

## ASSIGNMENT 6, due date April 11th, 2014

1. This question requires from you to pull together all the knowledge about structure formation.

Let us consider matter-dominated flat cosmology with  $H_0 = 67.3 \text{ km/s/Mpc}$ . Let us have present day magnitude of inhomogeneities at cluster scales  $\sigma(8h^{-1} \text{ Mpc}) = 0.83$ . Let us also suppose that  $\sigma^2(R) \propto 1/R$  at scales between cluster scales and galactic scales (this is actually close to reality).

Assuming that galaxy has formed from density enhancements that are  $\delta \approx 2\sigma(R_{\text{galactic}})$ , find for the galaxy with  $M = 10^{11} M_\odot$

- (a) Questions about typical galaxy of  $10^{11} M_\odot$ .
  - i. at what redshift  $z_c$  the halo of such galaxy has formed
  - ii. at what redshift the halo of such galaxy 'turned around' and got decoupled from overall expansion
  - iii. what is the average density of the galactic halo.
- (b) Questions about relatively rare cluster of galaxies of  $10^{14} M_\odot$ . Now answer the same questions for a galactic cluster of  $M = 10^{14} M_\odot$ , but assuming that it is a more rare event, of  $\delta = 3\sigma(R_{\text{cluster}})$ .
- (c) How many clusters of  $M = 10^{15} M_\odot$  are expected in the observable volume of such Universe ? (Definition: cluster has formed and exists if it already collapsed by the present time)
- (d) Within the spherical collapse model, compute the physical size (radius) of the halos for the  $M = 10^{14} M_\odot$  galaxy cluster and  $M = 10^{11} M_\odot$  galaxy.

As a hint: investigations should start with determining  $R_{\text{galaxy}}$  and  $R_{\text{cluster}}$  from the mass of the galaxy and the cluster, respectively.

2. Features in the CMB angular spectrum. This is a list of related questions (which are best addressed in order). Some questions may require numerical integration. It is useful to recall the early assignments, where we (you) investigated the model with radiation and matter in detail as well as the notion of angular-diameter distance. Lambda term is fairly negligible at the time of photon last scattering !

Consider flat Lambda-CDM model with  $H_0 = 67.3 \text{ km/s/Mpc}$ ,  $\Omega_b = 0.049$ ,  $\Omega_{\text{cdm}} = 0.266$  (Planck values). Temperature of the CMB at the present time is  $2.725 \text{ K}$ . Assume decoupling of photons and baryons happened at  $z = 1088$ .

- (a) Find the sound speed in the photon-baryon plasma at the moment when photons decouple the baryons (last scattering moment)
- (b) Determine the physical and comoving sizes of the sound horizon at the moment of last scattering.
- (c) Determine the angular size the sound horizon at last scattering subtends on the present day observed sky
- (d) Estimate the multipole separation between peaks in the angular power spectrum of CMB temperature fluctuations.