ASSIGNMENT 2, due date Fri Feb 7th

The problem

In this problem you solve for evolution of the scale factor in the spatially flat universe filled by radiation and matter. So we consider Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(\frac{\varepsilon_{m0}}{a^3} + \frac{\varepsilon_{r0}}{a^4}\right) \tag{1}$$

Analytical solutions exists if one uses conformal time η as the variable. Let us develop the full theory

1. Rewrite Friedman equation using conformal time and density paramters Ω_m and Ω_r. Is there a relation between the two ? How many parameters define the problem ?
2. Introduce dimensionless conformal time $\tilde{\eta} = H_0\eta$ and search for solution in the form

$$a(\tilde{\eta}) = a_0\tilde{\eta}(\tilde{\eta} + \tilde{\eta}_0) \tag{2}$$

   where $\tilde{\eta}_0$ and a_0 are yet unknown constants. Determine them, assuming $a(\eta_{now}) = 1$
3. Well, find what value of $\tilde{\eta}_{now}$ corresponds to the present time !
4. What is the size of the present day horizon in such Universe, as a function of H_0 and Ω_m ?
5. Find at what time moment the energy density in matter and radiation were equal.
6. What was the comoving and proper sizes of the horizon at the moment of matter-radiation equality ? How many horizons at equality fit into the present day horizon ?
7. Add numerical evaluation of the previous three questions for $\Omega_r = 10^{-4}$