Adaptive Stable PID Controller with Parallel Feedforward Compensator

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Abstract **- In this paper, a new design method of adaptive PID controller is proposed. The method utilizes the so- called almost strict positive realness (ASPR) of the plant so that the stability of the adaptive PID control system is guaranteed by use of K-Y lemma and Lyapunov's stability theorem. An application of the proposed basic design concept to a practical design of adaptive tracking PID control system is also discussed. The result is applied to the design of PID control system of the first order with time delay system. The effectiveness of the proposed method is examined through simulations and experiments**

Index Terms – PID, Adaptive PID, Time-Delay System, Process Control, PFC

I. INTRODUCTION

Most PID parameter tuning procedures are done through the so-called off line tuning work. Further the region of stability with respect to PID controller parameters receives constraints because of the luck of enough number of tuning parameters. Therefore it is interesting and important to consider the following two problems: (1) automatic adjusting or self tuning of near optimal PID controllers, and (2) guarantee of the stability for the control system with 3 adjustable PID controller parameters. In fact, automatic tuning and stabilizing of PID controllers have over the years been objects for a great amount of research. The proposed methods proposed in the past concerning auto tuning of PID controllers have been stated in the well known book written by Astrom and Hagglund [1]. Auto tuned PID controller does not necessarily behaves to cover the stability of any type of controlled plant. In other words, PID control system involves complex stability phenomena because of its limitation concerning the small number of controller parameters compared to the order of the plant. As to the stability analysis and synthesis of PID control system, it was shown that Hermite-Biehler Theorem can be used not only to derive conditions for the existence of the set of stabilizing controllers but also as a convenient analytical method to design compensators[2,3]. However these conditions do not contain the clear information concerning the improvement of the control performance at present.

In this paper, a new approach is proposed concerning the design of PID control system. This approach utilizes the special process characteristics called ASPRness (almost strict positive realness) [4,5]. It is known that the linear time invariant system can be stabilizable by output feedback if the controlled plant is ASPR [5]. The specific features of this approach are as follows: (1) it always gives stable PID control system so that it does not need to consider the constraints for stability region as to PID controller parameters and (2) the stability of the closed-loop system is essentially guaranteed by proportional feedback. In this sense, it has the same feature as that of the simple adaptive control (SAC) [5, 6, 7]. However, the redundancy concerning the integral and derivative terms contributes the improvement of the control performance. The effectiveness of the proposed method is examined by simulations and experiments using the first order with time delay process model.

II. BASIC CONCEPT OF STABLE PID CONTROLLER FOR ASPR PLANT

Let us consider the n-th order controllable and observable SISO plant:

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= c^T x \end{aligned} \tag{2.1}$$

We assume that eq. (2.1) satisfies the following assumption.

[*Assumption 1*]

Eq. (2.1) is ASPR. That is, there exist positive definite matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that

$$\begin{aligned} (A - k_p^* bc^T)^T P + P(A - k_p^* bc^T) &= -Q \\ Pb &= c \end{aligned} \tag{2.2}$$

for all $k_p^* \geq {}^{\exists}k_{10} > 0$ [5].

Then we have the following lemma.

[*Lemma 1*]

Suppose that the assumption 1 holds. Then plant (2.1) can be stabilized by the following PID controller:

$$\begin{aligned} u^* &= -k_P^* y - k_D^* \dot{y} - k_i^* w \\ \dot{w} &= y \end{aligned} \tag{2.3}$$

where

$$k_p^* \geq {}^{\exists}k_{10} > 0, k_D^* > 0, k_i^* > 0 \tag{2.4}$$

(Proof of Lemma 1)

Substituting eq. (2.3) into eq. (2.1) leads to the following equation:

$$\dot{x} = A^* x - k_D^* b\dot{y} - k_i^* bw \qquad y = c^T x \tag{2.5}$$

where

$$A^* = A - k_p^* bc^T$$

Let

$$V = x^T Px + k_p^* y^2 + k_i^* w^2 \tag{2.6}$$

be a candidate of Lyapunov function. Then, from the assumption 1, we can evaluate its derivative along the trajectory of eq. (2.5) as follows.

$$\dot{V} \le -x^T Qx \le 0 \tag{2.7}$$

It means $\lim_{t\to\infty} x(t) = 0$. From this, we can conclude $\lim_{t\to\infty} y(t) = 0$ and the boundedness of $w(t)$. Q.E.D.

