Assignment 4, due March 14th

Problem I We discussed the nucleosynthesis from the modern perspective where we now the properties of CMB. However, the theory of hot Universe as the mechanism for creating of heavy elements was proposed by George Gamow when CMB was not yet observed. Indeed CMB and, importantly, its temperature, was a prediction of this theory.

In this problem I ask to to reproduce the prediction by George Gamow (1942) of the existence and *present-day temperature* of the relict radiation from early hot Universe.

Gamow based his prediction on the idea that cosmological nucleosynthesis is responsible for production of chemical elements. Gamow realized the of nucleosynthesis should be brief to retain some deuterium.

What did Gamow knew and you can use

Known facts:

Nuclear physics

- Nucleosynthesis starts with deuterium fusion

$$p + n \leftrightarrow D + \gamma \tag{1}$$

- One can roughly use Saha formula for n_n, n_p, n_D (where n_p, n_n are number densities of free protons an neutrons) to determine when reaction starts, defining this moment as $n_n = n_D$. As we derived

$$\frac{n_D}{n_n} \approx 6 n_p \left(\frac{m_p kT}{\pi \hbar^2} \right)^{-3/2} e^{\frac{B_D}{kT}} \tag{2}$$

  Of course he knew the binding energy of Deuterium $B_D = 2.22 Mev$ and all fundamental constants.

- Clearly there is significantly less neutrons than protons before nucleosynthesis (otherwise we would have been left without any hydrogen !), so at nucleosynthesis time one can roughly put

$$n_p \approx n_{baryons} \tag{3}$$

- Gamov knew statistical physics and could relate the number density and energy of black body photons to the temperature

$$n_\gamma = 0.243 \left(\frac{kT}{\hbar c} \right)^3 \tag{4}$$

$$e_\gamma = \frac{\pi^2 \hbar c}{15} \left(\frac{kT}{\hbar c} \right)^4 \tag{5}$$

- in 1942 some nuclear data became available, and Gamow had an idea about the reaction rate of the deuterium fusion. He concluded that $T_{nuc} \approx 10^9\ K$.

Astrophysics

- Deuterium is observed, so nucleosynthesis must cease practically at the same time as it starts.
- Universe expands and present day expansion rate is

$$H_0 \approx 100 km/s/Mpc \tag{6}$$

  (From original Edwin Hubble's results Gamow may have been using larger value)

- There were no indication at the time for the dark matter or the dark energy, so a natural assumption for Gamow was to consider flat Universe, where the present day matter is all baryons.

So, given these knowns, derive the prediction for present day temperature of the Microwave Background, T_{now}.

Problem II Compute idealized freeze out time for Deuterium fusion reaction. Idealized, since it does not take neutron decay in the account. Adopt $\eta = 5.5 \times 10^{-10}$. According to modern data, *crossection* for neutron capture by the proton is

$$\sigma_{n+p} = 0.33 \times 10^{-28} m^2 \times \left(\frac{2200\ m/s}{v} \right) \tag{7}$$

where v is the relative velocity of the neutron and proton target.