MPC Monitoring: Detection and Diagnosis of Model Errors

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Credits: Bruce Wilson and Foon Szeto (Suncor Energy Inc.)
Outline

1. Introduction
2. Motivation
3. Assessing the need for re-identification
4. Detection and Isolation of Mismatch
5. Case Studies
6. Concluding Remarks
Introduction

- MPC performance is affected by model quality
- Model quality is influenced by changes in the plant and/or operating conditions
- Online model maintenance:
  - Monitoring of model quality
    - Mismatch detection and isolation
    - Assessment of need for re-identification
  - Only update the model if necessary
    - Identification under closed-loop
- Performance after maintenance
Motivation: Retune or Re-identify?

Tuning Problem

Model – Plant Mismatch
Motivation: Locate Significant MPM
Do we need to worry about MPM?
If yes, when?
Can we quantify the effect of MPM on performance?
A measure of degradation in controller performance – How far is the achieved performance from the designed performance?
Relate MPM and this measure
Model Predictive Control

Internal Model Control (IMC) Structure
Designed and Achieved Performances

\[ \Delta = G - \hat{G} \]

G: Actual Process

\( \hat{G} \): Model

\( r' \) \hspace{1cm} Q \hspace{1cm} u_d \hspace{1cm} \hat{G} \hspace{1cm} \Delta \hspace{1cm} \hat{y} \hspace{1cm} y_d \\
\]

\( r' \) \hspace{1cm} Q \hspace{1cm} u \hspace{1cm} G \hspace{1cm} \hat{G} \hspace{1cm} \hat{y} \hspace{1cm} y \\

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Important Closed-loop Term - 1

Achieved Performance = \[ \frac{1}{1 + \Delta Q} \]

\[ r - y_{\text{achieved}} \]
\[ (\text{MV})_{\text{achieved}} \]
\[ (\text{Sensitivity})_{\text{achieved}} \]

Designed Performance

\[ r - y_{\text{designed}} \]
\[ (\text{MV})_{\text{designed}} \]
\[ (\text{Sensitivity})_{\text{designed}} \]

\[ \left\| \frac{1}{1 + \Delta Q} \right\|_{\infty} > 1 \Rightarrow \text{Achieved is worse than designed} \]

- Indicates presence of MPM
- MPM ‘mishandled’ by controller
Important Closed-loop Term - 2

\[ \| \Delta Q \|_\infty < 1 \]

\[ \| \Delta Q \|_\infty \rightarrow \text{Measure of Robustness} \]

- Sufficient condition for stability
- Smaller the value, the more robust is the closed-loop to the current MPM
Important Closed-loop Term - 3

\[ S_{\text{designed}} = \left[ 1 - \hat{G}_Q \right] \]

**Designed Sensitivity**

Sensitivity: Closed-loop relationship between the CV and the disturbance

- A measure of the designed performance
- Depends on controller tuning
- A large peak (> 2) indicates poor designed performance – Issues with tuning

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Key Observations

- All terms can be estimated from routine operating data – SPs, CVs and model residuals are required.

- Variability: \[ \frac{\text{var}(e_{ach})}{\text{var}(e_{des})} \leq \left\| \frac{1}{1+\Delta Q} \right\|_\infty^2 \]

- Degraded performance:

<table>
<thead>
<tr>
<th>[ \left| \frac{1}{1+\Delta Q} \right|_\infty ]</th>
<th>[ | \Delta Q |_\infty ]</th>
<th>[ | S_{designed} |_\infty ]</th>
<th>Diagnosis</th>
<th>Recommendation</th>
</tr>
</thead>
<tbody>
<tr>
<td>\approx 1</td>
<td>\approx 0</td>
<td>&gt; 2</td>
<td>Tuning problem</td>
<td>Retune</td>
</tr>
<tr>
<td>&gt; 1</td>
<td>\geq 1</td>
<td>&gt; 2</td>
<td>Severe MPM</td>
<td>Retuning may help</td>
</tr>
<tr>
<td>&gt; 1</td>
<td>\geq 1</td>
<td>&lt; 2</td>
<td>Severe MPM</td>
<td>Re-identify</td>
</tr>
<tr>
<td>\approx 1</td>
<td>\approx 0</td>
<td>&lt; 2</td>
<td>Disturbances</td>
<td></td>
</tr>
</tbody>
</table>
Illustrative Example: SISO MPC