III. Adaptive PID Controller

If the plant (2.1) satisfies the assumption 1, we can obtain stable PID controller as shown in Lemma 1. Based on the result in Lemma 1, we can show that it can be possible to construct a PID controller even if the plant parameters are unknown.

[*Theorem 1*]

Let us assume that the plant (2.1) satisfies the assumption 1. Then the controller:

$$u(t) = -k_p(t)y - k_D(t)\dot{y} - k_i(t)w \qquad \dot{w} = y \tag{3.1}$$

stabilizes the plant (2.1). Here the variable PID gains $k_p(t), k_D(t), k_i(t)$ are tuned according to the following adaptive gain tuning laws:

$$\dot{k}_p(t) = \gamma_1 y(t)^2 \qquad \dot{k}_D(t) = \gamma_2 y(t)\dot{y}(t) \qquad \dot{k}_i(t) = \gamma_3 y(t)w(t) \tag{3.2}$$

where $\gamma_1, \gamma_2, \gamma_3$ are positive constants.

(Proof of Theorem 1) Let us define the following parameter estimation error vector:

$$\varsigma(t) = k(t) - k^* \tag{3.3}$$

In eq. (3.3), $k(t)$ is the adaptive PID gain vector and the k^* is the constant PID controller parameter vector which satisfies Lemma 1, where

$$k(t) = [k_p(t), k_D(t), k_i(t)]^T \qquad k^* = [k_p^*, k_D^*, k_i^*] \tag{3.4}$$

Define the regression vector as follows.

$$z(t) = [y(t), \dot{y}(t), w(t)] \tag{3.5}$$

Then, the equation of the closed- loop system can be derived by substituting eq. (3.1) into eq. (2.1). This equation is given by

$$\dot{x} = A^* x - k_D^* b\dot{y} - k_i^* bw - b\varsigma(t)^T z(t) \qquad y = c^T x \tag{3.6}$$

Let

$$V = x^T Px + k_D^* y^2 + k_i^* w^2 + \varsigma(t)^T \Gamma^{-1}\varsigma(t) \qquad \Gamma = \Gamma^T = diag[\gamma_i] > 0 \tag{3.7}$$

be the candidate of the Lyapunov function. Then we have

$$\frac{dV}{dt} = x^T(A^{*T}P + PA^*)x - 2k_D^*\dot{y}b^T Px - 2k_i^* wb^T Px - 2\varsigma^T zb^T Px + 2k_D^* y\dot{y} + 2k_i^* w\dot{w} + 2\varsigma^T zy$$

Taking into the assumption 1, we have

$$\frac{dV}{dt} = -x^T Qx < 0, x \ne 0 \tag{3.8}$$

From eq. (3.8), $\lim_{t\to\infty} x(t) = 0$ holds and the boundedness of other variables are guaranteed. Q.E.D.

(c.f.) It is noted that that the adaptive gain tuning laws are sometimes not robust for unmodeled dynamics. In such a case, robust adaptive gain tuning rules such as the so-called the adaptive gain tuning with σ-modification term [5,6] given in eq.(3.9) are often used.

$$\dot{k}_p(t) = \gamma_1 y(t)^2 - \sigma k_p(t) \qquad \dot{k}_D(t) = \gamma_2 y(t)\dot{y}(t) - \sigma k_D(t) \qquad \dot{k}_i(t) = \gamma_3 y(t)w(t) - \sigma k_i(t) \qquad \sigma > 0 \tag{3.9}$$

IV. Application to the Design of Stable Adaptive Tracking PID Control Systems

In this section, the basic design concept of stable adaptive PID Controller is applied to the design of stable adaptive tracking PID control system. Let us consider the following SISO n-th order plant:

$$\dot{x}(t) = Ax(t) + bu(t) + b_1 d(t) \qquad y(t) = c^T x(t) \tag{4.1}$$

where $d(t)$ denotes the input disturbance. The problem to be considered here is a design of stable adaptive tracking PID control system which achieves the tracking of the output $y(t)$ to the reference input $r(t)$. Suppose that the input $r(t)$ satisfies the following differential equation which is known as the internal model:

$$D(s)r(t) = 0 \qquad D(s) = s^p + d_1 s^{p-1} + \cdots + d_p \tag{4.2}$$

where " s " denotes the differential operator. Define $z(t), v(t)$ and tracking error $e(t)$ as follows.

