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THE TWO-SECTOR MODEL AND PRODUCTION TECHNIQUE IN THE SHORT RUN AND THE LONG RUN*

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IN the standard Heckscher–Ohlin model of international trade, capital moves instantly between sectors to equate rates of return. Doubts about the short run validity of this assumption led to the development of a literature which combined short run capital specificity with the standard two-sector model and examined its impact on factor prices and the composition of output. Examples of this analysis can be found in Mayer (1974), Lapan (1976), Mussa (1974, 1978, 1982), Neary (1978, 1982), and Schweinberger (1980). Less attention has been given in the literature to the Heckscher–Ohlin model's assumption of rapid factor substitutability. When physical capital is constructed, it is embedded with a particular production technique requiring the use of a specific ratio of capital to labour. Altering this technique in response to a change in relative factor prices requires an investment of resources and, therefore, may not be instantaneous.

Econometric evidence on the degree to which the technique of production adjusts slowly is provided in two recent empirical studies. Helliwell and Chung (1986) estimate a macroeconomic model from which the speed of adjustment of the capital-energy ratio can be determined. They find that a change in relative factor prices results in only 18 percent of the existing capital–energy bundle being adapted to the new optimum ratio each year. Sneessens and Dreze (1986) estimate a model in which the capital–labour ratio adjusts slowly to changes in the factor price ratio. (They assume that in the short run the technology of production is Leontief while in the long run it is Cobb–Douglas.) Their results indicate that "only 27 percent of the change in the optimal technical coefficients . . . implied by a change in relative factor costs is realized within a year" (p. S108). In contrast to the standard assumption of the Heckscher–Ohlin model, these two results suggest that the technique of production adjusts relatively slowly. The purpose of the analysis which follows is to incorporate this slow adjustment of the technique of production into a standard two-sector model and examine its impact on short run and long run factor prices and their adjustment paths.

The slow adjustment of production technique and the slow movement of capital between sectors, though similar in motivation, yield quite different results. In the sector specific capital model, because labour is intersectorally mobile, a relative price change causes an immediate change in the factor use ratio and the composition of output. Furthermore, since only labour can

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respond to a relative price change in the short run, a price increase raises the wage, irrespective of whether or not the industry whose price has risen is labour intensive. The long run reallocation of capital between sectors in the specific factors model is caused by a divergence in rentals across sectors following a relative price change. By inducing a decline in the derived demand for either labour or capital, this reallocation always causes one factor to become worse off.

In contrast, as the analysis to follow will show, given a slowly adjusting technique of production a relative price change will not cause a sectoral reallocation of output or employment on impact. While the usual Stolper–Samuelson result holds in the short run as well as the long run, both factors may benefit from adjustment to the new long run equilibrium, or one could benefit while the other loses. In addition, it is shown that the costs of adjustment have an impact on the path factor prices take from the short run to the long run equilibrium. These results suggest that neither the usual Stolper–Samuelson predictions, nor the results from specific factors models, can be relied upon to determine which factors benefit from adjustment and which path factor prices will follow during adjustment. This is important when analyzing the effect of policy changes since the direction of factor price movements is likely to determine which factors support and which oppose adjustment.[1]

The paper is divided into two principal sections. The first examines the comparative static relationship between the short run equilibrium associated with a fixed coefficients production structure and a long run equilibrium in which factors are substitutable. In the second section, the cost of changing the technique of production is made explicit and the path of adjustment from a short run unemployment equilibrium to a long run full employment equilibrium is analyzed.

1. The static framework

The model

Production uses two factors—labour and capital. The economy has a fixed endowment of both factors, and their full employment is guaranteed at all times by the perfect flexibility of factor prices. Capital and labour move freely between sectors, and thus factor prices are continually equalized across sectors.[2]

The economy produces two goods, X and Y, the prices of which, P_x and P_y, are exogenously determined in world markets. In the long run firms can use any technique of production corresponding to the linear homogeneous production functions:

$$X = F(L_x, K_x), \qquad F_L > 0, \qquad F_K > 0, \qquad F_{LL} < 0, \qquad F_{KK} < 0, \tag{1}$$

$$Y = G(L_y, K_y), \qquad G_L > 0, \qquad G_K > 0, \qquad G_{LL} < 0, \qquad G_{KK} < 0, \tag{2}$$

where it is assumed that, at all wage-rental ratios, sector X is more capital intensive than sector Y.

In the short run, the ratio of capital to labour in each sector is fixed and, as a result, the short run production functions must take the Leontief form:

$$X = \min\,[aL_x,\, bK_x], \tag{3}$$

$$Y = \min\,[cL_y,\, dK_y], \tag{4}$$

where a/b and c/d are the fixed short run capital–labour ratios in sectors X and Y respectively. If and only if a/b and c/d equal the capital–labour ratios associated with the two long run production functions for a given wage-rental ratio will the short run and long run production functions yield identical levels of output. As illustrated in Fig. 1, the short run production possibility frontier (Y_0AX_0) is completely within the long run production possibility frontier except at A where the long run and short run capital–labour ratios in each sector are identical.[3] The two long run

