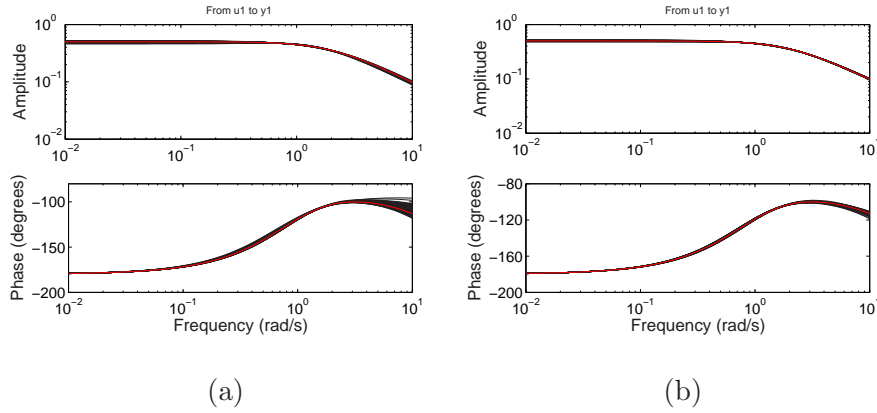


Fig. 8. Bode diagram of the 100 Monte Carlo estimates (a)Least square estimates (b) Instrumental variable estimates (example 2).

model for both least square (LS) and instrumental variable (IV) estimation. It is seen that although the quality of the estimates is satisfactory for both cases the IV estimates are better than the LS estimates.

3.2.2 Example 3: Nonlinear Unstable Bioreactor

Continuous bioreactors are typical nonlinear unstable processes for which open loop identification is not possible. The processes have significant time delays arising from the measurement procedure. In this example a linear transfer function model with time delay is estimated from a closed loop step response of the nonlinear model of a bioreactor. The following dynamic equations, steady

state models and the corresponding parameters of a bioreactor are considered:

$$\frac{dx}{dt} = (\mu - D)x \quad (68)$$

$$\frac{ds}{dt} = D(s_F - s) - \frac{\mu}{y_0}x \quad (69)$$

$$\mu = \frac{\mu_m s}{K_s + s + s^2/K_i} \quad (70)$$

Here, x and s are the concentrations of the cell and substrate, respectively, μ is the specific growth rate, μ_m is the maximum specific growth rate, y_0 is the yield, K_s and K_i are the constants of the substrate inhibition model and D is the dilution rate which is the manipulated variable to control the concentration of cell in the reactor. The values of the parameters are

$$s_F = 4g/g \quad \mu_m = 0.53h^{-1} \quad y_0 = 0.4g/g \quad K_s = 0.12g/g$$

A delay of $1h$ is considered in the measurement of x . The reactor exhibits an unstable steady state at $(x = 0.9951, s = 1.5122)$ for a nominal value of dilution rate $D = 0.36h^{-1}$. The closed loop response of x , for a change in the setpoint from 0.9951 to 1.1941, is obtained for a PID controller $k_c(1 + \frac{1}{\tau_I s} + \tau_D s)$ with $k_c = -0.7356$, $\tau_I = 4$ and $\tau_D = 0.2$. Here to avoid derivative kick, the derivative action is applied only to the output and not to the error signal. The model and model parameters of the bioreactor are taken from (Ananth and Chidambaram, 1999; Agrawal and Lim, 1986) A first order plus time delay

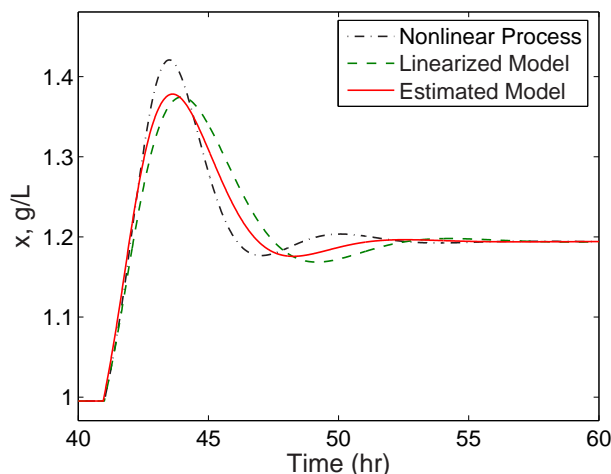


Fig. 9. Closed loop step response of the nonlinear bioreactor model, linearized model and the estimated linear model (example 3).

model with an unstable pole was estimated from the measured closed loop response. The closed loop response of the estimated model is plotted along with the closed loop response of the nonlinear model in Fig. 9. The figure also



Fig. 10. Photograph of the mixing process.

shows the closed loop response of the linearized model given in (Ananth and Chidambaram, 1999). It can be seen from the figure that the match between the response of the estimated model and that of the nonlinear model is better than the match between the response of the linearized and nonlinear model.