Plant:

\[ G = \frac{4 e^{-25s}}{50s+1} \]

Controller:

MPC : Design based on the model \( G \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction Horizon</td>
<td>40</td>
</tr>
<tr>
<td>Control Horizon</td>
<td>3</td>
</tr>
<tr>
<td>Error Weight</td>
<td>1</td>
</tr>
</tbody>
</table>

Disturbance: Integrated white noise of std. deviation 0.005
Case-1: Tuning Problem, Perfect Model

Simulation
- Aggressively tuned controller
- Model is perfect

Achieved is as good as designed:
No model-plant mismatch

Peak in the designed sensitivity:
Tuning Problem
Case-2: Good Tuning, Model-Plant Mismatch

Simulation

- Well-tuned controller
- Model-plant mismatch: Gain is underestimated by 50%
Case-3: Bad Tuning, Model-Plant Mismatch

Simulation

- Aggressive controller
- Model-plant mismatch: *Gain is Underestimated by 50%*

**Poor Robustness**
Case-4: Degradation due to disturbance

Simulation
- Well tuned controller
- Perfect Model
- Oscillatory disturbance in the process

Small peak in the designed sensitivity: Tuning is alright!
Achieved is as good as designed:
Model is perfect!
Degradation in performance is due to disturbance.
## Summary

<table>
<thead>
<tr>
<th>Case</th>
<th>$\frac{\text{var}(e_{ach})}{\text{var}(e_{des})}$</th>
<th>$| \frac{1}{1+\Delta Q} |_\infty$</th>
<th>$| \Delta Q |_\infty$</th>
<th>$| S_{designed} |_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuning problem</td>
<td>0.99</td>
<td>1.015</td>
<td>0.005</td>
<td>2.75</td>
</tr>
<tr>
<td>MPM</td>
<td>1.26</td>
<td>1.21</td>
<td>0.84</td>
<td>1.4</td>
</tr>
<tr>
<td>MPM + Tuning problem</td>
<td>6.10</td>
<td>6.46</td>
<td>1.32</td>
<td>2.77</td>
</tr>
<tr>
<td>Disturbances</td>
<td>1.03</td>
<td>1.02</td>
<td>0.002</td>
<td>1.36</td>
</tr>
</tbody>
</table>
Isolate that part/subsystem where model-plant mismatch is significant
MPM in MPC

\[
\begin{align*}
 e_1 &= \Delta_{11}MV_1 + \Delta_{12}MV_2 + v_1 \\
 e_2 &= \Delta_{21}MV_1 + \Delta_{22}MV_2 + v_2
\end{align*}
\]

\[
G - \hat{G} = \begin{bmatrix}
\Delta_{11} & \Delta_{12} \\
\Delta_{21} & \Delta_{22}
\end{bmatrix}
\]
Correlations amongst the MVs

Correlated MVs confound regular correlation analysis between model residuals and MVs – Misleading information regarding presence (or absence) of significant mismatch

Absence of setpoint activity – \( e = -Q^{-1}u \)

Detection of mismatch is not possible

In practice, setpoint activity is quite common
Problem and Approach

- **Problem**: A methodology for the detection and isolation of plant-model mismatch for MPC using closed-loop data.

- **Approach**: Analysis of partial correlations between the model residuals and the inputs.
Partial Correlations - Introduction

- Partial correlation analysis is a method used to describe the relationship between two variables whilst taking away the effects of other variables.

