

ECON 399- W08 MIDTERM

1) Economic Model

- Theoretical statement of relationship between y & x variables

(ie) $Utility = F(\text{income, hamburgers, intellect})$

TRUE Econometric Model

- Mathematical statement of relationship between y & x variables

(ie) $Utility_i = \beta_0 + \beta_1 \text{income}_i^2 + \beta_2 \text{hamburgers}_i + \beta_3 \ln(\text{intellect}_i) + u_i$

ESTIMATED Econometric Model

- Estimated Mathematical relationship between AVAILABLE data

(ie) $\hat{Utility}_i = 6.7 + 1.3 I_i^2 + 0.4 H_i$

$$\textcircled{a} \quad \hat{\beta}_1 = \frac{\sum (A - \bar{A})(B - \bar{B})}{\sum (A - \bar{A})^2} = \frac{\sum (A - \bar{A})B}{SST_A} = \frac{75}{25} = 3$$

$$\begin{aligned} \hat{\beta}_0 &= \bar{B} - \hat{\beta}_1 \bar{A} \\ &= 12 - 3(8) \\ &= -12 \end{aligned}$$

$$\begin{aligned} \textcircled{b} \quad SSR &= SST_B - SSE \\ &= 45 - 15 = 30 \end{aligned}$$

$$R^2 = \frac{SSR}{SST_B} = \frac{30}{45} = \frac{2}{3}$$

$$\textcircled{c} \quad \boxed{\Delta \hat{B} = 3 \Delta \hat{A}} \text{ if } \Delta u = 0$$

$$\textcircled{d} \quad \hat{\sigma}^2 = \frac{SSR}{n-k-1} = \frac{30}{62-2} = 0.5$$

$$\textcircled{e} \quad \hat{\phi}_1 = \frac{\sum AB}{\sum A} = \frac{32}{8(62)} = 0.0645$$

③^a IF Aspartame has no TRUE effect

→ No negative implications of exclusion

→ Including increases multicollinearity

④ IF Aspartame has a TRUE effect,

→ This will cause bias in the model

(invalidate all results) unless Aspartame is
uncorrelated w/ ALL other explanatory

variables

→ Omitting Aspartame will put it
into the error, increasing error

variance

→ Omitting Aspartame will generally

fight multicollinearity unless

it is uncorrelated w/ all x 's.

$$(4) \sum \hat{e}^2 = \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i + \hat{\beta}_2 x_i + \hat{\beta}_3 x_i)^2$$

$$\frac{\partial \sum \hat{e}^2}{\partial \hat{\beta}_0} = \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i + \hat{\beta}_2 x_i + \hat{\beta}_3 x_i) = 0 \Rightarrow \sum \hat{e} = 0$$

$$\frac{\partial \sum \hat{e}^2}{\partial \hat{\beta}_1} = \sum x_i \quad " \quad \sum x_i = 0 \Rightarrow \sum \hat{e} x_i = 0$$

$$\frac{\partial \sum \hat{e}^2}{\partial \hat{\beta}_2} = \sum x_i \quad " \quad \sum x_i = 0 \Rightarrow \sum \hat{e} x_i = 0$$

$$\frac{\partial \sum \hat{e}^2}{\partial \hat{\beta}_3} = \sum x_i \quad " \quad \sum x_i = 0 \Rightarrow \sum \hat{e} x_i = 0$$

(5) ① Bias - Purposeful poor selection

② Sample Selection Problem - Accidental poor selection

③ Small Sample - not representative

④ Large Sample - ie > 80% of pop

6a) $\ln p = \log(\text{price})$
 $\ln q = \log(\text{quantity})$
 $\ln i = \log(\text{income})$

OLS $\ln p \ln q \ln i$ Advertising

b) Test $\ln i = 0$

⑦ Partialling out analyzes x_i 's effect on y by first removing x_i 's correlation w/ all other x 's

STEP 1: Regress $x_i = F(\text{other } x\text{'s})$ } This removes
Obtain residuals $\hat{r}$ } correlation