4 Experimental evaluation

4.1 Open loop identification

A number of step tests from different unsteady initial conditions are performed in a laboratory scale mixing process. The set-up consists of a continuous stirred tank used as a mixing chamber having two input streams fed from two feed tanks. A salt solution and pure water run from the feed tanks and mixed together in the mixing chamber. A constant volume and a constant temperature of the solution in the mixing tank are maintained. Also the total inlet flow is kept constant. The input to the process is the flow rate of the salt solution as fraction of total inlet flow. The output is the concentration of salt in the mixing tank. We assume here a uniform concentration throughout the solution in the tank. The concentration is measured in terms of the electrical conductivity of the solution.

Figures 11a, 11b and 11c show the concentration profile of the salt solution in the tank resulting from a step change in the feed flow rate. It can be seen from the figures that the initial concentration in the tank is not at steady state when the step changes are made. Figure 12 show the step response and frequency responses of the three models estimated from the three sets of data. It can be concluded from the responses that the different data sets result almost the

same estimated models.

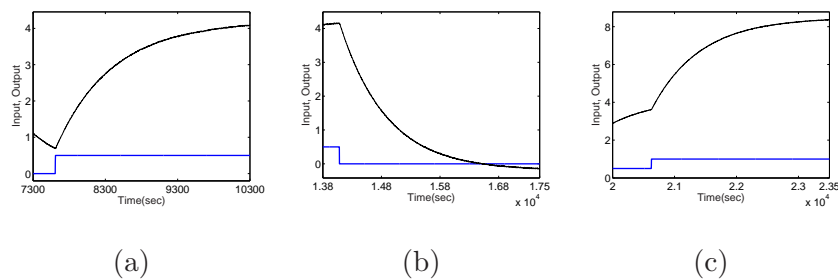


Fig. 11. Step response of the mixing process with different initial conditions.

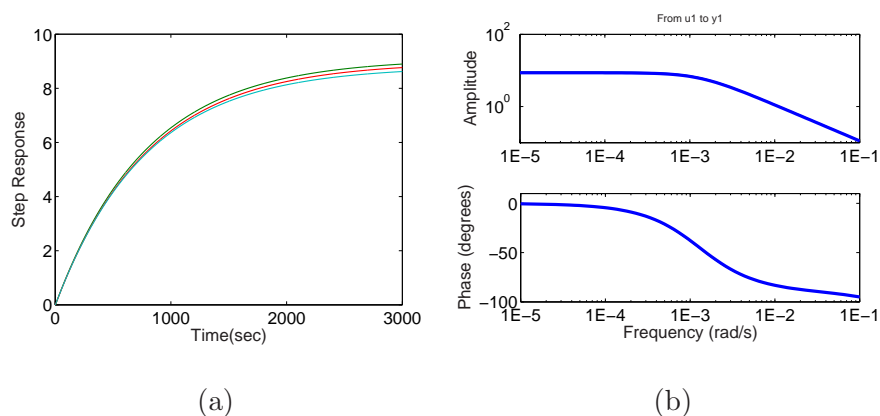


Fig. 12. Step and frequency responses of the three identified model of the mixing process.

4.2 Identification under closed loop conditions

The proposed closed loop identification technique is applied for the identification of a continuous stirred tank heating (CSTH) process. A photograph of the process is shown in figure 13. The cylindrical glass tank is equipped with steam coil with a controlled input facilitating the manipulation of steam flow to control temperature of water in the tank. Also the level of water is controlled by manipulating the inlet water flow. The water outlet and condensate flow is controlled only manually. A number of thermocouples are placed at different distance of the tank outlet flow line that introduce time delay in the system. The set-up is under Emersons Delta-V distributed control system (DCS).