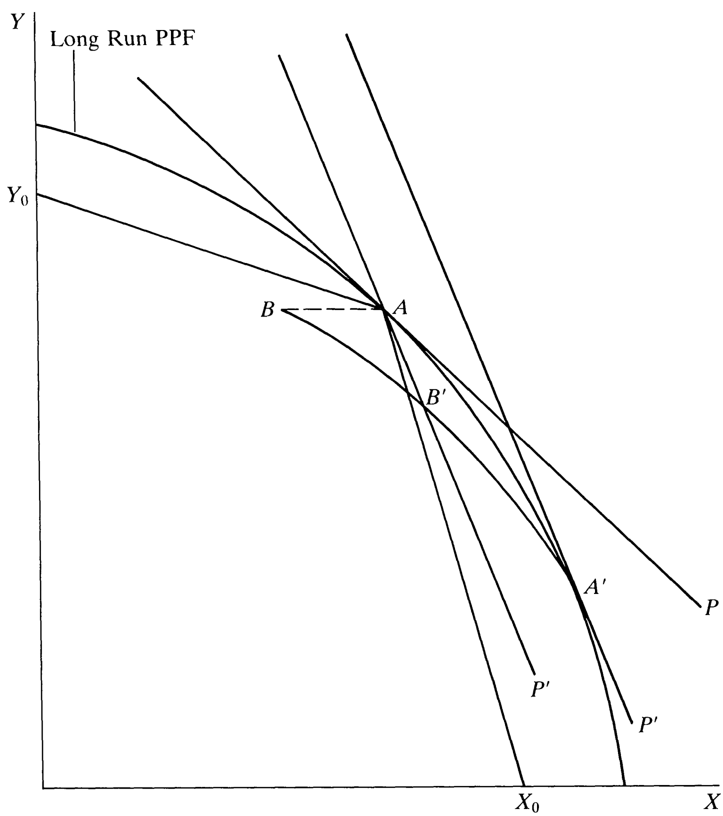

FIG. 1

production functions form the envelope of the short run production functions for different values of a/b and c/d.

This description of production technology suggests that the short and long run production functions are related through a set of blueprints which are constant (no technical change) and known to the firm at all points in time. These blueprints represent all the techniques of production embodied in the production functions (1) and (2). In the short run firms in each sector are committed to using just one blueprint, while in the long run they can choose any blueprint consistent with the two long run production functions. This does not mean that there exists some point in time at which firms can instantly choose any blueprint they desire. Rather, at every point in time firms are restricted to one blueprint, but through time they can adjust their technique of production so that, in the long run, the blueprint which they are constrained to use and that which they desire are identical.[4]

A relative price change

Assume initially that the relative price of X in terms of Y is P, and the economy is producing at A in Fig. 1. A fall in the price of Y which causes P to rise to P' will shift output to A' in the long run. (This relative price change could be the result of the removal of a tariff protecting the labour intensive industry.) However, because the capital–labour ratio is fixed in the short run, the equilibrium composition of output does not change on impact, and the economy continues to produce at A.[5] Thus, the entire short run effect of the price change must be reflected in factor price adjustments.

The short run change in factor prices can be determined by examining the behavior of the short run unit cost curves (C_y and C_x in Fig. 2) corresponding to the zero-profit conditions:[6]

$$C_x : P_x = \frac{w}{a} + \frac{r}{b}, \tag{5}$$

$$C_y : P_y = \frac{w}{c} + \frac{r}{d}. \tag{6}$$

The short run cost curves of both sectors are linear since they correspond to a fixed coefficients type production technology, and the slope of each gives the ratio of capital to labour in its respective sector.

The initial equilibrium in Fig. 2 is at A with wage w_0 and rental r_0. The decline in P_y shifts sector Y's unit cost curve down proportionately from C_y^0 to C_y^1. On impact, the wage falls from w_0 to w_1 and the rental rises from r_0 to r_1. As predicted by the Stolper–Samuelson Theorem, since the relative price change favours the capital intensive sector, the rental rises and the wage falls in terms of both commodities. (If the wage fell to w', it would have fallen just in proportion to P_y.)

If factors are substitutable to some degree in the long run, firms will adjust their technique of production in response to the change in relative

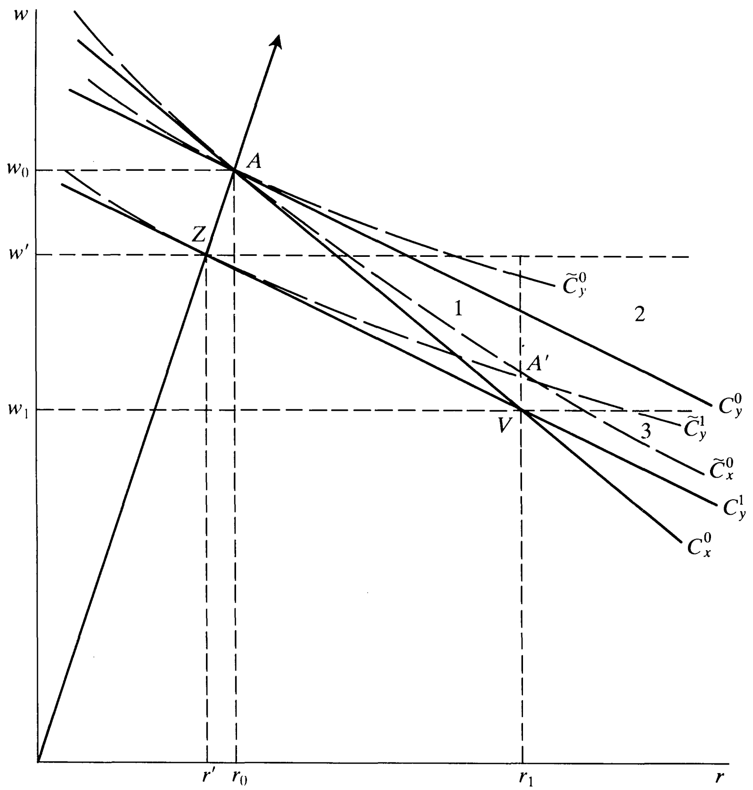

FIG. 2

factor prices. As a consequence, the short and long run effects of a relative price change on the wage and rental will differ. The pattern of this difference can be seen in Fig. 2 by utilizing the long run unit cost curves, $\tilde{C}_y^1$ and $\tilde{C}_x^0$. These follow from the long run zero-profit conditions:

$$\tilde{C}_x : P_x = \tilde{C}_x(w, r), \tag{7}$$

$$\tilde{C}_y : P_y = \tilde{C}_y(w, r), \tag{8}$$

where $\tilde{C}_x(w, r)$ and $\tilde{C}_y(w, r)$ are the duals of the two long run production functions.