STEP 2: Calculate $\hat{\beta}_i = \frac{\sum \hat{r} y}{\sum \hat{r}^2}$

$$\begin{aligned} \textcircled{8} \text{ Bias} &= E(\hat{\beta}_j) - \beta_j \\ &= \beta_j + E\left(\frac{\bar{x} - \bar{y}}{\text{Var}(u)}\right) - \beta_j \\ &= (\bar{x} - \bar{y}) E\left(\frac{1}{\text{Var}(u)}\right) \end{aligned}$$

$$\text{Bias} = 0 \text{ when } \bar{x} - \bar{y} = 0$$

$\bar{x} = \bar{y}$

⑨ Unbiasedness = sample
Consistent = population

19a)
$$\hat{UTIL} = \underset{(17.33)}{-6.34} + \underset{(0.18)}{3.62} \text{Social}_i + \underset{(0.99)}{2.96} \text{Mark}_i - \underset{(0.25)}{2.44} \text{Work}_i$$

$$+ \underset{(0.98)}{0.198} \text{Class}_i + \underset{(0.098)}{0.00572} \text{Build}_i$$

 $R^2 = 0.7684$
 $N = 168$

$$\hat{UTIL} = \underset{(5.59)}{-5.46} + \underset{(0.18)}{3.62} \text{Social}_i + \underset{(0.97)}{2.99} \text{Mark}_i - \underset{(0.25)}{2.44} \text{Work}_i$$

 $R^2 = 0.768$
 $N = 168$

b) $H_0: \beta_4 = 0$
 $H_a: \beta_4 \neq 0$

$$t = \frac{\hat{\beta}_4}{SE(\hat{\beta}_4)} = \frac{0.198}{0.98} = 0.20$$

 $t_{\alpha=0.05, df=\infty}^* = 1.960$
 $t < t^*$, DO NOT reject H_0 ; class doesn't matter @ $\alpha = 5\%$.

c) $H_0: \beta_5 \leq 0$
 $H_a: \beta_5 > 0$

$$t = \frac{\hat{\beta}_5}{SE(\hat{\beta}_5)} = \frac{0.00572}{0.098} = 0.584$$

 $t_{\alpha=0.1, df=\infty}^* = 1.282$
 $t < t^*$, DO NOT reject H_0 ; build doesn't have a +ve effect on Util @ $\alpha = 10\%$.

d) $H_0: \beta_4 = \beta_5 = 0$
 $H_a: \text{Not } H_0$

$$F = \frac{(R_{UR}^2 - R_L^2)/q}{(1 - R_{UR}^2)/(n - k - 1)} = \frac{(0.7684 - 0.768)/2}{(1 - 0.7684)/(168 - 6 - 1)}$$

$$= \frac{0.0001/2}{0.2316/161} = 0.035$$

 $F^*_{df=2, 161} = 4.61$

Therefore, since $F < F^*$, DO NOT reject H_0 ; class & build are jointly insignificant @ $\alpha = 1\%$.

e) $H_0: \beta_0 = \beta_1 = \dots = \beta_5 = 0$ (Model insignificant)
 $H_a: \text{Not } H_0$
 $F_{df=6, 161}^* = 2.80$

$$F = \frac{R^2/k}{(1 - R^2)/(n - k - 1)} = \frac{0.7684/6}{(1 - 0.7684)/(168 - 6 - 1)}$$

 $= 89.0$
 $F > F^*$, reject H_0 ; model is significant @ $\alpha = 0.01$.

$$\textcircled{PF} \quad CI = \hat{\beta}_j \pm t^* se \hat{\beta}_j$$

$$t_{\alpha=0.1, df=2}^* = 1.645$$

$$CI_{\hat{\beta}_1} = 3.62 \pm 1.645(0.18) \\ = [3.323, 3.916]$$

$$CI_{\hat{\beta}_0} = 2.99 \pm 1.645(0.97) \\ = [1.39, 4.59]$$

$$CI_{\hat{\beta}_3} = -2.44 \pm 1.645(0.25) \\ = [-2.85, -2.03]$$