For the purpose of this exercise, the set-point for the temperature of water in the tank is changed from $30^{\circ}C$ to $40^{\circ}C$ and the temperature of the outlet water



Fig. 13. Part of the CSTH process.

was measured and recorded at 5 seconds intervals. A PI controller having a gain of 4.85 and reset time of 100 seconds was in place for the control loop to manipulate the steam valve. The level of water in the tank was controlled to be constant. A second order plus time delay (SOPTD) model was estimated as the open loop transfer function between temperature and steam flow. Fig.

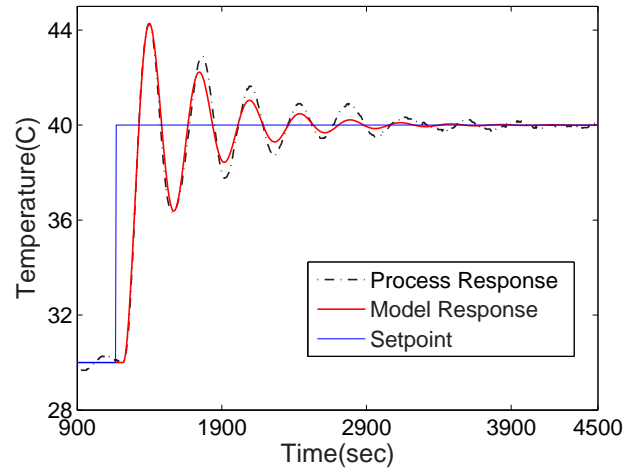


Fig. 14. Closed loop response of the heating tank process and its estimated model for a step change in steam flow.

14 shows the closed loop response of the process and the estimated model for the PI controller. It can be concluded that the two responses match quite well.

5 Conclusion

Identification from step response is a popular and commonly used method. In spite of this there are challenges in applying these methods in real life implementations. In this article two specific issues have been addressed. Detailed mathematical derivations have been presented to show how, under both open loop and closed loop framework, process model parameters and the delay can be estimated from raw data even when the process is not initially at steady state. Formulation of the estimation equation in terms of raw form of the variables is a unique feature of the proposed algorithms that allows the use of industrial data without much preprocessing. Through simulations, the applicability of the methods has been demonstrated for a diverse group of processes. Finally the performance of the methods is evaluated by experimental data under both open loop and closed loop conditions.

References

- Agrawal, P. and H. C. Lim (1986). Analysis of various control schemes for continuous bioreactors. *Advances in Biochemical Engineering/Biotechnology* **30**, 61–90.
- Ahmed, S., B. Huang and S. L. Shah (2006). Parameter and delay estimation of continuous-time models using a linear filter. *Journal of Process Control* **16**(4), 323–331.
- Ananth, I. and M. Chidambaram (1999). Closed-loop identification of transfer function model for unstable systems. *Journal of the Franklin Institute* **336**, 1055–1061.
- Ba Hli, F. (1954). A general method for time domain network synthesis. *IRE Transactions - Circuit Theory* **1**(3), 21–28.
- Bi, Q., W. Cai, E. Lee, Q. G. Wang, C. C. Hang and Y. Zhang (1999). Robust identification of first-order plus dead-time model from step response. *Control Engineering Practice* **7**, 71–77.
- Chen, C. L. (1989). A simple method for online identification and controller tuning. *AIChE Journal* **35**, 2037.
- Coelho, F. S. and P. R. Barros (2003). Continuous-time identification of first-order plus dead-time models from step response in closed loop. *Proc. 13th IFAC symposium on System Identification, Rotterdam, the Netherlands, August 2003* pp. 413–418.
- Garnier, H., M. Gilson and W. X. Zheng (2000). A bias-eliminated least-squares method for continuous-time model identification of closed-loop systems. *International Journal of Control* **73**(1), 38–48.