The long run cost curves must cross (point A'), and the new long run equilibrium wage-rental combination must lie, somewhere within regions 1, 2 or 3 of Fig. 2. (This implies that, relative to the initial equilibrium, the Stolper–Samuelson result must hold in the long run as well.) By the homogeneity of degree one of the short run and long run unit cost functions, C_y^1 and $\tilde{C}_y^1$ must be tangent at Z on the ray from the origin through the initial long run equilibrium. As a result, since $\tilde{C}_y^1$ cannot be positively sloped if negative inputs are ruled out, the upper bound on feasible regions 1 and 2 is given by the wage w'. Furthermore, since $\tilde{C}_x^0$ incorporates some degree of factor substitutability, it must be to the right of C_x^0, except at the initial point of tangency (A). This follows because substitutability facilitates the more efficient use of labour and capital, implying the association of a higher wage and rental with a given price. As a consequence, the left hand boundary of region 1 is given by C_x^0 while, for similar reasons, C_y^1 forms the lower bound on region 3.

Which region contains the new long run equilibrium will depend upon the relative long run substitutability of factors in the two sectors. The less substitutable the two factors in the production of a good, the less curved the long run unit cost curve. As can be seen from Fig. 2, if the long run degree of substitutability between capital and labour is less in sector X, the capital intensive sector, than in sector Y, the labour intensive sector, the new long run wage-rental combination is more likely to reside in region 1 with a higher wage and lower rental than in the short run (that is, on impact, factor prices overshoot). Conversely, if capital and labour are less substitutable in sector Y than in sector X in the long run, the new equilibrium is more likely to occur in region 3. If both sectors have similar long run elasticities of substitution, both factor prices may rise during adjustment as both sectors gain (relative to the impact equilibrium) from operating more efficiently. (This would be the case if the long run production functions were both Cobb–Douglas.)

These results can be explained by reference to the Stolper–Samuelson Theorem's prediction that a relative price change increases the relative price of the factor used intensively in the sector which benefits from the price change. In Fig. 2, it can be seen that the benefit to a sector from being able to adjust its technique of production in response to a change in the wage-rental ratio is equivalent to a price increase with the production technique held constant. Since a relative price increase and a relatively higher degree of long run factor substitutability are equivalent, the Stolper–Samuelson result can be applied directly to the current problem. That is, adjustment will tend to lead to an increase in the relative price of the factor used intensively in the sector which has the greater degree of long run factor substitutability.

Whether or not a factor price overshoots or undershoots its long run equilibrium on impact depends upon the direction of the relative price change and the relative long run degree of substitutability in the two sectors. For example, if the relative price change is in favour of the labour intensive sector, factor prices will tend to undershoot if this sector's long run degree of factor substitutability is greater than that of the capital intensive sector. This follows because the relatively greater degree of long run substitutability in the labour intensive sector is equivalent to this sector benefitting from a relative price change while holding its production technique constant. Thus, by the Stolper–Samuelson Theorem, adjustment will re-enforce the effect on factor prices of the initial price change.

Conversely, if the long run degree of factor substitutability in the capital intensive sector is greater than that in the labour intensive sector, factor prices will tend to overshoot on impact. In this case, adjustment of the production technique is equivalent to a relative price change in favour of the capital intensive sector holding the technique of production constant and, as a result, adjustment counteracts the effect of the initial price change on factor prices. More generally, overshooting of factor prices will tend to occur when the factor used intensively in the sector which can adjust its technique of production to the greatest extent, and which, therefore, will benefit from adjustment, is negatively affected by the initial shock.[7]

2. The dynamic framework

The comparative static analysis of Section 1 abstracted from the specifics of the adjustment process. The technique of production was taken to be fixed in the short run due to unspecified adjustment costs and, as a consequence, each sector's short run production set was Leontief in form. In the long run, any technique encompassed by the long run production functions, equations (1) and (2), could be chosen. In this section the cost of adjustment is made explicit and the path of adjustment between two long run equilibrium positions is characterized.

The dynamic analysis is made tractable, and the nesting of the short run fixed coefficients production function within its long run counterpart facilitated, by assuming that the long run production function in each sector takes the Cobb–Douglas form:

$$X = L_x^{\alpha} K_x^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{9}$$

$$Y = L_y^{\beta} K_y^{1-\beta}, \qquad 0 < \beta < 1, \tag{10}$$

where β must be larger than α since the production of X is assumed to be more capital intensive than that of Y.

An infinite number of different capital–labour ratios (techniques of production) are consistent with equations (9) and (10). However, at any point in time only one technique can be used in each sector. Thus, for any two techniques of production, k_x and k_y, the production functions (9) and (10) take the following Leontief form:

$$X = \min\left[k_x^{1-\alpha} L_x, \frac{K_x}{k_x^{\alpha}}\right], \tag{11}$$

$$Y = \min\left[k_y^{1-\beta} L_y, \frac{K_y}{k_y^{\beta}}\right], \tag{12}$$

where k_x and k_y are the capital–labour ratios in sectors X and Y respectively.

