ASSIGNMENT 1, due date Fri. January 31st.

Note: this assignment will have an extra low weight in the final mark, unless you do it perfectly !

Problem I: While in class we have used a metric for homogeneous and isotropic Universe in the form

$$ds^2 = c^2 dt^2 - a^2(t)\left(dr^2 + S_k^2(r) d\Omega^2\right) \tag{1}$$

where time variable t measures the proper time of a stationary observer ($dr = d\Omega = 0 \to ds = cdt$), this metric is often useful to write be using using so-called conformal time η, related to t by differential relation $d\eta = cdt/a(t)$. In conformal time, interval is given by

$$ds^2 = a^2(\eta)\left(d\eta^2 - dr^2 - S_k^2(r) d\Omega^2\right) \tag{2}$$

Rederive the relation between the scale factors of emission $a(\eta_e)$ and absorption $a(\eta_o)$ of the light and the redshift z it experiences while propagating radially, by working exclusively in conformal time coordinates. Start with the presentation of an accurate space-time diagram for propagation of light between emitter and observer in this coordinate frame.

Problem II: GR refresher

For the Universe with flat spatial sections, with metric specified by the interval

$$ds^2 = a^2(\eta)\left(d\eta^2 - dx^2 - dy^2 - dz^2\right) \tag{3}$$

(I have denoted the spatial coordinates as $x^1 = x, x^2 = y, x^3 = z$ in Cartesian tradition) calculate

1. Inverse contravariant metric tensor $g^{\alpha\beta}$
2. Ricci tensor component R_0^0

Note: we use Latin letters for spatial indexes. On my website there is a summary of useful formulae. Be careful with Einstein summation convention