- Gustavsson, I. (1973). Survey of applications of identification in chemical and physical processes. *Proc. 3rd IFAC Symposium, the Hague/Delft, the Netherlands* pp. 67–85.
- Huang, C. and M. Huang (1993). Estimation of the second-order parameters from the process transients by simple calculation. *Industrial & Engineering Chemistry Research* **32**, 228–230.
- Huang, C. and W. C. Clement (1982). Parameter estimation for the second-order-plus-dead-time model. *Industrial & Engineering Chemistry Process Design Development* **21**, 601–603.
- Hwang, S. and S. Lai (2004). Use of two-stage least-squares algorithms for identification of continuous systems with time delay based on pulse response. *Automatica* **40**, 1591–1568.
- Ingimundarson, A. (2003). Dead-time compensation and performance monitoring in process control. PhD thesis. Dep. Automatic Control, Lund Institute of Technology. Lund, Sweden.
- Ingimundarson, A. and T. Hagglund (2000). Closed-loop identification of a first-order plus dead-time model with method of moments. *Proc. IFAC International Symposium on Advanced Control of Chemical Processes, Pisa, Italy*.
- Jutan, A. and E. S. RodriguezII (1984). Extension of a new method for on-line controller tuning. *The Canadian Journal of Chemical Engineering* **62**, 802–807.
- Kavdia, M. and M. Chidambaram (1996). On-line controller tuning for unstable systems. *Computers & Chemical Engineering* **20**, 301–305.
- Küpfmüller, K. (1928). Über die dynamik der selbsttatigen verstärkungsregler. *ENT* **5**, 459–467.
- Lee, J. (1989). On-line PID controller tuning from a single, closed-loop test. *AIChE Journal* **35**(2), 329–331.
- Lee, J., W. Cho and T.F. Edgar (1990). An improved technique for PID controller tuning from closed-loop test. *AIChE Journal* **36**(12), 1891–1895.
- Li, S. Y., W. J. Cai, H. Mei and Q. Xiong (2005). Robust decentralized parameter identification for two-input two-output processes from closed-loop step responses. *Control Engineering Practice* **13**, 519–531.
- Nishikawa, Y., N. Sannomiya, T. Ohta and H. Tanaka (1990). A method for auto-tuning PID control parameters. *Automatica* **20**(3), 321–332.
- Oldenbourg, R. C. and H. Sartorius (1948). *The Dynamics of Automatic Control*. The American Society of Mechanical Engineers. New York, USA.
- Rake, H. (1980). Step response and frequency response methods. *Automatica* **16**, 519–526.

- Rangaiah, G. P. and P. R. Krishnaswamy (1994). Estimating second-order plus dead time model parameters. *Industrial & Engineering Chemistry Research* **33**, 1867–1871.
- Seborg, D. E., T. F. Edgar and D. A. Mellichamp (1989). *Process Dynamics and Control*. John Wiley & Sons.
- Strejc, V. (1959). Approximation aperiodischer übertragungscharakteristiken. *Regelungstechnik* **7**, 124–128.
- Sundaresan, K. R., C. C. Prasad and P. R. Krishnaswamy (1978). Evaluating parameters from process transients. *Industrial & Engineering Chemistry Process Design and Development* **17**(3), 237–241.
- Taiwo, O. (1993). Comparison of four methods of on-line identification and controller tuning. *IEE Proc.-D: Control Theory and Applications* **140**(5), 323–327.
- Åström, K. J. and T. Hägglund (1995). *PID Controllers: Theory, Design and Tuning*. 2nd ed.. Instrument Society of America. North Carolina.
- Unbehauen, H. and G. P. Rao (1987). *Identification of Continuous Systems*. first ed.. North-Holland.
- Viswanathan, P. K. and G. P. Rangaiah (2000). Process identification from closed-loop response using optimization methods. *Trans. IChemE - Part A* **78**, 528–541.
- Wang, L. and W. R. Cluett (1995). Building transfer function models from noisy step response data using Laguerre network. *Chemical Engineering Science* **50**(1), 149–161.
- Wang, L., P. Gawthrop, C. Chessari, T. Podsiadly and A. Giles (2004). Indirect approach to continuous time system identification of food extruder. *Journal of Process Control* **14**, 603–615.
- Wang, Q. G. and Y. Zhang (2001). Robust identification of continuous systems with dead-time from step response. *Automatica* **37**(3), 377–390.
- Young, P. C. (1970). An instrumental variable method for real time identification of noisy process. *Automatica* **6**, 271–287.
- Yuwana, M. and D. E. Seborg (1982). A new method for on-line controller tuning. *AIChE Journal* **28**(3), 434–440.