As in Section 1 above, factor prices are assumed to be perfectly flexible and (ruling out specialization for simplicity) this implies that labour and capital are fully employed at all points in time. As a result, for any two production techniques, the quantities produced of X and Y are:

$$X = \frac{k_x^{1-\alpha}(\bar{K} - k_y\bar{L})}{k_x - k_y}, \tag{13}$$

$$Y = \frac{k_y^{1-\beta}(k_x\bar{L} - \bar{K})}{k_x - k_y}, \tag{14}$$

where $\bar{K}$ and $\bar{L}$ are the economy's endowments of capital and labour respectively.

The slow adjustment of the technique of production is assumed to result from an adjustment process in which some quantity of a good must be sacrificed in order to change the technique used to produce that good. Specifically, it is assumed that the cost of adjustment (the output sacrificed) in each sector takes the following quadratic form:

$$\tilde{X} = h(\dot{k}_x)^2 X, \tag{15}$$

$$\tilde{Y} = g(\dot{k}_y)^2 Y, \tag{16}$$

where g and h are both positive scalars and $\dot{k}_x$ and $\dot{k}_y$ are the time derivatives of k_x and k_y (and, therefore, represent the speed at which each short run Leontief isoquant moves along its envelope, the Cobb–Douglas isoquant).

The adjustment cost functions given in equations (15) and (16) are consistent with a large literature (see, for example, Gould (1968), Brechling (1975), Purvis (1976) and, for an application to trade, Mussa (1982)). Adjustment costs arise only when the technique of production is changing, that is when $\dot{k}_x$ and $\dot{k}_y$ are non-zero, and depend upon the size of the plant which must be altered. In addition, the quadratic form of (15) and (16) implies that the marginal cost of adjustment is increasing, thereby precluding instantaneous adjustment.[8]

The planner's problem

The optimal adjustment path of the technique of production can be determined by choosing $\tilde{X}$ and $\tilde{Y}$ to maximize the present discounted value of domestic income.[9] More specifically, this involves choosing $\tilde{X}$ and $\tilde{Y}$ to maximize the objective function

$$\int_0^\infty e^{-\rho t}(P_x X + P_y Y - P_x\tilde{X} - P_y\tilde{Y})\,dt \tag{17}$$

(where ρ is the social discount rate) subject to (13), (14), (15), (16), $X > 0$ and $Y > 0$.

Substitution of equations (13), (14), (15), and (16) into (17), yields the current valued Hamiltonian:

$$H_{cv}=\frac{P_x k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}{k_x-k_y}+\frac{P_y k_y^{1-\beta}(k_x\bar{L}-\bar{K})}{k_x-k_y}-\frac{P_x h u^2 k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}{k_x-k_y}$$
$$-\frac{P_y g v^2 k_y^{1-\beta}(k_x\bar{L}-\bar{K})}{k_x-k_y}+\lambda_x u+\lambda_y v, \qquad u=\dot{k}_x, \qquad v=\dot{k}_y, \quad (18)$$

where λ_x and λ_y represent the marginal benefit of an increase in the capital intensity of production in sectors X and Y respectively evaluated at the point in time when the increase takes place.

Along the optimal path u and v are chosen to maximize the current valued Hamiltonian. The first order conditions for this problem are:

$$\frac{\partial H_{cv}}{\partial u}=\frac{-2P_x h u k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}{k_x-k_y}+\lambda_x=0,$$
$$\frac{\partial H_{cv}}{\partial v}=\frac{-2P_y g v k_y^{1-\beta}(k_x\bar{L}-\bar{K})}{k_x-k_y}+\lambda_y=0.$$

These can be solved for the optimal values of u and v:

$$u^*=\frac{\lambda_x(k_x-k_y)}{2P_x h k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}, \quad (19)$$
$$v^*=\frac{\lambda_y(k_x-k_y)}{2P_y g k_y^{1-\beta}(k_x\bar{L}-\bar{K})}. \quad (20)$$

Substitution of (19) and (20) into (18) gives the concentrated Hamiltonian:

$$H_{cv}^*=\frac{P_x k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}{k_x-k_y}+\frac{P_y k_y^{1-\beta}(k_x\bar{L}-\bar{K})}{k_x-k_y}+\frac{\lambda_x^2(k_x-k_y)}{4P_x h k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}$$
$$+\frac{\lambda_y^2(k_x-k_y)}{4P_y g k_y^{1-\beta}(k_x\bar{L}-\bar{K})}. \quad (21)$$

Differentiation of H_{cv}^* with respect to k_x, k_y, λ_x and λ_y yields the canonical equations:

$$\frac{\partial H_{cv}^*}{\partial k_x}=\frac{(\bar{K}-k_y\bar{L})}{(k_x-k_y)^2}[P_y k_y^{1-\beta}-\alpha P_x k_x^{1-\alpha}-(1-\alpha)P_x k_y k_x^{-\alpha}]$$
$$+\frac{\lambda_x^2(k_y+\alpha(k_x-k_y))}{4P_x h k_x^{2-\alpha}(\bar{K}-k_y\bar{L})}-\frac{\lambda_y^2(\bar{K}-k_y\bar{L})}{4P_y g k_y^{1-\beta}(k_y\bar{L}-\bar{K})^2}=-\dot{\lambda}_x+\rho\lambda_x, \quad (22)$$