$$z(t) = D(s)x(t) \qquad v(t) = D(s)u(t) \qquad e(t) = y(t) - r(t) \tag{4.3}$$

Then, operating $D(s)$ from both sides of (4.2) and taking eq.(4.1) into consideration lead to the following equation:

$$D(s)e(t) = c^T z(t) \tag{4.4}$$

Further we assume that the disturbance $d(t)$ also satisfies the following disturbance model:

$$D(s)d(t) = 0 \tag{4.5}$$

From eqs.(4.1)-(4.5), we can obtain the following equation:

$$\frac{d}{dt}\bar{x}(t) = \bar{A}\bar{x}(t) + \bar{b}v(t) \qquad (4.6)$$
$$\bar{y}(t) = \bar{c}^T \bar{x}(t)$$

where

$$\bar{x} = \begin{bmatrix} z \\ e \\ \vdots \\ e^{(p-1)} \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 & \cdots & 0 \\ 0^T & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0^T & \cdots & 0 & 1 \\ c^T & -d_p & \cdots & -d_1 \end{bmatrix}, \bar{b} = \begin{bmatrix} b \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \bar{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\bar{y} = e(t)$$

The input and the output of this system are $v(t)$ and $\bar{y}(t)$, respectively. However,

$$\bar{c}^T \bar{b} = 0 \tag{4.7}$$

holds in eq.(4.6). It means that the relative degree of the system is equal or greater that 2 so that the system (4.6) is not ASPR. To improve this situation, we introduce the following $n_f - th$ order parallel feedforward compensator (PFC) [7,8].

$$\frac{d}{dt}x_f(t) = A_f x_f(t) + b_f v(t)$$
$$y_f(t) = c_f^T x(t) \qquad (4.8)$$
$$c_f b_f > 0$$

By combining eq. ((4.6) and eq. (4.8), the following extended system is obtained.

$$\frac{d}{dt}x_a(t) = A_a x_a(t) + b_a v(t) \qquad (4.9)$$
$$y_a(t) = \bar{y}(t) + y_f(t) = c_a^T x(t)$$

where

$$x_a = \begin{bmatrix} \bar{x} \\ x_f \end{bmatrix}, A_a = \begin{bmatrix} \bar{A} & 0 \\ 0 & A_f \end{bmatrix}, b_a = \begin{bmatrix} \bar{b} \\ b_f \end{bmatrix}, c_a = \begin{bmatrix} \bar{c} \\ c_f \end{bmatrix}$$

[*Assumption 2*]
Extended system (4.9) is ASPR. That is, eq. (4.9) satisfies the assumption 1.

Assumption 2 means that the PFC (4.8) should be designed so as to the resultant extended system (4.9) becomes ASPR. Then we have the following theorem.

[*Theorem 2*]
Assume that the assumption 2 holds. Then, the following PID controller:

$$v(t) = -k_p(t)y_a(t) - k_d(t)\dot{y}_a(t) - k_i(t)w(t) \qquad (4.10)$$
$$\dot{w}(t) = y_a(t)$$

stabilizes the closed-loop system, where

$$\dot{k}_p(t) = \gamma_1 y_a(t)^2$$
$$\dot{k}_D(t) = \gamma_2 y_a(t)\dot{y}_a(t) \cdot \qquad (4.11)$$
$$\dot{k}_i(t) = \gamma_3 y_a(t)w(t)$$

(Proof of theorem 2)
It is apparent from the proof of theorem 1. (Q.E.D.)

It is noted that in this case $\lim_{t\to\infty} x_a(t) = 0$ holds. This relation includes $\lim_{t\to\infty} e(t) = 0$. It means that the output tracking to the reference input $r(t)$ is attained. In practical application, σ modification term is added to eq.(4.11) to realize the robustness of the algorithm. A schematic block diagram of the control system is derived in Fig.1.

