ASSIGNMENT 4, due date March 14th

Problem I In this problem you are asked to draw maps of isocontours of line-of-sight component of rotation velocity V_{ls} ("spider" diagram) for three galaxies that are observed under inclination angle $i = 30°$ and have the following rotation curves

1. $V(R) = V_0 = const$ (flat rotation curve)
2. $\Omega(R) = V(R)/R = \Omega_0 = V_0/R_0 = const$ (solid body rotation)
3. $L(R) = RV(R) = L_0 = R_0V_0 = const$ (hypothetical case of constant angular moment at every orbit)

We shall consider overall motion of the galaxies with respect to observer to be subtracted out.

First some **theory**:

Observer's coordinate system Let us introduce (see Fig. 2) the following coordinate system, related to observer which sees distant spiral galaxy at some inclination angle, so that the image is elliptical

- Origin of the coordinates is at the center of the image
- x direction is in the plane of the image along the longest dimension of the image
- y direction is in the plane of the image along the shortest dimension of the image
- z direction is along the line of sight of the observer, *pointing towards the observer.* I chose this direction of z in order x, y, z be a normal, right-handed coordinate system.

In this coordinate system our task is to find z-component of the velocity of the star which is on a circular orbit in the plane of the galaxy, as a function of image coordinates x, y. i.e $V_{ls}(x, y) = -V_z(x, y)$. Minus sign is since in our measurement we consider velocity positive when it is away from us, but we took z-axis pointing at observer.

Galactic coordinate system Let us first consider the galactic cartezian coordinates x_g, y_g, z_g where x_g, y_g are in the plane of the disk, while z_g is orthogonal to the the galactic plane (Fig. 1). We need to discuss two vectors. First, $\vec{r}_g$ - the position of a star in the disk, and the second $\vec{V}(\vec{r}_g)$ - its velocity. The star is considered to be on a circular orbit with radius R have circular velocity $V(R)$, so

$$\begin{aligned} \vec{r}_g &= (x_g, y_g, z_g) = (R\cos\phi, R\sin\phi, 0) \\ \vec{V}(\vec{r}_g) &= (V_{gx}, V_{gy}, V_{gz}) = (V(R)\sin\phi, -V(R)\cos\phi, 0) \\ where\ \ R &= \sqrt{x_g^2 + y_g^2} \end{aligned} \tag{1}$$

Relation between galactic and observers coordinates . Galaxy is observed at inclination angle i, so, taking x_g direction in the disk to coincide with x direction on the image, the transformation between galactic and observer coordinates is that of a rotation around x-axis by angle $90° - i$ (see Fig. 3), which mixes y and z coordinates, but leaves x unchanged. Thus,

$$\begin{aligned} x &= x_g \\ y &= y_g \sin i + z_g \cos i \\ z &= -y_g \cos i + z_g \sin i \end{aligned} \tag{2}$$

For the star, which has $z_g = 0$, image coordinates are $x = x_g$ and $y = y_g \sin i$.

rotation velocity in observers coordinates The vector of the velocity of a star in observer's coordinates is obtained by the same rotation as the position vector

$$\begin{aligned} V_x &= V_{gx} \\ V_y &= V_{gy} \sin i + V_{gz} \cos i \\ V_z &= -V_{gy} \cos i + V_{gz} \sin i \end{aligned} \tag{3}$$

Line-of-sight velocity as function of x, y . The line of sight rotational velocity is $V_{ls} = -V_z$, so we have

$$
\begin{aligned}
x &= R\cos\phi \\
y &= R\sin\phi\sin i \\
V_{ls} &= -V(R)\cos\phi\cos i
\end{aligned} \tag{4}
$$

where in the last formula we must express ϕ and R via x, y. Making some substitutions

$$
\begin{aligned}
V_{ls}(x,y) &= -\frac{V(R)}{R} x \cos i \\
R^2 &= x^2 + (y/\sin i)^2
\end{aligned} \tag{5}
$$

This is our main formula. Now to the problem !

Problem and the hints You have to draw the lines of constant line-of-sight velocity in the image plane, i.e lines of $V_{ls}(x,y) = C$ for different values of C for three galaxies we have defined in the beginning. Choose useful dimensionless units for x, y and velocities, normalizing them appropriately by V_0 and R_0. Make the plots carefully. Discuss the qualitative difference between the graphs you obtained. What is the difference between the flat and the falling rotation curve ? The flat and the rising one ?

Problem II : The Sagittarius dwarf galaxy is about 20 kpc from the Galactic center: find the mass of the Milky Way within that radius, assuming that the rotation curve remains flat with $V(R) \approx 200\ km/s$. Show that this dwarf galaxy would need the mass $M \sim 6 \times 10^9 M_\odot$ to keep the stars at 5 kpc from being tidally teared off. What mass-to-light ratio M/L in the V band will it require for the Sagittarius dwarf ? (It comes to be much larger than those listed in Table 4.2.)