$$\frac{\partial H_{cv}^*}{\partial k_y}=\frac{(\bar{K}-k_x\bar{L})}{(k_x-k_y)^2}[P_x k_x^{1-\alpha}-\beta P_y k_y^{1-\beta}-(1-\beta)P_y k_y^{-\beta}k_x]$$
$$-\frac{\lambda_x^2(\bar{K}-k_x\bar{L})}{4P_x h k_x^{1-\alpha}(\bar{K}-k_y\bar{L})^2}-\frac{\lambda_y^2((1-\beta)k_x+\beta k_y)}{4P_y g k_y^{2-\alpha}(k_x\bar{L}-\bar{K})}=-\dot{\lambda}_y+\rho\lambda_y, \quad (23)$$

$$\frac{\partial H_{cv}^*}{\partial \lambda_x}=\frac{\lambda_x(k_x-k_y)}{2P_x h k_x^{1-\alpha}(\bar{K}-k_y\bar{L})}=\dot{k}_x, \quad (24)$$

$$\frac{\partial H_{cv}^*}{\partial \lambda_y}=\frac{\lambda_y(k_x-k_y)}{2P_y g k_y^{1-\beta}(k_x\bar{L}-\bar{K})}=\dot{k}_y. \quad (25)$$

The path the economy follows from one long-run equilibrium to another is determined by these four non-linear differential equations. To make the characterization of this path feasible, it is necessary to reduce the model to a system of two differential equations.[10] This can be done by analyzing the special case in which the adjustment cost parameter g in sector Y approaches infinity. This effectively prevents adjustment in this sector (see equation (25)) and, as a consequence, fixes k_y. This makes it possible to analyze the determination and adjustment of the technique of production in sector X using equation (24) and the following modified version of equation (22):

$$\frac{(\bar{K}-k_y\bar{L})}{(k_x-k_y)^2}[P_y k_y^{1-\beta}-\alpha P_x k_x^{1-\alpha}-(1-\alpha)P_x k_y k_x^{-\alpha}]+\frac{\lambda_x^2(k_y+\alpha(k_x-k_y))}{4P_x h k_x^{2-\alpha}(\bar{K}-k_y\bar{L})}$$
$$=-\dot{\lambda}_x+\rho\lambda_x. \quad (22')$$

With $\dot{\lambda}_x$ and $\dot{k}_x$ set equal to zero, equations (22′) and (24) are illustrated in Fig. 3.[11] Their intersection determines the long run equilibrium value of the capital–labour ratio in sector X which corresponds to a point such as A in Fig. 1. The form of the $\dot{k}_x=0$ line is determined by the exogenous technology of the adjustment cost function. As shown in equation (24), the marginal supply price of more capital intensive techniques of production, λ_x, depends upon the speed of adjustment and the current technique. The current technique enters this expression because it determines the quantity of X produced and, therefore, has a role in the determination of adjustment costs. The $\dot{\lambda}_x=0$ curve represents the marginal benefit of a more capital intensive technique if that technique is expected to last forever. Thus, λ_x must equal zero at the optimal long run technique.

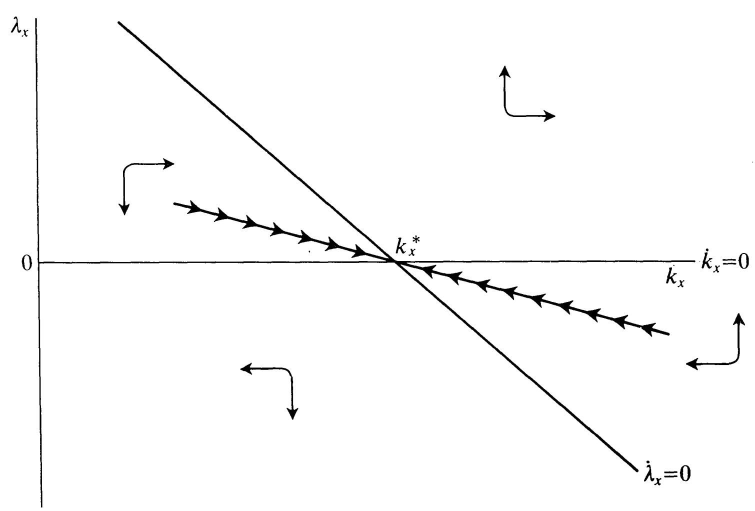

FIG. 3

The arrows in Fig. 3 represent the forces of motion implied by the differential equations (22′) and (24). The determinant of the matrix of partial derivatives of these two equations with respect to k_x and λ_x is negative when evaluated at the long run equilibrium indicating that the two roots of the system have opposite signs. This implies that adjustment to the long run equilibrium by the jump variable λ_x and the state variable k_x takes place along a saddle path as illustrated.

A price shock

A decline in either price leaves the $\dot{k}_x = 0$ curve unaffected since the technique of production is not changing along this curve and, as a consequence, a change in the cost of adjustment has no effect on it. On the other hand, by altering the relative price of X and Y, a fall in the price of Y alters the relative demand for labour and capital and, thereby, the long run wage-rental ratio. The relative price change in favour of the capital intensive sector causes the long run wage-rental ratio to fall and the marginal benefit of increasing the capital–labour ratio to become negative. This change in the marginal benefit of using the existing technique of production shifts the $\dot{\lambda}_x = 0$ curve to $\dot{\lambda}'_x = 0$ in Fig. 4.

On impact λ_x jumps to the negative value λ'_x on the stable arm passing through the new long run equilibrium technique k_x^{**}. Only this value of λ_x induces the appropriate change in the supply of the production technique that leads to the gradual movement down the stable trajectory to k_x^{**}.