- Extensively used in social sciences.

- Generally linear regression is used to de-correlate variables.
Partial Correlation Analysis for Problem at hand

- We wish to analyze partial correlations in a dynamic sense because the variables under consideration (MVs and model residuals) are time series variables.
  - Use lagged variables
- Disturbances may confound the analysis of partial correlations
  - Replace linear regression by PEM based models
Methodology based on Partial Correlations

1. Choose data (model residuals and MVs) from the period where there is sufficient setpoint excitation in the process.

2. Identify relationship between $\text{MV}_i$ and rest of the MVs and evaluate associated prediction error, $E_{\text{MV}_i}$.

3. Identify relationship between residual, $e_j$ and all MVs except $\text{MV}_i$ and evaluate associated error, $E_{e_j}$.

4. Evaluate correlation between $E_{\text{MV}_i}$ and $E_{e_j}$ – Large correlation indicates significant mismatch in channel $\text{MV}_i$-$\text{CV}_j$. 
Case Study 1: Simulations on a 3x3 system

- Simulation of 3 CVs X 4 MVs example
- MPC controller parameters:
  - Prediction Horizon = 30
  - Control Horizon = 10
  - Sampling Time = 1 min
  - All CVs are equally weighted
  - All MVs are equally weighted with move supression factor = 0 or 1
  - MV4 is constrained at its lower limit – 3CVs X 4MVs controller
- Each MV is a setpoint to a PID loop
Case Study 1 – Gain mismatch

- MV1- CV1, CV2 gains underestimated by 50%

- Full and partial correlations analyzed

- Channels corresponding to residual-MV pairs with significant partial correlations contain mismatch
Input-Output Data

![Graphs showing input-output data]

- Outputs
  - y1
  - y2
  - y3

- Inputs
  - u1
  - u2
  - u3

Time (min)
Case Study-1: Gain Mismatch – Full and partial correlation coefficients

Correlation coefficients between residuals and inputs

<table>
<thead>
<tr>
<th></th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>-0.62 (0)</td>
<td>0.28 (0)</td>
<td>-0.88(0)</td>
</tr>
<tr>
<td>e2</td>
<td>-0.62 (0)</td>
<td>0.28 (0)</td>
<td>-0.88(0)</td>
</tr>
<tr>
<td>e3</td>
<td>-0.006 (0.81)</td>
<td>-0.02 (0.42)</td>
<td>0.002 (0.92)</td>
</tr>
</tbody>
</table>

Partial correlation coefficients between residuals and inputs

<table>
<thead>
<tr>
<th></th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>0.15 (0)</td>
<td>-0.01 (0.57)</td>
<td>0.08 (0.16)</td>
</tr>
<tr>
<td>e2</td>
<td>0.38 (0)</td>
<td>-0.02 (0.33)</td>
<td>-0.04 (0.13)</td>
</tr>
<tr>
<td>e3</td>
<td>0.001 (0.97)</td>
<td>-0.01 (0.61)</td>
<td>-0.02 (0.48)</td>
</tr>
</tbody>
</table>

Mismatch correctly located in channels u1-y1,y2
Case Study-1: Gain Mismatch – Correlation plots

Full correlation plots

Partial correlation plots
Case Study 1 – Delay mismatch in u1-y1, y2, y3

- Delay is underestimated by 2

Partial correlation coefficients between residuals and inputs

<table>
<thead>
<tr>
<th></th>
<th>MV1</th>
<th>MV2</th>
<th>MV3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>0.45 (0)</td>
<td>-0.001 (0.45)</td>
<td>7.5e-4 (0.64)</td>
</tr>
<tr>
<td>$e_2$</td>
<td>-0.2 (0)</td>
<td>-2e-4(0.43)</td>
<td>-2e-4(0.52)</td>
</tr>
<tr>
<td>$e_3$</td>
<td>0.18 (0)</td>
<td>-1.3e-4(0.61)</td>
<td>-1.4e-3(0.39)</td>
</tr>
</tbody>
</table>