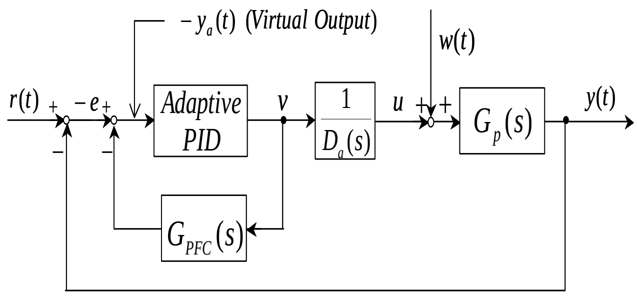

Fig.1 Schematic Diagram of Adaptive PID Control System with PFC.

V. A Concrete Design Scheme of PFC

Throughout the above consideration, we have assumed that the existence of PFC (4.8) which realizes the ASPRness of the extended system (4.9). A systematic procedure of constructing such a PFC was proposed by Iwai et al.[7] for minimum phase plant. Let γ be the relative degree of the plant (4.6) and define PFC $G_f(s)$ as follows:

$$G_f(s) = \sum_{i=1}^{\gamma-1} \delta^i G_{fi}(s) \tag{5.1}$$

$$G_{fi}(s) = \frac{\beta_i}{d_i(s)} \tag{5.2}$$

$$d_i(s) = \prod_{l=1}^{j} (s + \alpha_l), l = 1, \cdots, \gamma - 1 \tag{5.3}$$

β_i : coefficients of the following Hurwitz polynomial

$$\beta_{\gamma-1}s^{\gamma-1} + \cdots + \beta_1 s + \beta_0, \tag{5.4}$$

where β_0 is the leading coefficient of (4.6).

δ : small positive constant

Then, there exists a positive constant δ_0 such that the transfer function $G_a(s)$ of eq. (4.9) becomes ASPR for $\delta_0 > \delta > 0$. A structure of above stated PFC (ladder network structure PFC[7]) is shown in Fig.2.

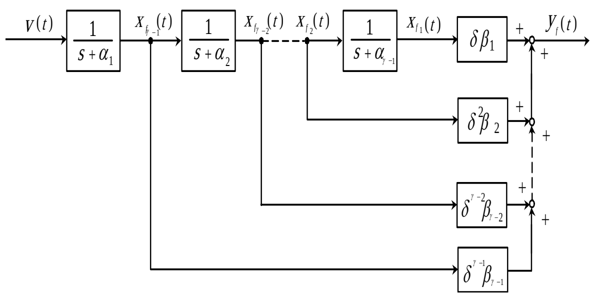

Fig.2 Ladder Network type PFC.

It is noted that PFC can be designed even if the original plant is non-minimum phase system though, in this case, the systematic approach of PFC has not existed yet.

VI. Application to the Control of the First order with Time-delay System.

In the following, the proposed method is examined by applying it to a thermal pilot plant experimental system shown in Fig.3.

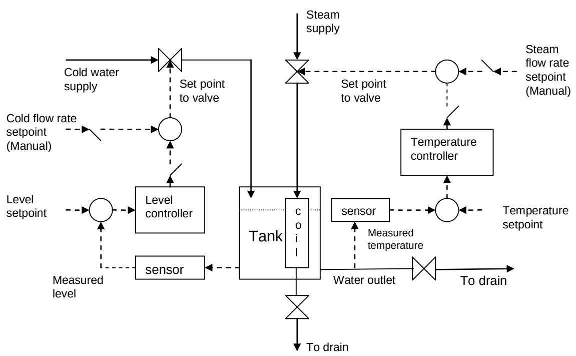

Fig.3 A schematic diagram of a temperature control system

In this case, the transfer function between steam (input) and temperature of the tank (output) can be approximated by first order plus time delay model or the so-called three parameter model [1, 2]:

$$G(s) = \frac{K}{Ts+1}e^{-Ls} \tag{6.1}$$

When the process can be approximated as in the form of eq.(6.1), many PID controller parameter tuning rules have been proposed[2]. In this example, plant model (6.1) is used as a typical batch test model of the PID controller. Further, we assume that the reference command is step input, and the form of the disturbance added on the control input is also step disturbance because many actual processes can be modelled by (6.1) and typical PID tuning algorithm methods are based on this process model with step input command and disturbance [1, 2].