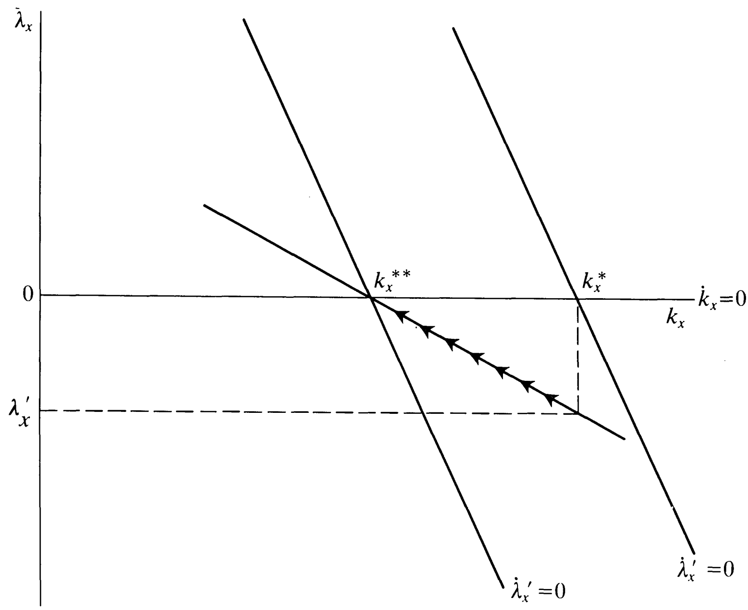

FIG. 4

(Constraining the economy to follow this unique path to the new steady state is equivalent to imposing the transversality conditions (Abel, 1981).)

As can be seen from equation (24), the increase in λ_x along the saddle path from λ_x' causes the rate of change of the technique of production to slow. (The fall in k_x adds to this effect by raising X and, thereby, the cost of adjustment). As λ_x rises toward zero, the benefit from adjusting falls and, as a result, the cost that should be borne in order to adjust the technique of production must also fall. This can only be done by slowing the speed of adjustment. An increase in the h parameter of the adjustment cost function, implying an increase in the cost of adjustment, will slow the speed of adjustment along the whole adjustment path. Similarly, an increase in the discount rate warrants a slower speed of adjustment by reducing the benefit of more efficient future production.

On impact adjustment costs cause the point in time welfare of the economy to be lower than it would have been had adjustment not taken place and adjustment costs not borne. That is, the output available for consumption immediately falls to B from A in Fig. 1 as $\tilde{X}$ suddenly becomes non-zero following the price shock. During the process of adjustment actual production moves along the long run production possibilities frontier from A to A' while the output available for consumption slowly increases along the path from B to A'. The gap between gross production and production net of adjustment costs narrows monotonically since the speed of adjustment gradually slows and fewer resources are committed to adjustment. However, only part way through the adjustment process (at B' in Fig. 1) does the value of net production equal what gross production would have been if adjustment had not taken place.

The effect of a fall in P_y on factor prices and the distribution of output during the adjustment process can be analyzed using equations (13) and (14), which determine X and Y, and the following expressions for the two factor prices:

$$w = \left[\frac{k_x^{1-\alpha}k_y^{1-\beta}(k_x^{\alpha}P_y - P_x k_y^{\beta})}{k_x - k_y}\right] + \frac{\lambda_x^2(k_x - k_y)}{4P_x h k_x^{1-\alpha}(\bar{K} - k_y\bar{L})^2}, \tag{26}$$

$$r = \left[\frac{P_x k_x^{1-\alpha} - P_y k_y^{1-\beta}}{k_x - k_y}\right] - \frac{\lambda_x^2(k_x - k_y)}{4P_x h k_x^{1-\alpha}(\bar{K} - k_y\bar{L})^2}, \tag{27}$$

which are derived from the zero profit conditions.[12] Equations (26) and (27) indicate that the wage and rental depend upon the speed of adjustment. This follows because the costs of adjustment contribute to the total cost of production and, therefore, alter the relative demand for labour and capital. (In addition, because marginal and average cost are the same at all points in time, including the period of adjustment, setting price equal to marginal cost does not imply that firms make negative profits during adjustment.)

The process of adjustment causes the wage to rise and the rental to fall so that the factor used intensively in the sector which is adjusting bears the costs of adjustment. This follows from the fact that adjustment raises the costs which must be borne by that sector and (as with any increase in costs) the effect of this on factor prices is equivalent to a negative price shock (as can be seen by examining the zero-profit conditions). Thus, the factor used intensively in the adjusting sector must lose during adjustment as it would from a negative price shock. In the case under study, since X is capital intensive, the process of adjustment tends to cause the rental to fall and the wage to rise. The magnitude of these effects depends upon the optimal speed of adjustment. The faster is adjustment, the larger the costs capital must bear and the greater the benefits to labour.

