ASSIGNMENT 2, due date Feb 8

1. Show that if the mass distribution of stars in the range of masses from the lower value M_l to the upper one M_u is described by Salpeter mass function

$$\xi(M)dM = \xi_0 M^{-2.5} dM \tag{1}$$

and $M_l \ll M_u$, then the total number and mass of all stars is determined mostly by M_l, but the total luminosity – by M_u. Write down explicitly expressions for these three quantities. (Note: recall that Salpeter mass function is an initial mass function, thus refers to stars on the main sequence, you will need relation between the luminosity and the mass for the main sequence stars)

Taking $M_l = 0.3M_\odot$ and $M_u \gg 5M_\odot$, find what fraction of all stars has (a) $M > 5M_\odot$, (b) $M > M_\odot$.

2. Assuming number density of stars in the disk is described by

$$n(R, z, S) = n(0, 0, S) \exp\left[-R/h_R(S)\right] \exp\left[-|z|/h_z(S)\right] \tag{2}$$

(a) find expression for the surface density $\Sigma(R, S) = \int_{-\infty}^{\infty} n(R, z, S)dz$ (we also called surface density column density, indeed $\Sigma = n_c$ that we had).

(b) Assuming every star of a given spectral class S has the same luminosity $L(S)$, find the expression for the surface brightness $I(R, S)$ of the disk (viewed face on). (Note: as we have learned from Problem 5, Assignment 1, the surface brightness I can be described either as flux per $arcsec^2$ or, equivalently, as the luminosity per unit area on the surface of the galaxy. Here you should use the second description of the I).

(c) Assuming both scale heights h_z, h_R independent on the type S, relate the total luminosity of the disk to the surface brightness at the center $I(R = 0)$

(d) For the Milky Way, $L_D = 1.5 \times 10^{10} L_\odot$ and $h_R = 3kpc$. Find the surface brightness at the Sun position ($R = 8kpc$).

3. Here we refer to two Figures, 2.2 and 2.14 (2.13 in the first edition) of the textbook. They give two HR diagrams, one for stars in the vicinity of the Sun, which we extrapolate to be valid to all stars in the galactic disk. The other is HR diagram for stars in globular clusters. Note, that these diagrams are given in "observers" coordinates - magnitude versus color. Be careful (and this is one point of the assignment) - the bands chosen are different. In Figure 2.2 we plot M_V (V band) versus $B - V$ colour, while in Figure 2.14 (upper panels) we plot *apparent* m_R (R band) versus $B - R$ colour.

Under important assumption that stars in the disk and $M30$ globular cluster are similar, answer the following questions:

(a) What is the absolute magnitude M_V of a disk star at $B - V = 0.4$? How far away must it be to have apparent magnitude $m_V = 20$? Express your answer both as distance modulus and in parsecs.

(b) Using the results from the previous question, estimate the distance to the globular cluster $M30$ using *upper-right* panel of Figure 2.14. Here is a piece of additional information needed - assume that *the bluest stars still on main sequence* in $M30$ have $B-V = 0.4$ (let us say we have additional measurements of $B-V$ colour for $M30$). Again, the main sequence stars in disk and in $M30$ are assumed similar in all respects (mainly - metallicity, which can affect colours).

For this question *lower* panels of Fig 2.14 may provide a check, but you need to get an answer only from upper, apparent magnitude, data.

(c) How bright could a disk star be if it has $B - V = 1.5$? How far away would it be at $m_v = 20$? Let us say that in M30 a star with $B - V \approx 1.5$ corresponds to $B - R \approx 2$. What are the possible values for M_V ? Is there a whole possible range ? How distant are those stars if $m_v = 20$?