(1) Nominal values of the plant:

$$T = 82.38, K = 1.075, L = 25.185 \tag{6.2}$$

Reference input and disturbance are assumed to be

$$r(t) = 6, t \geq 0 \text{ and } d(t) = 20\% change, t_2 \geq t \geq t_1 > 0 \tag{6.3}$$

(2) Controller design of the proposed plant.

(a) Reference input model and disturbance model.

$$D_a(s) = s \tag{6.4}$$

(b) Design of PFC

To design PFC, the time-delay is approximated by the most simple $0/1$-order Pade-approximation:

$$e^{-Ls} \approx \frac{1}{1+Ls} \tag{6.5}$$

so that the approximated plant model for eq. (6.1) becomes

$$\hat{G}(s) = \frac{K}{(1+Ts)(1+Ls)} \tag{6.6}$$

Then the augmented plant with reference model is given by

$$G_a(s) = \frac{1}{D_a(s)}G_p(s) \approx \frac{1}{D_a(s)}\hat{G}(s) = \hat{G}_a(s) \tag{6.7}$$

and the PFC is designed by using this third-order approximated model. Since eq. (6.7) is minimal phase and its relative degree is 3, we can introduce the following second-order PFC:

$$G_{PFC}(s) = \frac{\beta_1(s+\beta_2)}{(s+\alpha_1)(s+\alpha_2)} \tag{6.8}$$

Note that eq. (6.7) is minimum phase with relative degree 3 so that we can apply the systematic design procedure of PFC described in the section V [6, 7]. In this case, we choose the following PFC parameters:

$$\alpha_1 = 0.5, \alpha_2 = 1, \beta_1 = 0.3, \beta_2 = 600 \tag{6.9}$$

This corresponds to

$$\delta = 0.01, \alpha_1 = 9, \alpha_2 = 1, \beta_1 = 1400, \beta_2 = 300$$

in the ladder network form.

(c) Adaptive parameter adjusting law.

The control input (3.1) with parameter adjusting law (3.9) is used where

$$\gamma_1 = 50, \gamma_2 = 2, \gamma_3 = 5, \sigma = 0.01 \tag{6.10}$$

(d) Approximation of the differential element.

In the following, derivative action is approximated as the output of the following element:

$$\frac{s}{T_r s + 1}, 1 >> T_r > 0 \tag{6.11}$$

(3) Simulation results

Fig.4 shows the result when we add step reference input whit height 6.

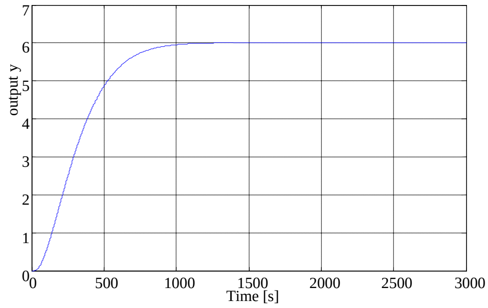

Fig.4 Output y(t) for the plant (6.2)

Fig.5 shows that the result when we give 50% parameter change to the plant. That is,

$$K = 1.6125, T = 41.19, L = 37.28 \qquad (6.12)$$

It is noted that we used the same controller parameters and PFC as those of the case in Fig.4.

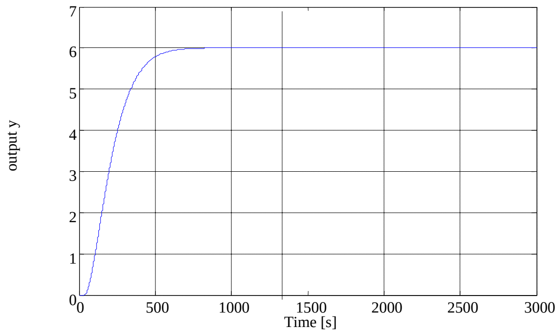

Fig.5 Output y(t) with 50% plant parameter change

Fig.6 shows the result when 20% step disturbance is added in the case of eq.(6.12).