The adjustment paths of the two factor prices and the two goods are illustrated in Fig. 5. The XX and YY curves represent the values of X and Y corresponding to different values of k_x. ww and rr represent the values of the wage and rental corresponding to the initial relative price for different values of k_x under the assumption that λ_x equals zero, while $w'w'$ and $r'r'$ are the corresponding curves following the change in P_y.[13]

The decline in the price of Y leaves both XX and YY unaffected since the

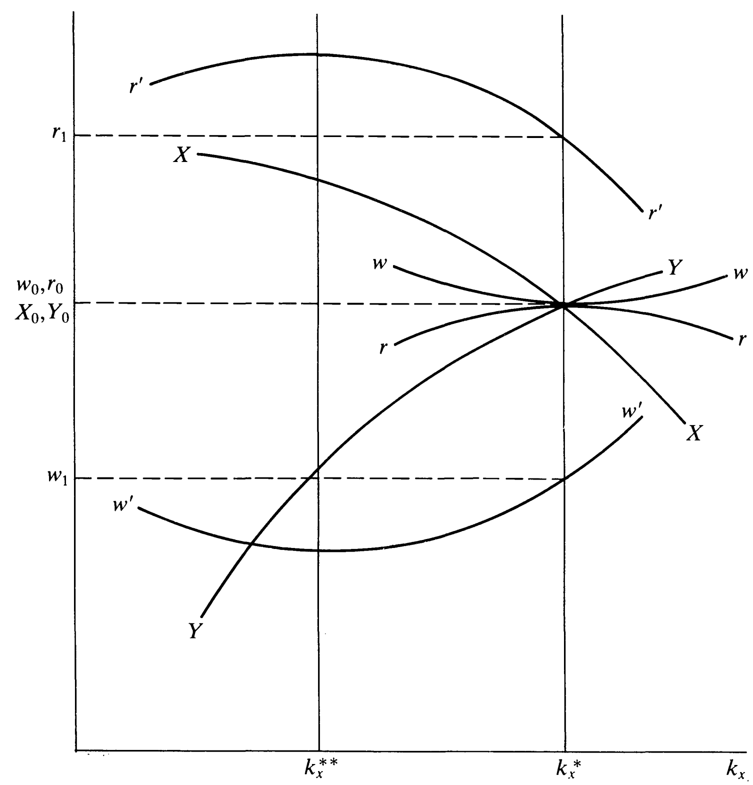

FIG. 5

relationship between the technique of production and the level of output is purely technical and does not depend on the price level. Following the initial price change, the technique of production gradually adjusts to its new long run level k_x^{**}. As this adjustment takes place, the production of X rises monotonically and that of Y declines. Because of the convexity of the production possibilities set, the output of Y falls more rapidly and the output of X rises more slowly as the capital–labour ratio approaches k_x^{**}.

The change in P_y shifts ww down to $w'w'$ and rr up to $r'r'$ since the relative price change favours the capital intensive sector. (Neither rr and $r'r'$ nor ww and $w'w'$ can cross since, with a change in the price of Y, the same capital–labour ratio cannot be associated with the same factor prices.) If the cost of adjustment had no impact on factor prices, the decline in P_y would immediately cause w to fall to w_1 and r to rise to r_1. (Note the correspondence between w_1 and r_1 in Figs 2 and 5.) During adjustment, r would rise up $r'r'$ until k_x reached k_x^{**} and w would gradually fall along $w'w'$.

As noted above, however, the existence of positive adjustment costs alters both the wage and rental and, as a consequence, adjustment does not occur along $r'r'$ and $w'w'$. Rather, the initial impact of the exogenous shock will cause wages to jump to some point above w_1 and the rental to jump to some point below r_1. If the initial level of adjustment costs is large, r_1 could lie below r_0 and w_1 could lie above w_0. In other words, while capital must benefit from adjustment in the long run and labour must lose, the costs of adjustment may be such as to cause capital to lose in the short run and labour to benefit.

Following the initial response, λ_x will gradually fall. This implies that the speed and costs of adjustment decline and, as a result, actual r converges to $r'r'$ from below while w converges to $w'w'$ from above. The rental reaches $r'r'$ and the wage $w'w'$ when the technique of production reaches its new long run level at k_x^{**} and λ_x goes to zero.[14]

Conclusion

This paper has developed a two-sector model in which a short run fixed coefficients production structure is combined with a long run production structure in which factors are substitutable. Given this type of production technology, a relative price change does not alter the composition of output on impact and the Stolper–Samuelson theorem holds on impact and in the long run. However, through adjustment from the impact equilibrium to the new long run equilibrium both factor prices could increase, or one could rise while the other falls. Which result obtains depends upon the relative long run substitutability of factors in the two sectors. If factors are relatively more substitutable in one sector than in the other, the factor used intensively in the first sector tends to gain the most from adjustment. If the initial price change goes against the sector in which factors are relatively more substitutable, factor prices tend to overshoot their long run values on impact since the factor used intensively in the sector which is negatively affected by the initial shock benefits from adjustment.

The predictions of the model with a slowly adjusting technique of production differ from those of specific factor models in three key ways. The Stolper–Samuelson Theorem holds in the short run, a relative price change has no immediate impact on output, and both factors could benefit from adjustment.

An example is given in which the cost of adjustment is made explicit and the path from a short run to a long run equilibrium is characterized for an exogenous shock. Adjustment immediately attains its maximum speed and then slows as movement toward the new long run equilibrium proceeds. The existence of adjustment costs alters both the wage and rental and, if large enough, could cause factor prices to move on impact in a direction opposite to that which they would take in the absence of adjustment costs.

From the analysis above it is clear that there exist a wide variety of potential short and long run factor price combinations following a relative price change. Detailed knowledge of relative long run elasticities of substitution, not just relative factor intensities, and the size of possible adjustment costs is required to determine which factor will benefit from adjustment. This is important from a policy perspective since the direction of factor price movements may indicate which factors will support adjustments from the impact equilibrium to the new long run equilibrium following a change in trade policy, and which will oppose it, and whether or not this support will continue throughout the entire adjustment process.