Mismatch correctly located in channels u1-y1, y2, y3
Case Study 1 – Delay mismatch in u1-y1, y2, y3
**Example: Shell Control Problem**

**Controlled Outputs:**
- \( y_1 \) Top End Point
- \( y_2 \) Side Endpoint
- \( y_3 \) Bottom Reflux Temp.

**Manipulated Inputs:**
- \( u_1 \) Top Draw
- \( u_2 \) Side Draw
- \( u_3 \) Bottom Reflux Duty

**Unmeasured Disturbances:**
- \( d_1 \) Upper reflux
- \( d_2 \) Intermediate reflux
\[ y(s) = G_u(s)u(s) + G_d(s)d(s) \]

\[ G_u(s) = \begin{bmatrix}
4.05e^{-27s} & 1.77e^{-28s} & 5.88e^{-27s} \\
50s + 1 & 60s + 1 & 50s + 1 \\
5.39e^{-18s} & 5.72e^{-14s} & 6.9e^{-15s} \\
50s + 1 & 60s + 1 & 40s + 1 \\
4.38e^{-20s} & 4.42e^{-22s} & 7.2e^{-19s} \\
33s + 1 & 44s + 1 & 19s + 1
\end{bmatrix} \]

- System with large time delays and significant multivariable interactions
- Time delay matrix is assumed to be known a-priori
Unmeasured Disturbance Dynamics

\[ G_d(s) = \begin{bmatrix} \frac{1.2e^{-27s}}{45s+1} & \frac{1.44e^{-27s}}{40s+1} \\ \frac{1.52e^{-15s}}{25s+1} & \frac{1.83e^{-15s}}{20s+1} \\ \frac{1.14}{27s+1} & \frac{1.26}{32s+1} \end{bmatrix} \]

All disturbance inputs assumed to be piecewise constant

\[ \begin{bmatrix} d_1(z) \\ d_2(z) \end{bmatrix} = \begin{bmatrix} I \\ z-0.95 \end{bmatrix} \begin{bmatrix} w_1(z) \\ w_2(z) \end{bmatrix} \]

mean\( (w_i) = 0 \); \( \sigma (w_i) = 0.0075 \) for \( i = 1,2 \)

Measurement Noise

mean\( (v_i) = 0 \); \( \sigma (v_i) = 0.005 \) for \( i = 1,2,3 \)
Shell Control Problem – Scenario-1

- 10% gain mismatch and 10% mismatch in time constant was added in all channels except channel $u_1-y_1$, where a larger mismatch of 50% (underestimated gain) was added.

- Challenge is to identify the channel with significant mismatch.
Shell Control Problem: Scenario - 1

Regular correlation plots

Significant partial correlation between $e_1$ and $u_1$ implies significant mismatch in channel MV$_1$-CV$_1$
Shell Control Problem: Scenario - 2

- A mismatch in delay (underestimate) of 5 samples was introduced in MV1-CV1,CV2,CV3

- 10 % gain mismatch in all channels
Shell Control Problem: Scenario - 2

Significant partial correlations between \( e_1, e_2, e_3 \) and \( u_1 \) imply significant mismatches in channels \( MV_1-CV_1, CV_2, CV_3 \).
Regular correlation analysis does not help

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KHU – Partial correlations methodology

Channels with significant mismatch
Concluding Remarks

1. Proposed a methodology for assessing the need for re-identification / re-tuning.

2. Demonstrated the efficacy on a SISO MPC simulation case study.

3. Proposed a technique based on partial correlations analysis for detection and isolation of mismatch.

4. Applied the technique successfully to simulated examples and industrial data.
Ongoing Work

- Assessing need for re-identification – Extension to the multivariable case

- Cancellation of effect of MPM – Re-tuning guidelines for MPC
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