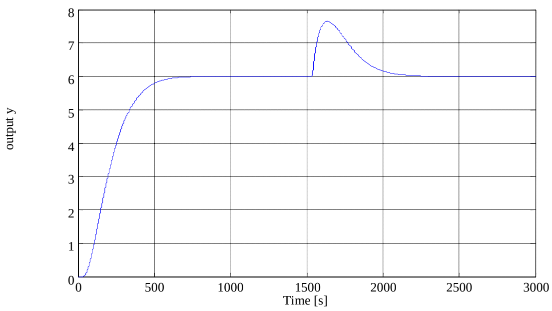

Fig.6. Effect of step disturbance with 50% plant parameter change

In Fig.7, the result for the change of the input is given where

$$\begin{aligned} &0 \le t < 1500, r(t) = 6 \\ &1500 \le t < 3000, r(t) = -1 \\ &3000 \le t < 4500, r(t) = 4 \end{aligned} \qquad (6.13)$$

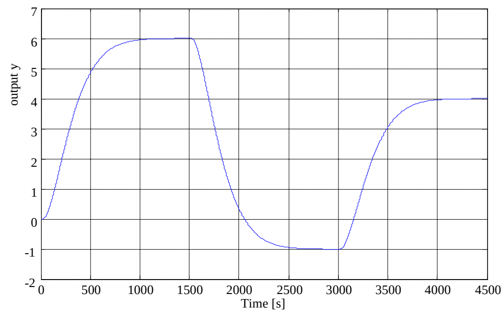

Fig.7 Output y(t) for the change of r(t)

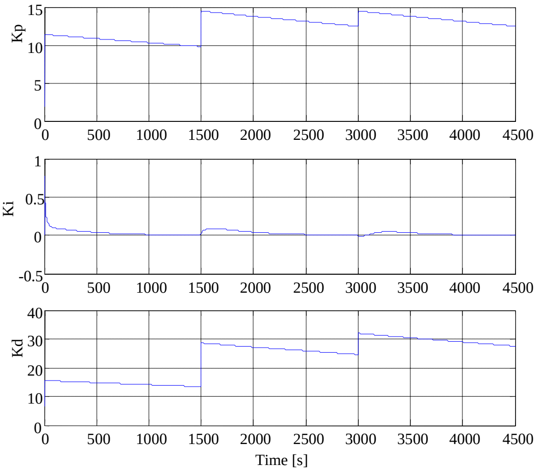

Fig.8. Change of adaptive PID gains at Fig.7

(4) Experimental Result

Finally, an experimental result of the plant is shown. This pilot plant is settled at Department of Chemical Engineering, University of Alberta. Schematic diagram of the plant is shown in Fig.3. Actual control input is the steam flow rate and the output is a temperature of the tank. All controller parameter values and reference input level are the same as those of the parameters and reference input used in the above stated simulation. The result of the output is given in Fig.9 and the control input is shown in Fig.10.

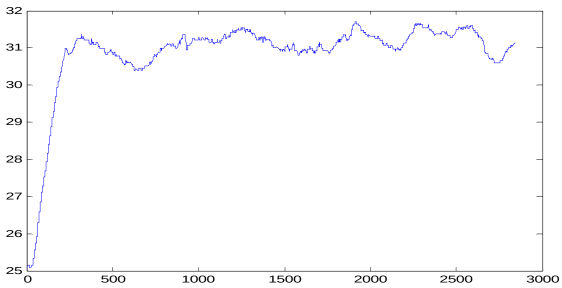

Fig.9 Experimental result (Temperature)

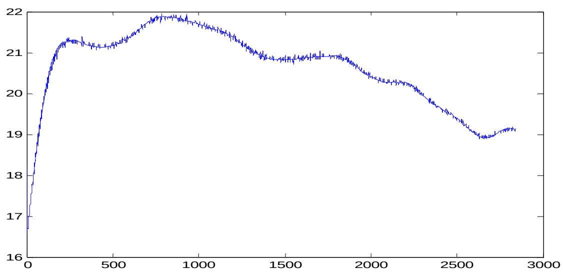

Fig.10 Experimental result (Steam flow rate)

VII. CONCLUSION

In this report, a new adaptive stable PID control design scheme based on the ASPRness of the plant is proposed. The idea is applied to the design of adaptive PID tracking control of the first-order with time delay system. The effectiveness and robustness of the proposed method is examined through simulations and experiments using the pilot scale experimental plant.

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