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[1] The analysis presented below also bears a resemblance to the putty-clay literature (see, for example, Phelps (1963), Bliss (1968) and Abel (1981)). Standard putty-clay and vintage capital models assume that at the time of construction a unit of capital can be chosen to embody any production technique, but, once built, it is characterized by that technique forever. This implies that the only method of changing the average production technique of the existing capital stock is through the construction of new capital and the depreciation of old. The focus of most of the analysis which uses this framework has been on savings, growth and the evolution of productivity. In contrast, the present analysis concentrates on the retooling and refitting of existing capital by assuming the size of the capital stock is fixed in both the short run and the long run, but the technique of production is fixed in the short run only. (Ignoring capital accumulation allows the role of factor substitutability to be emphasized.) Thus, unlike putty-clay models, capital is never completely malleable at a point in time. Furthermore, this framework is used to examine different issues than those of interest to the putty–clay literature. These include the relationship between the short run impact of an exogenous shock and its long run impact on the distribution of output and factor prices, and the movement these variables take during adjustment.

[2] For the model to yield a unique factor price equilibrium, all that is required is that it take less time to shift capital and labour between sectors, at the margin, than to alter the technique of production. The perfect mobility assumption simplifies the analysis.

[3] The short run production possibility frontier being everywhere within the long run production possibility frontier, except at one point, is a characteristic of specific factor models as well. The short run production possibility frontier for the Leontief case is not necessarily contained within the short run production possibility frontier of a specific capital model.

[4] McCurdy (1984) uses a similar framework.

[5] For simplicity the possibility of specialization is ignored.

[6] The dual of the two-sector model is analyzed in Woodland (1977) and Mussa (1979).

[7] The constraint on the adjustment of the technique of production does not alter the prediction of the Rybczynski theorem that an increase in the endowment of one factor increases the output of the good which uses that factor intensively and decreases the output of the other good. This must be the case because the change in factor endowments does not affect output prices. As a result, it has no impact on relative factor prices and, consequently, on the optimal technique of production. Since there is no pressure on the factor use ratio to change when the economy's endowment of factors is altered, the constraint on changing the technique of production is non-binding. This contrasts with sector specific factors models in which the Rybczynski theorem does not hold in the short run (Neary (1978)).

[8] Neary (1982) takes an alternative approach by simply imposing an adjustment function. While this avoids the problem of choosing an adjustment cost function, it abstracts from the choice of adjustment speed as part of the economy's optimization problem.

[9] This optimal planning problem yields an optimum which is identical to that given by a competitive equilibrium model as shown by Becker (1981) and Mussa (1982).

[10] This problem has been faced by others as well. See Sampson (1976) for an example.

[11] The slope of the $\dot{\lambda}_x = 0$ curve is

$$\left.\frac{d\lambda_x}{dk_x}\right|_{\dot{\lambda}_x=0} = \left[\frac{-\partial^2 H^*_{cv}}{\partial k_x^2}\right]\left[\frac{\lambda_x(\alpha(k_x - k_y) + k_y)}{(\bar{K} - k_y\bar{L})P_x 2hk_x^{2-\alpha}} - \rho\right]^{-1}.$$

This is negative when evaluated at the long run equilibrium since in equilibrium λ_x equals zero and $\partial^2 H^*_{cv}/\partial k_x^2$ is negative by the second order conditions.

[12] The zero profit conditions for sectors X and Y are:

$$P_x = \frac{w}{k_x^{1-\alpha}} + k_x^{\alpha} r + P_x h\dot{k}_x^2,$$

$$P_y = \frac{w}{k_y^{1-\beta}} + k_y^{\beta} r.$$

[13] The slopes of the cures in Fig. 5 (with $\lambda_x = 0$) are:

$$\left.\frac{dX}{dk_x}\right|_{XX} = \frac{-(\bar{K} - k_y\bar{L})(\alpha k_x + (1-\alpha)k_y)}{k_x^{\alpha}(k_x - k_y)^2} < 0$$

$$\left.\frac{dY}{dk_x}\right|_{YY} = \frac{(k_y^{1-\beta}(\bar{K} - k_y\bar{L})}{(k_x - k_y)^2} > 0$$

$$\left.\frac{dw}{dk_x}\right|_{ww} = \frac{k_y[(1-\alpha)k_x^{-\alpha}k_y P_x + \alpha k_x^{1-\alpha}P_x - k_y^{1-\beta}P_y]}{(k_y - k_x)^2} \quad \begin{matrix} >0 & \text{if} & k_x > k_x^* \\ =0 & \text{if} & k_x = k_x^* \\ <0 & \text{if} & k_x < k_x^* \end{matrix}$$

$$\left.\frac{dr}{dk_x}\right|_{rr} = \frac{[k_y^{1-\beta}P_y - (1-\alpha)k_x^{-\alpha}k_y P_x - \alpha k_x^{1-\alpha}P_x]}{(k_x - k_y)^2} \quad \begin{matrix} <0 & \text{if} & k_x > k_x^* \\ =0 & \text{if} & k_x = k_x^* \\ >0 & \text{if} & k_x < k_x^* \end{matrix}$$

where k_x^* is the long run value of k_x.

[14] The impact effect of adjustment costs on the wage and rental can also be illustrated using the unit cost curves of Fig. 2. In the absence of adjustment costs, the wage and rental jump to w_1 and r_1 immediately following a decline in P_y. However, since firms in sector X cover their adjustment as well as production costs, the unit cost curve for sector X, C_x^0, shifts to the left when λ_x becomes non-zero. As a result, the impact wage and rental will be on C_y^1 to the left of

point V with a lower rental and higher wage than in the case without adjustment costs. On impact it is possible for the rental to fall below what it would have been had the decline in the price of the labour intensive good never occurred.

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